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New residual generation design for fault detection

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Abstract: A new design procedure of a reduced order unknown input observer (UIO) is proposed to generate residuals for fault detection isolation (FDI). The originality of this work consists in the adopted approach for the procedure implementation. Indeed the kernel of the actuator fault distribution matrix is generated thanks to generalized inverses. The Kronecker product is used to solve a Sylvester equation which appears in the equations of an UIO. Residuals generated by bank of observers allow on the one hand the detection isolation of every actuator fault and on the other hand the isolation between actuator faults and sensor faults.

1. INTRODUCTION

Unknown input observers (UIOs) are usually used when limitation disturbance effects are targeted. This approach has been carried out as early as the 80s when researchers in control field paid great attention to UIOs (Kudva et al. [1980], Yang and Wilde [1988]). Model-based fault diagnosis techniques, mathematical description and definitions are detailed in Chen and Patton [1999]. Many works in the field of FDI are based on the design of a full order UIO (Chen and Patton [1999], Demetriou [2005]). In fact, there are more degrees of freedom available for the design of structured residuals. However, in most cases, a reduced order observer is suitable for a FDI approach (Koenig and Mammar [2001]).

A simple reduced-order UIO design procedure is described in this article. The observer leads to structured residuals in order to detect and isolate actuator faults from sensor faults.

In this procedure we use generalized inverses to design the observer, and it enables to introduce some arbitrary parameters which are very useful. Indeed, they offer some opportunities in the design. Several works use a generalized inverse to treat the equation which ensures that the residual is insensitive to the actuator faults. The arbitrary matrix which appears by solving a linear system with generalized inverse is used, when it is possible, to assign the observer dynamics. See for instance Kudva et al. [1980], Kurek [1983] and Darouach et al. [1994] who define existence conditions by using this approach too.

In this article, we also solve a linear system by finding the kernel of an application. Another particular point of our approach is that we set the structure of the matrix which provides the dynamics of the observer. An arbitrary matrix introduced in the resolution allows to ensure the compatibility of the linear system we finally have to solve to design the UIO. Existence conditions are given by several tests in the presented procedure.

In order to describe our purpose, notations are presented. A linear time invariant system is considered, where r actuator faults and m sensor faults can occur on the system. The system model is described as

$$\dot{x}(t) = Ax(t) + Bu(t) + K_a f_a \quad (1)$$

$$\begin{aligned} y(t) &= Cx(t) + K_s f_s \\ &= [\, I_m \;\; 0 \,]\, x(t) + K_s f_s, \end{aligned} \quad (2)$$

where for every time t, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $f_a \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^m$ and $f_s \in \mathbb{R}^m$ are respectively the state, known input, actuator fault assimilated as unknown input, output of the system and the sensor fault. Matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$ and $C \in \mathbb{R}^{m\times n}$ are known constant matrices constituting the state space model. Notice from (1) that a special structure for the sensor matrix C is assumed which can always be fulfilled under hypothesis of independent sensors. $K_a \in \mathbb{R}^{n\times r}$ and $K_s \in \mathbb{R}^{m\times m}$ are respectively the distribution matrices of actuator and sensor faults. It is usually admitted that faults are constant additive terms.

With a FDI approach, and following Chen and Patton [1999], it is possible to write (1,2) as

$$\dot{x}(t) = Ax(t) + Bu(t) + \sum_{i=1}^{r} K_{a_i} f_{a_i} \quad (3)$$

$$y(t) = Cx(t) + \sum_{i=1}^{m} K_{s_i} f_{s_i} \quad (4)$$

$$= [\, I_m \;\; 0 \,]\, x(t) + \sum_{i=1}^{m} K_s f_s, \quad (5)$$

which means that the distribution matrix K_a can be split in r distribution vectors and that the distribution matrix K_s can be split in m distribution vectors. So, f_{a_i} fits the i-th actuator fault and K_{a_i} is its distribution vector. That is the same for f_s which fits the i-th sensor fault and K_s the associated distribution vector.

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For every matrix $X \in \mathbb{R}^{m\times n}$, let us denote

$$X = \begin{bmatrix} X_{11} & \cdots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \cdots & X_{mn} \end{bmatrix} = [\, X_1 \; X_2 \; \cdots \; X_n \,],$$

with $X_{ij} \in \mathbb{R}$ and $X_i \in \mathbb{R}^{m\times 1}$, we can write

$$\begin{aligned}\text{vect}(X) &= [\, X_{11} \; \cdots \; X_{m1} \; \cdots \; X_{1n} \; \cdots \; X_{mn} \,]^T \\ &= [\, X_1^T \; X_2^T \; \cdots \; X_n^T \,]^T,\end{aligned}$$

where X^T stands for the transpose.

A generalized inverse of the $(r \times q)$ matrix X is defined as a $(q \times r)$ matrix denoted $X^{\{1\}}$ (Ben-Israel and Greville [1974]) such that

$$XX^{\{1\}}X = X.$$

For every matrix X, a generalized inverse exists and the set of generalized inverses of the matrix X is given by

$$X^{\{1\}} + Y - X^{\{1\}}XYXX^{\{1\}}, \tag{6}$$

where $X^{\{1\}}$is a particular generalized inverse for X and Y is an arbitrary $(q \times r)$ matrix. Moreover if

$$X = \begin{bmatrix} I_\rho & 0_{\rho,q-\rho} \\ L & 0_{r-\rho,q-\rho} \end{bmatrix},$$

where $\rho = \text{rank } X$ and L is a given $(r - \rho) \times \rho$ matrix, then we can choose

$$X^{\{1\}} = \begin{bmatrix} I_\rho & 0_{\rho,r-\rho} \\ 0_{q-\rho,\rho} & 0_{q-\rho,r-\rho} \end{bmatrix}.$$

Two equivalent conditions ensure the existence of a solution for $M = N\Gamma$

$$\left\{\text{rank } N = \text{rank } \begin{bmatrix} M \\ N \end{bmatrix}\right\} \text{ or } \left\{M(I_q - N^{\{1\}}N) = 0\right\}.$$

When these conditions are fulfilled, a general solution for $M = N\Gamma$ is given by

$$\Gamma = MN^{\{1\}} + Z(I - NN^{\{1\}}),$$

where Z is an arbitrary matrix.

The Kronecker product, defined in Brewer [1978], $C(ms \times nt)$ of two matrices $A(m \times n)$ and $B(s \times t)$ is defined by

$$C = A \otimes B = \begin{bmatrix} A_{11}B & \cdots & A_{1n}B \\ \vdots & & \vdots \\ A_{m1}B & \cdots & A_{mn}B \end{bmatrix}.$$

2. RESIDUAL GENERATION

The aim of this section is to provide a procedure to design a reduced order UIO. The fundamental issue of this design is the generation of a residual which is insensitive to only one actuator fault ($K_{a_i}f_{a_i}$ term in (3)) or insensitive to all actuator faults (K_af_a term in (1)). To simplify the notations, we will consider in the following matrix K_a. However if a residual insensitive to only one fault has to be designed, we have to replace K_a by K_{a_i}.

2.1 Residual and UIO design

Using the basic principles of functional observers design (Franck and Wünnenberg [1989], Chen and Patton [1999]), residual $r(t)$ can be estimated by

$$r(t) = G_1z(t) + G_2y(t), \tag{7}$$

when $r(t) \in \mathbb{R}$ and $z(t) \in \mathbb{R}^q$ are the residual and the observer state vector respectively. The observer state vector $z(t)$ is governed by

$$\dot{z}(t) = Nz(t) + Qu(t) + Ly(t), \tag{8}$$

with $G_1 \in \mathbb{R}^{1\times q}$, $G_2 \in \mathbb{R}^{1\times m}$, $N \in \mathbb{R}^{q\times q}$, $Q \in \mathbb{R}^{q\times r}$, $L \in \mathbb{R}^{q\times m}$ and $T \in \mathbb{R}^{q\times n}$. The estimation error is defined by

$$e(t) = z(t) - Tx(t). \tag{9}$$

Dimensions q and g and matrices namely G_1, G_2, N, Q, L and T have to be determined to obtain $\lim_{t\to\infty} e(t) = 0$ in the fault free case. Thus, the following conditions can be deduced (Tsui [2004])

$$N \text{ is Hurwitz} \tag{10}$$

$$Q = TB \tag{11}$$

$$LC = TA - NT. \tag{12}$$

By considering these conditions, the state estimation error is governed by

$$\dot{e} = Ne - TK_af_a + LK_sf_s.$$

When the system is subjected to fault and by substituting $y(t)$, defined in (2), and $z(t)$, defined in (9), in (7), we get

$$\begin{aligned} r(t) &= G_1(e(t) + Tx(t)) + G_2Cx(t) \\ &= G_1e(t) + (G_1T + G_2C)x(t) + G_2K_sf_s. \end{aligned}$$

Firstly, $r(t)$ must be independent of the state vector, so

$$G_1T + G_2C = 0, \tag{13}$$

which constitutes a new constraint for the design of the dynamic system observer-residual, as constraints (10), (11) and (12).

Furthermore, in steady state and considering f_a constant, $r(t)$ verifies the following condition

$$\begin{aligned} r = \lim_{t\to\infty} r(t) &= G_1N^{-1}TK_af_a \\ &- \left(G_1N^{-1}LK_s - G_2K_s\right)f_s. \end{aligned} \tag{14}$$

In order to make the residual r insensitive to fault f_a, matrix T should be orthogonal to K_a. It follows that

$$TK_a = 0 \Leftrightarrow K_a^TT^T = 0 \Leftrightarrow T^T \in Ker(K_a^T). \tag{15}$$

Moreover, to obtain the residual r, defined in (14), sensitive to sensor faults f_s, the following necessary condition must be satisfied

$$G_2K_s - G_1N^{-1}LK_s \neq 0. \quad (16)$$

Indeed, as seen in (**?**]), if this condition is not fulfilled the residual will be insensitive to sensor faults, and so as $TK_a = 0$, the residual will always be equal to zero.

To treat (7), it is interesting to set $T^T = VX$ where $V \in \mathbb{R}^{n\times n}$ and $X \in \mathbb{R}^{n\times q}$. Matrix V is such that its columns span $Ker(K_a^T)$. So, V verifies

$$\begin{aligned}\text{rank}(V) &= \dim(Ker(K_a^T)\\ &= n - \dim(Im(K_a^T))\\ &= n - \text{rank}(K_a) = q. \quad (17)\end{aligned}$$

K_a and n are known *a priori*, (17) defines the observer order. In fact K_a reflects the effects of actuator faults on the system, so we claim that $K_a = B$. To design V, equation $TK_a = 0$ is solved.

Remark 1. In the case where only one fault is considered ($K_a \equiv K_{a_i}$), observer order q is such that $q = n - 1$.

As said previously

$$T^T = VX, \quad (18)$$

with

$$V = (I_m - (K_a^T)^{\{1\}}K_a^T), \quad (19)$$

and X is an arbitrary matrix such that $X \in \mathbb{R}^{n\times q}$.

By studying T^T, (18) in (12), we get

$$A^TVX - VXN^T = C^TL^T. \quad (20)$$

Equation (20) is a Sylvester one. By using the Kronecker product, this equation can be written as

$$[(I_q \otimes A^TV) - (N \otimes V)]\text{vect}(X) = \text{vect}(C^TL^T). \quad (21)$$

To ensure condition (10), N is chosen diagonal $N = \underset{i=1}{\overset{q}{diag}}\{n_i\}$, where each n_i is nonzero and must have a negative real part. In fact, each n_i represents an eigenvalue of N or a dynamic of the observer. With this consideration on N, (21) can be written as

$$\left(\underset{i=1}{\overset{q}{diag}}\{A^TV\} - \underset{i=1}{\overset{q}{diag}}\{n_iV\}\right)\text{vect}(X) = \text{vect}(C^TL^T). \quad (22)$$

By using the notations established before, $(X)_i$ denotes the $i-th$ column of X. So (22) leads to solve q independent linear equations. They are expressed for $i = 1,\dots,q$, as

$$(A^TV - n_iV)X_i = (C^TL^T)_i. \quad (23)$$

Due to the form of C described in (2) and by substituting it in (23) we obtain

$$(A^TV - n_iV)X_i = \begin{bmatrix}(L^T)_i\\ 0\end{bmatrix}. \quad (24)$$

Matrices L and X are determined by solving these q systems. In order to generate a residual with (7) G_1 and G_2 have to be determined. By multiplying (13) with K_a, we find

$$G_2CK_a = 0 \Longleftrightarrow K_a^TC^TG_2^T = 0 \Longleftrightarrow G_2^T \in Ker(K_a^TC^T).$$

To solve equation $K_a^TC^TG_2^T = 0$ with generalized inverses, we get

$$G_2 = W^TV_2^T, \quad (25)$$

with

$$V_2 = (I_m - (K_a^TC^T)^{\{1\}}K_a^TC^T), \quad (26)$$

where $V_2 \in \mathbb{R}^{m\times m}$ and W is an arbitrary matrix such that $W \in \mathbb{R}^{g\times m}$.

Matrix G_2 is thus determined and (27) allows to write G_1 as a solution of

$$G_1T = -V_2WC. \quad (27)$$

2.2 Procedure

The presented design can be summarized in the following procedure:

Step 1 : With (17)

$$n - \text{rang}(K_a) = q,$$

calculate the order of the observer ($q > 0$). If $q = 0$ go to step 10.

Step 2 : Calculate matrix V with (19)

$$V = (I_m - (K_a^T)^{\{1\}}K_a^T).$$

Step 3 : Choose q observer dynamics denoted n_i for $i = 1,\dots,q$. N is defined by

$$N = \begin{bmatrix} n_1 & 0 & \cdots & 0\\ 0 & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & n_q\end{bmatrix}.$$

Step 4 : Solve q linear systems (24)

$$\text{for } i = 1,\dots,q,\ (A^TV - n_iV)X_i = \begin{bmatrix}(L^T)_i\\ 0\end{bmatrix},$$

to find X and L.

Step 5 : Calculate matrix V_2 with (26)

$$V_2 = (I_m - (K_a^TC^T)^{+}K_a^TC^T).$$

Step 6 : With (25)

$$G_2 = W^TV_2^T,$$

express G_2 according to W.

Step 7 : Solve (27)

$$G_1T = -V_2WC,$$

to determine G_1 and W with constraint (16)

$$W^TV_2^TK_s - G_1N^{-1}LK_s \neq 0,$$

and from W find out G_2.

Step 8 : Verify (16)

$$G_2K_s - G_1N^{-1}LK_s \neq 0.$$

If (16) is not fulfilled then go to step 10.

Step 9 : Compute residual generator $r(t)$. End.

Step 10 : No UIO can be designed with this procedure. End.

3. EXAMPLE

Let us consider the following faulty machine paper system presented in Kailath [1980]. This system is more precisely by (1-2) where

$$A = \begin{bmatrix} -0.2 & 0.1 & 1 \\ -0.05 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \\ 1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$K_a = \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \\ 1 & 0 \end{bmatrix} \text{ and } K_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{28}$$

Both actuators and sensors can be subjected to actuator faults (f_{a_1} and f_{a_2}) and sensor faults (f_{s_1} and f_{s_2}) respectively.

A bank of observers allows to detect and locate f_{a_1} or f_{a_2} and detect f_{s_1} and f_{s_2}. To design the bank of observers we have to:

- design an UIO which generates a residual insensitive to f_{a_1} ;
- design an UIO which generates a residual insensitive to f_{a_2} ;
- design an observer which generates a residual sensitive to all faults.

3.1 Residual insensitive to f_{a_1}

We consider : $K_a = K_{a_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and by applying the described procedure, the residual is designed by:

Step 1 : As rank$[K_a] = 1$, then rank$[T] = q = 2$.

Step 2 : Matrix V is determined with (18)

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Step 3 : As $q = 2$, we have to choose 2 arbitrary dynamics for the observer, namely in -1 and -2. N can be written as

$$N = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix},$$

Step 4 : As $q = 2$, matrix X is defined by

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \\ X_{31} & X_{32} \end{bmatrix},$$

So, (24) can be written as follows

$$\begin{cases} (A^T V - n_1 V)X_1 = (C^T L^T)_1 \\ (A^T V - n_2 V)X_2 = (C^T L^T)_2 \end{cases},\ \text{and becomes:}$$

$$\begin{cases} (A^T V - n_1 V)\begin{bmatrix} X_{11} \\ X_{21} \\ X_{31} \end{bmatrix} = (C^T L^T)_1 \\ (A^T V - n_2 V)\begin{bmatrix} X_{12} \\ X_{22} \\ X_{32} \end{bmatrix} = (C^T L^T)_2 \end{cases}.$$

Each sub-system is defined by

$$\left(\begin{bmatrix} 0.8 & -0.05 & 0 \\ 0.1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right)\begin{bmatrix} X_{11} \\ X_{21} \\ X_{31} \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_{12} \\ 0 \end{bmatrix} \tag{29}$$

and

$$\left(\begin{bmatrix} 1.8 & -0.05 & 0 \\ 0.1 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right)\begin{bmatrix} X_{12} \\ X_{22} \\ X_{32} \end{bmatrix} = \begin{bmatrix} L_{21} \\ L_{22} \\ 0 \end{bmatrix} \tag{30}$$

By solving (29) and (30) we get

$$\begin{cases} X_{11} = X_{12} = 0 \\ L_{11} = -0.05X_{21} \\ L_{12} = X_{21} \\ L_{21} = -0.05X_{22} \\ L_{22} = 2X_{22} \end{cases}.$$

Therefore, by choosing $X_{21} = 1$ and $X_{22} = 2$, we obtain

$$\begin{cases} L_{11} = -0.05 \\ L_{12} = 1 \\ L_{21} = -0.05 \\ L_{22} = 2 \end{cases}.$$

Finally N, X, and L are written as

$$N = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$
$$X = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ X_{31} & X_{32} \end{bmatrix}$$
$$L = \begin{bmatrix} -0.05 & 1 \\ -1 & 4 \end{bmatrix}.$$

Step 5 : With (26) V_2 is defined by

$$V_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Step 6 : By setting

$$W = \begin{bmatrix} W_{11} \\ W_{21} \end{bmatrix}.$$

So, G_2 is

$$G_2 = [\, W_{11}\ W_{21} \,].$$

Step 7 : From (27) and keeping in mind that $T = (VX)^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$, we obtain

$$[\, G_{1_{11}}\ G_{1_{12}} \,]\begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$
$$= -[\, W_{11}\ W_{21} \,]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned}&[\,0\ G_{1_{11}}+2G_{1_{12}}\ \ 0\,]\\&=-[\,W_{11}\ W_{21}\ 0\,].\end{aligned}$$

which directly gives

$$G_2=[\,0\ G_{1_{11}}+2G_{1_{12}}\,].$$

Equation (16) becomes

$$-0.05G_{1_{11}}-0.05G_{1_{12}}\neq 0.$$

By setting $[\,G_{1_{11}}\ G_{1_{12}}\,]=[\,1\ 2\,]$, the previous condition is fulfilled.

Step 8 : Equation (16) is fulfilled due to

$$G_2K_s-G_1N^{-1}LK_s=-0.15\neq 0.$$

Step 9 : Residual is then defined by

$$r=\lim_{t\to\infty} r(t)=\lim_{t\to\infty}([\,1\ 2\,]\,z(t)+[\,0\ 5\,]\,y(t)),$$

and is insensitive to f_{a_1}.

3.2 Residual insensitive to f_{a_2}

We now consider that $K_a=K_{a_2}=\begin{bmatrix}1\\0.7\\0\end{bmatrix}$. To design this residual, we proceed in the same way we presented before. We choose 2 observer dynamics in -1 and -1.1, and by choosing $G_1=[\,1\ 1\,]$, we get the following observer:

$$\begin{aligned}N&=\begin{bmatrix}-1&0\\0&-1.1\end{bmatrix}\\Q&=\begin{bmatrix}1&0\\-1&0\end{bmatrix}\\L&=\begin{bmatrix}0&0\\0.0971&-0.1471\end{bmatrix}\\T&=\begin{bmatrix}0&0&1\\0.1&-0.1429&-1\end{bmatrix}\\G_1&=[\,1\ 1\,]\\G_2&=[\,-0.1\ 0.1429\,].\end{aligned}$$

In order to complete the FDI problem, a residual sensitive to all faults (f_{a_1}, f_{a_2}, f_{s_1} and f_{s_2}) has to be designed. This residual could be generated thanks to a Luenberger observer.

3.3 Residual sensitive to all faults

To generate this residual, we design the following Luenberger observer (Luenberger [1971])

$$\begin{cases}\dot{\hat{x}}(t)=(A-KC)\,\hat{x}(t)+Bu(t)+Ky(t)\\ \hat{y}(t)=C\hat{x}(t)\end{cases}$$

The state estimation error is then

$$\begin{aligned}\tilde{x}(t)&=x(t)-\hat{x}(t)\\\dot{\tilde{x}}(t)&=\dot{x}(t)-\dot{\hat{x}}(t)\\&=(A-KC)\,\hat{x}(t)+K_af_a(t)-KK_sf_s(t).\end{aligned}$$

In a fault free case, in order that the state estimation error will approach zero asymptotically, matrix $A-KC$ must be stable. K will be determined as a consequence.In order to arbitrarily choose eigenvalues of $A-KC$, pair (A,C) has to be observable (Borne et al. [2000]). As this condition is fulfilled in this example, we choose to set the eigenvalues, which are the observer dynamics in -1, -2 and -3. Eventually K is such that

$$K=\begin{bmatrix}0.1&0.38\\0.95&2\\1&1\end{bmatrix}.$$

The residual generated with this observer is the output estimation error defined by

$$\begin{aligned}\tilde{y}(t)&=y(t)-\hat{y}(t)\\&=Cx(t)-C\hat{x}(t)+K_sf_s(t)\\&=C\tilde{x}(t)+K_sf_s(t).\end{aligned}$$

3.4 Simulation results

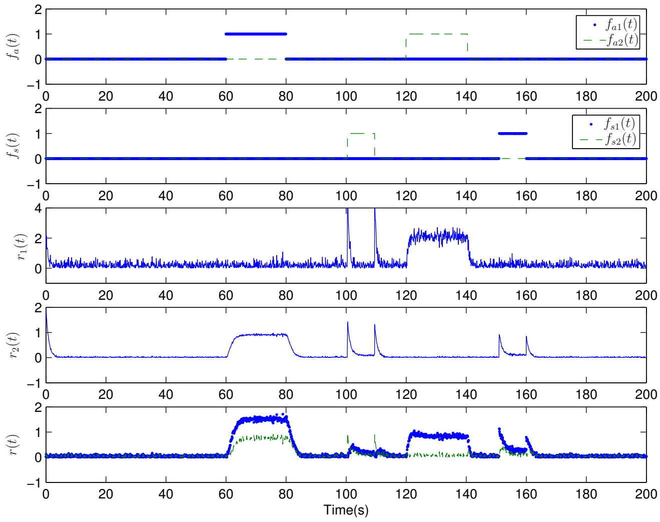

Fig. 1. Residuals $r_1(t)$, $r_2(t)$ and $r(t)$

The simulation, which results are given in fig. 1, is defined by:

- system (1), where constitutive matrices are given in (28) ;
- 2 actuators faults f_{a_1} and f_{a_2}. f_{a_1} appears when $t\in[60;80]$ and f_{a_2} appears when $t\in[120;140]$;
- 2 sensor faults f_{s_1} and f_{s_2}. f_{s_1} appears when $t\in[100;110]$ and f_{s_2} appears when $t\in[150;160]$;
- residual generator, called $r_1(t)$, insensitive to f_{a_1} ;
- residual generator, called $r_2(t)$, insensitive to f_{a_2} ;
- residual generator, called $r(t)$, sensitive to all faults (f_{a_1}, f_{a_2}, $f_{s_1}$and f_{s_2}).

Thus, as shown in fig. 1, faults f_{a_1} and f_{a_2} can be detected and located.

In fact, an analysis of the residual $r(t)$ allows to detect the time when a fault (f_{a_1}, f_{a_2}, f_{s_1}or f_{s_2}) occurs in the system.

Therefore, after this detection, it becomes necessary to analyze:

- $r_1(t)$ signal: if $r_1(t) \to 0$, then a fault occurs on actuator 1 (f_{a_1}), else the fault is either f_{a_2} or f_{s_1}or f_{s_2};
- $r_2(t)$ signal: if $r_2(t) \to 0$, then a fault occurs on actuator 2 (f_{a_2}), else the fault is either f_{a_1} or f_{s_1}or f_{s_2};
- if $r_1(t) \neq 0$ and $r_2(t) \neq 0$, then a sensor fault occurs: f_{s_1} or f_{s_2}.

In order to be complete in the FDI problem, sensor fault can be isolated as seen in Park et al. [1994].

4. CONCLUSION

From a FDI perspective, a new residual design procedure based on the design reduced-order unknown input observer is presented in this work. The starting point of the proposed procedure is the study of the kernel of the fault distribution vector. This property allows us to determine at first the observer order. Then, thanks to Kronecker product, we can propose a design procedure to generate a residual insensitive to one or several faults. In order to complete the FDI problem, we have briefly reminded the method to generate a residual sensitive to all faults. Thus, thanks to a basic detection logic, we can distinguish the actuator faults among them and categorize actuator faults and sensor faults.

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