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Mathematical Modeling, Simulation and Control of Flexible Vehicles

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Abstract: Possible approaches to the mathematical description of different types of flexible vehicles in view of structural oscillations, oscillations of fluid in tanks and moving masses inside the vehicle are observed. Elastic bending of a body surface in interaction with a surrounding medium in a broad band of speed variation are taken into account. Problems of regulator's synthesis and effective damping of elastic oscillations are solved for nonstationary flexible controlled plant. In the process of regulator's synthesis the local aerodynamic effects, dynamics and metering errors of sensors, time lag of engines and other elements of control system are considered. Methods of varying eigenfrequencies estimation in the real time are used for the control quality increasing. Principles of universal software design were developed for solving these complex problems of dynamic properties research, simulations of elastic vehicles motion and synthesis of perfect control law. Functioning of the program package is demonstrated and outcomes of calculations are presented.

1. INTRODUCTION[1]

Control systems of elastic mechanical objects have a number of specific features. Firstly, these features are connected with the mathematical description of elastic objects' motion by partial differential equations; the main method of such objects research is the Finite Element Method. Secondly, flexibility determines complexity of interaction of object and environment. Oscillations of a surface result in the origination of local angles of attack and origination of the local forces synchronized with these oscillations. There are such particular problems in research of control systems for elastic objects as choice of quantity and places of arrangement of sensors, in particular inertial ones, necessity for the registration of bending in the points of engines attachment, oscillations of liquids, etc. The design of elastic objects control system demands special approaches because of the capability of origination of unstable vibratory movement on frequencies of the first modes of natural oscillations at the extension of control system passband. This fact demands the special attention, as the parameters of elastic motion are considerably changed during the flight, and the precise information on eigenfrequencies is absent.

In such conditions there are two principal different approaches to control algorithms design. In the first one it is proposed to use the adaptive control with real time identification of the lowest eigenfrequencies. These identified parameters are used for turning parameters in filters of controller during the flight.

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This approach leads to complex controller algorithms. The second approach is connected with the application of robust control algorithms, which are simpler than adaptive algorithms and sometimes provide sufficient accuracy for all stages of flight. The prospective path to the problem of elastic object control solution consists of arrangement of several sensors in different places of object. Accuracy of a parameter estimation of elastic vibrations and, therefore, control efficiency depends on selection of the points of sensors installation.

An original design technique and the analysis of properties of control systems for elastic mechanical objects in time and frequency domains are offered in the paper.

2. METHODS OF THE PROBLEM SOLUTION

A flexible aerospace vehicle will be considered in the paper as an example, but the suggested method could be applied to any kind of flexible plant. Increasing requirements of the maneuverability of flying vehicles with vehicle weight restriction results the development of flexible properties which are significant for motion control. One must take into account all these effects which are essentially important for control of space stations, probes, airplanes and other mobile objects having the considerable dynamic loads because of functioning of engines and resistance of the air environment. Presence of flexibility determines the capability of appearance of oscillations in control system at different resonant frequencies. Many cases when flexibility of controlled plant was a reason of control system instability are known, resulting in development of oscillations and finally in a structural failure. Creation of effective regulators is precluded with complexity of obtaining the certain information about flexible properties of object, the significant dependence on natural frequencies to the varying

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mass, velocity and drag. Last two parameters largely depend on a flight path, which is frequently unknown beforehand. Complexity of obtaining the information about the local aerodynamic loads on a surface of object sophisticates the control system design.

Many types of vehicles considerably change mass and aerodynamic characteristics during flight. From the point of view of control theory such vehicles are the typical non-linear and non-steady plants. The aim of vehicles designers is to create lightest constructions. It results in straining of such objects in flight; flexible properties of their bodies getting are manifested. The elastic longitudinal and lateral oscillations of the composite shape increase and their frequencies change during the flight. Flexible vibrations are usually described by partial differential equations or ordinary differential equations of high order. Deformations of a body result in appearance of local angles of attack and slips. As a result of it the local forces and the moments of forces appear which are synchronized with changes of local angles of attack and slips. The local forces and moments cause amplification or attenuation of flexible vibrations. At excessive development of flexible vibrations the structural failure occurs. Oscillations of fuel and oxidant in tanks result in origination of forces and moments of forces concerning all three axes of the vehicle. Natural frequencies and oscillation frequencies of liquid depend on the shape of tanks and their location in the vehicle, a degree of filling of tanks by a liquid. Development of liquid oscillations in tanks depends on motion of object and in turn influences motion of object, in particular on flexible component oscillations. For this reason it is necessary to include the model of liquid oscillations in a structure of the generalized model of vehicle motion. Technical complexity and sophistication of state-of-the-art vehicles result in the necessity of dividing the processes of vehicle designing into some stages.

In the first design stage the vehicle is considered as a rigid body of variable mass. In this stage the problems of vehicle rational aerodynamic configuration and required efficiency of actuators for control system are solved. Possible methods and approaches of motion stabilization for vehicles with a rigid body are considered. Usually in such design stage the methods of aerodynamics, flight dynamics, automatic control, and also specialized and universal programs, such as MATLAB, are used. If the vehicle is unstable in this stage the synthesis of the elementary control system which ensures a steady motion along a desirable path is fulfilled. During the simulation the state vector is saved in the file as a reference path. Information about reference path is used as basis for linearization of complex models and control systems in the next stages of investigations. It is supposed, that peculiar properties of elastic object unaccounted in this design stage do not result in large deviations from a reference path and the possibility of using the linear model is saved.

In the next design stage flexibility of vehicle and oscillation of liquids in tanks are taken into account. Local forces and moments of forces as functions of time and coordinates along a centerline of a vehicle are computed. Analytical and semi-graphical methods of calculation in this design stage yield only approximated outcomes. For research of such elastic systems the special programs, for example, ANSYS, NASTRAN, COVENTOR, Structural Dynamics Toolbox for use with MATLAB and FEMLAB were designed. In these programs the Finite Element Method is used. This method could be well recommended for calculations concerning the simple designs. For calculation of dynamic processes as functions of time and more so for simulation of elastic vibrations for aerospace vehicles consisting of hundreds and thousands of details of composite form such approach is unsuitable. These programs do not allow solving all complex problems, which appear during designing of actual vehicles. The indicated reasons determine the necessity of development of the specialized program for simulating the motion of flexible objects of the composite form, the analysis of their dynamic properties and design of control systems (Nebylov, Panferov and Brodsky, 2005a,b,c; Rauw, 2001).

In the present paper other approach to modeling and control system design for flexible objects is observed. It is known that the flexible object is described by partial differential equations. The control theory of such objects is complex, bulky and presently is insufficiently designed analytically. There are numerical methods of calculation of the arbitrary quantity of harmonics of flexible vibrations and replacements of partial differential equations by ordinary differential equations of high dimension. For automation of analytical derivation of such mathematical models of flexible aerospace vehicle, for control law synthesis, for analysis and simulation of controlled flight, and also for representation of outcomes of modeling in the two-dimensional and three-dimensional space, the authors have developed the specialized software package.

3. MATHEMATICAL MODELS OF PHYSICAL PHENOMENA HAVING PLACE AT FLIGHT

3.1 Solid Dynamics

The rigid part of mathematical model of vehicle is allocated into the separate block, in which the system of differential non-linear equations of vehicle spatial motion is integrated. These equations in vector form in body-axes can be written as

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{F}}{m} - \Omega \times \mathbf{V}\,, \tag{1}$$

$$\frac{d\Omega}{dt} = \mathbf{I}^{-1}\left(\mathbf{M} - \Omega \times \left(\mathbf{I} \cdot \Omega\right)\right) \tag{2}$$

These equations express the motion of a rigid body relative to an inertial reference frame. Here $\mathbf{V}$ is velocity vector at the center of gravity (CG), Ω is angular velocity vector about the c.g., $\mathbf{F}$ is total external force vector, $\mathbf{M}$ is total external moment vector, $\mathbf{I}$ is inertia tensor of the rigid body.

Outputs of this subsystem are parameters of vehicle motion as a rigid body.

3.2 Flexibility

Equation of elastic line flexible displacements from the longitudinal neutral axis looks like

$$\Delta \mathbf{M} \ddot{\mathbf{q}} + \Delta \Xi \dot{\mathbf{q}} + \mathbf{q} = \Delta \mathbf{f} \,, \qquad (3)$$

where $\mathbf{q}(t)$ is deflection of elastic line from the longitudinal axis; Δ is symmetrical stiffness matrix; $\mathbf{M}$ is diagonal mass matrix; Ξ is symmetrical structural damping matrix; $\mathbf{f}$ is distributed load.

This equation describes only the flexible displacements of object points in the body-fixed coordinates. The distributed loads resulting to longitudinal moving of object and its rotation are filtered by a matrix of rigidity, do not result in deformation and are taken into account only in the equations of object motion as a solid body. Damping and elastic forces do not affect the moving vehicle as a rigid body, because the condition of dynamic balance is satisfied.

Full equation of flexible displacement in generalized coordinate $\mathbf{z}$ is set as:

$$\mathbf{G}\,\mathbf{V}\,\ddot{\mathbf{z}} + \mathbf{G}\,\mathbf{D}\,\mathbf{V}\,\dot{\mathbf{z}} + \mathbf{V}\,\mathbf{z} = \sqrt{\mathbf{M}}\Delta \mathbf{f} \,, \qquad (4)$$

where $\mathbf{z}(t)$ is vector of generalized coordinates (modes of flexible oscillations); $\mathbf{G} = \sqrt{\mathbf{M}}\,\Delta\sqrt{\mathbf{M}}$; $\mathbf{D} = (\sqrt{\mathbf{M}})^{-1}\,\Xi\,(\sqrt{\mathbf{M}})^{-1}$; Λ is diagonal matrix of eigenvalues of symmetric matrix $\mathbf{G}$; $\mathbf{V}$ is orthogonal matrix of eigenvectors: $\mathbf{V}' = \mathbf{V}^{-1}$. Eigenvalues equal to zero correspond to motion of solid body (Mishin, 1990).
The eigenfrequency of *i*-mode of free bending oscillation is defined as:

$$\omega_i = \lambda_i^{-1/2} \,. \qquad (5)$$

The relation between displacements of elastic line $\mathbf{q}(t)$ and generalized coordinates $\mathbf{z}(t)$ looks like

$$\mathbf{q}(t) = \sum_i \mathbf{h}^{<i>} z_i(t) \,, \qquad (6)$$

where $\mathbf{H}$ is matrix of shapes $\mathbf{h}^{<i>}$ of free bending oscillations $\mathbf{H} = (\sqrt{\mathbf{M}})^{-1}\mathbf{V}$.

For the flexible discrete system having the definite number of point mass particles the number of eigenfrequencies accords to the number of particles and can be defined by the equation (6) dimension. Shapes and eigenfrequencies for this system can be found as the exact solutions. For the continuous object, when the tolerance of forms and eigenfrequencies evaluation is given, the sampling frequency sets the number of bending eigenfrequencies.

Simulation of complex model with a big number of modes allows modes with small magnitudes. Elimination of those modes practically not influence on transient processes. For this reason it is advisable to limit the number of modes when simulating the distributed flexible object dynamics by dominant harmonics. Reduced equation of flexible displacement in generalized coordinate $\mathbf{z}$ is:

$$\mathbf{G}\,\mathbf{V}_{\{K\}}\ddot{\mathbf{z}} + \mathbf{G}\,\mathbf{D}\,\mathbf{V}_{\{K\}}\dot{\mathbf{z}} + \mathbf{V}_{\{K\}}\mathbf{z} = \sqrt{\mathbf{M}}\Delta \mathbf{f} \,, \qquad (7)$$

where K is dominant modes numbers: $K = \{\, i_1, i_2, \cdots i_k \,\}$, $k < n$; $\mathbf{V}_{\{K\}}$ is matrix that consists of K columns of matrix $\mathbf{V}$.

The equation (7) in the matrix form describes the singular system of differential equations that cannot be expressed by the highest order derivative. The transformation

$$\ddot{\mathbf{z}} = \left(\mathbf{G}\,\mathbf{V}_{\{K\}}\right)^{+} \left\{- \mathbf{G}\,\mathbf{D}\,\mathbf{V}_{\{K\}}\dot{\mathbf{z}} - \mathbf{V}_{\{K\}}\mathbf{z} + \sqrt{\mathbf{M}}\,\Delta \mathbf{f}\right\} \qquad (8)$$

is used for its numerical integration.

The elastic line displacements can be divided into two components $\mathbf{q} = \hat{\mathbf{q}} + \tilde{\mathbf{q}}$, as:

$$\hat{\mathbf{q}} \in L\left((\sqrt{\mathbf{M}})^{-1}\mathbf{V}_{\{K\}}\right), \qquad (9)$$

$$\tilde{\mathbf{q}} \perp L\left((\sqrt{\mathbf{M}})^{-1}\mathbf{V}_{\{K\}}\right). \qquad (10)$$

In the equation (9) the elastic line displacements $\hat{\mathbf{q}}$ and its derivatives $\dot{\hat{\mathbf{q}}}$ and $\ddot{\hat{\mathbf{q}}}$ are taken into account.

Normalized time response of stable and unstable flexible vehicle as reaction on impulse applied on the 0.2, 1st, 10th, … , 60th seconds of flight are shown in Fig. 1.

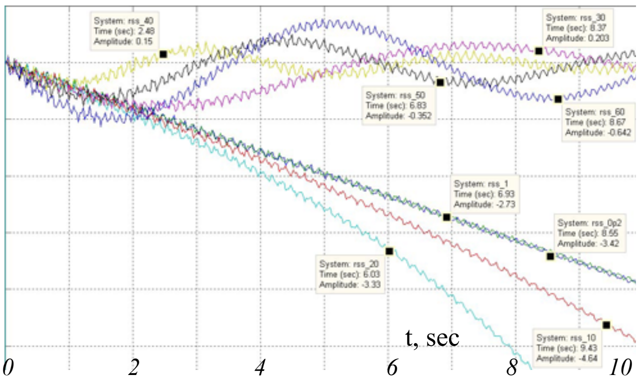

Fig. 1. Flexible oscillations during the first second after impulse applying

3.3 Aerodynamics and local loads

The distributed and concentrated forces appear because of formation and a break-down of a vortex on the vehicle surface.

Local aerodynamic effects substantially depend on the velocity and altitude of flight, the form of a mobile object, angular orientation and flexible deformations of a body. Even at a constant velocity of flow on the vehicle surface the vortices are generated. It results in the composite and time-varying distribution pattern of local loads on a surface of object. At high speeds of flight there are local spikes of pressure in the separate parts of vehicle. For their modeling it is important to define zones of the applying of large local loads and their time history. Usually these zones are arranged close to transitions from conical to cylindrical surface forms or to places of joints of surfaces with more composite form. In designing of vehicle the aim to avoid such connections is usually set, but it is not possible to remove them completely. Here the models for description of the most typical local loads from vortices are resulted.

Large local loads arise near to junctions of separate structural members. Usually these are places of transition from a conic surface to cylindrical, places of connection of cylinders of miscellaneous diameter. Vortex flows will be produced a little bit below streamwise places of details bonding, intensity of vortexes and frequency of their separation largely depends on conditions of flight. More particularly these models are described in (Brodsky, Nebylov and Panferov, 2004; Nebylov, Panferov and Brodsky, 2005a, b; Caldwell, B.D., R.W. Pratt, R. Taylor and R.D. Felton, 2000).

Distributed and integral aerodynamics forces are calculated in this program block. Parameters of vehicle motion as rigid body and bending oscillations for each flight moment are taken into account. Distributed coefficients are evaluated for each point along the longitudinal axis of vehicle. Distributed aerodynamic coefficients $C_n(x)$ and allocated values C_{n_i} are linked with integral coefficients C_n, $C_m(x_{cg})$, $C_{mq}(x_{cg})$ at arbitrary disposition of center gravitation x_{cg}, by the equations:

$$C_n = \sum_i C_{n_i} = \int_0^l c_n(x)dx\ , \tag{11}$$

$$C_m(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg}) = \int_0^l c_n(x)(x - x_{cg})dx, \tag{12}$$

$$C_{mq}(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg})^2 = \int_0^l c_n(x)(x - x_{cg})^2 dx. \tag{13}$$

Local angle of attack a_i^* at a point with coordinate x_i on the line of vehicle longitudinal axis with account of flexible oscillations is

$$a_i^* = a + \frac{x_{cg} - x_i}{V_i}\dot{\vartheta} - \frac{\dot{q}_i}{V_i} + \frac{\partial q_i}{\partial x_i}\ , \tag{14}$$

where $\frac{\partial q_i}{\partial x_i}$ is slope of elastic line in current time; $\dot{q}_i$ is velocity of shape of elastic line. Here V_i is local air velocity; a is angle of attack of solid body; ρ is air density.

The force f_i, distributed along longitudinal axis of vehicle and integral drag force F_x are evaluated by the block as:

$$f_i = \rho \frac{V_i^2}{2} C_{n_i} a_i^* \qquad F_x = \frac{\rho \cdot V^2}{2} C_d\, a\ . \tag{15}$$

Aerodynamic effects of jet exhaust stream turn are also taken into account.

3.4 Sloshing effects

Presence of the cavities filled with a liquid (fuel and oxidant) inside object results in appearance of additional forces and moments of forces influencing vehicle motion. Studying of oscillations of a liquid in tanks is one of the problems of classical hydrodynamics where either Lagrange variables or Eulerian variables are used for the description of liquid motion. Lagrange variables determine motion of the fixed liquid particle; they depend on time and coordinates of this particle in the initial moment of time. Studying of motion of a liquid by means of these variables consists in the analysis of changes, which undergo various vector and scalar values (for example, speed, pressure etc.), describing motion of some fixed particle of a liquid depending on time. Variables of Euler characterize a motion condition of particles of the liquid located in the different moments of time t in a given point of the space with coordinates x, y, z. In other words, various vector and scalar elemental motions are considered as a function of a point of the space and time that means the functions of four arguments: x, y, z, t.

In studying oscillations of a liquid the following assumptions were accepted:

1. A liquid in a cylindrical tank is ideal and incompressible.
2. Movements and speeds of all particles of a liquid and walls of a tank are small values in the sense that products and squares of them can be neglected.
3. Motion of a liquid in coordinate system $Oxyz$ has potential of speeds. Believing initial motion of a liquid vortex-free and a field of mass forces potential, on the basis of Lagrange's theorem a conclusion can be made.
4. The total acceleration vector of a field of mass forces g in any the time of motion makes a small angle with this axis.

As equations of oscillations of a free surface are similar to equations of mathematical pendulum oscillations in the analysis of vehicles dynamics, naturally, there is a question of replacement of a varying liquid with a system of mathematical pendulums. Mathematic description of this problem is adduced in (Nebylov, Brodsky and Panferov, 2005c).

4. STRUCTURE OF CONTROL SYSTEM

In the design stage the state-space model reduction for stabilization and guidance systems is used. The complexity of the models used in describing the aeroelastic effects via the equations of motion discussed previously makes design of stabilization and guidance systems an extremely difficult problem.

To start this process it is necessary to linearize the system dynamics near the nominal trajectory. Reduction of the linear model is then performed and the desired manner in which the reduced-order linear model approximates the full-order model. At the stage of control system synthesis it is important to represent accurately the system frequency response in the passband of the closed loop system. There are frequencies both above and below the critical frequency range which may not need to be well modeled. The frequency range of interest is very important for applying model simplification.

There are many methods by which the linear elastic vehicle models can be simplified. Several of these methods are used in the software package. The purpose of these simplifications is to design the robust controller. Truncation deletes some of the modes or states from the full-order model. Residualization accounts only the effects of some modes or states whose dynamics is not crucial. Balanced reduction minimizes frequency response error and has the certain advantages associated with obtaining the desired accuracy. Symbolic simplification addresses the impact of various physical parameters on the system responses and ignores those ones that have a little influence. Some advantages and disadvantages for each of these methods exist. After designing the robust or adaptive controller for simplified model, the analysis of real accuracy with wholeness is executed.

5. DESIGNING OF THE GUIDANCE SYSTEMS

The nonlinear flight dynamics equations of high dimension are not convenient for designing the control law for vehicle maneuvers. It is more acceptable to divide this problem into two stages. In the first stage the control law for damping of the certain flexible oscillations is synthesized. Usually these oscillations are in the passband of actuator. The plant with such control law is considered as a rigid plant and its mathematical model is simplified. This simplified model of the plant is used in the second stage of the control system synthesis.

It is possible to write in the common case for arbitrary small interval of time for SISO system after linearization:

$$W(s)=\frac{k(s+\omega_{11})...(s+\omega_{1i})(s^2+2\xi_{21}\omega_{21}s+\omega_{21}^2)...(s^2+2\xi_{2k}\omega_{2k}s+\omega_{2k}^2)}{(s+\omega_{31})...(s+\omega_{3j})(s^2+2\xi_{41}\omega_{41}s+\omega_{41}^2)...(s^2+2\xi_{4l}\omega_{4l}s+\omega_{4l}^2)}$$

where k, ξ_{mn}, ω_{mn} are constant numbers.

Separated multipliers in numerator and denominator of transfer function (TF) describe separated components of the plant motion and separated modes of flexible oscillations. This TF can be transformed in the sum of simple fractions

$$W(s)=\frac{r_1}{s-p_1}+\frac{r_2}{s-p_2}+...+\frac{r_n}{s-p_n}+r(s)\cdot \qquad (16)$$

For real plants the order of TF numerator is less than order of a denominator and for this reason $r(s)=0$.

The real poles p_i correspond to aperiodic components of plant movement. The complex pairs of poles correspond to oscillatory movement of object or modes of elastic oscillations. These pairs of poles are combined for deriving the TF of oscillatory parts with the real factors. Further, each TF will be transformed to system of the differential equations of the first order, and the obtained equations are united in uniform system of the equations of a following kind:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}A_1 & 0 & 0 & 0\\ 0 & A_2 & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & A_k\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}+\begin{bmatrix}b_1\\ b_2\\ \vdots\\ b_n\end{bmatrix}u$$

$$\dot{x}(t)=Ax(t)+Bu(t)+Gw(t) \qquad (17)$$

$$y(t)=Cx(t)+v(t) \qquad (18)$$

Physical interpretation of all components of state vector is very clear and it allows selecting the weighting coefficients for damp the separated flexible oscillations in the following functional

$$J=M\{\int_0^\infty [x^T(t)Q_x x(t)+u^T(t)R_u u(t)dt\} \qquad (19)$$

where matrixes Q_x and R_u are matrixes of weighting coefficients.

The optimal control law is well known and can be written in the following form

$$u(t)=-R_u^{-1}B^T S\,\hat{x}(t), \qquad (20)$$

where matrix S is the positive-definite solution of Riccati matrix algebraic equation

$$SA+A^T S-SBR_u^{-1}B^T S+Q_x=0 \qquad (21)$$

and the estimation of state vector is calculated in real time by integration of the following equations at the known initial conditions:

$$\dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t)+L(t)(y(t)-C\hat{x}(t));$$
$$L(t)=R^{-1}(t)CP(t);$$
$$\dot{P}(t)=AP(t)+P(t)A^T-P(t)C^T R^{-1}(t)CP(t)+GQ(t)G^T.$$

Three modes of flexible oscillations for vehicle without controller are shown in fig.2a. The decrements for separate modes are realized by choosing the weighting coefficients in matrix Q_x. Fig.2b displays the result of decreasing only the first mode and in fig.1c - the first and second modes. Such method can be used for elimination each next mode of flexible oscillations.

Dependence of vehicle dynamic characteristics from time and stage of flight hampers realization of this approach. Dependence of the vehicle dynamic characteristics from time presents in Fig. 3. In this figure some open loop Bode diagrams for different stages of motion of the flexible vehicle with control system are shown. Real eigenfrequencies during the flight are unknown values. For this reason for realization of the control law it is necessary to use methods of adaptive or robust control (Nebylov, 2004).

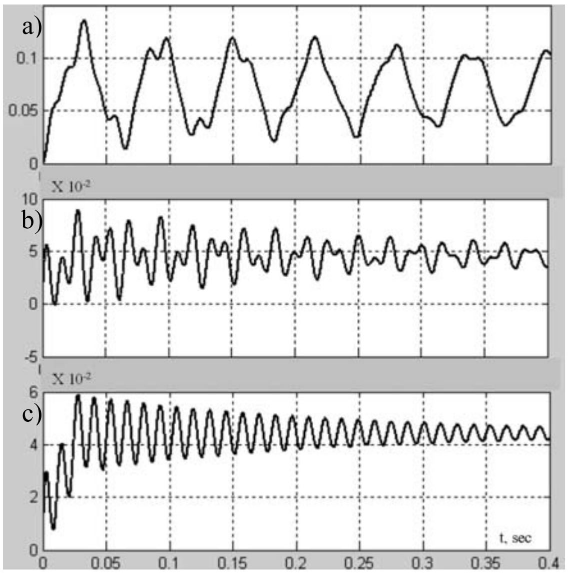

Fig. 2. Flexible oscillations: a) all three modes; b) the first mode is damped; c) the first and second modes are damped

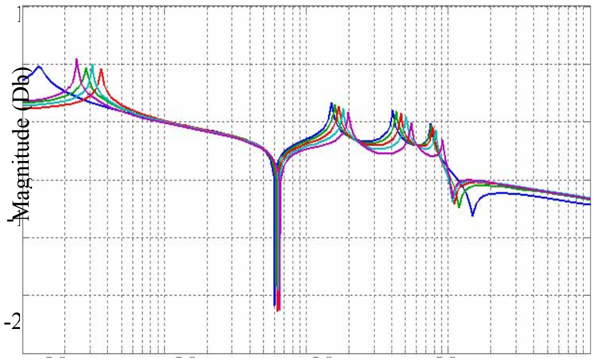

Fig. 3. Open loop Bode diagrams for different moments of time

For realization of adaptive control it is necessary to solve the problem of eigenfrequencies identification in real time. This problem can be solved separately and added to the basic control algorithm.

6. CONCLUSIONS

The uniform mathematical model is suggested for simulation of flexible aerospace vehicle flight. It consists of some particular models describing such phenomena as solid dynamics, flexibility, aerodynamics and local loads, sloshing effects and other factors.

The approach to regulator synthesis for elastic object control is offered. The procedure of synthesis is divided into two stages. In the first stage the control law for damping of the certain flexible oscillations is synthesized. In the second stage the plant is considered as rigid and it is possible to use any known method for regulator synthesis.

On the basis of suggested mathematical models the program was designed. This program allows solving the following problems:

-input of initial constructive data of vehicle,
-determination of controllability and observability for the full and simplified model of vehicle,
- choice of flight program and control law,
- automatic linearization for arbitrary trajectory,
- robust control system synthesis,
- determination of bending modes and sloshing characteristics,
- choice of sensors and actuators characteristics,
- bode plots construction,
- choice of flight program and control law at simplified nonlinear model,
- determination of the elastic vibrations of body and oscillation modes of liquid in tanks,
- study of control system sensitivity to vehicle parameters change,
- stability margins ranking.

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