

# Adaptive Control of Multi-fingered Robot Hand Using Quaternion

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**Abstract:** This paper presents an adaptive control for a multi-fingered robot hand with rolling contact to a grasped rigid object. In the proposed controller, the dynamic parameters of both the object and multi-fingered robot hand are estimated adaptively. The orientation error of the object is described around the relative rotational axis using quaternion. The asymptotic convergence of object motion and contact force was proven by the Lyapunov-like Lemma. An experiment of object grasping by the human-type robot hand using three fingers is shown.

Keywords: Adaptive Control; Quaternion; Multi-fingered Robot Hand.

## 1. INTRODUCTION

Many researchers have studied the control of grasp and manipulation of an object by a multi-fingered robot hand (Yoshikawa (2000); Bicchi (2000)). In general, most orientation control schemes of object manipulation are based on the Euler angles feedback concept. Modern orientation control schemes of satellites and so on tend to use quaternion feedback instead of Euler angles feedback. The unit quaternion is used in a singularity-free representation of orientation. The quaternion control enables the orientation change along the shortest path by matching the object moment to the eigen-axis, which is not possible with Euler angle control because Euler angles are based on the concept of sequential rotations. The research on object manipulation tends to use quaternion, too. For instance, a passivity-based object-level impedance control using quaternion for a multi-fingered hand has been presented (Wimboeck et al. (2006)). The researchers have implemented it on the DLR Hand II, and the experimental results shown confirmed its performance. We also proposed a controller using relative rotational axis, which is possible to express by quaternion (Ueki et al. (2006)).

On the Euler angle feedback concept of the trajectory tracking problem, for the dynamical model of the robot hand and the object, the tracking control of the object motion and the internal force has been considered (Cole et al. (1989); Sarkar et al. (1997)). However, most of the object manipulation controls were studied under the condition that the dynamic parameters of the object and the robot fingers are known. It is well known that accurately identifying the dynamic parameters of an object and the robot fingers is very difficult. Moreover, the dynamic parameters of the object often vary according to the task, which is variable. In order to solve this problem, we have proposed adaptive control of a multi-fingered robot hand with rolling contact (Ueki et al. (2005)).

In this paper, adaptive control of a multi-fingered robot hand with rolling contact using quaternion is proposed which extends our previous work. In the controller, the



Fig. 1. Grasped object and multi-fingered hand coordinate system

dynamic parameters of both the object and the robot hand are estimated. The asymptotic convergence of object motion and contact force is proven by the Lyapunov-like Lemma. The results of the experiment on object grasping by a human-type robot hand using three fingers are shown.

## 2. TARGET SYSTEM

Consider the robot hands with  $k(\geq 3)$  fingers with 3 DOF(degree of freedom) manipulating a rigid object in three-dimensional space, as shown in Fig. 1, in which the *i*-th robot finger contacts the object at point  $C_i$ . The coordinate systems are defined as follows:  $\Sigma_p$  is the task coordinate system,  $\Sigma_o$  is the object coordinate system fixed on the object, and  $\Sigma_i$  is the *i*-th fingertip coordinate system fixed on the *i*-th fingertip. We also use notations defined as follows:  $\mathbf{p}_o \in R^3$  is a position vector of the origin of the object coordinate system  $\Sigma_o$  with respect to  $\Sigma_p$ ,  $\mathbf{q}_o \in R^4$  is the unit quaternion for orientation of the origin of the object coordinate system  $\Sigma_o$  with respect to  $\Sigma_p$ ,  $\boldsymbol{\omega}_o \in R^3$  is the angular velocity vector of the origin of the

object coordinate system  $\Sigma_o$  with respect to  $\Sigma_p$ ,  $q \in R^{3k}$ is the joint angle vector of robot fingers,  ${}^o p_{oc_i} \in R^3$  is the position vector from  $\Sigma_o$  to the contact point  $C_i$  with respect to  $\Sigma_o$ , and  ${}^i p_{fc_i} \in R^3$  is the position vector from  $\Sigma_i$  to the contact point  $C_i$  with respect to  $\Sigma_i$ .

To facilitate the dynamic formulation, the following assumptions are made.

- (A1) All the finger-tips contact the common object at one point with frictional point contact, and the frictional force at each contact point follow Coulomb's law.
- (A2) The object and finger-tip surfaces are described by twice continuously differentiable hypersurfaces.
- (A3) The constraint at each contact point is described by the rolling contact. The force generated by the constraint do not work on the system (*d'Alembert's principle*).

## 2.1 Rolling constraint

When the *i*-th finger manipulates the object at a condition of rolling contact without slip, the following relation on the rolling velocities of contact positions on both surfaces with respect to  $\Sigma_p$ , is obtained:

$$\boldsymbol{R}_{o}(\boldsymbol{q}_{o}) \,^{o} \dot{\boldsymbol{p}}_{oc_{i}} = \boldsymbol{R}_{i}(\boldsymbol{q}_{i}) \,^{i} \dot{\boldsymbol{p}}_{fc_{i}} \tag{1}$$

where,  $\mathbf{R}_o$  is a rotation matrix from  $\Sigma_p$  to  $\Sigma_o$ ,  $\mathbf{R}_i$  is a rotation matrix from  $\Sigma_p$  to  $\Sigma_i$ , and  $\mathbf{q}_i \in \mathbb{R}^3$  is a joint angle vector of the *i*-th robot finger. Therefore, the relation between the object velocity and angular velocity of the finger joint is given by (Murray et al. (1994))

$$\boldsymbol{W}^{T}(\boldsymbol{R}_{o},^{o}\boldsymbol{p}_{oc_{1}},\cdots,^{o}\boldsymbol{p}_{oc_{k}})\boldsymbol{v}_{o} = \boldsymbol{J}_{c}(\boldsymbol{q},^{1}\boldsymbol{p}_{fc_{1}},\cdots,^{k}\boldsymbol{p}_{fc_{k}})\dot{\boldsymbol{q}}$$
(2)

where  $\boldsymbol{v}_o = \left[ \dot{\boldsymbol{p}}_o^T \ \boldsymbol{\omega}_o^T \right]^T \in R^6$  is the velocity vector of the object with respect to the task coordinate system  $\Sigma_p$ ,  $\boldsymbol{W} \in R^{6 \times 3k}$  is a grasp form matrix, and  $\boldsymbol{J}_c \in R^{3k \times 3k}$  is a Jacobian matrix at the contact point.

#### 2.2 Dynamic equation

By assumption (A3) and rolling constraint (2), the dynamic equation of the object and the robot fingers are given by (Murray et al. (1994)):

$$\boldsymbol{M}_{o}(\boldsymbol{R}_{o})\dot{\boldsymbol{v}}_{o} + \boldsymbol{C}_{o}(\boldsymbol{R}_{o},\boldsymbol{\omega}_{o})\boldsymbol{v}_{o} + \boldsymbol{g}_{o}(\boldsymbol{R}_{o}) = \boldsymbol{W}\boldsymbol{f}_{c} \qquad (3)$$

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \boldsymbol{J}_{c}^{T}\boldsymbol{f}_{c}$$
 (4)

where  $M_o \in R^{6\times 6}$  is the inertia matrix of the object,  $C_o v_o \in R^6$  is the Coriolis and centrifugal vector of the object,  $g_o \in R^6$  is the gravity force term of the object,  $M \in R^{3k\times 3k}$  is the inertia matrix of the robot fingers,  $C\dot{q} \in R^{3k}$  is the Coriolis and centrifugal vector of the robot fingers,  $g \in R^{3k}$  is the gravity force term of the robot fingers, and  $\tau \in R^{3k}$  is the input joint torque.

The dynamic equations (3) and (4) are characterized by the following structural properties, which are utilized in our controller design.

(P1)  $M_o$  and M are symmetric positive definite matrix.

- (P2) Suitable definition of  $C_o$  and C makes matrix  $M_o 2C_o$  and  $\dot{M} 2C$  skew-symmetric.
- (P3) The dynamic equations are linear with respect to the dynamic parameter vector, as follows:

$$M_{o}\dot{\boldsymbol{v}}_{or} + \boldsymbol{C}_{o}\boldsymbol{v}_{or} + \boldsymbol{g}_{o} = \boldsymbol{Y}_{o}(\boldsymbol{R}_{o},\boldsymbol{\omega}_{o},\boldsymbol{v}_{or},\dot{\boldsymbol{v}}_{or})\boldsymbol{\sigma}_{o} \quad (5)$$
$$M\ddot{\boldsymbol{q}}_{r} + \boldsymbol{C}\dot{\boldsymbol{q}}_{r} + \boldsymbol{g} = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_{r},\ddot{\boldsymbol{q}}_{r})\boldsymbol{\sigma} \quad (6)$$

where  $\boldsymbol{\sigma}_o \in R^{\alpha_o}$  is a dynamic parameter vector of the object,  $\boldsymbol{Y}_o \in R^{6 \times \alpha_o}$  is a regressor with respect to the dynamic parameters  $\boldsymbol{\sigma}_o$ ,  $\alpha_o$  is the number of the object dynamics parameters,  $\boldsymbol{\sigma} \in R^{\alpha}$  is a dynamic parameter vector of the robot finger,  $\boldsymbol{Y} \in R^{3k \times \alpha}$  is a regressor with respect to the dynamic parameters  $\boldsymbol{\sigma}$ , and  $\alpha$  is the number of the robot finger dynamic parameters.

#### 2.3 Quaternion

In this subsection, we describe a quaternion. A quaternion  $\boldsymbol{q}_a,$  composed of a scalar part and vector part, is defined as

$$\boldsymbol{q}_{a} = \begin{bmatrix} \boldsymbol{q}_{a.v}^{T} & \boldsymbol{q}_{a.s} \end{bmatrix}^{T}$$
(7)

where  $q_{a.s}$  is a scalar part of  $q_a$ , and  $q_{a.v}$  is a vector part of  $q_a$ .

Quaternion multiplication, designated by  $\otimes$ , is defined as

$$\boldsymbol{q}_{a} \otimes \boldsymbol{q}_{b} = \begin{bmatrix} \boldsymbol{q}_{a.v} \times \boldsymbol{q}_{b.v} + q_{a.s} \boldsymbol{q}_{b.v} + q_{b.s} \boldsymbol{q}_{a.v} \\ -\boldsymbol{q}_{a.v}^{T} \boldsymbol{q}_{b.v} + q_{a.s} q_{b.s} \end{bmatrix}$$
(8)

Quaternion inverse in case of unit quaternion, designated by  $^{-1}$ , is defined as

$$\boldsymbol{q}_{a}^{-1} = \begin{bmatrix} -\boldsymbol{q}_{a.v}^{T} & q_{a.s} \end{bmatrix}^{T}$$
(9)

#### 3. THE PROPOSED CONTROLLER

The control objective is to provide a set of input joint torques such that the object trajectory and the contact force converge to desired values asymptotically for the case in which the dynamic parameters of the object and the robot fingers are unknown. To propose control law, the following assumptions are made:

- (A4)  $\boldsymbol{p}_o, \boldsymbol{q}_o, \boldsymbol{v}_o, \boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \boldsymbol{f}_{c_i}, {}^i\boldsymbol{p}_{fc_i}, \text{ and } {}^i\dot{\boldsymbol{p}}_{fc_i}$  are measurable,
- (A5) Desired trajectory  $p_{od}$ ,  $q_{od}$ ,  $v_{od}$ , and  $\dot{v}_{od}$  of the object are bounded and uniformly continuous at time.
- (A6) The properties of force-closure and manipulable are satisfied, and there exists an internal force.

#### 3.1 Desired contact force

Let us define a position and orientation error of the object by

$$\boldsymbol{e}_{o} = \begin{bmatrix} \Delta \boldsymbol{p}_{o} \\ \Delta \boldsymbol{\theta}_{o} \end{bmatrix}$$
(10)

where  $\Delta p_o = p_{od} - p_o$  and  $\Delta \theta_o$  is selected from the following definitions:

$$\Delta \boldsymbol{\theta}_{o} = 2\boldsymbol{q}_{o}^{T}\boldsymbol{q}_{od}(q_{o.s}\boldsymbol{q}_{od.v} - q_{od.s}\boldsymbol{q}_{o.v} + \boldsymbol{q}_{o.v} \times \boldsymbol{q}_{od.v})$$
$$= 2\boldsymbol{R}_{o}q_{oe.s}\boldsymbol{q}_{oe.v} \tag{11}$$

or

$$\Delta \boldsymbol{\theta}_{o} = 2(q_{o.s}\boldsymbol{q}_{od.v} - q_{od.s}\boldsymbol{q}_{o.v} - \boldsymbol{q}_{o.v} \times \boldsymbol{q}_{od.v})$$
$$= 2\boldsymbol{R}_{o}\boldsymbol{q}_{oe.v}$$
(12)

 $q_{oe.s}$  and  $\boldsymbol{q}_{oe.v}$  are the scalar part and vector part of the quaternion as follows:

$$\boldsymbol{q}_{oe} = \boldsymbol{q}_o^{-1} \otimes \boldsymbol{q}_{od} \tag{13}$$

Equations (11) and (12) differ in magnitude. Both schemes exhibit the same behaviour for small orientation errors.

Using the above definition of the object error, let us define a reference velocity of the object by

$$\boldsymbol{v}_{or} = \boldsymbol{v}_{od} + \boldsymbol{\Lambda} \boldsymbol{e}_o \tag{14}$$

where  $\mathbf{\Lambda} = block \ diag \left[\rho_p \mathbf{I}_3 \ \rho_o \mathbf{I}_3\right] \in \mathbb{R}^{6 \times 6}, \ \rho_p > 0$  and  $\rho_o > 0$  are scalar constants, and  $\mathbf{I}_n \in \mathbb{R}^{n \times n}$  is an identity matrix. Then the following desired external force  $\mathbf{F}_{od} \in \mathbb{R}^6$  is generated by:

$$\boldsymbol{F}_{od} = \boldsymbol{Y}_{o}(\boldsymbol{R}_{o}, \boldsymbol{\omega}_{o}, \boldsymbol{v}_{or}, \dot{\boldsymbol{v}}_{or}) \hat{\boldsymbol{\sigma}}_{o} - \begin{bmatrix} \boldsymbol{K}_{op} & \boldsymbol{0} \\ \boldsymbol{0} & \gamma \boldsymbol{I}_{3} \end{bmatrix} \boldsymbol{s}_{o} \quad (15)$$

where  $\hat{\boldsymbol{\sigma}}_{o}$  is a parameter estimate of  $\boldsymbol{\sigma}_{o}$ ,  $\boldsymbol{K}_{op} > 0 \in R^{3\times 3}$ is a symmetric feedback gain matrix of the translational element,  $\gamma > 0$  is a feedback gain scalar of the rotational element, and  $\boldsymbol{s}_{o} \in R^{6}$  is a residual error given by

$$\boldsymbol{s}_o = \boldsymbol{v}_o - \boldsymbol{v}_{or} \tag{16}$$

An adaptive law of the dynamic parameter of the object is given by

$$\dot{\hat{\boldsymbol{\sigma}}}_o = -\boldsymbol{\Gamma}_o \boldsymbol{Y}_o^T(\boldsymbol{R}_o, \boldsymbol{\omega}_o, \boldsymbol{v}_{or}, \dot{\boldsymbol{v}}_{or}) \boldsymbol{s}_o$$
(17)

where  $\Gamma_o > 0 \in R^{\alpha_o \times \alpha_o}$  is a symmetric adaptive gain matrix.

A desired contact force  $\mathbf{f}_{cd} \in \mathbb{R}^{3k}$  is generated using  $\mathbf{F}_{od}$ . The desired contact force should satisfy

$$\boldsymbol{F}_{od} = \boldsymbol{W} \boldsymbol{f}_{cd} \tag{18}$$

Moreover, the force at contact points generate the internal force in the object. Hence, the general solution of the desired contact force is given by

$$\boldsymbol{f}_{cd} = \boldsymbol{W}^{+} \boldsymbol{F}_{od} + (\boldsymbol{I}_{3k} - \boldsymbol{W}^{+} \boldsymbol{W}) \boldsymbol{f}_{intd}$$
(19)

where  $f_{intd}$  is a bounded vector to generate the desired internal force, and  $W^+$  is a pseudo inverse of W given by

$$\boldsymbol{W}^{+} = \boldsymbol{W}^{T} (\boldsymbol{W} \boldsymbol{W}^{T})^{-1}.$$
 (20)

The second term of (19) is a homogeneous solution representing the internal force that gives no effect on external force. This term represents the arbitrariness of the contact force.

## 3.2 Input joint torque

Like the reference (Yuan (1997)), without using a raw contact force variable, let us handle  $\nu$  having the following relation:

$$\dot{\boldsymbol{\nu}} + \kappa \boldsymbol{\nu} = \kappa \Delta \boldsymbol{f}_c \tag{21}$$

where  $\kappa > 0$  is a design constant, and  $\Delta f_c = f_{cd} - f_c$ . Let us define a reference velocity of the robot finger by

$$\dot{\boldsymbol{q}}_r = \boldsymbol{J}^{-1} (\boldsymbol{W}^T \boldsymbol{v}_{or} + \boldsymbol{\Omega} \boldsymbol{\nu} + \boldsymbol{\Psi} \boldsymbol{\eta})$$
(22)

where  $\mathbf{\Omega} > 0$  is a symmetric gain matrix,  $\mathbf{\Psi} > 0$  is a symmetric gain matrix, and  $\boldsymbol{\eta}$  is given by  $\boldsymbol{\eta} = \int_0^t \Delta \boldsymbol{f}_c dt$ . Then a control law of the robot finger is given by

 $\boldsymbol{\tau} = \boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r) \hat{\boldsymbol{\sigma}} - \boldsymbol{K}\boldsymbol{s} + \boldsymbol{J}^T \boldsymbol{f}_{cd} + \beta \boldsymbol{J}^T \Delta \boldsymbol{f}_c$  (23) where  $\hat{\boldsymbol{\sigma}}$  is an estimate of  $\boldsymbol{\sigma}, \, \boldsymbol{K} > 0 \in R^{3k \times 3k}$  is a symmetric feedback gain matrix,  $\beta > 0$  is a force feedback gain scalar constant, and  $\boldsymbol{s}(= \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_r)$  is a residual error between the reference velocity and the actual velocity, which is rewritten by

$$\boldsymbol{s} = \boldsymbol{J}^{-1} (\boldsymbol{W}^T \boldsymbol{s}_o - \boldsymbol{\Omega} \boldsymbol{\nu} - \boldsymbol{\Psi} \boldsymbol{\eta})$$
(24)

An adaptive law of the parameter estimate of the robot finger is given by

$$\dot{\hat{\boldsymbol{\sigma}}} = -\boldsymbol{\Gamma} \boldsymbol{Y}^T (\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r) \boldsymbol{s}$$
(25)

where  $\Gamma > 0 \in R^{\alpha \times \alpha}$  is a symmetric adaptive gain matrix.

## 3.3 Stability analysis

It is easy to show the following equations:

$$\boldsymbol{Y}_{o}\Delta\boldsymbol{\sigma}_{o} - \Delta\boldsymbol{F}_{o} - \boldsymbol{K}_{o}\boldsymbol{s}_{o} = \boldsymbol{M}_{o}\dot{\boldsymbol{s}}_{o} + \boldsymbol{C}_{o}\boldsymbol{s}_{o} \qquad (26)$$

$$\boldsymbol{Y}\Delta\boldsymbol{\sigma} - \boldsymbol{K}\boldsymbol{s} + (\beta + 1)\boldsymbol{J}_{c}^{T}\Delta\boldsymbol{f}_{c} = \boldsymbol{M}\dot{\boldsymbol{s}} + \boldsymbol{C}\boldsymbol{s}.$$
 (27)

where  $\mathbf{K}_o = block \ diag [\mathbf{K}_{op} \ \gamma \mathbf{I}_3], \ \Delta \boldsymbol{\sigma}_o = \hat{\boldsymbol{\sigma}}_o - \boldsymbol{\sigma}_o$  is an estimate error vector of the object dynamic parameters,  $\Delta \mathbf{F}_o = \mathbf{F}_{od} - \mathbf{W} \mathbf{f}_c$  is an error vector of the object external force, and  $\Delta \boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}$  is an estimate error vector of the robot finger dynamic parameters. In case of selecting definition (11), consider as a candidate for a Lyapunov function the following equation

$$V = \frac{1}{2} \{ (\beta + 1) (\boldsymbol{s}_{o}^{T} \boldsymbol{M}_{o} \boldsymbol{s}_{o} + \Delta \boldsymbol{\sigma}_{o}^{T} \boldsymbol{\Gamma}_{o}^{-1} \Delta \boldsymbol{\sigma}_{o} + 2\rho_{p} \Delta \boldsymbol{p}_{o}^{T} \boldsymbol{K}_{op} \Delta \boldsymbol{p}_{o} + 8\rho_{o} \gamma \boldsymbol{q}_{oe.v}^{T} \boldsymbol{q}_{oe.v} + \boldsymbol{\eta}^{T} \boldsymbol{\psi} \boldsymbol{\eta} + \frac{1}{\kappa} \boldsymbol{\nu}^{T} \boldsymbol{\Omega} \boldsymbol{\nu}) + \boldsymbol{s}^{T} \boldsymbol{M} \boldsymbol{s} + \Delta \boldsymbol{\sigma}^{T} \boldsymbol{\Gamma}^{-1} \Delta \boldsymbol{\sigma} \}$$
(28)

where we referred to reference (Bullo et al. (1999)) for  $q_{oe.v}^T q_{oe.v}$ . Also, in case of selecting definition (12), it adds  $4\rho_o\gamma(\beta+1)(q_{oe.s}-1)^2$  to the above equation (28). A time derivative along the solution of the error equation gives as the following equation using  $\dot{M}_o - 2C_o$  and  $\dot{M} - 2C$  are skew-symmetric,  $s_o = v_o - v_{or}$ ,  $s = \dot{q} - \dot{q}_r$  (17), (25), (26), (27), (24), (21),  $\dot{\eta} = \Delta f_c$ ,  $\Delta F_o = W_o \Delta f_c$ , and  $s_o = v_o - v_{od} - \Lambda e_o$ 

$$\dot{V} = -(\beta + 1)\Delta \boldsymbol{v}_{o}^{T}\boldsymbol{K}_{o}\Delta \boldsymbol{v}_{o} - (\beta + 1)(\boldsymbol{\Lambda}\boldsymbol{e}_{o})^{T}\boldsymbol{K}_{o}\boldsymbol{\Lambda}\boldsymbol{e}_{o} -\boldsymbol{s}^{T}\boldsymbol{K}\boldsymbol{s} - (\beta + 1)\boldsymbol{\nu}^{T}\boldsymbol{\Omega}\boldsymbol{\nu} \leq 0$$
(29)

This shows that V is the Lyapunov function; hence,  $s, s_o$ ,  $\Delta p_o, q_{oe.v}, \Delta \sigma, \Delta \sigma_o, \eta$ , and  $\nu$  are bounded. Because  $\sigma$ 

and  $\boldsymbol{\sigma}_{o}$  are constant,  $\hat{\boldsymbol{\sigma}}$  and  $\hat{\boldsymbol{\sigma}}_{o}$  are bounded.  $\boldsymbol{p}_{o}$  is the bounded form  $\Delta \boldsymbol{p}_{o} = \boldsymbol{p}_{od} - \boldsymbol{p}_{o}$  and the boundness of  $\boldsymbol{p}_{od}$ .  $q_{oe.s}$  and  $\boldsymbol{q}_{oe}$  are bounded from  $\boldsymbol{q}_{oe}^{T}\boldsymbol{q}_{oe} = 1$  and boundness of  $\boldsymbol{q}_{oe.v}$ .  $\boldsymbol{q}_{o}$  is bounded from  $\boldsymbol{q}_{o} = \boldsymbol{q}_{od} \otimes \boldsymbol{q}_{oe}^{-1}$ . Therefore,  $\Delta \boldsymbol{\theta}_{o}$  is bounded.  $\boldsymbol{e}_{o}$  is bounded from the boundness of  $\Delta \boldsymbol{p}_{o}$  and  $\Delta \boldsymbol{\theta}_{o}$ .  $\boldsymbol{v}_{or}$  is bounded from the boundness of  $\boldsymbol{v}_{od}$  and  $\boldsymbol{e}_{o}$ .  $\boldsymbol{v}_{or}$  is bounded from the boundness of  $\boldsymbol{s}_{o}$ . is bounded from the boundness of  $\boldsymbol{s}_{o}$ .  $\dot{\boldsymbol{e}}_{o}$  is bounded from the boundness of  $\boldsymbol{s}_{o}$ .  $\dot{\boldsymbol{e}}_{o}$  is bounded from the boundness of  $\boldsymbol{q}_{oe}$ ,  $\Delta \boldsymbol{v}_{o}$ ,  $\boldsymbol{\theta}$ , and  $\boldsymbol{\omega}_{o}$ . Therefore,  $\dot{\boldsymbol{v}}_{or}$  is bounded. These results yield the bounding of  $\boldsymbol{Y}_{o}(\boldsymbol{r}_{o}, \dot{\boldsymbol{r}}_{o}, \boldsymbol{v}_{or}, \dot{\boldsymbol{v}}_{or})$  and  $\boldsymbol{F}_{od}$ .  $\boldsymbol{f}_{cd}$  is bounded from the boundness of  $\boldsymbol{F}_{od}$  and the second term of (19).  $\dot{\boldsymbol{q}}_{r}$  is bounded from (22).  $\dot{\boldsymbol{q}}$  is bounded from  $\boldsymbol{s} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{r}$ .  ${}^{o}\dot{\boldsymbol{p}}_{oci}$  and  ${}^{i}\dot{\boldsymbol{p}}_{fci}$  are bounded from assumption (A2) and the boundness of  $\boldsymbol{v}_{o}$  and  $\dot{\boldsymbol{q}}$  (the details are omitted). Therefore,  $\dot{\boldsymbol{W}}_{o}$  and  $\dot{\boldsymbol{J}}_{c}$  are bounded.

Next, the boundness of  $\Delta f_c$  is shown. We can derive the following equation by multiplying  $JM^{-1}$  to Eq. (27) from the left side and using differentiating (24), (26), differentiating (22), and (21),

$$\boldsymbol{A}\Delta\boldsymbol{f}_{c} = \boldsymbol{b} \tag{30}$$

where

$$\boldsymbol{A} = (\boldsymbol{J}_{c}\boldsymbol{M}^{-1}\Delta\boldsymbol{M}\boldsymbol{J}_{c}^{-1} + \boldsymbol{I})(\kappa\boldsymbol{\Omega} + \boldsymbol{\psi}) \\ + (\beta + 1)\boldsymbol{J}_{c}\boldsymbol{M}^{-1}\boldsymbol{J}_{c}^{T} + \boldsymbol{W}_{o}^{T}\boldsymbol{M}_{o}^{-1}\boldsymbol{W}_{o}$$
(31)

$$\boldsymbol{b} = \boldsymbol{W}_{o}^{T} \boldsymbol{M}_{o}^{-1} \{ \boldsymbol{Y}_{o}(\boldsymbol{R}_{o}, \boldsymbol{\omega}_{o}, \boldsymbol{v}_{or}, \dot{\boldsymbol{v}}_{or}) \Delta \boldsymbol{\sigma}_{o} - (\boldsymbol{K}_{o} + \boldsymbol{C}_{o}) \boldsymbol{s}_{o} \} \\ + \boldsymbol{J}_{c} \boldsymbol{M}^{-1} (\boldsymbol{K} \boldsymbol{s} + \boldsymbol{C} \boldsymbol{s} - \boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{r}, \boldsymbol{a}) \Delta \boldsymbol{\sigma}) \\ + (\dot{\boldsymbol{W}}_{o}^{T} \boldsymbol{s}_{o} + \kappa \boldsymbol{\Omega} \boldsymbol{\nu} - \dot{\boldsymbol{J}}_{c} \boldsymbol{s})$$
(32)

and  $\boldsymbol{a} = \boldsymbol{J}_c^{-1} (\boldsymbol{W}_o^T \boldsymbol{\dot{v}}_{or} + \boldsymbol{\dot{W}}_o^T \boldsymbol{v}_{or} - \kappa \boldsymbol{\Omega} \boldsymbol{\nu} - \boldsymbol{\dot{J}}_c \boldsymbol{\dot{q}}_r)$ . To make an analysis of the stability of the proposed controller, the following assumption is made.

(A7) If  $\beta$  is sufficiently large and  $\kappa \boldsymbol{\Omega} + \boldsymbol{\psi}$  is sufficiently small, the matrix A approaches to  $(\beta+1)\boldsymbol{J}_c\boldsymbol{M}^{-1}\boldsymbol{J}_c^T + \boldsymbol{W}_o^T\boldsymbol{M}_o^{-1}\boldsymbol{W}_o$ , which is nonsingular.

Moreover,  $\pmb{b}$  is bounded (the details are omitted). Hence,  $\Delta \pmb{f}_c$  is bounded.

 $\boldsymbol{f}_c$  is bounded from the boundness of  $\Delta \boldsymbol{f}_c$ .  $\Delta \boldsymbol{F}_o$  is bounded from  $\Delta \boldsymbol{F}_o = \boldsymbol{W}_o \Delta \boldsymbol{f}_c$ .  $\dot{\boldsymbol{s}}_o$  is bounded from (26).  $\dot{\boldsymbol{v}}_o$  is bounded by differentiating (16).  $\dot{\boldsymbol{\nu}}$  is bounded from (21).  $\ddot{\boldsymbol{q}}_r$  is bounded by differentiating (22).  $\boldsymbol{\tau}$  is bounded from (23).  $\ddot{\boldsymbol{q}}$  is bounded from (4).  $\dot{\boldsymbol{s}}$  is bounded by differentiating (24).

Differentiating (29) with respect to time give

$$\ddot{V} = -2((\beta+1)\Delta \dot{\boldsymbol{v}}_{o}^{T}\boldsymbol{K}_{o}\Delta\boldsymbol{v}_{o} + (\beta+1)(\boldsymbol{\Lambda}\dot{\boldsymbol{e}}_{o})^{T}\boldsymbol{K}_{o}\boldsymbol{\Lambda}\boldsymbol{e}_{o} +\dot{\boldsymbol{s}}^{T}\boldsymbol{K}\boldsymbol{s} + (\beta+1)\dot{\boldsymbol{\nu}}^{T}\boldsymbol{\Omega}^{T}\boldsymbol{\nu})$$
(33)

 $\ddot{\boldsymbol{v}}$  is bounded because of the boundness of  $\Delta \dot{\boldsymbol{v}}_o$ ,  $\Delta \boldsymbol{v}_o$ ,  $\dot{\boldsymbol{e}}_o$ ,  $\boldsymbol{e}_o$ ,  $\dot{\boldsymbol{s}}$ ,  $\boldsymbol{s}$ ,  $\dot{\boldsymbol{\nu}}$ , and  $\boldsymbol{\nu}$ . This means that  $\dot{\boldsymbol{V}}$  is uniformly continuous. It is shown that  $\dot{\boldsymbol{V}} \to 0$  as  $t \to \infty$  from the Lyapunov-like Lemma (Slotine et al. (1991)). This implies that  $\boldsymbol{e}_o \to \mathbf{0}$ ,  $\boldsymbol{v}_o \to \boldsymbol{v}_{od}$ ,  $\boldsymbol{s} \to \mathbf{0}$ , and  $\boldsymbol{\nu} \to \mathbf{0}$  as  $t \to \infty$ .

Moreover, these results lead to uniformly continuous of  $\boldsymbol{A}$  and  $\boldsymbol{b}$  (the detail is omitted). Therefore,  $\Delta \boldsymbol{f}_c$  is uniformly continuous.  $\dot{\boldsymbol{\nu}}$ ,  $\dot{\boldsymbol{s}}_o$  and  $\dot{\boldsymbol{s}}$  are uniformly continuous by the uniformly continuous of  $\Delta \boldsymbol{f}_c$ .  $\dot{\boldsymbol{\nu}} \to \mathbf{0}$ ,  $\dot{\boldsymbol{s}}_o \to \mathbf{0}$ , and  $\dot{\boldsymbol{s}} \to \mathbf{0}$ 

as  $t \to \infty$  using Barbalat's Lemma.  $\Delta f_c \to 0$  as  $t \to \infty$  from (21).

From the preceding discussion, the following theorem is proven.

Theorem 1. Consider a rigid object grasped by  $k \geq 3$  robot fingers, each robot finger having 3 DOF. For systems (3) and (4) with motion constraint (2) using the control law (23) with the adaptive law (25), in which the desired external force of the object is given by (15), the closed-loop system satisfies

(I) 
$$\boldsymbol{p}_o \to \boldsymbol{p}_{od}, \, \boldsymbol{q}_o \to \boldsymbol{q}_{od}, \, \text{and} \, \boldsymbol{v}_o \to \boldsymbol{v}_{od} \, \text{as} \, t \to \infty,$$
  
(II)  $\boldsymbol{f}_c \to \boldsymbol{f}_{cd} \, \text{as} \, t \to \infty.$ 

## 4. EXPERIMENT

A ball handling experiment by the human-type robot hand named Gifu-Hand III (Mouri et al. (2002)) was performed to show the effectiveness of the proposed control method in the case of selecting definition (11). In the experiment, as shown in Fig. 2, three fingers grasp the ball, which has a radius of 0.0625 (m) and a mass of 0.05 (kg). The joint angle of the robot hand is measured by a rotary encoder, and the contact force is measured by a 6-axis force sensor (NANO sensor, BL AUTOTEC, LTD.). The position and orientation of the object is measured by a 3-D position measurement device (OPTRAK, Northern Digital Co.). The resolution of this device is 0.1 (mm), and the sampling frequency is 500 (Hz). The contact position  ${}^{i}\boldsymbol{p}_{fc_{i}}$  is calculated using contact force by reference (Bicchi et al. (1993)). The joint angle velocity, object velocity, object angular velocity, and contact point velocity are calculated using the measured value by a differential filter. The control sampling is 1000 (Hz). The experiment conditions are as follows: the initial values of the unknown dynamic parameters are set to zero. The desired trajectory of the ball is given repeatedly by a 5 order polynomial in time with the initial  $\boldsymbol{p}_{od}(0) = \boldsymbol{p}_{o}(0), \boldsymbol{q}_{od}(0) = \boldsymbol{q}_{o}(0)$ and terminal  $\boldsymbol{p}_{od}(1.5) = \boldsymbol{p}_{o}(0) + [0.03, 0, 0]^{T}, \boldsymbol{q}_{od}(1.5) = \boldsymbol{q}_{o}(0) \otimes [0, 1, 0, -\pi/9]^{T}$ .  $\boldsymbol{f}_{intd}$  is given as follows

$$m{f}_{intd} = 1.5 \left[ rac{m{p}_{fc_1}^T}{\|m{p}_{fc_1}\|} \; rac{m{p}_{fc_2}^T}{\|m{p}_{fc_2}\|} \; rac{m{p}_{fc_3}^T}{\|m{p}_{fc_3}\|} 
ight]^T$$

The controller gains were selected to be:

The experiment results are shown in Figs. 3, 4, 5, 6, 7, and 8. Fig. 3 shows the desired object position  $p_{od}$  and the actual object position  $p_o$ . Fig. 4 shows the norm of the object position error  $\|\Delta p_o\|$ . These show that the



Fig. 2. Experiment system consisting of human-type robot hand and 3-D position measurement device



Fig. 3. Trajectory of object position



Fig. 4. The norm of object position error  $\|\Delta \boldsymbol{p}_o\|$ 

actual object position tracks the desired object position well, and the object position error decreases by repetition of motion. Fig. 5 shows the desired object orientation  $\boldsymbol{q}_{od}$  and actual object orientation  $\boldsymbol{q}_o$ . Fig. 6 shows the norm of the object orientation error  $\|\boldsymbol{q}_{oe}\|$ . These show that



(d) scalar part of quaternion

Fig. 5. Trajectory of object orientation using quaternion

the actual object orientation tracks the desired object orientation well, and that the object orientation error decreases greatly compared with the object position error by repetition of motion. Fig. 7 shows the desired contact force  $\boldsymbol{f}_{cd}$  and actual contact force  $\boldsymbol{f}_c$  of the 3rd finger. Fig. 8 shows the norm of the contact force error  $\|\Delta \boldsymbol{f}_c\|$ .



Fig. 6. The norm of object orientation error  $\|\boldsymbol{q}_{oe}\|$ 



Fig. 7. Trajectory of contact force of 3rd finger



Fig. 8. The norm of contact force error  $\|\Delta f_c\|$ 

These show that the x and y elements of the actual contact force track the desired contact force well, and that the contact force error decreases by repetition of motion. However, although the actual z element of the contact force converges to the desired contact force, the tracking capability is not sufficient. Also, the tendency of trajectory of the contact force of the 3rd finger and other fingers is the same. The z element of contact force corresponds to the internal force, and there are several reasons the tracking capability of the z element of the contact force is not sufficient: the dynamics of the mechanism, such as the flexibility of the joint, is not modeled; the controller is not a continuous-time control system but a discrete-time control system whose accuracy of trajectory depends on the sampling cycle; the difference in response occur by the difference in the properties of individual fingers.

# 5. CONCLUSION

An adaptive control method for multi-fingered robot hands manipulating an object using quaternion has been proposed. In the proposed controller, the dynamic parameters of both the object and fingers are estimated adaptively. The asymptotic convergence of the object motion and contact force was proven by the Lyapunov-like Lemma. The experiment results show that the adaptation was successful, and the control objective was almost achieved. However, a problem at the convergence speed of the contact force that corresponds to the internal force arose. This problem must be solved in a future study because the internal force is important for keeping a grasp on the object.

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