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Fault detection based on uncertain models with bounded parameters and bounded parameter variations

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Abstract: This paper deals with a fault detection method taking into account model uncertainties described by bounded variables. A parity space approach is used for generating testable redundancy relations in which each uncertain parameter is defined by an interval containing all its feasible values. Consistency tests consist in evaluating these set-membership relations and lead to convex sets containing the feasible free-fault behaviours of the supervised system. The objective is to improve fault detection performance by taking into account constraints on variations of uncertain parameters, which do not randomly vary.

1. INTRODUCTION

Fault Detection (F.D.) methods often use analytical redundancy based on a mathematical model of the supervised physical system (Patton *et al.*, 1989; Rajaraman et *al*., 2004; Frank, 1990; Staroswiecki *et al.*, 1991). Redundancy relation generation consists in structuring model equations in order to make this information exploitable in the form of relations sensible to faults which must be detected. A major drawback lies in the fact that a model only defines an approximate description of the physical system because of modeling errors. Thus, to avoid confusing a modeling error with a fault, the model inaccuracy, represented in our case by structured parameter uncertainties, has to be taken into account.

The knowledge of some model parameters is often not complete. Instead of representing an uncertain parameter by a constant nominal value, it is defined as a bounded variable. In other words, its real value is unknown, but it belongs to a set of feasible values defined as an interval whose bounds are known. Because of model inaccuracy, residuals may thus be different from zero in the fault free case and describe a set of behaviors representing the normal operation domain of the supervised system. Built by using interval analysis according to uncertainty amount, this domain naturally defines the adaptive thresholds of the F.D. method by determining whether sensor observations are consistent with the reference model. An inconsistency thus reveals a fault.

More important is model inaccuracy, larger are uncertainties and worse is fault detection quality. In previous works (Adrot *et al.*, 2000a, 2000b; Ploix *et al.*, 2006), only supports of uncertain model parameters are taken into account. Concerning variations of these parameters, only two opposite cases can be treated. These cases correspond respectively to uncertain constant parameters which do not vary in model time horizon and to uncertain time-variant parameters which can vary randomly on their interval supports. In the second case, this means that a parameter can be equal to one of its bound at time *t*, and can be equal to the other one (or any value belonging to this interval) at next time. Model parameters having a physical meaning generally have slower variations and do not randomly vary on their supports. To take into account the way in which uncertain parameters vary enables to increase fault detection quality.

Principles of analytical redundancy relation (A.R.R.) generation and consistency tests are presented in section 2 and 3. The section 4 explains how to consider uncertain parameter variations. An example illustrates the proposed method in section 5.

2. A.R.R. GENERATION

2.1. Model presentation

Uncertain structured models take into account the lack of knowledge on a physical system by indicating which parameters are uncertain. These uncertainties are described by normalized bounded variables, whose bounds are equal to −1 and 1. For example, a parameter υ whose value belongs to an interval defined by a lower bound $\underline{\upsilon}$ and an upper bound $\overline{\upsilon}$, will be written:

$$\upsilon = c(\upsilon) + w(\upsilon)\theta, \quad |\theta| \le 1 \quad \left(\text{i.e. } \theta \in [-1,1]\right),$$

$$\text{with } c(\upsilon) = \frac{\overline{\upsilon} + \underline{\upsilon}}{2} \text{ and } w(\upsilon) = \frac{\overline{\upsilon} - \underline{\upsilon}}{2}.$$

In the fault free case, considered dynamic systems are described by the general following linear discrete-time state equations:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{A}(\boldsymbol{\theta}_k, \boldsymbol{\vartheta})\boldsymbol{x}_k + \boldsymbol{B}(\boldsymbol{\theta}_k, \boldsymbol{\vartheta})\boldsymbol{u}_k \\ \boldsymbol{y}_k = \boldsymbol{C}(\boldsymbol{\theta}_k, \boldsymbol{\vartheta})\boldsymbol{x}_k \end{cases}, \qquad (1)$$

$$\boldsymbol{\theta} \in R^{S_\theta}, \boldsymbol{\vartheta} \in R^{S_\vartheta}, \boldsymbol{x} \in R^{S_x}, \boldsymbol{u} \in R^{S_u}, \boldsymbol{y} \in R^{S_y}.$$

The terms $\boldsymbol{x}_k$, $\boldsymbol{u}_k$ and $\boldsymbol{y}_k$, respectively define the state, actuator input and sensor output vectors at time *k*. Since the chosen parity space approach leads to mathematical relations with a finite time horizon, it is quite usual to consider that some uncertain parameters can be time-invariant over this time horizon. Thus, the bounded vectors $\boldsymbol{\theta}_k$ and $\boldsymbol{\vartheta}$ contain respectively uncertain time-variant parameters and uncertain parameters, which can be considered as constant. In this section, the components θ_k^i of $\boldsymbol{\theta}_k$ are represented by independent random variables with bounded realizations. At two different instants k and t, it is assumed that a same uncertain parameter θ^i is represented by two independent variables θ_k^i and θ_t^i with the same bounds. Moreover, the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are assumed to be linear in uncertainties.

2.2. Parity space approach

A major drawback of interval analysis is its explosive nature in case of set-membership recursive models (Armengol *et al.*, 1999). In order to avoid this problem known as wrapping effect, a parity space approach is chosen (Chow *et al.*, 1984; Massoumnia *et al.*, 1988; Nguang *et al.*, 2006). It consists in reformulating the dynamic model equations in the form of algebraic relations on a chosen time horizon $s\in N$. By stacking sensor observations on the time window $[k,k+s]$ according to initial state vector $\boldsymbol{x}_k$, a static representation is obtained where it is no need to integrate model equations in order to generate A.R.R. (Adrot *et al.*, 2000a):

$$\boldsymbol{O}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)\boldsymbol{x}_k=\left[-\boldsymbol{H}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)\ \boldsymbol{I}\right]\begin{bmatrix}\boldsymbol{u}_{k,s-1}\\ \boldsymbol{y}_{k,s}\end{bmatrix},\quad (2)$$

$$\underbrace{z_{k,s}}_{z\in\{\boldsymbol{u},\boldsymbol{y},\boldsymbol{\theta}\}}=\begin{bmatrix}z_k\\ \vdots\\ z_{k+s}\end{bmatrix},\ \boldsymbol{O}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)=\begin{bmatrix}\boldsymbol{C}_k\\ \boldsymbol{C}_{k+1}\boldsymbol{A}_k\\ \vdots\\ \boldsymbol{C}_{k+s}\boldsymbol{A}_{k+s-1}\ldots\boldsymbol{A}_k\end{bmatrix}$$

$$\boldsymbol{H}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)=\begin{bmatrix}0 & 0 & \cdots & 0\\ \boldsymbol{C}_{k+1}\boldsymbol{B}_k & 0 & \cdots & 0\\ \boldsymbol{C}_{k+2}\boldsymbol{A}_{k+1}\boldsymbol{B}_k & \boldsymbol{C}_{k+2}\boldsymbol{B}_{k+1} & \cdots & 0\\ \vdots & & & \\ \boldsymbol{C}_{k+s}\boldsymbol{A}_{k+s-1}\cdots\boldsymbol{B}_k & \boldsymbol{C}_{k+s}\boldsymbol{A}_{k+s-1}\cdots\boldsymbol{B}_{k+1}\cdots & & \boldsymbol{C}_{k+s}\boldsymbol{B}_{k+s-1}\end{bmatrix}$$

$$\boldsymbol{O}\in R^{(s+1)s_y\times n},\boldsymbol{H}\in R^{(s+1)s_y\times s.s_u},\boldsymbol{I}\in R^{s_y\times s_y}\ (\text{identity})$$
$$\text{with } \boldsymbol{A}_k=\boldsymbol{A}\left(\boldsymbol{\theta}_k,\boldsymbol{\vartheta}\right),\boldsymbol{B}_k=\boldsymbol{B}\left(\boldsymbol{\theta}_k,\boldsymbol{\vartheta}\right),\boldsymbol{C}_k=\boldsymbol{C}\left(\boldsymbol{\theta}_k,\boldsymbol{\vartheta}\right)$$

In the previous equality (2), the term on the left depends on unknown state variables whereas the term on the right groups together measured outputs and known inputs. In order to eliminate the unknown state vector $\boldsymbol{x}_k$, an uncertain parity matrix $\boldsymbol{W}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)$ orthogonal to $\boldsymbol{O}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)$ is sought:

$$\boldsymbol{W}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)\boldsymbol{O}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)=0.\quad (3)$$

The parity matrix can be written in the form of a polynomial matrix in uncertainties whose symbolic expression is given in (Adrot *et al.*, 2000b). A numeric method for parity matrix calculation is also given in (Ploix *et al.*, 2006). After multiplying the static form (2) by $\boldsymbol{W}$, the vector $\boldsymbol{r}_k$ of analytical redundancy relations is deduced:

$$\boldsymbol{r}_k\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)=\underbrace{\boldsymbol{W}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)\left[-\boldsymbol{H}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)\ \boldsymbol{I}\right]}_{\boldsymbol{P}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)}\begin{bmatrix}\boldsymbol{u}_{k,s-1}\\ \boldsymbol{y}_{k,s}\end{bmatrix},(4)$$

where $\boldsymbol{r}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)$ is polynomial in uncertainties and linear in known inputs and outputs. Moreover, this expression depends on all the uncertainties which initially affect the state representation (1).

The existence of redundancy relations depends on the existence of a parity matrix $\boldsymbol{W}$ satisfying (3) and so on the determination of a relevant time horizon s. If this horizon is too small, $\boldsymbol{W}$ can not be constructed or some relations miss and fault detection performance is limited. If the horizon is too large, the amount of computations increases without any benefits. The right determination of s is a key issue. An uncertain system defined by (1) is called regularly observable (Ploix *et al.*, 2006) if, for all $s\in N^+$, the rank of its observable subspaces $\boldsymbol{O}\left(\boldsymbol{\theta}_{k,s},\boldsymbol{\vartheta}\right)$ is independent of uncertainties. Assuming the system (1) is regularly observable, the method provided by the deterministic theory (Chow *et al.*, 1984; Massoumnia *et al.*, 1988) may be used to determine s. In this way, if the observation matrix $\boldsymbol{C}_k$ is full row rank, there are s_y redundancy relations: $\boldsymbol{W}\in R^{s_y\times(s+1)s_y},\boldsymbol{r}_k\in R^{s_y}$.

3. CONSISTENCY TESTS

3.1. Principle

Let us note $\boldsymbol{\upsilon}_k$ the vector composed of all normalized bounded variables contained in $\boldsymbol{r}_k$: $\boldsymbol{\upsilon}_k=\left[\boldsymbol{\theta}_{k,s}^T\ \ \boldsymbol{\vartheta}^T\right]^T$. At a given instant k, the physical system is normally operating if at least one value $\boldsymbol{\upsilon}$ of the uncertain vector $\boldsymbol{\upsilon}_k$ exists such that:

- the model is consistent with measurements, that implies $\boldsymbol{r}_k(\boldsymbol{\upsilon})=0$,
- $\boldsymbol{\upsilon}$ is a feasible value in the sense that $\|\boldsymbol{\upsilon}\|_\infty\le 1$.

To check consistency between sensor observations and model (1) consists in evaluating redundancy relations (4) according to model uncertainties and testing whether obtained set-membership residuals can be equal to zero. According to (4) and by noting the origin of the residual space O, consistency analysis leads to test if:

$$O\in S\left(\boldsymbol{r}_k\right)\ \text{with}\ S\left(\boldsymbol{r}_k\right)=\left\{\boldsymbol{r}_k\left(\boldsymbol{\upsilon}_k\right)/\left\|\boldsymbol{\upsilon}_k\right\|_\infty\le 1\right\}.$$

The value set $S(\boldsymbol{r}_k)$ defines all the feasible values of the uncertain residual vector $\boldsymbol{r}_k$, which are consistent with the chosen model according to sensor observations and constraints $\|\boldsymbol{\upsilon}_k\|_\infty\le 1$. Thus, a fault is detected if the origin O of the residual space does not belong to $S(\boldsymbol{r}_k)$, since in this case $\boldsymbol{r}_k$ can not be equal to zero.

Thus, the objective is to compute the value set of $\boldsymbol{r}_k$. Since $\boldsymbol{r}_k$ is non-linear in bounded variables $\boldsymbol{\upsilon}_k$, to exactly evaluate $S(\boldsymbol{r}_k)$ is generally impossible. The proposed solution is to compute an overestimation of $S(\boldsymbol{r}_k)$ by using the method detailed in (Adrot *et al.*, 2000b), which enables to obtain redundancy relations linear in uncertainties. Briefly, the principle is to replace each monomial of bounded variables occurring in $\boldsymbol{r}_k$ by a new independent variable with an adequate support. For example, by noting υ_k^i the i^{th}

component of $\boldsymbol{\upsilon}_k$, monomials $\upsilon_k^i \upsilon_k^j$ and $\left(\upsilon_k^i\right)^2$ are replaced by μ_k^j and $0.5+0.5\mu_k^l$, where μ_k^j and μ_k^l defines the j^{th} and l^{th} components of a normalized bounded vector $\boldsymbol{\mu}_k$. All occurrences of a given monomial are replaced by the same component of $\boldsymbol{\mu}_k$. This linearization is guaranteed because the value set $S(\boldsymbol{r}_{lin,k})$ of the linearized residual vector $\boldsymbol{r}_{lin,k}$ always contains the theoretic domain $S(\boldsymbol{r}_k)$ (Fig. 1).

By noting $\boldsymbol{\mu}_k$ a vector composed of all normalized bounded variables appearing in the linearized residual vector, $\boldsymbol{r}_{lin,k}$ is written as follows where the certain matrix $\boldsymbol{R}_\mu$ and the vector $\boldsymbol{r}_0$ are linear in measurements:

$$\boldsymbol{r}_{lin,k}(\boldsymbol{\mu}_k)=\boldsymbol{R}_\mu(\boldsymbol{y}_{k,s},\boldsymbol{u}_{k,s-1})\boldsymbol{\mu}_k+\boldsymbol{r}_0(\boldsymbol{y}_{k,s},\boldsymbol{u}_{k,s-1}).$$

Since $\boldsymbol{r}_{lin,k}$ is linear in $\boldsymbol{\mu}_k$, $S(\boldsymbol{r}_k)$ is overestimated by a convex zonotope $S(\boldsymbol{r}_{lin,k})$ centered in $\boldsymbol{r}_0$ and whose shape is imposed by $\boldsymbol{R}_\mu$. In other words, this zonotope is a domain delimited in the residual space by two by two parallel hyperplanes which can be defined by a set of inequality constraints. Therefore, $S(\boldsymbol{r}_{lin,k})$ can be exactly described by an inequality system:

$$\boldsymbol{M}\left(\boldsymbol{y}_{k,s},\boldsymbol{u}_{k,s-1}\right)\boldsymbol{r}_{lin,k} \le \boldsymbol{n}\left(\boldsymbol{y}_{k,s},\boldsymbol{u}_{k,s-1}\right), \qquad (5)$$

where the matrix $\boldsymbol{M}$ and the vector $\boldsymbol{n}$ are certain and can be computed by the method detailed in (Adrot *et al.*, 2000b). In this way, consistency tests for fault detection consist in verifying whether the inequality $0 \le \boldsymbol{n}\left(\boldsymbol{y}_{k,s},\boldsymbol{u}_{k,s-1}\right)$ holds. All these steps are resumed in Fig. 1.

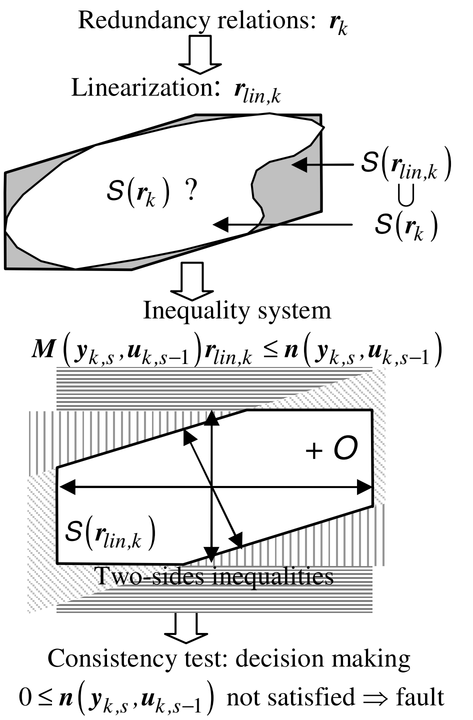

Fig 1. Steps of the fault detection method

Since $S(\boldsymbol{r}_{lin,k})$ is pessimistic and necessarily contains $S(\boldsymbol{r}_k)$ which represents all fault free behaviors, this method does not generate any false alarms other than those due to the no-completeness of the model. Thus, if the model is initially complete, an inconsistency necessarily guarantees the presence of a fault. On the contrary, a consistency does not assure the absence of a fault which may be masked by some uncertainties (problem only due to model inaccuracy) or by the use of $S(\boldsymbol{r}_{lin,k})$ instead of $S(\boldsymbol{r}_k)$ (pessimism due to linearization).

The interest of this method is that:

- the linearization can be done a priori,
- the computation of inequalities (5) is very fast,
- consistency tests are simple and very fast.

4. UNCERTAINTIES WITH BOUNDED VARIATIONS

4.1. Principle

In the method proposed in section 2, the support of a time-variant and uncertain model parameter θ^i is known a priori or identified (Adrot *et al.*, 2006), but its variations are not constrained (provided that it belongs to its interval support). In this way, θ^i can randomly vary on the time horizon s of redundancy relations (4) since it is represented by independent bounded variables θ_{k+j}^i, $j\in\{0,\ldots,s\}$ on the time horizon $[k,k+s]$. This means that the bounded parameter θ^i can be equal to one of its bound at time $k+j$, and can be equal to the other one (or any value belonging to $[-1,1]$) at time $k+j+1$. Generally, model parameters having a physical meaning (or any combination of them) have slower variations and do not vary randomly on their support. For example, two bounded parameters with bounded variations delimited by dashed lines are illustrated in Fig. 2. To take into account the way in which uncertain parameters vary, enables to increase fault detection quality by forbidding operating points which can not be physically reached, i.e. by reducing $S(\boldsymbol{r}_k)$.

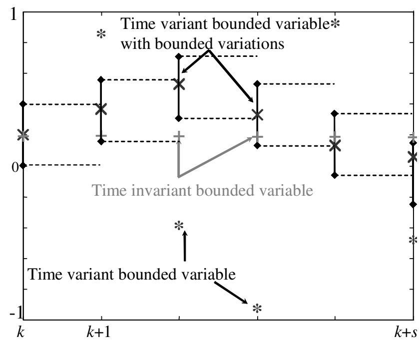

Fig. 2. Different types of bounded variables

The original objective of this work consists in considering that the variation of the parameter θ^i is limited (bounded in our case) between two consecutive instants. For a discrete time variable θ_k^i, this leads to impose the constraint:

$$\boldsymbol{\theta}_{k+1}^i=\boldsymbol{\theta}_k^i+\boldsymbol{\delta}^i\boldsymbol{\varepsilon}_k^i, \left\|\begin{bmatrix}\boldsymbol{\theta}_k^i & \boldsymbol{\theta}_{k+1}^i & \boldsymbol{\varepsilon}_k^i\end{bmatrix}^T\right\|_\infty \le 1, \boldsymbol{\delta}^i\in R^{*+}. \quad (6)$$

Geometrically, equation (6) defines a zonotope (grey zone) in the $\left(\theta_k^i,\theta_{k+1}^i\right)$-space:

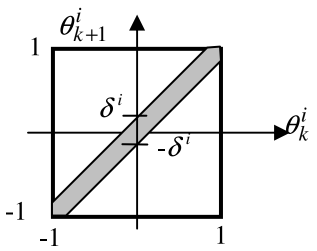

Fig. 3. Constraint on parameter variation

Remark. The case $\delta^i = 0$ is already treated by $\boldsymbol{\vartheta}$.

Remark. As shown in Fig. 3, to take a width δ^i larger than 2 leads to a zonotope which corresponds to the box $[-1,1]\times[-1,1]$. In other words, the constraint (6) becomes less restrictive than the constraint due to the normalization of bounded variables because a bounded parameter can be equal to one of its bound at time k, and can be equal to the other one at time k+1. In this case, equation (6) does not provide any additional information for fault detection.

For beginning, let us consider the following real function r_{nv} ('*nv*' standing for normalized variables) depending of an uncertain parameter θ expressed at two consecutive instants:

$$r_{nv}(\theta_k, \theta_{k+1}) = \theta_{k+1} - \theta_k . \qquad (7)$$

The bounded variables θ_k and θ_{k+1} being normalized, the value set of r_{nv} given by:

$$\mathcal{S}_{nv} = \left\{ r_{nv}(\boldsymbol{\theta}_k, \boldsymbol{\theta}_{k+1}) / |\boldsymbol{\theta}_k| \le 1, |\boldsymbol{\theta}_{k+1}| \le 1 \right\}, \qquad (8)$$

is evaluated by interval analysis (Moore, 1979; Neumaier, 1990) and leads to interval $[-2,2]$.

Taking into account constraint (6) gives a new expression r_{bv} ('*bv*' standing for bounded variation) of the function r_{nv}: $r_{bv}(\theta_k, \varepsilon_k) = \theta_k + \delta\varepsilon_k - \theta_k$. A well-known cause of pessimism of interval analysis comes from the multiple occurrences of bounded variables in a real function. Interval analysis can not take into account the dependence between several bounded variables (Moore, 1979; Neumaier, 1990) because it works on their bounds (supports) where this dependence does not appear. A solution consists in putting together identical bounded variables before using interval analysis: $r_{bv}(\varepsilon_k) = \delta\varepsilon_k$. The evaluation of the value set of r_{bv}:

$$\mathcal{S}_{bv} = \left\{ r_{bv}(\boldsymbol{\varepsilon}_k) / |\boldsymbol{\varepsilon}_k| \le 1, \boldsymbol{\delta} \in R^{*+} \right\} \qquad (9)$$

leads to interval $[-\delta,\delta]$. In case the value of δ is less than 2, constraint on parameter variations leads as expected to a more conservative set $\mathcal{S}_{bv}$ than $\mathcal{S}_{nv}$.

Let us note $\boldsymbol{r}_{ext}$ ('*ext*' standing for extended) the vector composed of r_{nv} and r_{bv}. The value set of $\boldsymbol{r}_{ext}$ satisfying both constraints on parameter supports (7) and constraints on parameter variations (8) given as:

$$\mathcal{S}_{ext} = \left\{ \begin{bmatrix} r_{nv}(\theta_k, \theta_{k+1}) \\ r_{bv}(\varepsilon_k) \end{bmatrix} \Big/ \begin{matrix} |\theta_k| \le 1, |\theta_{k+1}| \le 1 \\ |\varepsilon_k| \le 1, \delta \in R^{*+} \end{matrix} \right\},$$

is an interval vector (box) $[-2,2]\times[-\delta,\delta]$ because no bounded variable appears simultaneously in both component of $\boldsymbol{r}_{ext}$.

The same principle can be used for a vector of redundancy relations. Let us consider the following vector field $\boldsymbol{r}_{nv}$:

$$\boldsymbol{r}_{nv}(\theta_k, \theta_{k+1}) = \begin{bmatrix} \theta_{k+1} + \theta_k + \theta_{k+1}\theta_k \\ \theta_{k+1} - \theta_k + 2\theta_{k+1}\theta_k \end{bmatrix}.$$

Constraint (6) leads to:

$$\boldsymbol{r}_{bv}(\theta_k, \varepsilon_k) = \begin{bmatrix} 2\theta_k + \delta\varepsilon_k + (\theta_k)^2 + \delta\theta_k\varepsilon_k \\ \delta\varepsilon_k + 2(\theta_k)^2 + 2\delta\theta_k\varepsilon_k \end{bmatrix}.$$

By using the linearization method explained in section 3, the linearized vectors are respectively expressed as:

$$\boldsymbol{r}_{nv,lin}(\theta_k, \theta_{k+1}, \mu_k^1) = \begin{bmatrix} \theta_{k+1} + \theta_k + \mu_k^1 \\ \theta_{k+1} - \theta_k + 2\mu_k^1 \end{bmatrix}$$

$$\boldsymbol{r}_{bv,lin}(\theta_k, \varepsilon_k, \mu_k^2, \mu_k^3) = \begin{bmatrix} 2\theta_k + \delta\varepsilon_k + 0.5(1 + \mu_k^2) + \delta\mu_k^3 \\ \delta\varepsilon_k + 1 + \mu_k^2 + 2\delta\mu_k^3 \end{bmatrix}$$

where all bounded variables are independent and normalized. The value set $\mathcal{S}_{ext}$ of the vector $\boldsymbol{r}_{ext}$:

$$\boldsymbol{r}_{ext} = \begin{bmatrix} \boldsymbol{r}_{nv,lin}^T & \boldsymbol{r}_{bv,lin}^T \end{bmatrix}^T ,$$

is a zonotope in R^4-space. Some projections in different (r^i_{ext}, r^j_{ext})-spaces are shown on Fig. 4, where r^i_{ext} is the i^{th} component of $\boldsymbol{r}_{ext}$. The value set $\mathcal{S}_{ext}$ is generally a zonotope instead of a box. Working without limited parameter variations leads to consider only the left-up zonotope.

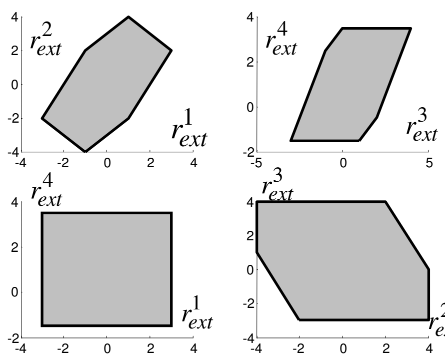

Fig. 4. Projections of $\mathcal{S}_{ext}$

Therefore, the principle is to construct additional redundancy relations associated to constraints on bounded parameter variation, and then to simultaneously apply consistency tests on all A.R.R.

4.2. Proposed fault detection method

The method initially proposed in section 2 enables to construct redundancy relations (4) satisfying constraints on parameter supports:

$$\boldsymbol{r}_{nv,k}(\boldsymbol{\theta}_{k,s}, \boldsymbol{\vartheta}) = \boldsymbol{r}_k(\boldsymbol{\theta}_{k,s}, \boldsymbol{\vartheta}) = \boldsymbol{P}(\boldsymbol{\theta}_{k,s}, \boldsymbol{\vartheta}) \begin{bmatrix} \boldsymbol{u}_{k,s-1} \\ \boldsymbol{y}_{k,s} \end{bmatrix} . (10)$$

In order to take into account bounded parameter variations, it is needed to generate additional redundancy relations from previous equations (10) by means of constraints (6). Each bounded variable θ_{k+j}^i of $\boldsymbol{\theta}_{k,s}$ (2) is expressed according to θ_k^i (beginning of time window) and new independent normalized variables ε_{k+h}^i, $h\in\{0,\ldots,j-1\}$:

$$\theta_{k+j}^i = \theta_k^i + \delta^i \sum_{h=0}^{j-1} \varepsilon_{k+h}^i, \quad \text{with} \left|\varepsilon_{k+h}^i\right| \le 1 .$$

In this way, all variations of the uncertain parameter θ_k^i are taken into account on the time horizon s of the static representation (2).

Additional redundancy relations are expressed as:

$$\boldsymbol{r}_{bv,k}\left(\boldsymbol{\theta}_k, \boldsymbol{\varepsilon}_{k,s-1}, \boldsymbol{\vartheta}\right) = \boldsymbol{P}\left(\boldsymbol{\theta}_k, \boldsymbol{\varepsilon}_{k,s-1}, \boldsymbol{\vartheta}\right) \begin{bmatrix} \boldsymbol{u}_{k,s-1} \\ \boldsymbol{y}_{k,s} \end{bmatrix}. \quad (11)$$

Finally, consistency tests proposed in section 3 are directly applied on the extended vector $\boldsymbol{r}_{ext,k}$:

$$\boldsymbol{r}_{ext,k}\left(\boldsymbol{\theta}_{k,s}, \boldsymbol{\varepsilon}_{k,s-1}, \boldsymbol{\vartheta}\right) = \begin{bmatrix} \boldsymbol{r}_{nv,k}\left(\boldsymbol{\theta}_{k,s}, \boldsymbol{\vartheta}\right) \\ \boldsymbol{r}_{bv,k}\left(\boldsymbol{\theta}_k, \boldsymbol{\varepsilon}_{k,s-1}, \boldsymbol{\vartheta}\right) \end{bmatrix}.$$

By construction, since $\boldsymbol{r}_{ext,k}$ contains all redundancy relations (4), the proposed fault detection method in this section can not lead to results worse than for the initial method resumed in section 2 and 3.

5. EXAMPLE

In order to illustrate previous developments, let us consider the following discrete-time free fault state space model:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{B} u_k = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.2+\rho^2\theta_k^2 \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k \\ \boldsymbol{y}_k &= \boldsymbol{C}_k \boldsymbol{x}_k = \begin{bmatrix} 1+\rho^1\theta_k^1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}_k \quad \text{with } \begin{matrix} \rho^1 = 0.05 \\ \rho^2 = 0.1 \end{matrix} \end{aligned} . (12)$$

Normalized bounded variables θ_k^i, $i\in\{1,2\}$, describe multiplicative uncertainties. The chosen time horizon (i.e. the smallest integer s for which the matrix $\boldsymbol{O}\left(\boldsymbol{\theta}_{k,s}, \boldsymbol{\vartheta}\right)$ is not full row rank) is $s = 1$. The static representation (2) is given as:

$$\begin{bmatrix} \boldsymbol{C}_k \\ \boldsymbol{C}_{k+1}\boldsymbol{A}_k \end{bmatrix} \boldsymbol{x}_k = \begin{bmatrix} -\begin{bmatrix} 0 \\ \boldsymbol{C}_{k+1}\boldsymbol{B} \end{bmatrix} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} u_k \\ \boldsymbol{y}_{k,1} \end{bmatrix}$$

$$\begin{bmatrix} 1+\rho^1\theta_k^1 & 0 \\ 0 & 1 \\ 0.8\left(1+\rho^1\theta_{k+1}^1\right) & 1+\rho^1\theta_{k+1}^1 \\ 0 & 0.2+\rho^2\theta_k^2 \end{bmatrix} \boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{y}_k \\ \boldsymbol{y}_{k+1} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_k$$

Let us note $\boldsymbol{w}^T$ a generic row of the parity matrix $\boldsymbol{W}$:

$$\boldsymbol{w}^T = \begin{bmatrix} w^1 & w^2 & w^3 & w^4 \end{bmatrix}.$$

Constraint (3) leads to:

$$\begin{cases} w^1 = -0.8\left(1+\rho^1\theta_{k+1}^1\right) \big/ \left(1+\rho^1\theta_k^1\right) w^3 \\ w^2 = -\left(1+\rho^1\theta_{k+1}^1\right) w_3 - \left(0.2+\rho^2\theta_k^2\right) w^4 \end{cases}.$$

Since the rank of the matrix $\boldsymbol{O}(0,0)$ is equal to 2, two redundancy relations must be generated. By choosing two arbitrary values of the pair (w^3,w^4):

$$\left(w^3 = 1+\rho^1\theta_k^1, w^4 = 0\right) \text{ and } \left(w^3 = 0, w^4 = 1\right),$$

the following uncertain parity matrix

$$\boldsymbol{W}\left(\boldsymbol{\theta}_{k,1}, \boldsymbol{\vartheta}\right) = \begin{bmatrix} -0.8\left(1+\rho^1\theta_{k+1}^1\right) & -\left(1+\rho^1\theta_{k+1}^1\right)\left(1+\rho^1\theta_k^1\right) & 1+\rho^1\theta_k^1 & 0 \\ 0 & -\left(0.2+\rho^2\theta_k^2\right) & 0 & 1 \end{bmatrix}$$

is obtained. Finally, A.R.R. are given as:

$$\boldsymbol{r}_{nv,k}\left(\boldsymbol{\theta}_{k,1}, \boldsymbol{\vartheta}\right) = \boldsymbol{W}\left(\boldsymbol{\theta}_{k,1}, \boldsymbol{\vartheta}\right) \left(\begin{bmatrix} \boldsymbol{y}_k \\ \boldsymbol{y}_{k+1} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_k \right). (13)$$

$$\boldsymbol{r}_{nv,k}\left(\boldsymbol{\theta}_{k,1}, \boldsymbol{\vartheta}\right) = \begin{bmatrix} -0.8y_k^1 - y_k^2 + y_{k+1}^1 + \rho^1\left(y_{k+1}^1 - y_k^2\right)\theta_k^1 \cdots \\ \cdots - \rho^1\left(0.8y_k^1 + y_k^2\right)\theta_{k+1}^1 - \left(\rho^1\right)^2 y_k^2\theta_{k+1}^1\theta_k^1 \\ \hline -0.2y_k^2 + y_{k+1}^2 - u_k - \rho^2 y_k^2\theta_k^2 \end{bmatrix}.$$

Then the linearization procedure is applied:

$$\boldsymbol{r}_{nv,lin,k}\left(\boldsymbol{\mu}_{nv,k}\right) = \begin{bmatrix} -0.8y_k^1 - y_k^2 + y_{k+1}^1 \\ -0.2y_k^2 + y_{k+1}^2 - u_k \end{bmatrix} + \cdots$$

$$\cdots + \begin{bmatrix} \rho^1\left(y_{k+1}^1 - y_k^2\right) & -\rho^1\left(0.8y_k^1 + y_k^2\right) & -\left(\rho^1\right)^2 y_k^2 & 0 \\ 0 & 0 & 0 & -\rho^2 y_k^2 \end{bmatrix} \boldsymbol{\mu}_{nv,k}$$

with $\left\|\boldsymbol{\mu}_{nv,k}\right\|_\infty \le 1$, where the bounded variables $\mu_{nv,k}^i$, $i\in\{1,\ldots,4\}$, represent the monomials θ_{k+1}^1, θ_k^1, $\theta_{k+1}^1\theta_k^1$, θ_k^2.

In equation (13), the first uncertain parameter θ_k^1 appears at times k and k+1. A constraint on its variations is imposed:

$$\theta_{k+1}^1 = \theta_k^1 + \delta^1\varepsilon_k^1, \ \left|\varepsilon_k^1\right| \le 1, \ \delta^1 = 0.1 .$$

Additional redundancy relations are expressed as:

$$\boldsymbol{r}_{bv,k}\left(\boldsymbol{\theta}_k, \boldsymbol{\varepsilon}_k, \boldsymbol{\vartheta}\right) = \begin{bmatrix} -0.8y_k^1 - y_k^2 + y_{k+1}^1 - \rho^1\left(0.8y_k^1 + y_k^2\right)\delta^1\varepsilon_k^1 \cdots \\ \cdots + \rho^1\left(-0.8y_k^1 - 2y_k^2 + y_{k+1}^1\right)\theta_k^1 \cdots \\ \cdots - \left(\rho^1\right)^2 y_k^2\left(\theta_k^1\right)^2 - \left(\rho^1\right)^2 y_k^2\delta^1\theta_k^1\varepsilon_k^1 \\ \hline -0.2y_k^2 + y_{k+1}^2 - u_k - \rho^2 y_k^2\theta_k^2 \end{bmatrix}$$

By representing respectively the monomials θ_k^1, ε_k^1, $\left(\theta_k^1\right)^2$, $\theta_k^1\varepsilon_k^1$, θ_k^2 by the bounded variables $\mu_{bv,k}^i$, $i\in\{1,\ldots,5\}$, the linearization procedure leads to:

$$\boldsymbol{r}_{bv,lin,k}\left(\boldsymbol{\theta}_k,\boldsymbol{\varepsilon}_k,\boldsymbol{\vartheta}\right)=\begin{bmatrix}-0.8y_k^1-y_k^2+y_{k+1}^1-0.5\left(\rho_1\right)^2y_k^2\\ -0.2y_k^2+y_{k+1}^2-u_k\end{bmatrix}+\cdots$$
$$\cdots+\left[\begin{matrix}\rho^1\left(-0.8y_k^1-2y_k^2+y_{k+1}^1\right) & -\delta^1\rho^1\left(0.8y_k^1+y_k^2\right)\\ 0 & 0\end{matrix}\cdots\right.$$
$$\left.\cdots\begin{matrix}-0.5\left(\rho^1\right)^2y_k^2 & -\delta^1\left(\rho^1\right)^2y_k^2 & 0\\ 0 & 0 & -\rho^2y_k^2\end{matrix}\right]\boldsymbol{\mu}_{bv,k}\ \text{with}\ \left\|\boldsymbol{\mu}_{bv,k}\right\|_\infty\leq 1$$

Figure 5 represents the variables simulated in this example. Two faults are considered. A bias of magnitude 0.5 affects the first sensor between times 10 and 40 and a second one of magnitude 0.15 affects the second sensor between times 60 and 90 (grey areas). The difference between the state variable x_k^1 and its measurement y_k^1 in fault free case comes from influence of uncertain parameter θ_k^1 on matrix $\boldsymbol{C}_k$ (12).

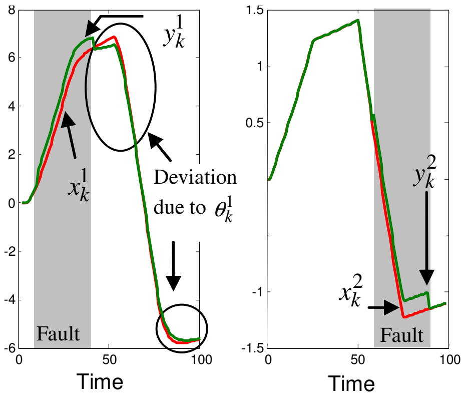

Fig. 5. States variables and measurements

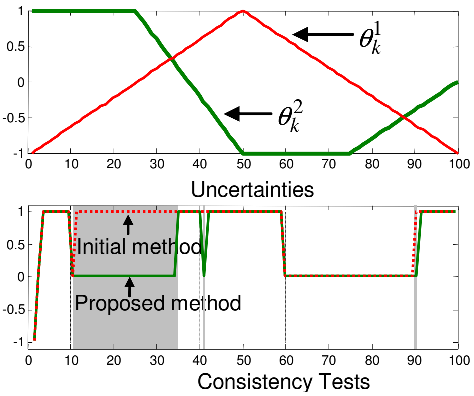

Fig. 6. Consistency tests

Variations of uncertain parameters and results of consistency tests are shown on Fig. 6. During F.D., only supports of uncertain parameters (and their variations) are assumed to be known. For consistency tests, the values 0 and 1 indicate respectively abnormal and normal behaviours. Since a time horizon is used (s = 1), tests can not be performed at the fist sample (value equal to −1). Initial and proposed methods correspond respectively to work on $\boldsymbol{r}_{nv,link,k}$ and on the extended vector composed of $\boldsymbol{r}_{nv,link,k}$ and $\boldsymbol{r}_{bv,link,k}$. Abnormal behaviour is detected when a fault is really present. Faults are still detected out of the faulty time period (at times 41, 91) because of time horizon equal to one sample time. The first fault is only detected by the proposed method. The second fault is perfectly detected by both methods. Miss-detections are essentially due to uncertainties (±5%, ±50%), which may mask a fault, but to take into account uncertainty variations increases fault detection performance (grey areas).

6. CONCLUSION

The objective of this paper is to improve previous works on fault detection using interval analysis (section 2) for handling uncertain dynamic systems where uncertainties can be multiplicative. A new method based on constraints on uncertainty variations (section 4) is explained. In case model parameters do not randomly vary on their support, this information leads to additional redundancy relations which may reduce miss-detections.

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