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Stochastic Switching Approach of Bilateral Teleoperation Systems: Part I - Theory

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Abstract: In this paper, new control strategies based on Linear Matrix Inequalities (LMIs) are proposed for bilateral teleoperation systems over networks with time delays and packet losses. The characteristics of the network are thoroughly incorporated in the design and are mainly discussed in two cases: random packet loss with constant time delays; and random packet loss with random time varying delays. Correspondingly, a stochastic switching control approach is designed for the system with random packet loss. The Markov Jump Linear Systems (MJLSs) framework is applied using time based controllers to guarantee Mean Exponential Stability. The tracking error is shown to be bounded by the rate of change of the external forces acting on the teleoperation system. Finally, simulation results with experimentally collected network data show the performance of the proposed scheme as well as how to fine tune the controller gain.

Keywords: Bilateral Teleoperation, Time Varying Delay, Packet Loss, Network, Stochastic Stability, Linear Matrix Inequalities (LMIs), Switching Control, Jump Linear Systems

1. INTRODUCTION

Bilateral teleoperation is a challenging problem due to the difficulty posed by the presence of a communication channel with random transmission delays (Pan *et al.*, 2006). The situation becomes more complicated when the communication channel in question has intermittent transmissions as well as random delays. Internet Protocol (IP) based networks like the Internet and wireless networks are the main alternatives that have been considered.

Many teleoperation efforts in recent times have been based on the wave variable methods developed in (Niemeyer and Slotine, 1991), including (Anderson and Spong, 1989). These models impose passivity constraints on the operator and the environment which cannot be met in all applications. Other methods require precise knowledge of the environment, the operator or both so that predictive methods can be used. These methods provide the ability to overcome some of the effects of the time delay, but require a great deal of information about elements that are often unknown.

Much work has been done in the field of Networked Control Systems (NCSs), including methods based on Linear Matrix Inequalities (LMIs) and Markov Jump Linear Systems (MJLS) that consider networks with packet loss and varying, bounded delays (Lin and Antsaklis, 2005). NCS have a different structure from teleoperation systems; where NCSs have one plant that must be controlled, teleoperation systems have two or more separate plants whose operation must be coordinated. Many teleoperation tasks consist of interactions with an environment in a complex manner. Remote manipulation often involves coupling the dynamics of the controlled device with the environment's dynamics. These can change abruptly and drastically depending on the particular task. In telesurgery, for example, the properties of bone are quite different from those of muscle.

Work has been done to adapt the controller to the changing environment (Heredia *et al.*, 1996) and others have sought to model the operator (Prokopiou *et al.*, 1998) and some have done both (Zhu and Salcudean, 2000). In the proposed paper, neither models for the operator nor for the environment are used when deriving the controllers to allow for general results that cover a wide range of scenarios and which would be robust to outside variations. Different design scenarios are addressed incorporating different levels of knowledge about the network. The first incorporates packet losses and formulates the teleoperation system as a Markov Jump Linear System (MJLS) with stochastic properties. The second deals with random, bounded delays as well as losses. The main contribution of this work is the successful integration of the ideas in various NCS approaches and their adaptation to the bilateral teleoperation problem. Packet losses and bounded time delays are considered to develop delay-dependent stability conditions and gain design procedures. The tracking error is shown to be bounded by the rate of change of the external forces acting on the teleoperation system.

2. SYSTEM MODELLING

In the teleoperation case, the controller would be updated both with received packets from the remote site as well

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as local sensor measurements. With control frequencies of 1kHz required for high quality haptic systems (Tanner and Niemeyer, 2004), our controller would essentially become a time based controller, therefore a time based approach is adopted in this paper. The various components are the master and slave manipulators, the human operator, the environment and the communication network.

2.1 The Master and Slave Manipulators

In this paper, a linear, single degree of freedom manipulator (Pan *et al.*, 2006) is modelled using state-space equations, where the master and slave manipulators are considered identical. The sign conventions depicted in Fig. 1 are adopted for the rest of this work, where u_m is the master control signal, u_s is the slave control signal, f_h is the operator force, f_e is the environmental force, x_m is the master position, and x_s is the slave position.

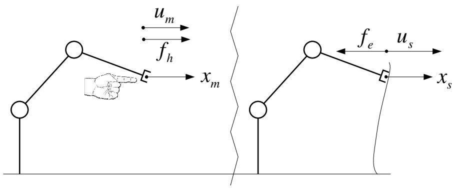

Fig. 1. Teleoperation Force Diagram

The manipulator model is considered as

$$\dot{\mathbf{x}}_m = A_{ct}\mathbf{x}_m + B_{ct}(u_m + f_h) \quad (1)$$
$$\dot{\mathbf{x}}_s = A_{ct}\mathbf{x}_s + B_{ct}(u_s - f_e) \quad (2)$$

where b is the damping factor, J is the inertia and

$$A_{ct} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{J} \end{bmatrix}, B_{ct} = \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix}, \mathbf{x}_m = \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}, \mathbf{x}_s = \begin{bmatrix} x_s \\ \dot{x}_s \end{bmatrix}.$$

The slave model is analogous to the master model, but the sign convention on f_e is opposite to f_h. These models are then discretized using a sample and hold discretization as

$$\mathbf{x}_m[k+1] = A\mathbf{x}_m[k] + B(u_m + f_h) \quad (3)$$
$$\mathbf{x}_s[k+1] = A\mathbf{x}_s[k] + B(u_s - f_e), \quad (4)$$

where $A = e^{A_{ct}T_s}$ and $B = \int_0^{T_s} e^{A_{ct}\tau} d\tau B_{ct}$.

2.2 The Operator and The Environment

The operator is not modelled explicitly here. The operator's forces are the input to the system. Although no model is considered, the input force is considered band limited based on (Ang *et al.*, 2004) where it was shown that voluntary motion is limited to 6Hz. Anything above that frequency can be considered an involuntary tremor and should be filtered electronically for improved precision. The limitation on the frequency of the operator's motion means that its derivative is also bounded. If we take the Fourier Transform of $f_h(t)$, $F_h(\omega)$, the frequency bound means that $F_h(\omega) = 0, \omega \in \{(-\infty, -\omega_{Max}) \cup (\omega_{Max}, \infty)\}$. Using the inverse Fourier transform, the derivative of the operator's force can be expressed as

$$\begin{aligned}\dot{f}_h(t) &= \int_{-\infty}^{\infty} -i\omega F_h(\omega) e^{-i\omega t} d\omega \\ &= \int_{-\omega_{Max}}^{\omega_{Max}} -i\omega F_h(\omega) e^{-i\omega t} d\omega. \end{aligned} \quad (5)$$

So long as $\int_{\omega_{Max}}^{\omega_{Max}} F_h(\omega) d\omega$ is finite, then $\dot{f}_h$ is bounded. This condition applies to all practical signals even though $F_h(\omega)$ itself may not be bounded at all points (i.e. for a DC signal where the transform is the Dirac Delta). The discrete-time difference function, $d_h[k] \triangleq f_h[k] - f_h[k-\tau]$ where τ is the time delay, is also bounded since it is the discrete time version of the derivative. This quality is essential to show that the system is stable.

In the simulations, a spring and damper system is used for the environment as,

$$f_e = [b_e \;\; k_e]\mathbf{x}_s, \quad (6)$$

where b_e is the environmental damping and k_e is the spring constant. Due to the generality of the proposed methods, any model can be used so long as its difference function, $d_e[k] \triangleq f_e[k] - f_e[k-\tau]$, is bounded. In the event that the environmental forces do not satisfy the necessary criterion, like with rigid surfaces, the forces can be separated into high and low frequency content (Tanner and Niemeyer, 2004). The high frequency elements are filtered at the slave side and transmitted as haptic events to the master where they are rendered with open-loop pulses. The transmitted low frequency signal would ensure $d_e[k]$ is bounded.

2.3 The Network

The network delay model will change in the various sections as the control assumptions are brought more in line with actual Internet-based teleoperation. The values used for the simulations will be derived from an experiment where Internet Control Message Protocol (ICMP) Echo Request packets were transmitted from Dalhousie University in Halifax, Nova Scotia, Canada to Google.com in Mountain View, California. This represents a round-trip distance of 12,000km and provides an illustrative example of the challenges of transmitting over significant distances. The packets were sent every second for a week using the hrPing software program from cFos (http://www.cfos.de). This program was selected due to its increased time resolution compared to the ping program included with Microsoft Windows. The elapsed time between the initial transmission and the response is measured using the number of CPU cycles, easily providing micro-second resolution. In this paper, it is assumed that the communication channel behaves identically in both directions.

3. CONTROL OVER NETWORKS WITH PACKET LOSS AND CONSTANT DELAY

The stochastic losses are modelled according to a two-state Markov Chain. The constant delay is defined as τ. The system is expressed as a Markov Jump Linear System (MJLS), switching between the open loop and closed loop configurations. A stability condition and control gain design method is derived.

3.1 *Modelling and Control Strategy*

Force information will be repeated when a packet is lost according to (7):

$$\hat{f}_h[k+1] = \begin{cases} f_h[k-\tau], & \text{packet received} \\ \hat{f}_h[k], & \text{packet lost.} \end{cases} \tag{7}$$

The system state is updated based on an assumption of constant velocity and is estimated as (8)

$$\hat{\mathbf{x}}_m[k+1] = \begin{cases} \mathbf{x}_m[k-\tau], & \text{packet received} \\ A_{loss}\hat{\mathbf{x}}_m[k], & \text{packet lost} \end{cases} \tag{8}$$

where $A_{loss} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$. Our closed loop controllers are the same as in the Section 3, repeated here as (9) and (10), and the open loop controllers are given by (11) and (12) as:

$$u_m[k] = K_1\big(\mathbf{x}_m[k] - \mathbf{x}_s[k-\tau]\big) - f_e[k-\tau] \tag{9}$$

$$u_s[k] = K_1\big(\mathbf{x}_s[k] - \mathbf{x}_m[k-\tau]\big) + f_h[k-\tau] \tag{10}$$

$$u_m[k] = K_2\big(\mathbf{x}_m[k] - \hat{\mathbf{x}}_s[k]\big) - \hat{f}_e[k] \tag{11}$$

$$u_s[k] = K_2\big(\mathbf{x}_s[k] - \hat{\mathbf{x}}_m[k]\big) + \hat{f}_h[k]. \tag{12}$$

3.2 *Markov Jump Linear Systems*

The system is modelled as a Markov Jump Linear System (MJLS), switching between the open loop and closed loop configurations depending on whether or not an update was received. The random variables are governed by a discrete-time, discrete-state Markov Chain (MC) $i_k \in \{1, 2\},\ k = \{1, 2, ...\}$ where State 1 represents a received packet and State 2 is a packet loss. MCs are stochastic processes where the transition probabilities depend only on the current state, as in (13).

$$\begin{aligned} T_{\alpha,\beta} &= P(i_{k+1} = \beta | i_k = \alpha, i_{k-1} = \alpha_1, ..., i_{k-n} = \alpha_n) \\ &= P(i_{k+1} = \beta | i_k = \alpha), \end{aligned} \tag{13}$$

where $T_{\alpha,\beta}$ is the probability that the system will transit from state α to state β. A detailed state history is not required to make a prediction and is a simple means of describing many phenomena, including network transmissions (Nilsson and Bernhardsson, 1996).

An MC of this nature is completely described by its transition matrix which is an organization of all the transition probabilities into matrix form. The transition matrix T in (14) represents a two state MC which will be used to model the packet losses in the communication channel. It would be a simple exercise to increase the number of states if a more complicated MC would better describe the network.

$$T = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix}. \tag{14}$$

For the rest of this work, the system stability is discussed in terms of stochastic stability. We will be using the definition of Mean Exponential Stability (MES), which is a type of second moment stability based on the expected value of the square of a random variable. MES is defined as follows (Boukas and Liu, 2002).

Definition 1. A system is MES if for some $\alpha > 0$, $\beta > 0$ and initial state i_0 and initial trajectory $\mathbf{e}_0 = \mathbf{e}[k], k = [-\tau, 0)$ if

$$E\left\{\|\mathbf{e}[k]\|^2 | \mathbf{e}_0, i_0\right\} \le \alpha \|\mathbf{e}_0\| e^{-\beta k}. \tag{15}$$

The error dynamics must be rewritten so that they can be put into the MJLS framework. By modifying the NCS formulation in (Kawka and Alleyne, 2006), we define the augmented state-vectors as (16) and the disturbance vector as (17).

$$\mathbf{s}_m[k] = \begin{bmatrix} \mathbf{x}_m[k] \\ \hat{\mathbf{x}}_m[k] \end{bmatrix},\ \mathbf{s}_s[k] = \begin{bmatrix} \mathbf{x}_s[k] \\ \hat{\mathbf{x}}_s[k] \end{bmatrix}, \tag{16}$$

$$\mathbf{d}_k = \begin{bmatrix} f_h[k] - f_h[k-\tau] \\ f_e[k] - f_e[k-\tau] \\ f_h[k] - \hat{f}_h[k] \\ f_e[k] - \hat{f}_e[k] \end{bmatrix}. \tag{17}$$

The disturbance vector is not directly related to the disturbance forces but rather their difference functions. The error expression is formed of the actual state error as well as the last known state error:

$$\mathbf{e}[k] = \mathbf{s}_m[k] - \mathbf{s}_s[k] = \begin{bmatrix} \mathbf{x}_m[k] - \mathbf{x}_s[k] \\ \hat{\mathbf{x}}_m[k] - \hat{\mathbf{x}}_s[k] \end{bmatrix} \tag{18}$$

If the actual state errors are bounded then the last known state errors will also be bounded.

Using this error definition, the closed loop error dynamics are (19) and the open loop error dynamics are (20). The two equations have the same form and so the MJLS framework can be applied.

$$\mathbf{e}[k+1] = A_1\mathbf{e}[k] + A_{1_d}\mathbf{e}[k-\tau] + D_1\mathbf{d}[k]\ ,\ \text{closed loop} \tag{19}$$

$$\mathbf{e}[k+1] = A_2\mathbf{e}[k] + A_{2_d}\mathbf{e}[k-\tau] + D_2\mathbf{d}[k]\ ,\ \text{open loop} \tag{20}$$

where

$$A_1 = \begin{bmatrix} (A+BK_1) & 0 \\ 0 & 0 \end{bmatrix}, A_{1_d} = \begin{bmatrix} BK_1 & 0 \\ I & 0 \end{bmatrix}, D_1 = \begin{bmatrix} B & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} (A+BK_2) & BK_2 \\ 0 & A_{loss} \end{bmatrix}, A_{2_d} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 & B & B \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

3.3 *Control Design With Constant Delay and Packet Loss*

Theorem 1. If there exists symmetric positive definite matrices $\hat{P}_i \in \mathcal{R}^{2n\times 2n}$, $\hat{Q} \in \mathcal{R}^{2n\times 2n}$, $\hat{R} \in \mathcal{R}^{2n\times 2n}$ and $\hat{S} \in \mathcal{R}^{2n\times 2n}$, block diagonal structured matrices $X_i \in \mathcal{R}^{2n\times 2n}$ and $Y_i \in \mathcal{R}^{2n\times 2m}$, positive scalar γ and arbitrary scalars θ_2 and θ_3 such that (21) holds for $i, j = 1, 2$ for the time delay τ and packet loss transition matrix T

$$\sum_j \sum_i p(\alpha = i, \beta = j) L_{MJLS}(i, j) < 0, \tag{21}$$

where $L_{MJLS}(\alpha, \beta)$ is

$$\begin{bmatrix} \Phi_{11} & \Phi_{21}^T & \Phi_{31}^T & (\tau+1)\hat{N}_{\alpha_1} & \gamma D_\alpha \\ \Phi_{21} & \Phi_{22} & \Phi_{32}^T & (\tau+1)\hat{N}_{\alpha_2} & \gamma\theta_2 D_\alpha \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & (\tau+1)\hat{N}_{\alpha_3} & \gamma\theta_3 D_\alpha \\ (\tau+1)\hat{N}_{\alpha_1}^T & (\tau+1)\hat{N}_{\alpha_2}^T & (\tau+1)\hat{N}_{\alpha_3}^T & -(\tau+1)\hat{R} & 0 \\ \gamma D_\alpha^T & \gamma\theta_2 D_\alpha^T & \gamma\theta_3 D_\alpha^T & 0 & -\gamma I \end{bmatrix}, \tag{22}$$

and

$$
\begin{aligned}
\Phi_{11} &= \hat{P}_\beta + (\tau+1)\hat{R} + \hat{N}_{\alpha_1} + \hat{N}_{\alpha_1}^T + X_\alpha^T + X_\alpha \\
\Phi_{21} &= -(\tau+1)\hat{R} + \hat{N}_{\alpha_2} - X_\alpha A_\alpha'^T - Y_\alpha A_\alpha''^T + \theta_2 X_\alpha^T \\
\Phi_{22} &= \hat{Q} - \hat{P}_\alpha + (\tau+1)\hat{R} - \theta_2 A_\alpha' X_\alpha^T - \theta_2 A_\alpha'' Y_\alpha^T \\
&\quad -\theta_2 X_\alpha A_\alpha'^T - \theta_2 Y_\alpha A_\alpha''^T + \hat{S}_\alpha \\
\Phi_{31} &= \hat{N}_{\alpha_3} - \hat{N}_{\alpha_1}^T - X_\alpha A_{\alpha_d}'^T - Y_\alpha A_{\alpha_d}''^T + \theta_3 X_\alpha^T \\
\Phi_{32} &= -\hat{N}_{\alpha_2}^T - \theta_2 X_\alpha A_{\alpha_d}'^T - \theta_2 Y_\alpha A_{\alpha_d}''^T \\
&\quad -\theta_3 A_\alpha' X_\alpha^T - \theta_3 A_\alpha'' Y_\alpha^T \\
\Phi_{33} &= -\hat{Q} - \hat{N}_{\alpha_3} - \hat{N}_{\alpha_3}^T - \theta_3 A_{\alpha_d}' X_\alpha^T - \theta_3 A_{\alpha_d}'' Y_\alpha^T \\
&\quad -\theta_3 X_\alpha A_{\alpha_d}'^T - \theta_3 Y_\alpha A_{\alpha_d}''^T,
\end{aligned}
$$

then the teleoperation system described by (19), and (20) will be MES stable with a bounded error for the control gains $K_i = Y_i'^T (X_i'^T)^{-1},\ i = 1, 2.$

Proof: The Lyapunov functional is designed as

$$V(\mathbf{e}, k, i_k) = V_1 + V_2 + V_3 \tag{23}$$

$$V_1 = \mathbf{e}_k^T P_{i_k} \mathbf{e}_k\ ,\ V_2 = \sum_{j=1}^{\tau} \mathbf{e}_{k-j}^T Q \mathbf{e}_{k-j}\ ,$$

$$V_3 = \sum_{i=-\tau}^{0} \sum_{j=i+k}^{k-1} \Delta\mathbf{e}_j^T R \Delta\mathbf{e}_j\ . \tag{24}$$

Define $\Delta V = V(\mathbf{e}, k+1) - V(\mathbf{e}, k)$. Considering a transition from State α to State β, $E\{\Delta V\}$ can be computed as (25)

$$E\{\Delta V\} = \sum_{\alpha,\beta} p(i_k = \alpha, i_{k+1} = \beta) E\{\Delta V | i_k = \alpha, i_{k+1} = \beta\} \tag{25}$$

The goal of this section is to derive a convenient expression for $E\{\Delta V | i_k = \alpha, i_{k+1} = \beta\}$. With $\mathbf{z}_k = [\mathbf{e}_{k+1}^T, \mathbf{e}_k^T, \mathbf{e}_{k-\tau}^T]^T$, $E\{\Delta V_1\}$ is developed as:

$$
\begin{aligned}
E\{\Delta V_1(\mathbf{e}, k, i_k) | i_k = \alpha, i_{k+1} = \beta\} &= \mathbf{e}_{k+1}^T P_\beta \mathbf{e}_{k+1} - \mathbf{e}_k^T P_\alpha \mathbf{e}_k \\
&= \mathbf{z}_k^T \begin{bmatrix} P_\beta & 0 & 0 \\ 0 & -P_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{z}_k.
\end{aligned}
$$

The terms ΔV_2 and ΔV_3 in the Lyapunov functional can be derived similar as in (Walker *et al.*, 2007), but the matrices M, N, and P are now state dependent. They are omitted here due to limited space. This yields (26)

$$
\begin{bmatrix}
\Phi_{11} & \Phi_{21}^T & \Phi_{31}^T & (\tau+1)\hat{N}_{\alpha_1} & \gamma D_\alpha \\
\Phi_{21} & \Phi_{22} & \Phi_{32}^T & (\tau+1)\hat{N}_{\alpha_2} & \gamma\theta_2 D_\alpha \\
\Phi_{31} & \Phi_{32} & \Phi_{33} & (\tau+1)\hat{N}_{\alpha_3} & \gamma\theta_3 D_\alpha \\
(\tau+1)\hat{N}_{\alpha_1}^T & (\tau+1)\hat{N}_{\alpha_2}^T & (\tau+1)\hat{N}_{\alpha_3}^T & -(\tau+1)\hat{R} & 0 \\
\gamma D_\alpha^T & \gamma\theta_2 D_\alpha^T & \gamma\theta_3 D_\alpha^T & 0 & -\gamma I
\end{bmatrix} < 0, \tag{26}
$$

where

$$
\begin{aligned}
\Phi_{11} &= \hat{P}_\beta + (\tau+1)\hat{R} + \hat{N}_{\alpha_1} + \hat{N}_{\alpha_1}^T + X_\alpha^T + X_\alpha \\
\Phi_{21} &= -(\tau+1)\hat{R} + \hat{N}_{\alpha_2} - X_\alpha A_\alpha^T + \theta_2 X_\alpha^T \\
\Phi_{22} &= \hat{Q} - \hat{P}_\alpha + (\tau+1)\hat{R} - \theta_2 A_\alpha X_\alpha^T - \theta_2 X_\alpha A_\alpha^T + \hat{S}_\alpha \\
\Phi_{31} &= \hat{N}_{\alpha_3} - \hat{N}_{\alpha_1}^T - X_\alpha A_{\alpha_d}^T + \theta_3 X_\alpha^T \\
\Phi_{32} &= -\hat{N}_{\alpha_2}^T - \theta_2 X_\alpha A_{\alpha_d}^T - \theta_3 A_\alpha X_\alpha^T \\
\Phi_{33} &= -\hat{Q} - \hat{N}_{\alpha_3} - \hat{N}_{\alpha_3}^T - \theta_3 A_{\alpha_d} X_\alpha^T - \theta_3 X_\alpha A_{\alpha_d}^T.
\end{aligned}
$$

To solve for K_1 and K_2, break down the system matrices as

$$A_1 = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix} \overline{K}_1 \triangleq A_1' + A_1'' \overline{K}_1 \tag{27}$$

$$A_{1_d} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix} \overline{K}_1 \triangleq A_{1_d}' + A_{1_d}'' \overline{K}_1 \tag{28}$$

$$A_2 = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} B & B \\ 0 & 0 \end{bmatrix} \overline{K}_2 \triangleq A_2' + A_2'' \overline{K}_2 \tag{29}$$

$$A_{2_d} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \overline{K}_2 \triangleq A_{2_d}' + A_{2_d}'' \overline{K}_2, \tag{30}$$

where

$$\overline{K}_i = \begin{bmatrix} K_i & 0 \\ 0 & K_i \end{bmatrix}\ ,\ i = 1, 2. \tag{31}$$

Define $Y_i \triangleq X_i \overline{K}_i^T$ where X_i and Y_i have the following block diagonal structure

$$X_i = \begin{bmatrix} X_i' & 0 \\ 0 & X_i' \end{bmatrix},\ \ Y_i = \begin{bmatrix} Y_i' & 0 \\ 0 & Y_i' \end{bmatrix}. \tag{32}$$

The final expression for $E\{\Delta V | i_k = \alpha, i_{k+1} = \beta\}$ is given in (33): $L_{MJLS}(\alpha, \beta)$ is

$$
\begin{bmatrix}
\Phi_{11} & \Phi_{21}^T & \Phi_{31}^T & (\tau+1)\hat{N}_{\alpha_1} & \gamma D_\alpha \\
\Phi_{21} & \Phi_{22} & \Phi_{32}^T & (\tau+1)\hat{N}_{\alpha_2} & \gamma\theta_2 D_\alpha \\
\Phi_{31} & \Phi_{32} & \Phi_{33} & (\tau+1)\hat{N}_{\alpha_3} & \gamma\theta_3 D_\alpha \\
(\tau+1)\hat{N}_{\alpha_1}^T & (\tau+1)\hat{N}_{\alpha_2}^T & (\tau+1)\hat{N}_{\alpha_3}^T & -(\tau+1)\hat{R} & 0 \\
\gamma D_\alpha^T & \gamma\theta_2 D_\alpha^T & \gamma\theta_3 D_\alpha^T & 0 & -\gamma I
\end{bmatrix}, \tag{33}
$$

where

$$
\begin{aligned}
\Phi_{11} &= \hat{P}_\beta + (\tau+1)\hat{R} + \hat{N}_{\alpha_1} + \hat{N}_{\alpha_1}^T + X_\alpha^T + X_\alpha \\
\Phi_{21} &= -(\tau+1)\hat{R} + \hat{N}_{\alpha_2} - X_\alpha A_\alpha'^T - Y_\alpha A_\alpha''^T + \theta_2 X_\alpha^T \\
\Phi_{22} &= \hat{Q} - \hat{P}_\alpha + (\tau+1)\hat{R} - \theta_2 A_\alpha' X_\alpha^T \\
&\quad - \theta_2 A_\alpha'' Y_\alpha^T - \theta_2 X_\alpha A_\alpha'^T - \theta_2 Y_\alpha A_\alpha''^T + \hat{S}_\alpha \qquad (34) \\
\Phi_{31} &= \hat{N}_{\alpha_3} - \hat{N}_{\alpha_1}^T - X_\alpha A_{\alpha_d}'^T - Y_\alpha A_{\alpha_d}''^T + \theta_3 X_\alpha^T \\
\Phi_{32} &= -\hat{N}_{\alpha_2}^T - \theta_2 X_\alpha A_{\alpha_d}'^T - \theta_2 Y_\alpha A_{\alpha_d}''^T - \theta_3 A_\alpha' X_\alpha^T \\
&\quad - \theta_3 A_\alpha'' Y_\alpha^T \\
\Phi_{33} &= -\hat{Q} - \hat{N}_{\alpha_3} - \hat{N}_{\alpha_3}^T - \theta_3 A_{\alpha_d}' X_\alpha^T - \theta_3 A_{\alpha_d}'' Y_\alpha^T \\
&\quad - \theta_3 X_\alpha A_{\alpha_d}'^T - \theta_3 Y_\alpha A_{\alpha_d}''^T. \qquad (35)
\end{aligned}
$$

We can now conclude that the overall system will be stable if (36) holds:

$$\sum_j \sum_i p(\alpha = i, \beta = j) L_{MJLS}(i, j) < 0. \tag{36}$$

Theorem 1 provides a means of incorporating the packet loss process into the design method. A stable system is guaranteed, with a bounded error.

3.4 Simulation Results

For the master and slave manipulators, the inertia term is $J = 0.0203$ kgm^2 and the damping term is $b = 0.0923$ Nms/rad. The ideal spring in (Prokopiou and Tzfestas, 1999), $k_e = 40$N/m, with an additional damping term, $b_e = 5$ Nms/rad, is used to simulate the environment.

We are using a sampling frequency of 1kHz to match the human sense of touch (Tanner and Niemeyer, 2004). γ is chosen as 1. The input is a sine wave with a frequency of 0.5Hz, satisfying the bounded derivative assumption. The time delay, τ, is 55 ms which is the average value of the delay of all received packets from the network experiment. The transition matrix was produced using the data from the experiment mentioned in the modelling section.

The data was scanned sequentially and the conditional probabilities were computed based on the fraction of the time that, given that the current packet was received (or lost), the next packet was received (or lost). This method identifies parameters of the MC associated with the observed process, which produced the transition matrix in (37). This transition matrix corresponds to an equivalent loss rate of approximately 7.5%.

$$T = \begin{bmatrix} 0.9258 & 0.0742 \\ 0.9200 & 0.0800 \end{bmatrix} \tag{37}$$

It is interesting to note that although packet loss often occurs in bursts on IP networks, the transition matrix shows that in this experiment the packet losses can be described by a Bernoulli process since the rows of the MC are almost identical.

Again, various values of θ_2 and θ_3 were considered to produce gains which yield acceptable errors. In Figure 2, a slight diagonal trough exists in which both position is minimized.

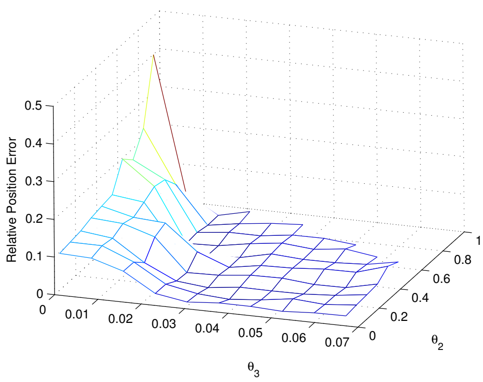

Fig. 2. Position Erorr vs. θ_2 and θ_3 with Packet Loss

Choosing $\theta_2 = 0.6$ and $\theta_3 = 0.02$, the control gains are $K_1 = [-33.0526 \;\; -11.1172]$ and $K_2 = [-3.0011 \;\; -3.1232]$. The system performance with those gains can be seen in Figure 3. The high frequency oscillations in the control signals represent the controller attempting to compensate for an accumulated error after the occurrence of packet losses. When the data is examined closely, a beat frequency can be observed as the effects of a packet loss are reflected back and forth between the master and slave.

In these simulation results, the error is once again bounded and is related to the magnitude of the disturbances. The designed system tracks both position and forces well.

4. CONTROL OVER LOSSY NETWORKS WITH BOUNDED DELAYS

Often in practical applications it is impossible to know the exact value of the delay, but we can frequently determine an upper and lower bound. In this section an upper bound, τ_M, and a lower bound, τ_m, are considered.

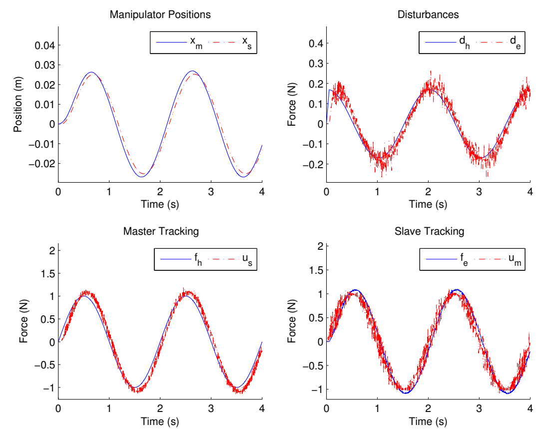

Fig. 3. Response with Proposed Controller

4.1 Control Design With Bounded Delay and Packet Loss

Theorem 2. If there exists symmetric positive definite matrices $\hat{P}_i \in \mathcal{R}^{2n\times 2n}$, $\hat{Q} \in \mathcal{R}^{2n\times 2n}$, $\hat{R} \in \mathcal{R}^{2n\times 2n}$, and $\hat{S} \in \mathcal{R}^{2n\times 2n}$, block diagonal structured matrices $X_i \in \mathcal{R}^{2n\times 2n}$ and $Y_i \in \mathcal{R}^{2n\times 2m}$, positive scalar γ and arbitrary scalars θ_2 and θ_3 with a lower bound τ_m on the delay and an upper bound τ_M and with packet loss transition matrix T such that (38) holds for $i = 1, 2$:

$$\sum_j \sum_i p(\alpha = i, \beta = j) L_{bounded}(i, j) < 0, \tag{38}$$

and $L_{bounded}(\alpha, \beta)$ is

$$\begin{bmatrix} \Phi_{11} & \Phi_{21}^T & \Phi_{31}^T & (\tau_M+1)\hat{N}_{\alpha_1} & \gamma D_\alpha \\ \Phi_{21} & \Phi_{22} & \Phi_{32}^T & (\tau_M+1)\hat{N}_{\alpha_2} & \gamma\theta_2 D_\alpha \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & (\tau_M+1)\hat{N}_{\alpha_3} & \gamma\theta_3 D_\alpha \\ (\tau_M+1)\hat{N}_{\alpha_1}^T & (\tau_M+1)\hat{N}_{\alpha_2}^T & (\tau_M+1)\hat{N}_{\alpha_3}^T & -(\tau_M+1)\hat{R} & 0 \\ \gamma D_\alpha^T & \gamma\theta_2 D_\alpha^T & \gamma\theta_3 D_\alpha^T & 0 & -\gamma I \end{bmatrix},$$

where

$$\begin{aligned} \Phi_{11} =& \hat{P}_\beta + (\tau_M+1)\hat{R} + \hat{N}_{\alpha_1} + \hat{N}_{\alpha_1}^T + X_\alpha^T + X_\alpha \\ \Phi_{21} =& -(\tau_M+1)\hat{R} + \hat{N}_{\alpha_2} - X_\alpha A_\alpha'^T - Y_\alpha A_\alpha''^T + \theta_2 X_\alpha^T \\ \Phi_{22} =& (\tau_M - \tau_m + 1)\hat{Q} - \hat{P}_\alpha + (\tau_M+1)\hat{R} - \theta_2 A_\alpha' X_\alpha^T \\ & - \theta_2 A_\alpha'' Y_\alpha^T - \theta_2 X_\alpha A_\alpha'^T - \theta_2 Y_\alpha A_\alpha''^T + \hat{S}_\alpha \\ \Phi_{31} =& \hat{N}_{\alpha_3} - \hat{N}_{\alpha_1}^T - X_\alpha A_{\alpha_d}'^T - Y_\alpha A_{\alpha_d}''^T + \theta_3 X_\alpha^T \\ \Phi_{32} =& -\hat{N}_{\alpha_2}^T - \theta_2 X_\alpha A_{\alpha_d}'^T - \theta_2 Y_\alpha A_{\alpha_d}''^T \\ & - \theta_3 A_\alpha' X_\alpha^T - \theta_3 A_\alpha'' Y_\alpha^T \\ \Phi_{33} =& -\hat{Q} - \hat{N}_{\alpha_3} - \hat{N}_{\alpha_3}^T - \theta_3 A_{\alpha_d}' X_\alpha^T - \theta_3 A_{\alpha_d}'' Y_\alpha^T \\ & - \theta_3 X_\alpha A_{\alpha_d}'^T - \theta_3 Y_\alpha A_{\alpha_d}''^T, \end{aligned} \tag{39}$$

then the teleoperation system described by (19) and (20) will be MES stable with a bounded error for the control gain $K_i = Y_i'^T (X_i'^T)^{-1}$, $i = 1, 2$.

The proof is omitted here due to the space limit. The control gain is $K_i = Y_i'^T (X_i'^T)^{-1}$, $i = 1, 2$. Theorem 2 does not require exact knowledge of the delay, unlike Theorem 1. Variability in the delay is permitted while ensuring stability with a bounded error.

4.2 Simulation Results

The original data has a minimum delay of 44 ms and it was found that roughly 97% of packets arrived in 64 ms or less so the lower and upper bounds were chosen to be $\tau_m = 44$ and $\tau_M = 64$. The original data is then processed to count any packets with delays above τ_M as lost. A sub-sample is then randomly chosen from the data to provide the random delay and packet loss sequences for the simulation. This technique was chosen since the measured packet losses were independent events. This results in the following packet loss transition matrix which has slightly higher loss rates than the original transition matrix in (37), with a loss rate of approximately 11%. The transition matrix is : $T = \begin{bmatrix} 0.8859 & 0.1141 \\ 0.8960 & 0.1040 \end{bmatrix}$. The probability density functions of the original experimental delays and the generated bounded delays are illustrated in Figure 4. Because packets with large delays were dropped for this simulation, the received packets have a smaller average delay of 51 ms. Both distributions have a long tail, which is typical in network communications.

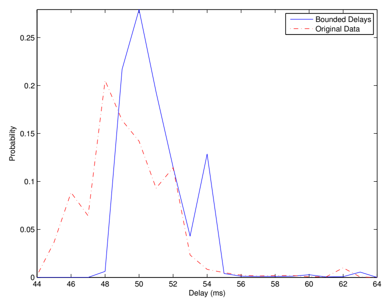

Fig. 4. Probability Density Function of Experimental and Simluation Data

Choosing $\theta_2 = 0.2$ and $\theta_3 = 0.004$, the position error is approximately 10%. For these values, the LMI produces the control gains $K_1 = [-13.7599 \quad -7.4230]$ and $K_2 = [-0.0049 \quad -0.0026]$. These gains result in the performance shown in Figure 5. The chattering of the control signals, u_m and u_s, is slightly worse than in Section 3.4 due to the varying time delay. Packets arriving out of order induce higher frequency components into the signal. The error bound is related to the disturbances and the system is stable, although the performance has degraded slightly over Section 3.4 because of the unreliable channel.

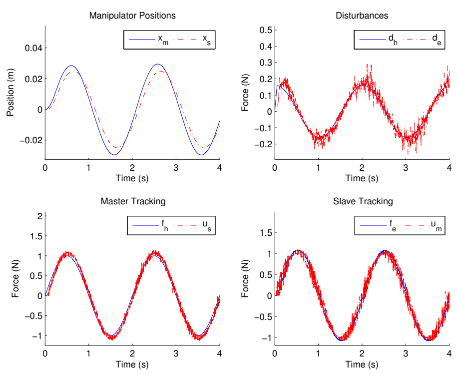

Fig. 5. Response with Proposed Controller

5. CONCLUSIONS AND FUTURE WORK

Moving from an ideal channel to one with delays and packet losses requires more modelling effort and has a negative impact on system performance. By incorporating the imperfections of the communication channel into our control design, stability can be assured under a variety of conditions and the performance can be optimized.

REFERENCES

Anderson, R. J. and M. W. Spong (1989). Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control* **34**(5), 494–501.

Ang, W. T., P. K. Pradeep and C. N. Riviere (2004). Active tremor compensation in microsurgery. In: *Proceedings of the 26th Annual International Conference of the IEEE EMBS*. San Fransico, CA. pp. 2738–2741.

Boukas, E. K. and Z. K. Liu (2002). *Deterministic and Stochastic Time Delay Systems*. Birkhser.

Heredia, E. A., V. Kumar and T. Rahman (1996). Adaptive teleoperation transparency based on impedance modeling. In: *Proc. SPIE Vol. 2901, p. 2-12, Telemanipulator and Telepresence Technologies III, Matthew R. Stein; Ed.* (M. R. Stein, Ed.). pp. 2–12.

Kawka, P. A. and A. G. Alleyne (2006). Stability and performance of packet-based feedback control over a markov channel. In: *Proceedings of the 2006 American Control Conference*. Minneapolis, Minnesota, USA. pp. 2807–2812.

Lin, H. and P. J. Antsaklis (2005). Stability and persistent disturbance attenuation properties for a class of networked control systems: Switched system approach. *International Journal of Control* **78**(18), 1447–1458.

Niemeyer, G. and J. J. Slotine (1991). Stable adaptive teleoperation. *IEEE Journal of Oceanographic Engineering* **16**(1), 152–162.

Nilsson, J. and B. Bernhardsson (1996). Analysis of real-time control systems with time delays. In: *Proceedings of the 35th IEEE Conference on Decision and Control*. Kobe, Japan.

Pan, Y. J., C. Canudas de Wit and O. Sename (2006). A new predictive approach for bilateral teleoperation with applications to drive-by-wire systems. *IEEE Transactions on Robotics* **22**(6), 1146–1162.

Prokopiou, P. A. and S. G. Tzfestas (1999). A novel scheme for human-friendly and time-delays robust neuropredictive teleoperation. *Journal of Intelligent and Robotic Systems* **25**, 311–340.

Prokopiou, P. A., W. S. Harwin and S. G. Tzafestas (1998). Fast, intuitive and time-delays-robust telemanipulator designs using a human arm model. In: *Proceedings 6th Symposium on Intelligent Robotic Systems*. Edinburgh, UK. pp. 7–16.

Tanner, N. A. and G. Niemeyer (2004). High-frequency acceleration feedback in wave variable telerobotics. *IEEE/ASME Transactions on Mechatronics*.

Walker, K., Y. J. Pan and J. Gu (2007). Control gain design for bilateral teleoperation systems using linear matrix inequalities. In: *Proceedings of the 2007 IEEE International Conference on Systems, Man, and Cybernetics*. Montreal, Canada.

Zhu, W. H. and S. E. Salcudean (2000). Stability guaranteed teleoperation: An adaptive motion/force control approach. *IEEE Transactions on Automatic Control* **45**(11), 1951–1969.