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Robust Adaptive Control of Time-Delay Nonlinear Systems via TS Recurrent Fuzzy CMAC Approach ⋆

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Abstract: This paper proposes an adaptive TS recurrent fuzzy CMAC (TS-RFCMAC) model based control of uncertain time-delay nonlinear systems. First, we introduce a TS-RFCMAC network and its application on system modeling. Next, a TS-RFCMAC controller is developed based on parallel distributed compensation and adaptive control laws. Even if uncertain local subsystem matrices and fuzzy sets exist in the model, asymptotic stability is assured by proper gain design and adaptive learning laws. Since all the weights (including recurrent weights) are also on-line adjusted, the proposed controller is more suitable for applying to uncertain time-delay systems. Finally, the simulation results show the expected performance.

Keywords: TS fuzzy model, recurrent CMAC, adaptive control, time-delay systems.

1. INTRODUCTION

The cerebellar model articulation controller (CMAC) is a non-fully connected perceptron-like associative memory network with an overlapping receptive-field (cf. Albus [1975]). The advantages of using CMAC have been reported in many practical applications in recent literature. Compared to the multi-layer percetron with back-propagation algorithms, the CMACs are widely adopted for the closed-loop control of complex dynamic systems because of its fast learning property, good generalization capability, and simple computation (Hwang et al. [1998], Shiraishi et al. [1995]). However, the traditional CMAC uses logic basis functions as the input sensors such that a discontinuous output exits. To avoid this drawback, some fuzzy CMAC (FCMAC) networks (e.g., Lin et al. [2004]-Su et al. [2006]) use fuzzy sensors to extension to more complex applications. Nevertheless, these FCMAC based control methods usually need complex learning laws. This drawback stems from the fact that some information about the controlled systems (e.g., the structure) is not involved into the FCMAC.

Many significant research efforts have been done for TS (Takagi-Sugeno) fuzzy controllers to guarantee control performance and system stability (Tanaka et al. [2001], Lian et al. [2001]). Most of the stability analysis and design methods are according to the LMI formulation (Boyd et al. [1994]). The advantage is providing an efficient and effective way for the controller design. However, these TS fuzzy model-based controllers will fail when considering both uncertain local subsystem matrices and fuzzy sets of premise variables. Moreover, the fuzzy rules and the fuzzy defuzzification are complicated. On the other hand, some TS fuzzy model-based control methods have solved the control problem of nonlinear time-delay systems as an extension with works considering linear time-delay systems, such as (Yi et al. [2002]). In addition, based on neural-network approaches, some researches have focused on dealing with the nonlinear time-delay systems (Huang et al. [2003], Cao et al. [2003]). Nevertheless, these control schemes cannot be straightforwardly applied to practical systems due to the high complexity.

To combine the advantages of the CMAC control (e.g., easy implementation) and the TS fuzzy control (e.g., unified formulation), this study develops a TS recurrent FCMAC (TS-RFCMAC) control scheme for controlling uncertain systems with time-delays. Inspired by the TS fuzzy model, we introduce a TS-RFCAMC network to represent general nonlinear systems. Based on the TS-RFCMAC model, the TS-RFCMAC model-based control is developed in a straightforward manner. The advantages of the proposed modeling network are: 1) the complex fuzzy defuzzification is dropped; and 2) the parallel distributed compensation concept can be used in the controller design (i.e., the advantage of the TS fuzzy model-based control). When there is uncertainty on the TS-RFCMAC model, the TS-RFCMAC controller is modified into an adaptive learning network, i.e., the control gains and fuzzy sensors are tuned on-line. Moreover, asymptotic stability is assured by proper learning laws. Therefore, the adaptive TS-RFCMAC controller achieves high robustness and easy implementation.

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2. T-S RECURRENT FUZZY CMAC NETWORK

This section proposes a recurrent fuzzy CMAC network, called *Takagi-Sugeno Recurrent Fuzzy CMAC* (TS-RFCMAC). This TS-RFCMAC is composed of the input layer, the recurrent fuzzified layer, the rule association layer, and the output layer. The detailed construction of the TS-RFCMAC is introduced as follows.

1) *Input Layer X*: This is the layer where the input is obtained from the raw data. Consider the input $\mathbf{x} = [\, x_1 \ x_2 \ \cdots \ x_{n_x} \,]^T \in R^{n_x}$, each input variable x_ℓ has an appropriate discussion region.
2) *Recurrent Fuzzified Layer F*: This layer fuzzifies all input variables to obtain fuzzy activated levels, while the outputs of this layer are back-propagated into the fuzzy sensors of this layer. When the sensors fuzzify the inputs, the activated level will be back-propagated into the premise variables of fuzzy rules (i.e., recurrent states) as
$$\bar{x}_\ell(N) = x_\ell(N) + h_{\ell i} F_{\ell i}(\bar{x}_\ell(N-1)) \tag{1}$$
for $\ell = 1, 2, ..., n_x$, where N denotes the number of iteration; $F_{\ell i}$ denotes a proper fuzzy membership function for the recurrent state $\bar{x}_\ell$ in the i-th fuzzy rule; and $h_{\ell i}$ is the recurrent weight associated to the (ℓ, i) membership function. Contrary to the traditional CMAC, the output of these sensors are real numbers from zero to 1. Moreover, the recurrent neuron can further provide more flexibility for functional approximation.
3) *Rule Association Layer R*: This layer is the rule layer, and each association cell represents a TS fuzzy rule. The AND operation and OR operation are carried out to activate appropriate rules. Consider the network associated to the following TS fuzzy rules:
Rule i : IF $\bar{x}_1(t)$ is F_{1i} and $\cdots$ and $\bar{x}_{n_x}(t)$ is $F_{n_x i}$ THEN
$$y_o = a_{0i} + a_{1i}x_1 + ... + a_{n_x i}x_{n_x}, \ i = 1, 2, ..., r$$
where r is the number of the fuzzy rules; y_o is the output of the network; and $a_{0i} \sim a_{n_x i}$ are tunable weights for the i-th TS fuzzy rule. Here all parameters of fuzzy rules are stored in a corresponding physical memory space. Then the activated weight of the i-th fuzzy rules is obtained as
$$w_i(\bar{\mathbf{x}}) = \prod_{\ell=1}^{n_x} F_{\ell i}(\bar{x}_\ell) \geq 0, \text{ for } i = 1, 2, ..., r \tag{2}$$
where $\bar{\mathbf{x}} = [\, \bar{x}_1 \ \bar{x}_2 \ \cdots \ \bar{x}_{n_x} \,]^T$.
4) *Output Layer O*: This layer is fully connected to the rule association layer. The output of TS-RFCMAC is the algebraic sum of the activated weights and is expressed as
$$y_o = \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})[a_i\mathbf{x} + a_{i0}] \tag{3}$$
where $a_i = [\, a_{1i} \ ... \ a_{n_x i} \,]$.

Therefore, the proposed TS-RFCMAC network has combined the concept of TS fuzzy rules and the structure of the recurrent CMAC network. To show the difference between the traditional TS fuzzy systems and the proposed TS-FRCAMC, we made the following note.

Remark 1: Without loss of generality, a TS fuzzy system consists of IF-THEN rules as follows:

$$\begin{aligned}
&\textit{Rule } i: \\
&\text{IF } x_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } x_{n_x}(t) \text{ is } F_{n_x i} \text{ THEN} \\
&y_o = a_{0i} + a_{1i}x_1 + ... + a_{n_x i}x_{n_x}, \ i = 1, 2, ..., r
\end{aligned}$$

By using the singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier, the output of the above fuzzy system is
$$y_o = \sum_{i=1}^{r} \mu_i(\mathbf{x}(t))[a_i\mathbf{x} + a_{i0}] \tag{4}$$
where $\mu_i(\mathbf{x}(t)) = \frac{w_i(\mathbf{x}(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))}$ with
$$w_i(\mathbf{x}(t)) = \prod_{\ell=1}^{n_x} F_{\ell i}(x_\ell(t)) \tag{5}$$
is regarded as a normalized weight. Making a comparison between (3) and (4), we find that they are the same if $h_{\ell i} = 0$ (no recurrent loop exists) and $\sum_{i=1}^{r} w_i(\mathbf{x}(t)) = 1$. In other words, since the output of the TS-RFCMA network is not fuzzy, the proposed TS-RFCMAC network does not require a complex defuzzification (e.g., a weighted average defuzzifier is not needed). Moreover, the recurrent neuron can further provides better approximation of complex functions, such as time-delay states. Thus, the above TS-RFCMAC has a simple structure, which is easily implemented in comparison of the traditional fuzzy CMAC. The validity of this in practice contains benefits with fast learning and good generalization of CMAC.

Due to the TS-RFCMAC structure, the dynamics of time-delay systems can potentially be modelled in a more flexible way than with a pure time-series approach. Accordingly, the TS-RFCMAC is more suitable for complex functional approximation and system modeling. In the following, we apply the proposed TS-RFCMAC to represent nonlinear systems.

3. TS-RFCMAC MODELING OF NONLINEAR SYSTEMS

Consider a general dynamic equation of nonlinear systems which contains time delays as follows:
$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{x}(t-\tau(t))) + g(\mathbf{x}(t), \mathbf{x}(t-\tau(t)))u, \tag{6}$$
where $\mathbf{x}(t) \in R^{n_x}$ denotes the state vector in a continuous-time system; $\tau(t)$ is a time-varying delay satisfying $\dot{\tau}(t) \leq \beta < 1$; $u(t) \in R^{n_u}$ is a control input of the system; and $f(\cdot), g(\cdot)$ are the nonlinear dynamic function vectors. According to the work (Lian et al. [2001]), if each scalar nonlinear term in $f(\cdot)$ is separable with a proper state variable x_ℓ, the system has the expression:
$$\begin{aligned}
\dot{\mathbf{x}}(t) = \ & \Phi_1(\mathbf{x}(t), \mathbf{x}(t-\tau(t)))\mathbf{x}(t) \\
& + \Phi_2(\mathbf{x}(t), \mathbf{x}(t-\tau(t)))\mathbf{x}(t-\tau(t)) \\
& + g(\mathbf{x}(t), \mathbf{x}(t-\tau(t)))u
\end{aligned}$$
where $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ are proper matrices. Based the above section, the TS-RFCMAC presenting the system is associated to the following rules:

Rule i :

IF $\bar{x}_1$ is F_{1i} and $\cdots$ and $\bar{x}_{n_x}$ is $F_{n_x i}$ THEN

$$\dot{\mathbf{x}}(t) = A_i\mathbf{x}(t) + A_{di}\mathbf{x}(t-\tau(t)) + B_i u \tag{7}$$

for $i = 1, 2, ..., r$, where $\bar{x}_\ell$ is the recurrent variable defined in (1); A_i, A_{di}, B_i are local subsystem matrices with proper dimensions. The inferred output of the TS-RFCMAC is

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_i\mathbf{x}(t) + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_{di}\mathbf{x}(t-\tau(t)) + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i u$$

with $w_i(\bar{\mathbf{x}}) = \prod_{\ell=1}^{n_x} F_{\ell i}(\bar{x}_\ell)$. If we properly choose the fuzzy membership function $F_{\ell i}(\bar{x}_\ell)$ and parameters A_i, A_{di}, B_i, the system (6) can be represented by the TS-RFCMAC as actual as possible. To choose appropriate fuzzy sets and parameters, some on-line learning schemes (Kim [2002])-(Su et al. [2006]) can be applied to solve this problem. As a result, the system (6) is rewritten in terms of the TS-RFCMAC as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_i\mathbf{x}(t) + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_{di}\mathbf{x}(t-\tau(t)) + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i(u + \psi(t, \mathbf{x}(t), \mathbf{x}(t-\tau(t)))) \tag{8}$$

where $\psi(\cdot)$ presents the modeling error and the system uncertainties. Since the matrices A_i and A_{di} may contain uncertainty, the remainder of this paper will discuss the controller design based on the TS-RFCMAC model.

4. IDEAL TS-RFCMAC MODEL-BASED CONTROL

Based on the TS-RFCMAC model (8), an ideal TS-RFCMAC model-based control scheme is developed below. Here, the ideal case is considered with an exactly known model, i.e., A_i and A_{di} are exactly known. First, we set the TS-RFCMAC controller with the same construction as the TS-RFCMAC model. Inspired by the parallel distributed compensation concept, the TS-FRCAMC controller is associated to the fuzzy rules:

Rule i :

IF $\bar{x}_1$ is F_{1i} and $\cdots$ and $\bar{x}_{n_x}$ is $F_{n_x i}$ THEN

$u = -(k_{0i} + k_{1i}x_1 + k_{2i}x_2 + ... + k_{n_x i}x_{n_x})$

where $k_{0i} \sim k_{n_x i}$ are controller gain vectors determined later. According to Section 2, the output of the TS-RFCMAC controller is

$$u(t) = -\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})[K_i\mathbf{x}(t) + k_{0i}]. \tag{9}$$

with $K_i = [\, k_{1i} \; ... \; k_{n_x i} \,]$. By substituting the control law (9) into the system (8), the closed-loop system is expressed as

$$\begin{aligned}\dot{\mathbf{x}}(t) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}})(A_i - B_iK_j)\mathbf{x}(t) \\ & + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_{di}\mathbf{x}(t-\tau(t)) + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i \\ & \times[\psi(t, \mathbf{x}(t), \mathbf{x}(t-\tau(t)) - \sum_{j=1}^{r} w_j(\bar{\mathbf{x}})k_{0j}]\end{aligned} \tag{10}$$

From an observation on the above equation, the controller gain K_i will be designed from the Lyapunov-Krasovskii stability method while the controller k_{0j} will be on-line adjusted to compensate the uncertainty $\psi(\mathbf{x}(t))$. According to the proposed TS-RFCMAC network, there exists an optimal parametric set k_{0j}^* (corresponding to k_{0j}, for $j = 1, 2, ..., r$) resulting into an approximation error of ψ which is defined as

$$\varepsilon = \psi(t, \mathbf{x}(t), \mathbf{x}(t-\tau(t)) - \sum_{j=1}^{r} w_j(\bar{\mathbf{x}})k_{0j}^*$$

The error ε can be made arbitrarily small by using appropriate fuzzy sets and parameters. Due to the unknown optimal parameter k_{0j}^*, we rewrite the error system (10) in the following form:

$$\begin{aligned}\dot{\mathbf{x}}(t) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}})[(A_i - B_iK_j)\mathbf{x}(t) + B_i\varepsilon] \\ & + \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_{di}\mathbf{x}(t-\tau(t)) \\ & - \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}})B_i\tilde{k}_{0j}\end{aligned} \tag{11}$$

where $\tilde{k}_{0j} = k_{0j} - k_{0j}^*$. Then, the design theorem is stated below.

Theorem 1: Consider an uncertain nonlinear system described by the TS-RFCMAC model (8) using the TS-RFCMAC controller (9) adjusted by the update law

$$\dot{k}_{0j} = \gamma w_j(\bar{\mathbf{x}}) \sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i^T P\mathbf{x}, \text{ for } j = 1, 2, ..., r \tag{12}$$

and $\gamma > 0$. If there exist symmetric positive-definite matrices $X = P^{-1}, Q_X$ and matrices M_i satisfying the following LMIs

$$X, Q_X > 0$$

$$\begin{bmatrix} \{A_iX + XA_i^T - B_iM_j - M_j^TB_i^T + \dfrac{1}{1-\beta}Q_X\} & A_{di} & B_i \\ A_{di}^T & -Q_X & 0 \\ B_i^T & 0 & -\dfrac{1}{\rho^2}I_{n_u} \end{bmatrix} < 0 \tag{13}$$

for given $\rho > 0$, $M_i = K_iX$ and all i, j, then asymptotic stability is assured with uniformly ultimate bound. Moreover, the closed-loop system achieves the following H^∞ performance:

$$\alpha \int_0^{t_f} \mathbf{x}^T(t)\mathbf{x}(t)dt \le V_1(0) + \frac{1}{\rho^2}\int_0^{t_f} \|\varepsilon(t)\|^2 dt. \tag{14}$$

Proof: Consider the Lyapunov-Krasovskii function $V_1 = \mathbf{x}^T(t)P\mathbf{x}(t) + \frac{1}{1-\beta}\int_{t-\tau(t)}^{t} \mathbf{x}^T(\nu)Q\mathbf{x}(\nu)d\nu + \frac{1}{\gamma}\sum_{i=1}^{r} \tilde{k}_{0j}^T\tilde{k}_{0j}$

with $P = P^T = X^{-1}$, $Q = Q^T = PQ_XP$, and $\gamma > 0$. Taking the time derivative of $V_1(t)$ along the error dynamics (11), we obtain

$$\begin{aligned}
\dot{V}_1 = & \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}}) \\
& \times\{\mathbf{x}^T(t)P[(A_i - B_iK_j)\mathbf{x}(t) + B_i\varepsilon] \\
& +[(A_i - B_iK_j)\mathbf{x}(t) + B_i\varepsilon]^T P\mathbf{x}(t)\} \\
& +2\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})\mathbf{x}^T(t)PA_{di}\mathbf{x}(t-\tau(t)) \\
& +\frac{2}{\gamma}\sum_{i=1}^{r}\tilde{k}_{0j}^T\dot{k}_{0j} + \{\frac{1}{1-\beta}\mathbf{x}^T(t)Q\mathbf{x}(t) \\
& -\frac{1-\dot{\tau}}{1-\beta}\mathbf{x}^T(t-\tau(t))Q\mathbf{x}(t-\tau(t))\} \\
& -2\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}})\mathbf{x}^T(t)PB_i\tilde{k}_{0j}
\end{aligned}$$

After applying the update law (12) and the stability condition

$$\begin{bmatrix} \begin{matrix}\{P(A_i - B_iK_j) + (A_i - B_iK_j)^T P \\ +\frac{1}{1-\beta}Q + \rho^2 PB_iB_i^T P\}\end{matrix} & PA_{di} \\ A_{di}^T P & -Q \end{bmatrix} < 0$$

the time derivative of V_1 satisfies $\dot{V}_1 \le -\alpha\mathbf{x}^T(t)\mathbf{x}(t) + \frac{1}{\rho^2}\|\varepsilon\|^2$ for some $\alpha > 0$. Accordingly, if the LMI (13) has a feasible solution, the closed-loop system achieves the robust criterion (14). In other words, the state error $\mathbf{x}(t)$ can be made arbitrarily small by a proper choose of the parameter ρ.

5. ROBUST ADAPTIVE TS-RFCMAC CONTROL

This section considers the worst cases of the control problem — the system matrices and fuzzy sets are unknown in the TS-RFCMAC model.

5.1 Uncertain system matrices

When the TS-RFCMAC model of the plant (8) has uncertainty on system matrices (here the fuzzy sets are exactly known), the controller cannot be designed straightforwardly according to the above section. A robust TS-RFCMAC model-based control is developed here. Assume that the system matrices can be expressed as $A_i = A_{ni} + \Delta A_i$ and $A_{di} = A_{dni} + \Delta A_{di}$ with the nominal parts A_{ni}, A_{dni} and uncertain parts $\Delta A_i, \Delta A_{di}$. The system has controllable pairs $\{A_i, B_j\}$ and $\{A_{ni}, B_j\}$ for all i, j. To overcome the uncertainty, the TS-RFCMAC controller is firstly set with the following rules:

Rule i :

IF $\bar{x}_1$ is F_{1i} and $\cdots$ and $\bar{x}_{n_x}$ is $F_{n_x i}$ THEN
$u = -(k_{d1i}x_1 + k_{d2i}x_2 + ... + k_{dn_xi}x_{n_x})$
$-(k_{0i} + k_{a1i}x_1 + k_{a2i}x_2 + ... + k_{an_xi}x_{n_x})$

where $k_{d1i} \sim k_{dn_xi}$ are fixed control gains; and $k_{0i}, k_{a1i} \sim k_{an_xi}$ are adaptive parameters. It yields the compact controller

$$u(t) = -\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})[K_{di}\mathbf{x}(t) + K_{ai}\mathbf{x}(t) + k_{0i}] \tag{15}$$

where $K_{di} = [\,k_{d1i}\ ...\ k_{dn_xi}\,]$ and $K_{ai} = [\,k_{a1i}\ ...\ k_{an_xi}\,]$. The closed-loop controlled system is further written as follows:

$$\begin{aligned}
\dot{\mathbf{x}}(t) = & \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\bar{\mathbf{x}})w_j(\bar{\mathbf{x}})(A_i - B_i(K_{dj} + K_{aj}^*))\mathbf{x}(t) \\
& +\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})A_{di}\mathbf{x}(t-\tau(t)) - \sum_{i=1}^{r} w_i(\bar{\mathbf{x}}) \\
& \times\sum_{j=1}^{r} w_j(\bar{\mathbf{x}})B_i[\widetilde{K}_{aj}\mathbf{x}(t) + \tilde{k}_{0j} + \varepsilon]
\end{aligned} \tag{16}$$

where $\widetilde{K}_{aj} = K_{aj} - K_{aj}^*$; and K_{aj}^* is an unknown optimal gain to assure the control stability and robust performance. Then, the overall adaptive TS-RFCMAC controller is given below.

Theorem 2: Consider a nonlinear system described by the TS-RFCMAC model (8) with uncertain system matrices A_i and A_{di} (for all i). The asymptotic stability with uniformly ultimate bound is assured if the TS-RFCMAC controller (15) using the control gain K_{dj} satisfying the LMI ($M_{dj} = K_{dj}X$)

$$X, Q_X > 0$$

$$\begin{bmatrix} \begin{matrix}\{A_{ni}X + XA_{ni}^T - B_iM_{dj} \\ -M_{dj}^TB_i^T + \frac{1}{1-\beta}Q_X\}\end{matrix} & A_{dni} & B_i \\ A_{dni}^T & -Q_X & 0 \\ B_i^T & 0 & -\frac{1}{\rho^2}I_{n_u} \end{bmatrix} < 0 \tag{17}$$

for given $\rho > 0$ and all i, j, and the update laws

$$\dot{k}_{0j} = \gamma w_j(\bar{\mathbf{x}})\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i^T P\mathbf{x}, \tag{18}$$

$$\dot{K}_{aj} = \gamma_a w_j(\bar{\mathbf{x}})\sum_{i=1}^{r} w_i(\bar{\mathbf{x}})B_i^T P\mathbf{x}\mathbf{x}^T \tag{19}$$

for $P = X^{-1}$, $j = 1, 2, ..., r$, and $\gamma, \gamma_a > 0$.

As similar as the proof of Theorem 1, the proof can be derived by using the Lyapunov-Krasovskii function

$$\begin{aligned}
V_2 = & \mathbf{x}^T(t)P\mathbf{x}(t) + \frac{1}{1-\beta}\int_{t-\tau(t)}^{t}\mathbf{x}^T(\nu)Q\mathbf{x}(\nu)d\nu \\
& +\frac{1}{\gamma}\sum_{i=1}^{r}\tilde{k}_{0j}^T\tilde{k}_{0j} + \frac{1}{\gamma_a}\sum_{i=1}^{r} tr(\widetilde{K}_{aj}^T\widetilde{K}_{aj})
\end{aligned}$$

with $P = P^T = X^{-1}$, $Q = Q^T = PQ_XP$, and $\gamma, \gamma_a > 0$. For the solution of the LMI (17), i.e., P, Q, and K_{dj}, there are ideal control parameters K_{aj}^* satisfying

$$\begin{bmatrix} \begin{matrix}\{A_{ni}X + XA_{ni}^T - B_iM_{dj} \\ -M_{dj}^TB_i^T - B_iK_{aj}^*X \\ -X^TK_{aj}^{*T}B_i^T + \frac{1}{1-\beta}Q_X\}\end{matrix} & A_{di} & B_i \\ A_{di}^T & -Q_X & 0 \\ B_i^T & 0 & -\frac{1}{\rho^2}I_{n_u} \end{bmatrix} < 0$$

This implies that the closed-loop system is guaranteed with the robust criterion (14) under the above inequality. In other words, the asymptotic stability can be achieved via proper adaptation laws for K_{aj} and k_{0j} stated as (18), (19).

5.2 Uncertain system matrices and fuzzy sets

When considering both uncertain system matrices and fuzzy sets in the TS-RFCMAC model of the plant, the control problem becomes very difficult. To solve this problem, we let all fuzzy sets of the controller composed of Gaussian membership functions in the form

$$F_{\ell i}(\bar{x}_\ell) = exp\left(\frac{-(\bar{x}_\ell - m_{\ell i})^2}{\sigma_{\ell i}^2}\right)$$

for $i = 1, 2, \cdots, r$ and $\ell = 1, 2, ..., n_x$, where $m_{\ell i}$ is the center of the Gaussian function; and $\sigma_{\ell i}$ is the variance of the Gaussian function. In other words, the activated level of the i-th rule is

$$\begin{aligned} w_i(\bar{\mathbf{x}}) &= \prod\nolimits_{\ell=1}^{n_x} F_{\ell i}(\bar{x}_\ell) \\ &= exp\left(\sum\nolimits_{\ell=1}^{n_x} \frac{-(\bar{x}_\ell - m_{\ell i})^2}{\sigma_{\ell i}^2}\right) \end{aligned}$$

Then, the robust adaptive TS-RFCMAC model-based controller is designed below.

Theorem 3: Consider a nonlinear system described by the TS-RFCMAC model (8) with uncertain system matrices A_i, A_{di} (for all i) and unknown fuzzy sets $F_{\ell i}(\bar{x}_\ell)$. The asymptotic stability with uniformly ultimate bound is assured if the TS-RFCMAC controller (15) uses the control gain K_{dj} satisfying the LMI (17) and the update laws (18), (19),

$$\begin{aligned} \dot{m}_{i\ell} &= \gamma_m w_i(\bar{\mathbf{x}})\left(\frac{2(\bar{x}_\ell - m_{\ell i})}{\sigma_{\ell i}^2}\right)\eta_i \\ \dot{\sigma}_{i\ell} &= \gamma_\sigma w_i(\bar{\mathbf{x}})\left(\frac{2(\bar{x}_\ell - m_{\ell i})^2}{\sigma_{\ell i}^3}\right)\eta_i \\ \dot{\hat{h}}_{\ell i} &= \gamma_h \left(\frac{2(\bar{x}_\ell - m_{j\ell})}{\sigma_{\ell i}^2}\right)\eta_i \cdot F_{\ell i}(N-1) \\ \eta_i &= -\sum_{j=1}^{r} w_j(\bar{\mathbf{x}})\mathbf{x}^T P B_j(K_{di}\mathbf{x} + K_{ai}\mathbf{x} + k_{0i}) \end{aligned}$$

with update gains $\gamma_m, \gamma_\sigma, \gamma_h > 0$.

The above tuning laws are obtained from the gradient descent method such that the stability property derived in Theorem 2 is not affected when using the on-line tuning laws. Moreover, the above adaptive laws cope with an inappropriate initial selection of fuzzy membership functions and recurrent weights. Accordingly, the overall controlled system is illustrated in Fig. 1.

Remark 2: Except for removing complex fuzzy defuzzification, the proposed TS-RFCMAC controller keeps the advantage of the TS fuzzy model-based control, including LMI based gain design and parallel distributed compensation. Moreover, the adaptive algorithm copes with the uncertainty of fuzzy sets and parameters.

6. SIMULATION RESULTS

Consider the uncertain Duffing system described by the following dynamics:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= 1.1x_1(t) - x_1^3(t) - 0.4x_2(t) \\ &\quad +0.02x_1(t-\tau(t)) + 1.8\cos(1.8t) \end{aligned}$$

where $\tau(t) = 0.05$. Since the Duffing system has the nonlinear term $x_1^3(t)$, we choose the input variable $x_1(t)$ and a discussion region $d = \sup_{x\in\Omega} |x_1(t)| = 3$ for the TS-RFCMAC model (8). Set the nominal system matrices: $B_1 = B_2 = [\,0\ 1\,]^T$,

$$A_{n1} = A_{n2} = \begin{bmatrix} 0 & 1 \\ 1 & -0.3 \end{bmatrix}$$

$$A_{dn1} = A_{dn2} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0 \end{bmatrix}$$

According to Theorem 3, the TS-RFCMAC model-based controller is constructed with $\gamma = 500$, $\gamma_a = 500$, $\gamma_m = \gamma_\sigma = \gamma_h = 20$, and proper initial conditions. After solving the LMI (17), we obtain $K_{d1} = K_{d2} = [\,2.3125\ 0.6375\,]$ for $\rho = 1.1$. The control results are illustrated in Figs. 2 and 3, while the control input is given in Fig. 4. When the proposed controller is activated at 20 second, the states are quickly driven to zero. Therefore, the results have shown expected performances.

7. CONCLUSION

This paper has presented a TS-RFCMAC modeling method and its application on robust adaptive control. Based on the proposed TS-RFCMAC, the controller is designed in a straightforward manner and provides the same advantages as the TS fuzzy model-based control, e.g., the control gains are designed from LMI techniques. Moreover, the uncertain subsystem matrices and fuzzy membership functions are allowed, i.e., the TS-RFCMAC model-based control is more robust than the TS fuzzy model-based control. Different to traditional TS fuzzy model-based control, the uncertainties are compensated by the adaptive TS-RFCMAC scheme. As a result, the robust performance is achieved via the proposed control scheme.

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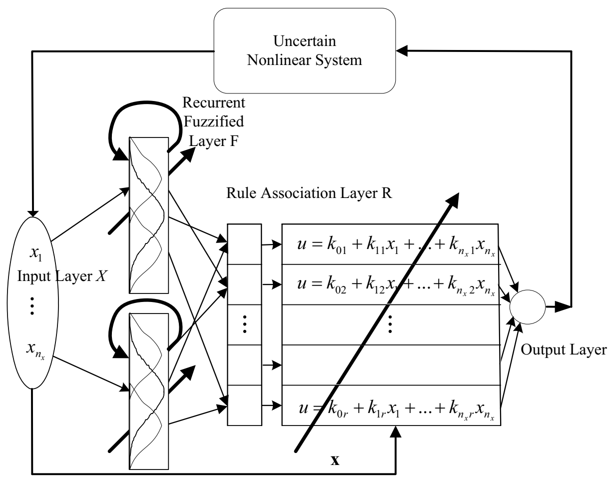

Fig. 1. The configuration of the TS-RFCMAC control.

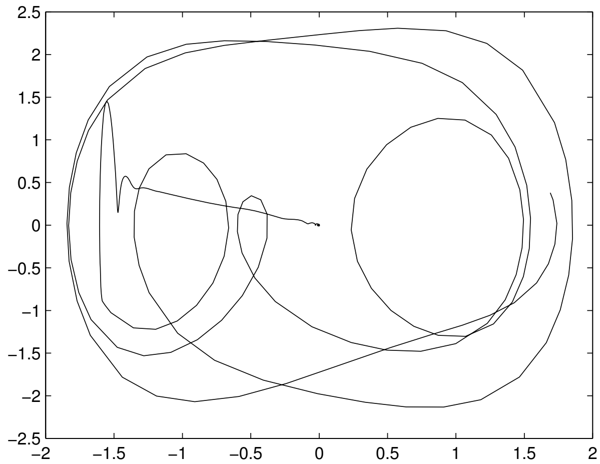

Fig. 2. The phase portrait of the controlled Duffing system.

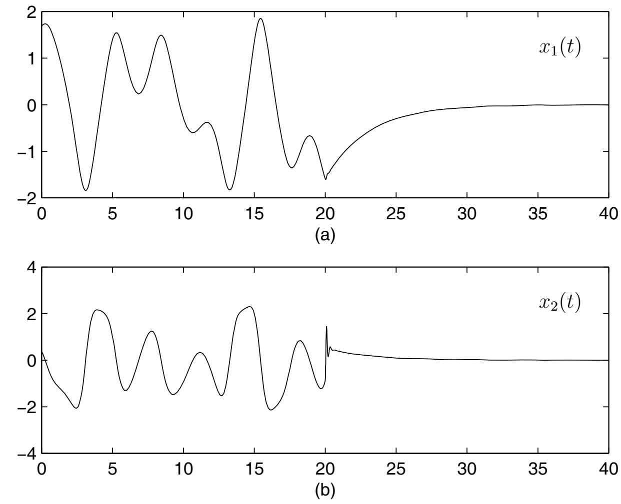

Fig. 3. The control response of (a) the state $x_1(t)$, and (b) the state $x_2(t)$.

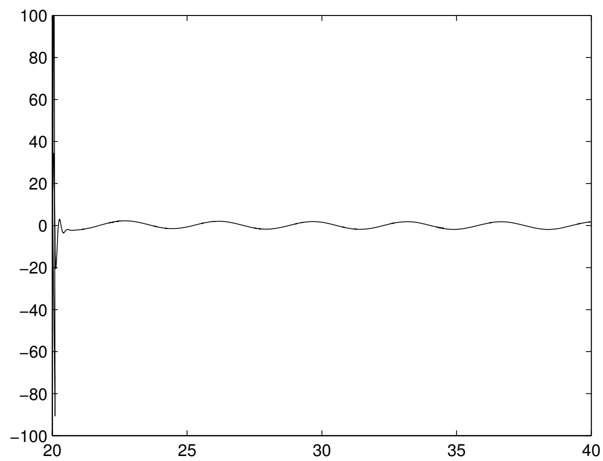

Fig. 4. The control force from the TS-RFCMAC.