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# Robust Fault Detection Linear Interval Observers Avoiding the Wrapping Effect

**Jordi Meseguer, Vicenç Puig, Teresa Escobet**

*Automatic Control Department (ESAII) - Campus de Terrassa*
*Universidad Politécnica de Cataluña (UPC)*
*Rambla Sant Nebridi, 10. 08222 Terrassa (Spain)*
*vicenc.puig@upc.edu*

**Abstract:** In model based fault detection is very important to analyze how the effect of model uncertainty is considered when determining the optimal threshold to be used in residual evaluation. In case of model uncertainty is located in parameters (interval model), an interval observer has been shown to be a suitable strategy to generate this adaptive threshold. However, interval observers can be affected by the wrapping effect when low computational algorithms, such as region-based approaches coming from the interval community, are used to determine the predicted output interval. This paper shows that the wrapping effect might be avoided forcing the observer gain to satisfy the isotonicity condition. Then, the effect of this observer condition on the time evolution of the residual sensitivity to a fault and the minimum detectable fault is analyzed in order to see whether the fault detection performance is enhanced or not. Finally, an example based on an industrial servo actuator will be used to illustrate the derived results.

## 1. INTRODUCTION

Most of the robust residual evaluation methods are based on an adaptive threshold changing in time according to the plant input signal and taking into account the model uncertainty. These last years the research of adaptive thresholding algorithms that use interval models for FDI has been a very active research area since the seminal work (Horak, 1988): (Armengol et al, 2000), (Puig et al, 2002), (Fagarasan el al, 2004) and (Ploix et al, 2006). In (Puig et al, 2003a) interval observers applied to robust fault detection have been introduced and in (Puig et al, 2003b), an interval simulation algorithm based on optimization through the set of possible real trajectories contained in the interval model is proposed. However, this trajectory based approach has a very high computational complexity. On the other hand, region (or set) based algorithms coming from the interval analysis (Kühn, 1998) are much less computational demanding but interval observers can suffer from the wrapping effect, if the model matrix does not fulfil the isotonicity property (Cugueró et al, 2002). The aim of this paper is to show how the wrapping effect can be avoided when an interval observer model is considered in spite a low computational algorithm is used to estimate the output interval time evolution. This will only be possible if the observer gain matrix satisfies a key condition. On the other hand, the effect of this condition on the observer fault detection performance is also analyzed to see whether it is enhanced or not. This paper continues the work developed in (Meseguer et al., 2007) which is focused on fault detection based on intervals observers. It shows the influence of the observer gain on the residual sensitivity to a fault and on the minimum detectable fault (Gertler, 1998) since, such as it was noticed by (Chen and Patton, 1999), the observer gain plays an important role in fault detection because it determines the time evolution of those fault detection properties.

The structure of the paper remainder is the following: in *Section 2*, fault detection concepts using interval observers are recalled and besides, the observer gain matrix design to avoid the wrapping effect is discussed. Then, (*Section 3*) the influence of avoiding the wrapping effect using the observer gain matrix on the observer fault detection performance is analyzed. In *Section 4*, an example based on an industrial smart actuator is used to illustrate the derived results. Finally, *Section 5* describes the paper conclusions.

## 2. FAULT DETECTION USING LINEAR INTERVAL OBSERVERS

### 2.1 Interval Observer Expression

Considering that the system to be monitored can be described by a MIMO linear dynamic model in discrete-time, its state-space form including faults is

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(\tilde{\boldsymbol{\theta}})\boldsymbol{x}(k) + \boldsymbol{B}(\tilde{\boldsymbol{\theta}})\boldsymbol{u}_0(k) + \boldsymbol{F}_a(\tilde{\boldsymbol{\theta}})\boldsymbol{f}_a(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}(\tilde{\boldsymbol{\theta}})\boldsymbol{x}(k) + \boldsymbol{F}_y(\tilde{\boldsymbol{\theta}})\boldsymbol{f}_y(k) \end{aligned} \tag{1}$$

where $\boldsymbol{y}(k) \in \Re^{ny}$, $\boldsymbol{u}_0(k) \in \Re^{nu}$, $\boldsymbol{x}(k) \in \Re^{nx}$ are the system output, input and the state-space vectors respectively; $\boldsymbol{A}(\tilde{\boldsymbol{\theta}})$, $\boldsymbol{B}(\tilde{\boldsymbol{\theta}})$, and $\boldsymbol{C}(\tilde{\boldsymbol{\theta}})$ are the state, the input and the output matrices respectively; $\tilde{\boldsymbol{\theta}}$ is the system parameter vector; $\boldsymbol{f}_y(k) \in \Re^{ny}$ and $\boldsymbol{f}_a(k) \in \Re^{nu}$ represent faults in the system output sensors and actuators respectively being $\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}})$ and $\boldsymbol{F}_a(\tilde{\boldsymbol{\theta}})$ their associated matrices. The system in Eq. (1) can be expressed in its input-output form using the shift operator, $q^{-1}$, and assuming zero initial conditions:

$$\boldsymbol{y}(k) = \boldsymbol{y}_0(k) + \boldsymbol{G}_{fa}(q^{-1},\tilde{\boldsymbol{\theta}})\boldsymbol{f}_a(k) + \boldsymbol{G}_{fy}(q^{-1},\tilde{\boldsymbol{\theta}})\boldsymbol{f}_y(k) \tag{2}$$

where $\boldsymbol{y}_0(k)$ (Eq. (3)) is the system output when the system is unaffected by faults, disturbances and noises.

$$\boldsymbol{y}_0(k) = \boldsymbol{C}(\tilde{\boldsymbol{\theta}})(q\boldsymbol{I} - \boldsymbol{A}(\tilde{\boldsymbol{\theta}}))^{-1}\boldsymbol{B}(\tilde{\boldsymbol{\theta}})\boldsymbol{u}_0(k) \tag{3}$$

Besides, $G_{fa}$ and $G_{fy}$ are the system transfer functions regarding the system faults $(f_a, f_y)$

$$G_{fa}(q^{-1},\tilde{\theta}) = C(\tilde{\theta})(qI - A(\tilde{\theta}))^{-1} F_a(\tilde{\theta}) \quad (4)$$

$$G_{fy}(q^{-1},\tilde{\theta}) = F_y(\tilde{\theta}) \quad (5)$$

The system described by Eq. (1) is monitored using a linear observer with *Luenberger* structure based on an ***interval model***. This type of model considers that model parameters $\theta$ are bounded by an interval set $\Theta = \left\{\theta \in \Re^{n\theta} \middle| \underline{\theta} \le \theta \le \overline{\theta}\right\}$. This set represents the uncertainty about the exact knowledge of real system parameters $\tilde{\theta}$. The resulting ***interval observer*** can be written as:

$$\begin{aligned} \hat{x}(k+1) &= (A(\theta) - LC(\theta))\hat{x}(k) + B(\theta)u(k) + Ly(k) \\ \hat{y}(k) &= C(\theta)\hat{x}(k) \end{aligned} \quad (6)$$

where $u$ is the measured system input vector, $\hat{x}$ is the estimated system space-state vector and $\hat{y}$ is the estimated system output vector. The observer gain matrix $L$ is designed to stabilise the matrix $A_o(\theta) = A(\theta) - LC(\theta)$ and to guarantee a desired performance regarding fault detection for all $\theta \in \Theta$. The effect of the uncertain parameters $\theta$ on the observer temporal response will be bounded using an interval: [ $\underline{\hat{y}}(k)$ , $\overline{\hat{y}}(k)$ ], where for each output:

$$\underline{\hat{y}}_i(k) = \min_{\theta \in \Theta}(\hat{y}_i(k,\theta)) \text{ and } \overline{\hat{y}}_i(k) = \max_{\theta \in \Theta}(\hat{y}_i(k,\theta)) \quad (7)$$

Considering $u$ might be affected by an input sensor fault, it can be expressed as:

$$u(k) = u_0(k) + F_u(\theta) f_u(k) \quad (8)$$

where $f_u(k) \in \Re^{nu}$ is the input sensor fault while $F_u(\theta)$ is its associated matrix. Conversely, $u_0(k)$ (Eq. (1)) is the real system input unaffected by faults and nuisance inputs. Thus, the observer given by Eq. (6) can be expressed in input-output form using the $q$-transform and considering zero initial conditions as it follows:

$$\hat{y}(k) = G(q^{-1},\theta)u_0(k) + H(q^{-1},\theta)y(k) + G_{fu}(q^{-1},\theta)f_u(k) \quad (9)$$

where:

$$G(q^{-1},\theta) = C(\theta)(qI - A_o(\theta))^{-1}B(\theta) \quad (10)$$

$$H(q^{-1},\theta) = C(\theta)(qI - A_o(\theta))^{-1}L \quad (11)$$

$$G_{fu}(q^{-1},\theta) = G(q^{-1},\theta)F_u(\theta) \quad (12)$$

*2.2 Fault Detection Using Interval Observers*

Fault detection is based on generating a residual comparing the measurements of physical variables $y(k)$ of the process with their estimation $\hat{y}(k)$ provided by the associated system model:

$$r(k) = y(k) - \hat{y}(k) \quad (13)$$

where: $r(k) \in \Re^{ny}$ is the residual set. According to *(Gertler, 1998)*, a generic form of a residual generator is given by

$$r(k,\theta) = -G(q^{-1},\theta)u(k) + (I - H(q^{-1},\theta))y(k) \quad (14)$$

Eq. (14) is known as the ***computational form*** of the residual which can be also expressed in terms of the effects caused by faults using Eq. (2) and Eq. (8). The resulting residual expression is known as the ***internal*** or ***unknown-input-effect form*** (*Gertler, 1998)*:

$$\begin{aligned} r(k,\theta) &= r_0(k,\theta) + (I - H(q^{-1},\theta))G_{fa}(q^{-1},\tilde{\theta})f_a(k) + \\ &+ (I - H(q^{-1},\theta))G_{fy}(q^{-1},\tilde{\theta})f_y(k) - G(q^{-1},\theta)F_u(\theta)f_u(k) \end{aligned} \quad (15)$$

where $r_0(k,\theta) = -G(q^{-1},\theta)u_0(k) + (I - H(q^{-1},\theta))y_0(k)$ (16)

would be the expression of the residual if the system were unaffected by faults and nuisance inputs being only caused by the parameter structured uncertainty. When considering model uncertainty located in parameters, the residual generated by Eq. (13) will not be zero even in a non-faulty scenario. Then, the fault detection test is based on propagating the parameter uncertainty to the residual *(Puig et al, 2002)* and checking if

$$0 \in [r(k)] = y(k) - [\hat{y}(k)] \text{ or } y(k) \in [\hat{y}(k)] \quad (17)$$

holds or not. In case it does not hold, a fault can be indicated.

*2.3 Designing the Observer Gain to Avoid the Wrapping Effect*

In (Puig et al. 2005), a classification of the algorithms used to compute the output predicted interval is given according to if they are based on: one step-ahead iteration based on previous approximations of the estimated state set (***region based approaches***) or on a set of point-wise trajectories generated by selecting particular values of $\theta \in \Theta$ using heuristics or optimisation (***trajectory based approaches***). But, when the undesired wrapping effect wants to be avoided, (Puig et al, 2005) shows the trajectory based approach must be used in spite of its high computational cost. However, if the observer matrix $A_o = A(\theta) - LC(\theta)$ is isotonic (Cugueró et al, 2002), the interval observer does not suffer from the wrapping effect when using the region-based approach, in spite of the non-isotonicity of $A(\theta)$ because of the existence of some negative elements of this matrix: $a_{ij} < 0$. Therefore, the clue to avoid this undesired problem is to design properly the observation gain matrix ($L$) so that $A_o(\theta)$ becomes isotonic. Thus, the observer matrix $A_o(\theta)$ achieves isotonicity only if the corresponding element $a_{oij}$ of this matrix is zero-valued.

$$a_{oij} = a_{ij} - (LC)_{ij} = 0 \qquad \forall i,j \text{ where } a_{ij} < 0 \quad (18)$$

where $(LC)_{ij}$ is the element of the resultant matrix $LC$ placed in the $i^{th}$-row and $j^{th}$-column. Conversely, condition given by Eq. (18) can be also expressed as it follows:

$$a_{ij} = \sum_{\alpha=1}^{ny} l_{i\alpha} c_{\alpha j} \qquad \forall i,j \text{ where } a_{ij} < 0 \quad (19)$$

where $l_{i\alpha}$ are the $i^{th}$-row elements of the observation gain matrix $L$ and $c_{\alpha j}$ are the $j^{th}$-column elements of the output matrix $C(\theta)$ associated to the observer model. Regarding the other elements $(LC)_{mn}\big|_{m\neq i, n\neq j}$, they do not have any effect on the isotonicity property of the observer matrix $A_o(\theta)$ and consequently, the elements $l_{m\alpha}$ ($1 \le \alpha \le ny$) associated to the observation gain matrix $L$ might be chosen freely to achieve the desired fault detection performance.

Regarding the observation gain $L$, this matrix can be partitioned in a matrix $L_-$ whose elements determine the observation gain values needed to force the isotonicity condition (19) and a matrix $L_+$ whose elements can be chosen freely to enhance the observer fault detection performance and to guarantee the observer stability. In line with the definition of matrices $L_-$ and $L_+$, the next expressions can be set:

$$L = L_+ + L_- \quad (20)$$

$$A_o(\theta) = A(\theta) - L_+C(\theta) - L_-C(\theta) \quad (21)$$

$$\left(\boldsymbol{L}_+\boldsymbol{C}(\boldsymbol{\theta})\right)_{mn} = \begin{cases} \sum_{\alpha=1}^{ny} l_{m\alpha} c_{\alpha n} & \forall m \neq i, n \neq j \text{ where } a_{mn} > 0 \\ 0 & \forall m = i, n = j \text{ where } a_{ij} < 0 \end{cases} \quad (22)$$

$$\left(\boldsymbol{L}_-\boldsymbol{C}(\boldsymbol{\theta})\right)_{mn} = \begin{cases} 0 & \forall m \neq i, n \neq j \text{ where } a_{mn} > 0 \\ a_{mn} & \forall m = i, n = j \text{ where } a_{ij} < 0 \end{cases} \quad (23)$$

Thus, the elements of matrix $\boldsymbol{L}_+\boldsymbol{C}$ are positive or zero-valued while those elements of matrix $\boldsymbol{L}_-\boldsymbol{C}$ are negative or zero-valued. Then, comparing the norm of the observer gain matrix $\boldsymbol{A}_o(\boldsymbol{\theta})$ (Eq. (21)) when the isotonicity condition (19) is forced or not, the next relation is set

$$\left\| \boldsymbol{A}_o(\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}=\boldsymbol{0}} \right\| \geq \left\| \boldsymbol{A}_o(\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}\neq\boldsymbol{0}} \right\| \quad (24)$$

In spite of the previous relation (Eq. (24)), it must be taken into account that condition (19) forces the negative elements of $\boldsymbol{A}_o(\boldsymbol{\theta})$ to be null what let also establish the next relation:

$$\left\| \boldsymbol{I} - \boldsymbol{A}_o(\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}=\boldsymbol{0}} \right\| \geq \left\| \boldsymbol{I} - \boldsymbol{A}_o(\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}\neq\boldsymbol{0}} \right\| \quad (25)$$

Alternatively, analyzing the non-zero-valued elements of matrix $\boldsymbol{L}_-$, they must fulfil the relation given by condition (19). Thereby, when all non-zero-valued elements of ***C(θ)*** are positive, the non-zero-valued elements of $\boldsymbol{L}_-$ must be negative while the non-zero-valued elements of $\boldsymbol{L}_+$ must be positive. In general, neither all elements of matrix $\boldsymbol{L}_-$ have to be negative nor all elements of matrix $\boldsymbol{L}_+$ have to be positive when forcing condition (19).

## 3. INFLUENCE OF THE ISOTONICITY CONDITION ON FAULT DETECTION

### *3.1 Influence on the Residual Sensitivity to a Fault*

The ***residual sensitivity*** (Gertler, 1998) to a fault is given by

$$\boldsymbol{S}_f(q^{-1}) = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{f}} \quad (26)$$

which is a transfer function that describes the effect on the residual, $\boldsymbol{r}$, of a given fault $\boldsymbol{f}$. Thereby, taking into account Eq. (26), the residual internal form (Eq. (15)) can be written in terms of the residual sensitivities to an output sensor fault $\boldsymbol{f}_y$, $\boldsymbol{S}_{fy}$; to an input sensor fault $\boldsymbol{f}_u$, $\boldsymbol{S}_{fu}$; and to an input sensor fault $\boldsymbol{f}_a$, $\boldsymbol{S}_{fa}$:

$$\boldsymbol{r}(k,\boldsymbol{\theta}) = \boldsymbol{r}_0(k,\boldsymbol{\theta}) + \boldsymbol{S}_{fa}(q^{-1},\boldsymbol{\theta})\boldsymbol{f}_a(k) + \boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta})\boldsymbol{f}_y(k) + \boldsymbol{S}_{fu}(q^{-1},\boldsymbol{\theta})\boldsymbol{f}_u(k) \quad (27)$$

where

$$\boldsymbol{S}_{fa}(q^{-1},\boldsymbol{\theta}) = (\boldsymbol{I} - \boldsymbol{H}(q^{-1},\boldsymbol{\theta}))\boldsymbol{G}_{fa}(q^{-1},\tilde{\boldsymbol{\theta}}) \quad (28)$$

$$\boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta}) = (\boldsymbol{I} - \boldsymbol{H}(q^{-1},\boldsymbol{\theta}))\boldsymbol{G}_{fy}(q^{-1},\tilde{\boldsymbol{\theta}}) \quad (29)$$

$$\boldsymbol{S}_{fu}(q^{-1},\boldsymbol{\theta}) = -\boldsymbol{G}(q^{-1},\boldsymbol{\theta})\boldsymbol{F}_u(\boldsymbol{\theta}) \quad (30)$$

In the following, the effect of condition (19) on these fault residual sensitivity time functions is analyzed. In this manner, according to Eq. (29), the residual sensitivity to an output sensor fault $\boldsymbol{f}_y$ is given by

$$\begin{aligned} \boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta}) &= (\boldsymbol{I} - \boldsymbol{H}(q^{-1},\boldsymbol{\theta}))\boldsymbol{G}_{fy}(q^{-1},\tilde{\boldsymbol{\theta}}) \\ &= (\boldsymbol{I} - \boldsymbol{C}(\boldsymbol{\theta})(q\boldsymbol{I} - \boldsymbol{A}_o(\boldsymbol{\theta}))^{-1}\boldsymbol{L})\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}}) \end{aligned} \quad (31)$$

Eq. (31) is a time function whose dynamics and steady-state gain are influenced by the observer gain $\boldsymbol{L}$. Thereby, when condition (19) is forced, the residual sensitivity time evolution is deeply affected. Its initial value at time instant $k=0$, i.e., when fault occurs, is

$$\boldsymbol{s}_{fy}(0) = \lim_{q\to\infty} \boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta}) = \boldsymbol{F}_y(\tilde{\boldsymbol{\theta}}) \quad (32)$$

independently of the observer gains and thus, it is unaffected by condition (19). Conversely, the steady-state value for an abrupt fault modelled as a unit-step function is given by

$$\boldsymbol{s}_{fy}(\infty) = \lim_{q\to 1} \boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta}) = (\boldsymbol{I} - \boldsymbol{C}(\boldsymbol{\theta})(\boldsymbol{I} - \boldsymbol{A}_o(\boldsymbol{\theta}))^{-1}(\boldsymbol{L}_+ + \boldsymbol{L}_-))\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}}) \quad (33)$$

Consequently, when forcing condition (19) and assuming that the non-zero valued elements of ***C(θ)*** are positive, the steady-state value of the output sensor residual sensitivity matrix (Eq.(33)) norm increases regarding the case where that condition is not forced.

$$\left\| \boldsymbol{s}_{fy}(\infty)\big|_{\boldsymbol{L}_-\boldsymbol{C}=\boldsymbol{0}} \right\| \leq \left\| \boldsymbol{s}_{fy}(\infty)\big|_{\boldsymbol{L}_-\boldsymbol{C}\neq\boldsymbol{0}} \right\| \quad (34)$$

In a general case where the elements of the output matrix ***C(θ)*** are not assumed to be positive, the relation given by Eq. (34) might be satisfied when the elements of the observation gain matrix $\boldsymbol{L}$ are required to be negative by the interval observer structure to place the model poles to obtain a proper fault detection performance or by the isotonicity condition (19) to avoid the wrapping effect. Concerning the residual sensitivity function to an input sensor fault and to an actuator fault, the same analysis could be done obtaining similar conclusions regarding the influence of the isotonicity condition (30) on their time evolution.

### *3.2 Influence on the Residual*

Analyzing the residual computational form given by Eq. (14) and taking into account Eq. (29) and Eq. (30), it is seen that the residual expression can be also written in terms of the output and input sensor fault residual sensitivity matrices ($\boldsymbol{S}_{fy}$, $\boldsymbol{S}_{fu}$) assuming $\boldsymbol{F}_y$ and $\boldsymbol{F}_u$ are equal to the identity matrix (Meseguer et al, 2007).

$$\boldsymbol{r}(k,\boldsymbol{\theta}) = \boldsymbol{S}_{fu}(q^{-1},\boldsymbol{\theta})\boldsymbol{u}(k) + \boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta})\boldsymbol{y}(k) \quad (35)$$

Then, derived from the conclusions obtained in *Section 3.1*, the time evolution of the residual is clearly influenced by condition (19). In this case, the next relation can be established assuming condition (19) forces the non-zero-valued elements of $\boldsymbol{L}_-$ to be negative:

$$\left[ \boldsymbol{r}(k,\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}=\boldsymbol{0}} \right] \subseteq \left[ \boldsymbol{r}(k,\boldsymbol{\theta})\big|_{\boldsymbol{L}_-\boldsymbol{C}\neq\boldsymbol{0}} \right] \quad (36)$$

Thus, according to Eq. (36) and the residual definition given by Eq. (13), when condition (19) is forced to avoid the wrapping effect, the output interval **[*y(k)*]** encloses the one generated if that condition would not be forced. Consequently, according to the fault detection condition given by Eq. (17) (*Section 2.2*), the interval observer will need bigger faults so that they can be detected.

### *3.3 Influence on the Minimum Detectable Fault Function*

According to *(Meseguer et al., 2007)* and derived from the "***triggering limit***" concept given by *(Gertler, 1998)*, the minimum detectable fault is a fault, $f_f^{min}(k)$, whose residual disturbance counteracts the interval observer adaptive threshold from its apparition time instant. Recalling shortly how the expression of $f_f^{min}(k)$ was obtained in *(Meseguer et al., 2007)*, the expression of the residual disturbance associated to $f_f^{min}(k)$ is given by

$$\boldsymbol{d}_f^{\min}(k) = \begin{cases} \boldsymbol{0} & if \quad k < t_0 \\ -\boldsymbol{r}_0(k) & if \quad k \geq t_0 \end{cases} \quad (37)$$

where $t_0$ is the fault occurrence time instant and $\boldsymbol{r}_0(k)$ is the interval adaptive threshold whose expression is given by Eq. (16). Then, according to the fault residual sensitivity concept

(Eq. (26)), the residual disturbance caused by a fault ***f(k)*** can be written as

$$\boldsymbol{d}_f(k,\boldsymbol{\theta}) = \boldsymbol{S}_f(q^{-1},\boldsymbol{\theta})\boldsymbol{f}(k)q^{-t_0} \qquad (38)$$

where $\boldsymbol{S}_f(k)$ is the residual sensitivity to that fault. Then, the minimum detectable fault function $\boldsymbol{f}_f^{min}(k)$ can be written as:

$$\boldsymbol{f}_f^{min}(k-t_0) = -\boldsymbol{S}_f(q^{-1})^{-1}\boldsymbol{r}_0(k) \qquad (39)$$

where $k \geq t_0$ and assuming $\boldsymbol{S}_f^{-1}$ exists because of the clearness of the derived conclusions. Thus, a fault ***f(k)*** producing a residual disturbance, $\boldsymbol{d}_f(k)$, bigger than the associated to $\boldsymbol{f}_f^{min}(k), \boldsymbol{d}_f^{min}(k)$, is always detected (***strong fault detection***) while a fault producing a smaller residual disturbance is never detected *(Meseguer et al., 2007)*.

In the following, the effect of forcing the isotonicity condition (19) on the minimum detectable function is analyzed considering the cases of an output sensor fault $\boldsymbol{f}_y$, an input sensor fault $\boldsymbol{f}_u$ and an actuator fault given by $\boldsymbol{f}_a$. Considering that the output sensor fault residual sensitivity, $\boldsymbol{S}_{fy}$, is given by Eq. (31), the minimum detectable fault function specified by Eq. (39) can be particularized for the output sensor fault case as:

$$\begin{aligned}&\boldsymbol{f}_{fy}^{min}(k-t_0) = -\boldsymbol{S}_{fy}(q^{-1})^{-1}\boldsymbol{r}_0(k) = \\ &-\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}})^{-1}\left(\boldsymbol{I} + \boldsymbol{C}(\boldsymbol{\theta})\left(q\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{\theta})\right)^{-1}(\boldsymbol{L}_+ + \boldsymbol{L}_-)\right)\boldsymbol{r}_0(k)\end{aligned} \qquad (40)$$

where $k \geq t_0$ and $\boldsymbol{S}_{fy}^{-1}$ is obtained using the *matrix inversion lemma.* Besides, it must be taken into account that $\boldsymbol{r}_0(\boldsymbol{k})$ given by the Eq. (16) is affected by condition (19) such as it is ***r(k)*** (*Section 3.2*) according to their expressions. At time instant $k=t_0$ when the fault occurs, the value of Eq. (40) is given by

$$\boldsymbol{f}_{fy}^{\min}(0) = -\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}})^{-1}\boldsymbol{r}_0(t_0) \qquad (41)$$

Then, given that forcing condition (19) widens the interval adaptive threshold regarding the case this condition is not used (Eq. (36)), the initial value of the minimum output sensor fault will increase its absolute value.

$$\left\| \boldsymbol{f}_{fy}^{\min}(0)\Big|_{\boldsymbol{L},\boldsymbol{C}=\boldsymbol{0}} \right\| \leq \left\| \boldsymbol{f}_{fy}^{\min}(0)\Big|_{\boldsymbol{L},\boldsymbol{C}\neq\boldsymbol{0}} \right\| \qquad (42)$$

Regarding the steady-state value of this minimum fault function (Eq.(40)), it can be demonstrated *(Meseguer et al, 2007)* that its expression, using some matrix product properties, is given by

$$\boldsymbol{f}_{fy}^{min}(\infty) = -\boldsymbol{F}_y(\tilde{\boldsymbol{\theta}})^{-1}(-\boldsymbol{C}(\boldsymbol{\theta})\left(\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{\theta})\right)^{-1}\boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}_0(\infty) + \boldsymbol{y}_0(\infty)) \qquad (43)$$

which does not depend on $\boldsymbol{L}$ and thus, it is unaffected by condition (19). Concerning the minimum detectable input sensor fault and the minimum detectable actuator fault, the same analysis could be done obtaining similar results.

In conclusion, when forcing the isotonicity condition, the minimum detectable fault functions are only affected during their transient-state and not once they reach their steady-state values. The transient-state values of these functions are bigger than the ones obtained when this condition is not used and consequently, this fact means that the interval observer requires a bigger fault during the residual transient state caused by the fault to start indicating the faulty situation.

### *3.4 Influence on the Fault Detection Persistency*

Derived from the minimum detectable fault concept, a fault is detected while its effect on the residual (residual disturbance $\boldsymbol{d}_f(\boldsymbol{k})$ given by Eq. (38)) surpasses the interval observer threshold $\boldsymbol{r}_0(\boldsymbol{k})$ which is originated by the effect of the model structured uncertainty on the residual (Meseguer et al, 2007). This condition can be written using the following equation:

$$\left[-\boldsymbol{d}_f(k,\boldsymbol{\theta})\right] \not\subset \left[\boldsymbol{r}_0(k,\boldsymbol{\theta})\right] \quad k \geq t_0 \qquad (44)$$

Conversely, when condition (19) is forced, both the interval observer threshold $\boldsymbol{r}_0(\boldsymbol{k})$ (*Section 3.2*) and the residual disturbance $\boldsymbol{d}_f(\boldsymbol{k})$ (*Section 3.1*) are affected increasing its values regarding the case where that condition is not used. However, it must be taken into account that $\boldsymbol{r}_0(\boldsymbol{k})$ is affected since $k=0$ while $\boldsymbol{d}_f(\boldsymbol{k})$ is from the fault occurrence time instant $k=t_0$ and is not fully affected until it reaches its steady-state. In consequence, the fault indication might be affected negatively during the transient-state caused by the fault requiring more time instants to start indicating the fault or/and indicating the fault during less time instants.

## 4. APPLICATION EXAMPLE

### *4.1 Description*

The application example proposed to illustrate the obtained results deals with an industrial smart actuator proposed as an FDI benchmark in the European DAMADICS project. Using physical modelling (Bartys, 2002) linearising around the operating point and a mixed optimization-identification algorithm as in (Ploix, 1999) the following linear interval model has been derived:

$$\begin{aligned}\hat{\boldsymbol{x}}(k+1) &= \boldsymbol{A}(\boldsymbol{\theta})\hat{\boldsymbol{x}}(k) + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}(k) \\ \hat{\boldsymbol{y}}(k) &= \hat{x}_3(k)\end{aligned} \qquad (45)$$

with: $\hat{\boldsymbol{x}}(k) = \left[\hat{x}_1(k) \quad \hat{x}_2(k) \quad \hat{x}_3(k)\right]'$,

$$\boldsymbol{A}(\theta) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \theta_3 & \theta_2 & \theta_1 \end{bmatrix}, \ \boldsymbol{B}(\theta) = \begin{bmatrix} 0 \\ 0 \\ \theta_4 \end{bmatrix} \text{ and } \boldsymbol{u}(k) = CVP(k-2)$$

where: $\hat{x}_3(k)$ is the valve position estimation, $\hat{\boldsymbol{y}}(k)$ is the estimation of this position measured by the displacement transducer (in Volt), ***CVP(k)*** is the command pressure (in Pascal) measured by a given input sensor and the uncertain parameters are bounded by their confidence intervals according to: $\theta_1$=[1.1417 1.1471], $\theta_2$=[0.3995 0.4103], $\theta_3$=[-0.5537 -0.5484], and $\theta_4$=[2.180e-4 2.183e-4]. In this application example, a constant command pressure whose value is $u(k) = 1Pa$ has been considered. Regarding the isotonicity property, $\boldsymbol{A}(\theta)$ does not satisfy it because some of their elements are negative and consequently, this model suffers from wrapping effect.

### *4.2 Input-Output Observer Expression*

Following from Eq. (45), the expression of the interval observer associated to the output is given by

$$\begin{aligned}\hat{\boldsymbol{y}}(k) &= \frac{\theta_4 q^{-1}}{1 + (k_1 - \theta_1)q^{-1} + (k_2 - \theta_2)q^{-2} + (k_3 - \theta_3)q^{-3}}\boldsymbol{u}(k) + \\ &+ \frac{k_1 q^{-1} + k_2 q^{-2} + k_3 q^{-3}}{1 + (k_1 - \theta_1)q^{-1} + (k_2 - \theta_2)q^{-2} + (k_3 - \theta_3)q^{-3}}\boldsymbol{y}(k)\end{aligned} \qquad (46)$$

where $k_1$, $k_2$ and $k_3$ are the observer gains used to avoid the wrapping effect and to enhance fault detection performance regarding the needed requirements. Conversely, in line with the actuator model (45), the isotonicity condition (19) is satisfied whether

$$k_3 = \theta_3 \tag{47}$$

Considering the parameterisation $k_i = l_i\theta_i$, Eq. (47) implies $l_3 = 1$. Concerning $l_1$ and $l_2$, the next values are used in this application example in order to guarantee the model stability:

$$l_1 = l_2 = 0.5 \tag{48}$$

Regarding the observer residual, its expression is given by

$$\begin{aligned} \boldsymbol{r}(k,\boldsymbol{\theta}) = & -\frac{\theta_4 q^{-1}}{1+(k_1-\theta_1)q^{-1}+(k_2-\theta_2)q^{-2}+(k_3-\theta_3)q^{-3}}\boldsymbol{u}(k)+ \\ & +\frac{1-\theta_1 q^{-1}-\theta_2 q^{-2}-\theta_3 q^{-3}}{1+(k_1-\theta_1)q^{-1}+(k_2-\theta_2)q^{-2}+(k_3-\theta_3)q^{-3}}\boldsymbol{y}(k) \end{aligned} \tag{49}$$

### *4.3 Avoiding the Wrapping Effect Using the Observer Gain*

The goal of this *Section* is to show how the output interval modelled by Eq. (46) is affected by the wrapping effect when it is computed using the region-based approach and the isotonicity condition given by Eq. (47) is not satisfied.

Thus, in *Figure 1*, the time evolution of the estimated output interval, its nominal value and the system output are plotted between the time instants $t_1=190$ and $t_2=220$ using $l_1=l_2=l_3=0.5$ and considering the region-based approach. Besides, a constant additive fault affecting the system output sensor occurring at time instant $t_0=200$ and whose value is given by $f= 0.01$ *Volt* has been considered. In this figure, it is seen that the estimated output interval is useless to indicate the fault because it suffers from unstable wrapping effect.

Conversely, when the isotonicity condition is applied ($l_3=1$) without changing the value of the others observer gains (*Fig. 2*), the region-based approach avoids the wrapping effect estimating the same output interval than the trajectory-based approach.

### *4.4 Influence of the Isotonicity Condition on the Sensitivity of the Residual to an Output Sensor Fault*

Taking into account Eq.(31) and considering the example application residual expression (Eq. (49)), the residual sensitivity to an output sensor fault associated to the example application is given by the following expression

$$\boldsymbol{S}_{fy}(q^{-1},\boldsymbol{\theta}) = \frac{1-\theta_1 q^{-1}-\theta_2 q^{-2}-\theta_3 q^{-3}}{1+(k_1-\theta_1)q^{-1}+(k_2-\theta_2)q^{-2}+(k_3-\theta_3)q^{-3}} \tag{50}$$

where $\boldsymbol{F}_y$ is assumed to be the identity matrix.

In this *Section,* the time evolution of the residual sensitivity to an output sensor fault (Eq. 50) is plotted in *Fig. 3* assuming an abrupt fault modelled as a unit-step function and considering an observer gain set that satisfies the isotonicity condition ($l_1=l_2=0.5$ and $l_3=1$) and another that does not ($l_1=l_2=0.5$ and $l_3=0$). Besides, for the clearness of the plot, instead of using the interval parameters $\theta_i$, their associated lower bounds $\underline{\theta}_i$ are used. Thereby, *Fig. 3* shows that when the isotonicity condition ($l_3=1$) is forced, the absolute value of the residual sensitivity steady-state value increases (Eq. (34)). Regarding its dynamics, this is deeply affected but it is difficult to say anything in general since it also depends on the system model.

### *4.5 Influence of the isotonicity condition on the observer adaptive threshold*

Following from Eq. (16), the interval observer threshold can be obtained from Eq. (49) assuming there are no faults affecting the input and output sensors. In *Fig. 4*, the time evolution of that threshold is plotted considering the two observer gain sets, ($l_1=l_2=0.5$ and $l_3=1$) and ($l_1=l_2=0.5$ and $l_3=0$) and using also the parameters $\underline{\theta}_i$. When the isotonicity condition is forced ($l_3=1$), the interval observer threshold increases its absolute value, such as it was indicated by Eq. (36), worsening the fault detection as bigger faults will be required in order to detect them.

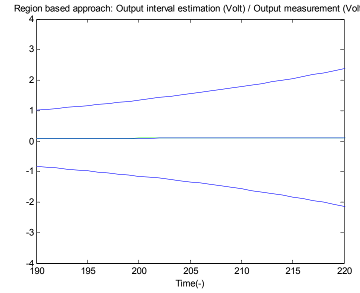


Fig. 1 Time evolution of the estimated interval output, its nominal value and the output sensor measurement.

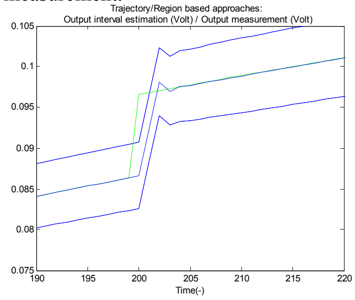


Fig. 2 Time evolution of the estimated interval output, its nominal value and the output sensor measurement.

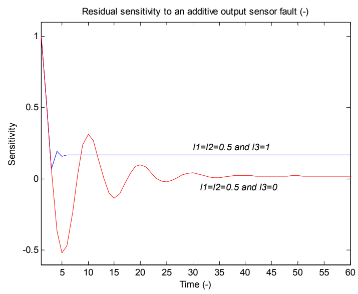


Fig. 3 Time evolution of the residual sensitivity

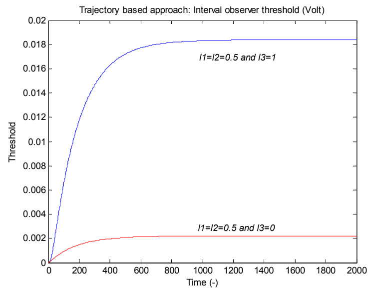


Fig. 4 Time evolution of the observer adaptive threshold.

### *4.6 Influence of the isotonicity condition on the minimum detectable output sensor fault*

The minimum detectable output sensor fault is given by Eq. (39) using the residual sensitivity (50) and the interval observer threshold given by *Fig. 4*. Thus, the considered function can be expressed as it follows:

$$f_{fy}^{min}(k-t_0) = -\frac{1+(k_1-\theta_1)q^{-1}+(k_2-\theta_2)q^{-2}+(k_3-\theta_3)q^{-3}}{1-\theta_1 q^{-1}-\theta_2 q^{-2}-\theta_3 q^{-3}} r_0(k) \quad (51)$$

where $k \geq t_0$. Considering the two observer gain sets and assuming the fault occurrence time instant, $t_0$, is 400, the time evolution of the minimum detectable fault function using the parameters $\underline{\theta}_i$ is drawn in *Fig. 5*. In this *Figure*, it is seen that forcing the isotonicity condition has no effect on the steady-state value of this function (Eq. (43)). In opposition, it is affected during its transitory-state as consequence of the influence on the interval observer threshold (Eq. (41) and Fig. 4) such as it was indicated in *Section 4.5.*

### *4.7 Influence of the Isotonicity Condition on Additive Output Sensor Fault Detection*

In this case, a fault occurring at time instant $t_0$=400 and whose value is given by $f$= *-0.06 Volt* is considered. In *Fig. 6*, the time evolution of the estimated interval output, its nominal value and the system output is plotted considering an observer gain set which does not satisfy the isotonicity condition ($l_1$=$l_2$=0.5 and $l_3$=0) and using the trajectory-based approach to avoid the wrapping effect. Besides, at the bottom of the figure, a fault indicator activated when the fault is detected is also plotted. On the other hand, in *Fig. 7,* the same faulty scenario is plotted but using an observer gain set which fulfils the isotonicity condition ($l_1$=$l_2$=0.5 and $l_3$=1). Comparing both cases, it is seen that the observer fault detection performance is worsened when the isotonicity condition is forced.

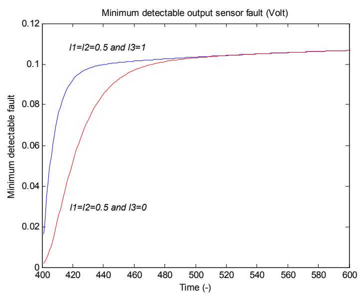


Fig. 5 Time evolution of the minimum detectable output sensor fault

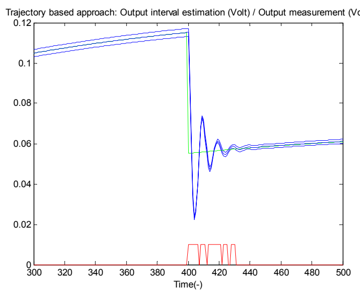


Fig. 6 Time evolution of the estimated interval output, its nominal value and the output sensor measurement using $l_1$=$l_2$=0.5 and $l_3$=0.

## 5. CONCLUSIONS

This paper shows a method to avoid the wrapping effect that is affecting an interval model when it is computed using a low computational algorithm ("region-based approach"). It is demonstrated that it is not necessary to use a high computational algorithm ("trajectory-based approach") to avoid this effect but it is enough to consider an interval observer model and to design properly the observation gain matrix $\boldsymbol{L}$. In fact, this method is only based on turning the non-isotonic model matrix into an isotonic one using the mentioned matrix $\boldsymbol{L}_-$ ($\boldsymbol{L}$= $\boldsymbol{L}_+$+$\boldsymbol{L}_-$). This paper also shows that designing $\boldsymbol{L}$ to avoid the wrapping effect worsens apparently the observer fault detection performance. However, analyzing the main observer fault detection properties, it can be seen that a proper design of $\boldsymbol{L}_+$ might counteract the negative effect of $\boldsymbol{L}_-$ regarding fault detection performance. This task is planned to carry out as a further research.

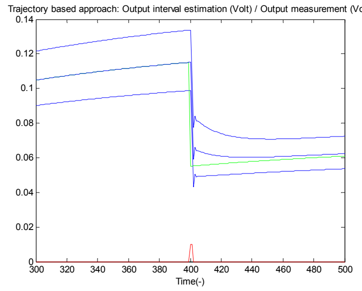


Fig. 7 Time evolution of the estimated interval output, its nominal value and the output sensor measurement using $l_1$=$l_2$=0.5 and $l_3$=1

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