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Fault Diagnosis using a Timed Discrete Event Approach based on Observers

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Abstract: This paper proposes a fault diagnosis method using a timed discrete-event approach based on interval observers which improves the integration of fault detection and isolation tasks. The interface between fault detection and fault isolation considers the degree of fault signal activation and the occurrence time of the diagnostic signals using a combination of several theoretical fault signature matrices which store the knowledge of the relationship between diagnostic signals and faults. As a novelty, this paper proposes to implement the fault isolation module using a timed discrete event approach in spite of using an analytical fault detection model. In this way, the diagnosis result will be enhanced since the occurrence of a fault generates a unique sequence of observable events (fault signals) that will be recognized by the isolation module implemented as a timed discrete event system. The states and transitions that characterize such a system can be inferred directly from the relation between fault signals and faults. The proposed fault diagnosis approach is applied to detect and isolate faults of the Barcelona's urban sewer system limnimeters (level meter sensors).

1. INTRODUCTION

When classifying models applied to the diagnostics of processes (systems), it is possible to distinguish between models applied to fault detection and models used for fault isolation (or system state recognition) (Kościelny et al. 2004a). Models used for fault detection (either qualitative used by DX community or analytical used by FDI community) describe relationships existing between the system inputs and outputs, and allow detecting inconsistencies caused by faults generating fault diagnostic signals (fault signals). A fault signal appears when the residual evaluation stage associated to the fault detection task concludes that the residual time evolution is caused by the effect of a fault (Chow et al., 1984). Thus, although the fault signal is characterized by a given dynamics, it can be considered as a discrete event caused by the fault effect on the monitored system. The goal of the fault detection model is to generate fault signals so that the fault can be isolated. The type of the model used in fault detection (qualitative or quantitative) in general depends on the system knowledge and the effort required to obtain an accurate model. If an accurate analytical model can be obtained using a reasonable effort, this type of models seems to be a better choice than the qualitative models. Otherwise, qualitative models seem to be better in fault detection.

On the other hand, models used for fault isolation (qualitative or analytical) define the relationship existing between observed diagnostic signals and faults. The basic idea of a fault diagnostic system is that the occurrence of a fault will generate a unique sequence of observable fault signals (events) that will establish the presence of a given fault. In general, the model type (qualitative or quantitative) used in fault isolation depends on the type of the fault detection model. However, since a fault signal can be seen as a discrete-time event with a given occurrence time instant, dynamics and duration, the use of those qualitative models known as timed discrete events models (Daigle et al, 2007) (Lunze et al. 2005) follows naturally. However, this kind of models is not very common when fault detection stage is modelled using an analytical model. In this paper, the proposed fault diagnosis approach will combine a qualitative timed discrete event model with an analytical model used in fault detection. The proposed method can be considered as a BRIDGE approach that tries to benefit from the best of the two fault diagnosis communities (FDI and DX): Fault signals are represented as a temporal sequence of discrete events using a qualitative DX approach while fault detection is based on an analytical model, as usual in FDI, which takes account of the model uncertainty using an interval associated to the parameter vector. In this way, it will be shown that all available and useful information for the fault detection and isolation tasks is considered. Normally, when a pure FDI or DX scheme is used, there is a loss of useful information either in the fault isolation or in the fault detection as a consequence of the type of model/representation used.

The aim of this paper is to show the fault isolation module can naturally be represented as a timed discrete event model in spite the monitored system is modelled analytically. This paper continues the research developed in (Puig et al., 2005), (Meseguer et al., 2006) and (Meseguer et al., 2007). (Puig et al, 2005) shows that the typical binary interface proposed in (Gertler, 1998) between fault detection and fault isolation can lead to inaccurate fault isolation results and shows that it can be improved when other fault signal properties are considered: the sign of the fault signal, the static fault residual sensitivity, the order occurrence of the fault signal and the fault signal occurrence time instant. In (Meseguer et al., 2006), the interface presented in (Puig et al., 2005) is used and the monitored system is modelled using an interval observer model. This last paper characterizes the influence of the fault detection stage on the fault isolation result. Thus, the observation gain matrix can be designed to enhance the fault detection and isolation results. (Meseguer et al., 2007) continues the work developed in both previous papers showing that the relationship between faults and the properties of the temporal sequence of fault signals can be obtained analytically using the interval observer model and it is stored in several fault signature matrices: one matrix for each property.

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Following the results obtained in the papers mentioned previously, this paper shows how it is possible to build a fault isolation model based on a timed discrete event system using the fault signatures matrices mentioned above which are obtained using the interval observer. Regarding this type of fault isolation model, (Daigle et al, 2007) uses a temporal labelled transition system which is built on the grounds of a temporal causal graph that models the behaviour of the monitored systems. Conversely, *T-DTS* method (Kościelny et al., 2004b) models the relationship between fault signals and faults using the called Fault Information System (*FIS*). The fault isolation algorithm used by this method is based on series inference where the occurrence of a new fault signal let narrow the possible fault hypothesis checking its observed properties and the information stored in the *FIS*.

Regarding the structure of the remainder, in next section, the passive robust fault detection using interval observers is recalled. Then, (*Section 3*) the interface between fault detection and fault isolation is also recalled showing how to obtain the theoretical fault signature matrices. In *Section 4*, the fault isolation algorithm based on a timed discrete event system is presented. Finally, in *Section 5* the interval observer-based fault diagnosis algorithm will be applied to the limnimeters of Barcelona's urban sewer system to assess the validity of the derived results..

2. FAULT SIGNAL GENERATION

2.1 Fault Detection Interval Observer

Considering that the system to be monitored can be described by a MIMO linear uncertain dynamic model in discrete-time and in state-space form as

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{x}(k) + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{x}(k) \end{aligned} \tag{1}$$

without considering faults, disturbances and noise and where $\boldsymbol{A}(\boldsymbol{\theta})$, $\boldsymbol{B}(\boldsymbol{\theta})$, $\boldsymbol{C}(\boldsymbol{\theta})$ are the state, the input and the output matrices respectively, $\boldsymbol{u}(k) \in \Re^{nu}$ and $\boldsymbol{y}(k) \in \Re^{ny}$ are the system input and output vectors, respectively. $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ is a set of interval bounded parameters representing the model uncertainty: $\boldsymbol{\Theta} = \left\{ \boldsymbol{\theta} \in \Re^{n\theta} \middle| \underline{\boldsymbol{\theta}} \le \boldsymbol{\theta} \le \overline{\boldsymbol{\theta}} \right\}$. This type of model is known as an ***interval model***.

Instead of using directly the system model given by (1) to detect faults, the following state observer will be used:

$$\begin{aligned} \hat{\boldsymbol{x}}(k+1) &= (\boldsymbol{A}(\boldsymbol{\theta}) - \boldsymbol{W}\boldsymbol{C}(\boldsymbol{\theta}))\hat{\boldsymbol{x}}(k) + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}(k) + \boldsymbol{W}\boldsymbol{y}(k) \\ \hat{\boldsymbol{y}}(k) &= \boldsymbol{C}(\boldsymbol{\theta})\hat{\boldsymbol{x}}(k) \end{aligned} \tag{2}$$

where $\boldsymbol{W}$ is the observer gain, designed to stabilize the matrix $A_o = \boldsymbol{A}(\boldsymbol{\theta}) - \boldsymbol{W}\boldsymbol{C}(\boldsymbol{\theta})$ and to guarantee a desired fault detection performance for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. The effect of the uncertain parameters $\boldsymbol{\theta}$ on the observer temporal response will be bounded using an interval: [$\underline{\hat{y}}(k)$, $\overline{\hat{y}}(k)$], where:

$$\underline{\hat{y}}_i(k) = \min_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} (\hat{y}_i(k, \boldsymbol{\theta})), \; \overline{\hat{y}}_i(k) = \max_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} (\hat{y}_i(k, \boldsymbol{\theta})) \tag{3}$$

This interval can be computed using the algorithm presented in (Puig et al. 2003).

2.2 Fault Signal Generation Using Interval Observers

Model-based fault detection is based on generating a residual comparing the measurements of physical variables $\boldsymbol{y}(k)$ of the system with their estimation $\hat{\boldsymbol{y}}(k)$ provided by a model:

$$\boldsymbol{r}(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k) \tag{4}$$

Then, when considering model uncertainty located in parameters, the residual generated by (4) will not be zero even in a non-faulty scenario. Then, the possible values of this residual could be bounded using an interval (Puig et al. 2002)

$$r_i^o(k) \in [\underline{r}_i^o(k), \overline{r}_i^o(k)] \tag{5}$$

where: $\underline{r}_i^o(k) = \underline{\hat{y}}_i(k) - \hat{y}_i^o(k)$ and $\overline{r}_i^o(k) = \overline{\hat{y}}_i(k) - \hat{y}_i^o(k)$ (6)

are computed considering the nominal observer output prediction $\hat{\boldsymbol{y}}^{\boldsymbol{o}}(k)$ obtained using $\boldsymbol{\theta} = \boldsymbol{\theta}^o \in \boldsymbol{\Theta}$ and the [$\underline{\hat{y}}(k)$, $\overline{\hat{y}}(k)$] given by (3). This residual interval provides an ***adaptive threshold***. When condition (5) is not fulfilled, a fault is indicated by the interval observer.

As it is proposed in (Puig et al. 2005), the fault diagnostic signal (fault signal) for each residual is calculated as in the DMP-approach (Petti et al., 1990) using the Kramer function:

$$\phi_i(k) = \begin{cases} \dfrac{(r_i^o(k)/\overline{r}_i^o(k))^4}{1+(r_i^o(k)/\overline{r}_i^o(k))^4} & if \quad r_i^o(k) \ge 0 \\ -\dfrac{(r_i^o(k)/\underline{r}_i^o(k))^4}{1+(r_i^o(k)/\underline{r}_i^o(k))^4} & if \quad r_i^o(k) < 0 \end{cases} \tag{7}$$

In this way, residuals are normalized to a metric between -1 and 1, $\phi_i(k) \in [-1,1]$, which indicates the satisfaction degree of every equation: 0 for perfectly satisfied, 1 for severely violated high and -1 for severely violated low. When there is no fault affecting to the monitored system, the values obtained using Eq. (7) satisfy the expression $|\phi_i(k)|<0.5$. Otherwise ($|\phi_i(k)| \geq 0.5$), these normalized residuals becomes fault signals indicating that a given fault is affecting the monitored system.

Because the occurrence of a fault signal can be caused by different faults, what let distinguish one fault from another are the fault signal dynamic properties since they should be different for each different fault. The theoretical dynamic properties of a fault signal $\phi_i(k)$ caused by a given fault f_j are set by the sensitivity of the associated residual $r_i(k)$ to this fault f_j. The concept of ***residual sensitivity to a fault*** (Gertler, 1998) is given by

$$\boldsymbol{S}_{\boldsymbol{f}}(q^{-1}) = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{f}} = \boldsymbol{G}_{\boldsymbol{f}}(q^{-1}) \tag{8}$$

where $\boldsymbol{G_f}(q^{-1})$ is the transfer function that describes the effect on the residual, $\boldsymbol{r}$, of a given a fault, $\boldsymbol{f}$. The residual sensitivity to fault is a dynamic time function that indicates how a fault is affecting the residual from its occurrence time instant. In (Meseguer et al. 2007) and (Meseguer et al. 2007b), this concept is obtained when using interval observers models and it is demonstrated its key influence on fault detection and isolation tasks.

3. FAULT DETECTION AND ISOLATION INTERFACE

3.1 Description

The used interface in this paper derives from the one used in (Puig et al., 2005). It is based on the concept of the theoretical fault signature matrix (***FSM***) which was introduced by (Gertler, 1998) considering only a binary interface between fault detection and fault isolation modules. This matrix has as many rows as residuals ($r_i(k)$) and as many columns as considered faults (f_j) to isolate. Thus, an element $\boldsymbol{FSM}_{ij}$ of this matrix is *'1'* when the sensitivity of the residual $r_i(k)$ to fault f_j is not null, otherwise, this element is *'0'*. In (Puig et al., 2005), the fault signature matrix concept is generalized since the binary interface is extended taking into account more fault signal properties. In this approach, there are as many ***FSM*** matrices as different properties are taken into account: Boolean property (***FSM***01), sign property (***FSMsign***), fault residual sensitivity property (***FSMsensit***), occurrence order property (***FSMorder***) and the occurrence time instant (***FSMtime***). Those matrices store the influence of the considered faults on the residual set: the element $\boldsymbol{FSM}_{ij}$ of a matrix contains the expected influence of fault f_j on r_i^0. (Meseguer et al, 2007) shows how to obtain matrices ***FSMsensit*** and ***FSMtime*** using the interval observer model of the monitored system while the other three matrices can clearly be derived from them. In the next two sections, it is recalled the way to obtain both ***FSM*** matrices.

3.1 FSMsensit: Evaluation of Fault Sensitivities

The element $\boldsymbol{FSM}sensit_{ij}$ (Meseguer et al, 2007) of matrix ***FSMsensit*** considers the time evolution of the sensitivity of residual r_i^0 to a fault f_j once the fault has occurred what determines the capacity of the residual to indicate the fault. This property can be computed as it follows:

$$\boldsymbol{FSM}sensit_{ij} = \begin{cases} \dfrac{S_{f_{ij}}(q^{-1})\eta(k-t_0)}{\left|\overline{r}_i^o(k)\right|} & if \quad r_i^o(k) \geq 0 \text{ and } k \geq t_0 \\ \dfrac{S_{f_{ij}}(q^{-1})\eta(k-t_0)}{\left|\underline{r}_i^o(k)\right|} & if \quad r_i^o(k) < 0 \text{ and } k \geq t_0 \\ 0 \quad if \quad k < t_0 \text{ or } S_{f_{ij}}(q^{-1}) = 0 \end{cases} \tag{9}$$

where $\eta(k)$ is an unitary abrupt step input, $S_{f_{ij}}$ is the sensitivity of the nominal residual $r_i^o(k)$ regarding the fault hypothesis f_j and t_0 is the fault occurrence time instant. When t_0 is unknown, it must be estimated using the occurrence time instant $k_{\phi i}$ of the first fault signal. Conversely, it must be taken into account that ***FSMsensit*** matrix has a time evolution once the fault occurs at t_0 time instant. Besides, according to the definition of ***FSM***01 and ***FSMsign***, both matrices can be derived from ***FSMsensit***.

The consistency between the observed fault signals $\phi_i(k)$ and the stored information in ***FSMsensit*** for the j^{th}-fault hypothesis can be evaluated computing $\boldsymbol{factor}sensit_j$ (Meseguer et al, 2007) as it follows:

$$\boldsymbol{factor}sensit_j(k) = \frac{\left|\sum_{i=1}^{n}\left(\phi_i(k)\boldsymbol{FSM}sensit_{ij}\right)\right|}{\sum_{i=1}^{n}\left|\boldsymbol{FSM}sensit_{ij}\right|}\boldsymbol{zvf}_j \tag{10}$$

$$\boldsymbol{zvf}_j = \begin{cases} 0, & \text{if } \exists\, i \in \{1,\ldots,n\} \text{ with } \boldsymbol{FSM}\text{sensit}_{ij} = 0 \text{ and } \left|\phi_i(k)\right| \geq 0.5 \\ 1, & \text{otherwise} \end{cases} \tag{11}$$

3.2 FSMtime: Evaluation of Fault Signal Occurrence Time Instant

The element $\boldsymbol{FSM}time_{ij}$ (Meseguer et al., 2007) of matrix ***FSMtime*** contains the time interval $[\underline{\varphi}_{ij}, \overline{\varphi}_{ij}]$ in which the fault signal ϕ_i caused by the fault f_j is expected to appear. This time interval is referred to the occurrence time instant of the first fault signal according to the fault hypothesis f_j as in most of the cases, the fault occurrence time instant t_0 is unknown. (Meseguer et al., 2007) shows the interval $[\underline{\varphi}_{ij}, \overline{\varphi}_{ij}]$ basically depends on the sensitivity of the residual $r_i^o(k)$ to fault f_j ($S_{f_{ij}}$), on the adaptive threshold $[\underline{r}_i^o(k), \overline{r}_i^o(k)]$ related to this residual and on t_0. Thus, the elements of matrix ***FSMtime*** are given by

$$\boldsymbol{FSM}time_{ij} = \begin{cases} [\underline{\varphi}_{ij}, \overline{\varphi}_{ij}] & if \quad S_{f_{ij}}(q^{-1}) \neq 0 \\ [-1,-1] & if \quad S_{f_{ij}}(q^{-1}) = 0 \end{cases} \tag{12}$$

Derived from ***FSMtime***, one of the most important parameters of the fault isolation algorithm can be obtained. This is the time window T_w which determines the maximum period of time required once the first fault signal is observed so that all fault signals can appear. In other words, T_w is the period of time needed, once the first fault signal is detected, to give an accurate fault diagnosis result unless there were only one fault hypothesis left supporting the observed fault signal temporal sequence before T_w would have elapsed. Thereby, T_w can be obtained as it follows:

$$T_w = \max_{\forall i,j}(\overline{\varphi}_{ij}) \tag{16}$$

Besides, according to the own definition of matrix ***FSMtime***, the fault signal occurrence order matrix ***FSMorder*** can also be derived. Each element $\boldsymbol{FSM}order_{ij}$ of ***FSMorder*** contains the theoretical occurrence order of fault signal ϕ_i when fault f_j occurs regarding all fault signals that will be caused by this fault. When for a given fault hypothesis, a fault signal is unaffected, then, $\boldsymbol{FSM}order_{ij}$=0.

4. TIMED DISCRETE EVENT FAULT ISOLATION ALGORITHM

4.1 Fault isolation Algorithm: Initial Version

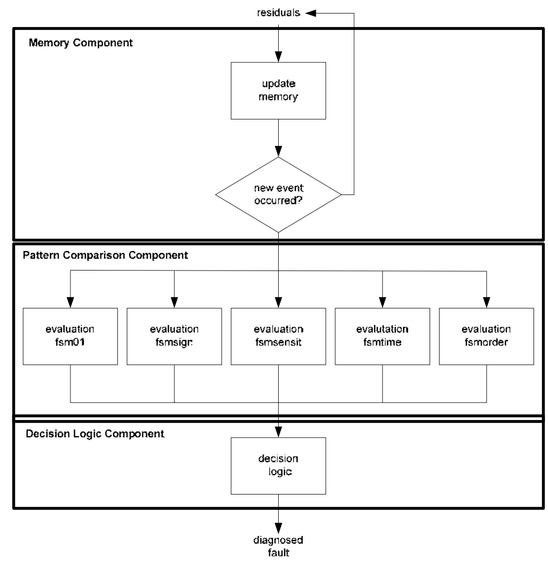

Fig. 1. Block diagram of the fault isolation algorithm initial version

In the original algorithm (see *Fig. 1*) proposed in (Puig et al., 2005), the first component between fault detection and fault isolation modules is a ***memory***. Once the first fault signal is observed, for each fault signal $\phi_i(k)$, the occurrence time instant ($k_{\phi i}$) and the fault signal value (ϕ_{imax}) whose absolute value is maximum are stored in this memory along the time window T_w. Then, at the end of this time window, the ***pattern comparison component*** compares the memory information with the theoretical one stored for each fault hypothesis in the different fault signature matrices ***FSM*** mentioned previously. This comparison allows computing the fault isolation factors ***factorsensit***$_j$, ***factortime***$_j$ ***factororder***$_j$, ***factor01***$_j$ and ***factorsign***$_j$ for each fault hypothesis f_j. Then, the last component of the algorithm, the ***decision logic component***, computes for each fault hypothesis a fault occurrence probability d_j using the previous factors, Thereby, the fault hypothesis with the highest occurrence probability d_j will be the given diagnostic.

Once the first fault signal is observed, this algorithm requires the occurrence of all fault signals before an accurate fault diagnostic can be given. Otherwise, wrong diagnosis results can be obtained. In this initial version, the value of the time window T_w (Eq. (16)) was not calculated.

4.2 Fault isolation Algorithm based on a Timed Discrete Event Model

The algorithm proposed here is an evolution of the one presented in the previous section. The main idea is that a given fault affecting to the monitored system will cause a unique temporal sequence of fault signals. This allows obtaining a diagnostic result comparing their observed dynamic properties with the ones stored for each fault hypothesis in the fault isolation matrices.

The fault isolation algorithm starts with the occurrence of the first fault signal ($|\phi_i(k)| \geq 0.5$) and ends when there is only one fault hypothesis supporting the observed temporal sequence of fault signals or when the diagnosis time window T_w has ended. Thereby, the first element of this algorithm is also the ***memory component*** (*Section 4.1*). The second element is the ***timed series inference component*** which compares the stored information of the new observed fault signal with the information stored in matrices ***FSM01***, ***FSMtime***, and ***FSM****order* for the non-rejected fault hypothesis. The result of this series inference component is the rejection of those fault hypotheses that do not support the observations. When there is only one fault hypothesis left, the algorithm ends giving that hypothesis as the fault diagnostic result. Otherwise, when the time window T_w has ended, the third element, the ***pattern comparison component***, computes ***factorsensit***$_j$ for those non-rejected fault hypotheses. Then, the last element, the ***logic decision component***, gives as a diagnostic result the fault with the biggest ***factorsensit***$_j$.

The main difference of this algorithm regarding the one presented in *Section 4.1* is based on the ***timed series inference component*** inspired in the ***T-DTS*** method (Kościelny et al., 2004b) and on the fact that each new fault signal allows rejecting those hypotheses that do not support the observations and in consequence, a diagnostic result can be given before the time window T_w ends.

4.3 Fault isolation algorithm based on a Timed LTS model

Analyzing the fault isolation algorithm presented in *Section 4.2* from a DX point of view, this algorithm can be seen as a ***Labelled Transition System*** (Daigle et al., 2007) model where the input is the fault signal temporal sequence, the states are given by the non-rejected fault hypotheses at each time instant and the transitions are given by the comparison of the fault signal observed properties with the information stored in ***FSM01***, ***FSM****order* and ***FSM****time*. Analyzing the structure of the ***FSM*** matrices and the information they store, a ***LTS*** model as the presented in (Daigle et al., 2007) could be build. Thereby, while the transitions between states proceed from a ***Temporal Causal Graph*** in (Daigle et al., 2007), they proceed from the ***FSM*** matrices obtained from the interval observer model in the approach presented in this paper,. Regarding the fault isolation algorithm presented in *Section 4.2*, the only logic which is not considered by a pure *fault isolation Timed LTS model* is the one included in the *comparison and logic decision components*. In *Section 5.3* the equivalent fault isolation model based on a ***Timed LTS model*** is obtained using the example application and then, a fault scenario case is considered.

5. CASE OF STUDY

5.1 Description

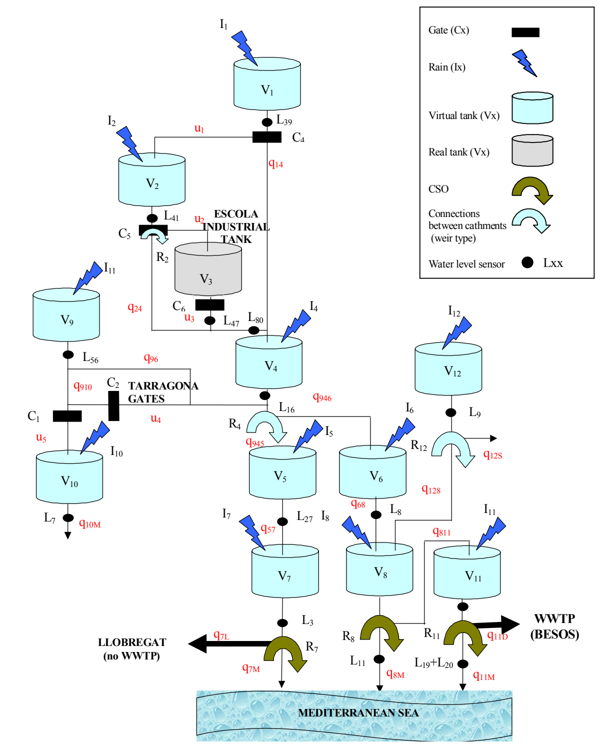

Fig. 2. Model of the considered part of the Barcelona's sewer network

To illustrate the approach proposed in this paper, a real case of study based on the Barcelona urban drainage system is used. In particular, the main focus is on detecting and isolating faults in the limnimeters used to measure the sewer levels. The sewer network is modelled using a simplified graph relating the main sewers and a set of virtual and real reservoirs (Cembrano, 2002). A ***virtual reservoir*** is an aggregation of a catchment of the sewer network which approximates the hydraulics of rain, runoff and sewage water retention using a mass balance:

$$\frac{dV(t)}{dt} = Q_{up}(t) - Q_{down}(t) + I(t)S \tag{17}$$

where: V is the volume of water accumulated in the catchment, Q_{up} and Q_{down} are flows entering and exiting the catchment, I is the rain intensity falling in the catchment and S its surface. Input and output sewer levels are measured using limnimeters and they can be related with flows using a linearised Manning relation: $Q_{up}(t) = M_{up}L_{up}(t)$ and $Q_{down}(t) = M_{down}L_{down}(t)$. Moreover, it is assumed that: $Q_{down}(t) = K_v V(t)$. Then, substituting those relations in Eq. (17) and considering that the measurement sampling time is

T_s = 300 s, the following discrete-time model for each limnimeter can be obtained:

$$L_{down}(k+1) = aL_{down}(k) + bL_{up}(k) + cI(k)) \tag{18}$$

where: $a = (1 - K_v \Delta t)$, $b = M_{up} K_v \Delta t / M_{down}$

and $c = SK_v / M_{down}$.

Using this modelling methodology, the model of the considered part of the Barcelona's sewer network is presented in *Fig. 2*. Thereby, this methodology let diagnose faults of a set $\boldsymbol{f_L}$ composed by 14 limnimeters (L_{03}, L_{07}, L_{08}, L_{09}, L_{16}, L_{27}, L_{39}, L_{41}, L_{45}, L_{47}, L_{53}, L_{56}, L_{80} and L_{54}) modelling (Eq. (17)) a set $\boldsymbol{L}$ of 12 limnimeters (L_{03}, L_{07}, L_{08}, L_{09}, L_{16}, L_{27}, L_{39}, L_{41}, L_{45}, L_{56}, L_{80} and L_{54}). The fault detection model for each limnimeter of the set $\boldsymbol{L}$ is given by an interval observer whose general expression is given by Eq. (2). As a example, the interval observer model associated to L_{03} is given by

$$\begin{aligned} \hat{L}_{03}(k) &= a_{03}(1-\lambda_{03})\hat{L}_{03}(k-1) + b_{03}L_{27}(k-1) + c_{03}I_{07}(k-1) + \\ &+ a_{03}\lambda_{03}L_{03}(k-1) \end{aligned} \tag{19}$$

where λ_{03} is the associated observer gain using the parameterization $w_{03}=\lambda_{03}a_{03}$ (λ_{03} =0, simulation; λ_{03} =1, prediction). I_{07} is the rain intensity measured by the rain gauge P_{14}. The model parameters are obtained using parameter estimation from experimental data. Considering that the uncertainty associated to the following intervals describes the possible values of each parameter: a_{03}*=[0.8816, 0.9084]*, b_{03}*=[0.0381, 0.0393]* and c_{03}*=[1.4469e4, 1.4910e4]*.

According to Eq. (4), the estimations given by limnimeter observer models of the set $\boldsymbol{L}$ allow obtaining a set $\boldsymbol{r}$ of 12 residuals which let obtain a diagnosis result. As example, the expression of the residual associated to L_{03} is given by

$$\begin{aligned} r^o_{03}(k) = {} & \frac{1-a^o_{03}q^{-1}}{1+a^o_{03}(\lambda^o_{03}-1)q^{-1}}L_{03}(k) - \frac{b^o_{03}q^{-1}}{1+a^o_{03}(\lambda^o_{03}-1)q^{-1}}L_{27}(k) \\ & -\frac{c^o_{03}q^{-1}}{1+a^o_{03}(\lambda^o_{03}-1)q^{-1}}I_{07}(k) \end{aligned} \tag{20}$$

Each residual of the set $\boldsymbol{r}$ determines a fault signal according to Eq. (7) being ϕ the set of all possible fault signals caused by the fault set $\boldsymbol{f_L}$. Then, a fault affecting a limnimeter of the set $\boldsymbol{f_L}$ will cause a temporal sequence of fault signals (a subset of ϕ**)** whose properties will let detect and isolate the fault using the new fault isolation algorithm presented in *Section 4.2* or its equivalent fault isolation model based on a ***Timed LTS model*** (Daigle et al, 2007) (*see Section 4.3*).

5.2 Fault Signature Matrices: FSMsensit *and* FSMtime

In this section, the value of the fault signature matrices ***FSM****sensit* and ***FSM****time* associated to the considered case of study is given. These matrices are computed as shown in (Meseguer et al., 2007) assuming the observer gains λ_i of all interval observers associated to the limnimeter set $\boldsymbol{L}$ are equal to *0.01* and the occurrence of the first fault signal is detected at time instant t_0*=4000 s*.

These matrices show the relationship between the limnimeter fault set $\boldsymbol{f_l}$ and the properties of the fault signal temporal sequence ϕ originated by these faults and consequently, they let isolate limnimeter faults using either the fault isolation algorithm shown in *Section 4.2* or its equivalent one shown in *Section 4.3*. As mentioned in *Section 3*, the other three matrices (***FSM****01,* ***FSM****sign* and ***FSM****order*) of the interface presented in this section are not given here because the lack of space but they can be easily obtained from ***FSM****sensit* and ***FSM****time* (Meseguer *et al*., 2007).

Regarding ***FSM****sensit* (see *Section 3.1*), it must be taken into account that each element of this matrix is a time function mainly based on the sensitivity of the fault signal associated residual to a given fault hypothesis (Eq.(9)). Thus, in the following, the elements of ***FSM****sensit* matrix showed in Table 1 are just the fault residual sensitivity steady-state values instead of the ones derived from the use of Eq. (9). Thus, f_{Lj} is a fault affecting the limnimeter L_j while ϕ_{Li} is the fault signal associated to the residual r_{Li} obtained using the interval observer model of L_i.

Table 1. *FSM**sensit***

FSMsensit Matrix

	f_{L03}	f_{L07}	f_{L08}	f_{L09}	f_{L16}	f_{L27}	f_{L39}	f_{L41}	f_{L45}	f_{L47}	f_{L53}	f_{L56}	f_{L80}	f_{L54}
ϕ_{L03}	0.921	0	0	0	0	-0.340	0	0	0	0	0	0	0	0
ϕ_{L07}	0	0.933	0	0	0	0	0	0	0	0	-0.150	0	0	0
ϕ_{L08}	0	0	0.951	0	-0.752	0	0	0	0	0	0	-0.406	0	-5.162
ϕ_{L09}	0	0	0	0.946	0	0	0	0	0	0	0	0	0	0
ϕ_{L16}	0	0	0	0	0.977	0	0	0	0	0	0	0	-0.255	0
ϕ_{L27}	0	0	0	0	0	0.789	0	0	0	0	0	0	0	0
ϕ_{L39}	0	0	0	0	0	0	0.964	0	0	0	0	0	0	0
ϕ_{L41}	0	0	0	0	0	0	-0.908	0.955	0	0	0	0	0	0
ϕ_{L45}	0	0	0	0	0	0	0	-57.243	0	97.484	0	0	0	0
ϕ_{L54}	0	0	0	0	0	0	0	0	0	0	0.265	-0.183	0	1
ϕ_{L56}	0	0	0	0	0	0	0	0	0	0	0	0.802	0	0
ϕ_{L80}	0	0	0	0	0	0	0	0	0	-14.085	0	0	1	0

Concerning ***FSM****time* (see *Section 3.2*), its value is given by

Table 2. *FSM**time***

FSMtime Matrix

	f_{L03}	f_{L07}	f_{L08}	f_{L09}	f_{L16}	f_{L27}	f_{L39}	f_{L41}	f_{L45}	f_{L47}	f_{L53}	f_{L56}	f_{L80}	f_{L54}
ϕ_{L03}	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[900,6300]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L07}	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[2100,5400]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L08}	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[600,4200]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[900,4200]	[-1,-1]	[300,4200]
ϕ_{L09}	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L16}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[300,2400]	[-1,-1]
ϕ_{L27}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L39}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L41}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[300,3900]	[0,0]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L45}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[300,1800]	[0,0]	[300,30600]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]
ϕ_{L54}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[0,0]	[-1,-1]	[0,0]
ϕ_{L56}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]
ϕ_{L80}	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]	[-1,-1]	[0,0]	[-1,-1]

where the fault occurrence time intervals are expressed in seconds and abrupt limnimeter faults have been considered. According to the previous value of ***FSM****time* and Eq. (16), the value of the diagnosis time window for this scenario is T_w *= 30600 s*.

5.3 Fault isolation Timed LTS model

In this section and for the considered case of study, a fault isolation algorithm based on a *Timed LTS model* as the one presented in (Daigle et al., 2007) will be obtained using ***FSM****01*, ***FSM****order* and ***FSM****time* (see *Section 4.2*). In this kind of model, the initial state is the non-faulty state, then, each fault hypotheses (set $\boldsymbol{f_l}$) have a Timed LTS representation which are connected to this initial state. The LTS representation related to a given fault hypothesis shows the fault signal temporal sequence caused by this fault. In each state transition, the properties of the new observed fault signal are compared with those stored in ***FSM****01*, ***FSM****order* and ***FSM****time* for this fault hypothesis. The present state of a fault hypothesis LTS representation just indicates that this fault hypothesis is still supporting the observed fault signal temporal sequence. When a new fault signal occurs, for each non-rejected fault hypotheses, the state transition starting in the present state is evaluated. If this evaluation fails, the fault hypothesis is rejected. At the end of the diagnosis time window T_w, those non-rejected fault hypotheses will determine the final fault diagnosis result. If there is more than one, a FDI logic as the implemented in the *comparison and* *decision logic components* (see *Section 4.2*) should be used in order to find the most probable fault hypothesis.

In *Figure 3*, the Timed LTS model associated to a subset (L_{16}, L_{27}, L_{39}, L_{41}, L_{03} and L_{54}) of the limnimeter fault set $\boldsymbol{f_l}$ is given. In this figure, the transition logic between two states is noted as $\phi_{Li}\boldsymbol{FSM}_{Lj}$ indicating the result of comparing the binary property, the occurrence time instant and the occurrence order of the new observed fault signal ϕ_{Li} (associated to the residual r_{Li} obtained using the interval observer model of L_i) with the information stored in matrices ***FSM01***, ***FSM****order* and ***FSM****time* for the fault hypothesis f_{Lj} (fault affecting limnimeter L_j) and for this fault signal.

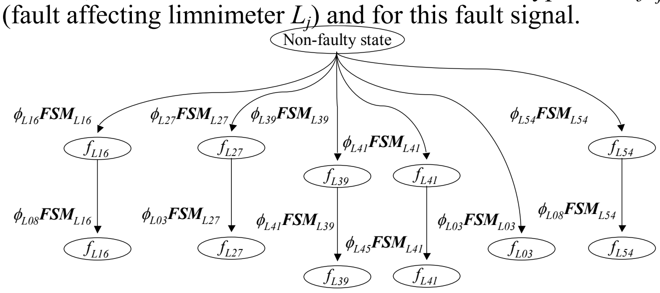

Fig. 3. Limnmeter fault isolation based on a Timed LTS model

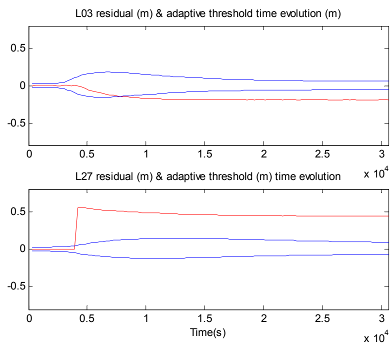

Fig. 4.Time evolution of the residuals and their adaptive thresholds.

The associated representation to this Timed LTS model can be seen as the integration of all the information stored in ***FSM01***, ***FSM****order* and ***FSM****time* in the same structure.

Focusing on a fault scenario where an abrupt fault affecting limnimeter L_{27} occurs at t_0= *4000s*, the time evolution of the affected residuals and their associated adaptive thresholds (Eq.(5)) are plotted in *Figure 4*.

Analyzing the time evolution of those residuals and according to Eq. (5) and Eq. (7), the first observed fault signal will be ϕ_{L27} (fault signal related to limnimeter L_{27} observer model) since time instant $t=t_0$. This event, according to the fault isolation model given by *Figure 3* and the information stored in ***FSM01***, ***FSM****order* and ***FSM****time* for this fault signal, will let reject all fault hypotheses except f_{L27} (fault affecting L_{27}). Afterwards, the fault signal ϕ_{L03} is observed supporting the LTS representation associated to f_{L27}

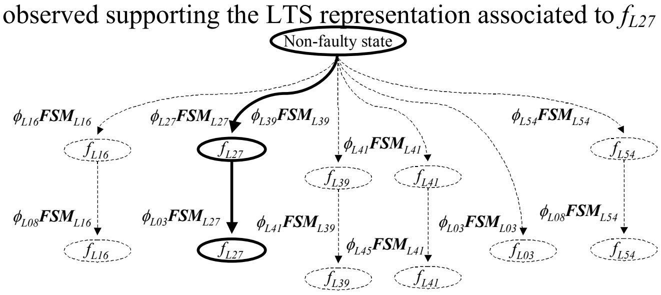

Fig. 5. Isolation of a fault affecting L_{27} using a Timed LTS model

Focusing on the *BRIDGE approach* presented in *Section 4.2*, there is not the need to use the structure of a ***LTS model*** as all the information needed to run the ***timed discrete event fault isolation algorithm*** is already structured in the different *Fault Signature Matrices*. Although the difference of structures between both approaches, the final result should be similar when the system model (Qualitative: DX approach; Quantitative: FDI approach) estimates accurately the system behaviour assuming the fault detection and isolation interface are taking into account the same fault signal properties.

6. CONCLUSIONS

This paper proposes a fault diagnosis method using a timed discrete-event approach. The novelty is that this approach is based on interval observers instead of using qualitative models. The interface between fault detection and fault isolation considers the degree of fault signal activation and the occurrence time of the diagnostic signals using a combination of several theoretical fault signature matrices which store the knowledge of the relationship between diagnostic signals and faults. This paper focuses on the discrete-time event nature of the fault signals generated by the fault detection module, what has led to the use of a timed discrete event model. In this way, the diagnosis result has been enhanced since the occurrence of a fault generates a unique sequence of observable events (fault signals) that can be recognized by the isolation module implemented as a timed discrete event system. The states and transitions that characterize such a model can be inferred directly from the relationship between fault signals and faults.

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