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Adaptive Attitude Tracking Control with L2-gain Performance for an Orbiting Flexible Spacecraft

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Abstract: This paper treats the problem of nonlinear adaptive attitude tracking control of an orbiting flexible spacecraft. It is assumed that the system parameters are unknown and the truncated model of the spacecraft has finite but arbitrary dimension. An adaptive sliding mode control law is derived for a three-axis stabilized spacecraft attitude tracking control system. The control gains are designed by solving a linear matrix inequality (LMI) problem to achieve a prescribed L2-gain performance criterion. The external torque disturbance/parametric error attenuation, with respect to the performance measure, along with control input penalty are ensured in the L2-gain sense. Lyapunov analysis is employed to show that the closed-loop system is asymptotically stable and the effect of the external disturbances/parametric error on the tracking error can be attenuated to any prescribed level. Simulation results show the effectiveness of the control scheme.

1. INTRODUCTION

One of the most important problems in spacecraft design is that of attitude stabilization and control. Many studies related to attitude control of flexible spacecraft have been performed. Control laws based on linearization and nonlinear inversion have been presented (Singh, 1988). Optimal and nonlinear control systems for the control of flexible spacecraft have been developed (Nagata, et al., 2001; Karray et al., 1997). Based on variable structure system theory, controllers for maneuvering large flexible space structures have been designed (Hu and Ma, 2005; Iyer and Singh, 1991; Singh, 1987; Oz and Mostafa, 1988). A nonlinear controller based on a neural network for the nonlinear slew maneuver of flexible spacecraft has been designed using state feedback (Nayeri et al. 2004). In these studies, it is assumed that the parameters of the spacecraft are exactly known. This is generally not the case with modern spacecraft, because it is very difficult to precisely know and model their complex nonlinear dynamics characteristics. The problem of model deficiencies can be dealt with by closing the control loop with a linear, robust controller, for example, an H ∞ controller (Show et al., 2003). However, the desired performance cannot be expected in the presence of gross errors in the spacecraft dynamics resulting from, for example, fuel usage and articulation. In these cases nonlinear adaptive control methods are called for. The problem of combining feedback linearization with an adaptive loop is presented (Zeng et al., 1999). Adaptive control based on variable structure techniques have also been used for designing controllers for flexible spacecraft (Singh and Zhang, 2004; Hu and Ma, 2006). For rigid spacecraft, many adaptive control strategies have been presented to compensate the unknown spacecraft inertia matrix (Jasim et al., 1998). Bošković et al. (2001) presented an adaptive variable structure tracking controller for rigid spacecraft in the presence of inertia uncertainties and external disturbances. However, most results reported in the literature suffer from at least one of the following substantial restrictions: (1) the proposed adaptive controllers ensure only local stability; (2) the external disturbances are assumed to be bounded, and the effect of the disturbances on the system performance are not considered. Although there are relatively many research papers on the disturbance attenuation control problem for robotics (Chiu and Lian, 2007; Chen, et al., 1997), there are few for the attitude control problem.

The contribution of this work lies in the derivation of a robust adaptive control law for attitude tracking control of flexible spacecraft in the presence of external disturbances, uncertainties and control input constraints. It is assumed that the moment of inertia matrix of the spacecraft is unknown to the designer. An adaptive sliding mode control law is designed such that the need to know the moment of inertia is eliminated. In the controller synthesis, the tracking performance is evaluated by an L2-gain constraint, with disturbance attenuation on both the tracking error and its derivative, and weighting on the control input. Also, the effect of parametric estimation on the error attenuation is considered in the design. Finally, the application to flexible spacecraft is investigated, with numerical simulation results used to show the expected performance.

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2. MATHEMATICAL MODEL OF SPACECRAFT AND THE CONTROL PROBLEM

2.1. Mathematical Model of a Flexible Spacecraft

In this work, the attitude tracking control of a flexible spacecraft is addressed. The attitude kinematics are represented by error quaternions, which are expressed in the reference frame $\Re$, fixed in the main body, with respect to a desired reference frame $\Re_d$, and given as follows

$$\begin{Bmatrix} \dot{e}_0 \\ \dot{e} \end{Bmatrix} = \frac{1}{2}\left[\mathcal{Q}\left(e_0, e\right)\right]^T \{\omega - \omega_d\} \tag{1}$$

where $e = \left[e_1 \;\; e_2 \;\; e_3\right]^T$, ω and ω_d are the angular velocity vectors of frames $\Re$ and $\Re_d$, respectively, and $\mathcal{Q}\left(e_0, e\right) = \left[-e, e^{\times} + e_0 I\right]$ with $e^{\times}$ satisfying

$$e^{\times} = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix} \tag{2}$$

Note that the error quaternion is subject to the unit constraint condition: $e^T e + e_0^2 = 1$.

The dynamic equations of a spacecraft with flexible appendage are given by (Hu and Ma, 2006)

$$J\dot{\omega} + \delta^T \ddot{\eta} = -\omega \times \left(J\omega + \delta^T \dot{\eta}\right) + u(t) + T_d \tag{3a}$$

$$\ddot{\eta} + \bar{C}\dot{\eta} + \bar{K}\eta + \delta\dot{\omega} = 0 \tag{3b}$$

where J is the symmetric inertia matrix of the whole structure, δ is the coupling matrix between the elastic and rigid structure, η is the modal coordinate vector, $u(t)$ is the control torque generated by reaction wheel, and T_d is the external disturbance torque. $\bar{C} = \text{diag}\left(\left[2\zeta_1\omega_1,\; 2\zeta_2\omega_2, \ldots, 2\zeta_n\omega_n\right]\right)$ is the damping matrix and $\bar{K} = \text{diag}\left(\left[\omega_{n1}^2,\; \omega_{n2}^2, \ldots, \omega_{nn}^2\right]\right)$ is the stiffness matrix, n the number of elastic modes considered, ω_{ni} are the natural frequencies, and ζ_i are the corresponding damping ratios.

If the terms $\delta^T \ddot{\eta}$ and $\omega \times \delta^T \dot{\eta}$ are considered as lumped perturbations to the rigid body dynamics, then Eq. (3a) can be rewritten as

$$J\dot{\omega} = -\omega \times J\omega + u(t) + T_d(t) + \Delta f(\eta, \ddot{\eta}, \omega) \tag{4}$$

where $\Delta f(\eta, \ddot{\eta}, \omega) = -\delta^T \ddot{\eta} - \omega \times \delta^T \dot{\eta}$ may be considered as the lumped perturbation. Let the angular velocity error be defined as $\omega_e = \omega - \omega_d$, then the error dynamics are given by

$$\begin{aligned} J\dot{\omega}_e = &-\left(\omega_e + \omega_d\right) \times J\left(\omega_e + \omega_d\right) - J\dot{\omega}_d \\ &+ u(t) + T_d(t) + \Delta f(\eta, \ddot{\eta}, \omega) \end{aligned} \tag{5a}$$

where

$$\begin{bmatrix} \dot{e}_0 \\ \dot{e} \end{bmatrix} = \frac{1}{2}\mathcal{Q}^T(e_0, e)\,\omega_e \tag{5b}$$

In order to develop the controller, the error equations (5) are transformed using the approach of Jasim et al. (1998) to give the following error dynamics

$$\begin{aligned} J^*\left(e, e_0\right)\ddot{e} + C^*\left(e, e_0, \dot{e}\right)\dot{e} + N^*\left(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d\right) + \Delta f^* \\ = u^* + d^* \end{aligned} \tag{6a}$$

$$\dot{e} = \frac{1}{2}\Xi\omega_e, \qquad J^* \triangleq P^T J P \tag{6b,c}$$

$$u^* \triangleq \frac{1}{2}Pu, \quad C^* \triangleq -J^*\dot{P}^{-1}P - 2P^T(JP\dot{e})^{\times}P \tag{6d,e}$$

$$\begin{aligned} N^* = &P^T\left[(P\dot{e})^{\times} J\omega_d\right] + P^T\left[(\omega_d)^{\times} JP\dot{e}\right] \\ &+ \frac{1}{2}P^T\left[(\omega_d)^{\times} J\omega_d\right] + \frac{1}{2}P^T J\dot{\omega}_d \end{aligned} \tag{6g}$$

$$\Delta f^* \triangleq \frac{1}{2}P\Delta f(\eta, \ddot{\eta}, \omega), \qquad d^* \triangleq \frac{1}{2}PT_d(t) \tag{6g,h}$$

where $\Xi \triangleq e^{\times} + e_0 I$ and $P \triangleq \Xi^{-1}$. Equations (6) are the general non-linear equations of motion for flexible spacecraft, which will be used for the controller synthesis.

Throughout the remainder of this paper, the following are assumed:

Assumption 1: The elastic oscillation and its rate are assumed to be bounded, that is to say, $\|\eta(t)\|$ and $\|\dot{\eta}(t)\|$ are bounded during the whole attitude tracking process by designing the proper control input, $u_p(t)$. The control input design will be discussed in a later section. The lumped perturbation $\Delta f(\eta, \ddot{\eta}, \omega)$ is assumed to be bounded and there exists a constant $\rho_{\Delta f} > 0$ such that $\|\Delta f(\eta, \ddot{\eta}, \omega)\| \le \rho_{\Delta f}$ is satisfied.

Assumption 2: The external disturbance $T_d(t)$ to the spacecraft system in Eq. (3) is assumed to be bounded.

2.2 The Control Problem

In this work, the objective of the control design is to achieve attitude tracking control and vibration reduction in the presence of possible uncertainty, external disturbance and control input constraints. Here the tracking performance criterion is given by an L_2-gain constraint in the controller synthesis. More specially, for a prescribed level of disturbance attenuation $1/\gamma > 0$ and the penalty matrices $Q \geq 0$ and $R \geq 0$ for the tracking errors and control input, respectively, there exists a control law such that the closed-loop system in Eq. (6a) satisfies the following: (a) all signals containing e, $\dot{e}$, $\ddot{e}$, σ and $\tilde{\theta}$ are bounded; (b) the control system achieves the performance criterion

$$\int_{t_0}^{T} z^T Qz dt + \int_{t_0}^{T} \bar{u}^T R\bar{u} dt \leq W(t_0) + \frac{\varepsilon}{\beta}\left(e^{-\beta t_0} - e^{-\beta T}\right) + \frac{1}{\gamma^2}\int_{t_0}^{T} \left\| d^*(t) \right\|^2 dt \tag{8}$$

where the definition of $\tilde{\theta}$, γ, ε, β, z, $\bar{u}$ and W will be given in later; (c) if d^* is L2-integrable, then e and $\dot{e}$ asymptotically converge to zero.

3. ADAPTIVE ATTITUDE CONTROL LAW DESIGN

In this section, a discontinuous attitude control law is derived based on sliding mode control theory, with assumptions 1 and 2, such that $e \to 0$ and $\omega_e \to 0$ as $t \to \infty$, in spite of uncertainties and external disturbances. By taking the error quaternion and its derivative vector, a linear sliding surface in vector form is defined as

$$\sigma = \dot{e} + \Lambda e \tag{9}$$

where Λ is a positive definite matrix.

We observe that the inertia parameters, $J_{i,j}$ for $i, j = 1,2,3,$, appear linearly in Eq. (6). To isolate these parameters, a linear operator is defined, $L : \mathbb{R}^3 \to \mathbb{R}^3 \times \mathbb{R}^6$, acting on $b = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}^T$ (Jasim et al., 1998) as

$$L(b) = \begin{bmatrix} b_1 & 0 & 0 & 0 & b_3 & b_2 \\ 0 & b_2 & 0 & b_3 & 0 & b_1 \\ 0 & 0 & b_3 & b_2 & b_1 & 0 \end{bmatrix} \tag{10}$$

Defining $\theta \triangleq \begin{bmatrix} J_{11} & J_{22} & J_{33} & J_{23} & J_{13} & J_{12} \end{bmatrix}^T$, it follows that

$$Jb = L(b)\theta \tag{11}$$

Using Eqs. (11), (6) and (7), we obtain

$$\Lambda\alpha J^* \dot{e} + \Lambda C^* e - N^* = \Phi(e, \dot{e}, \omega_d, \dot{\omega}_d)\theta \tag{12}$$

where $\Phi(e, \dot{e}, \omega_d, \dot{\omega}_d)$ is defined as

$$\begin{aligned} \Phi(e, \dot{e}, \omega_d, \dot{\omega}_d) = P^T \Big\{ & \Lambda L(P\dot{e}) - \Lambda JP\dot{P}^{-1}Pe \\ & + 2\Lambda(Pe)^{\times}(JP\dot{e}) - (P\dot{e})^{\times} L(\omega_d) - (\omega_d)^{\times} L(P\dot{e}) \\ & - \tfrac{1}{2}(\omega_d)^{\times} L(\omega_d) - \tfrac{1}{2} L(\dot{\omega}_d) \Big\} \end{aligned} \tag{13}$$

Then we have the following statements.

Theorem 1. Consider the system defined by Eq. (6) using the control law in Eq. (14) and the adaptation law in Eq. (15)

$$u^* = -\Phi(e, \dot{e}, \omega_d, \dot{\omega}_d)\hat{\theta} - K\sigma - \frac{\sigma \left\| \frac{1}{2} P \right\|^2 \rho_{\Delta f}^2}{\|\sigma\| \left\| \frac{1}{2} P \right\| \rho_{\Delta f} + \varepsilon e^{-\beta t}} \tag{14}$$

$$\dot{\hat{\theta}} = \Gamma \Phi^T(e, \dot{e}, \omega_d, \dot{\omega}_d)\sigma \tag{15}$$

where $\hat{\theta}$ is the estimate of θ, $\Gamma = \Gamma^T > 0$, ε is a small positive scalar control gain and β is a positive constant. Suppose there exist symmetric positive definite matrices K and P_1, and a positive constant γ satisfying the following LMI problem:

$$\begin{bmatrix} Q - \begin{bmatrix} \Lambda & I \end{bmatrix}^T \left(K - \frac{1}{4}\gamma^2 I\right) \begin{bmatrix} \Lambda & I \end{bmatrix} + \begin{bmatrix} I & 0 \end{bmatrix}^T \times P_1 \begin{bmatrix} 0 & I \end{bmatrix} + \begin{bmatrix} 0 & I \end{bmatrix}^T P_1 \begin{bmatrix} I & 0 \end{bmatrix} & \begin{bmatrix} \Lambda & I \end{bmatrix}^T K \\ K \begin{bmatrix} \Lambda & I \end{bmatrix} & -R^{-1} \end{bmatrix} \leq 0 \tag{16}$$

Then the closed-loop system has the following properties: (1) the signals containing e, $\dot{e}$, σ and $\tilde{\theta}$ are bounded; (2) the performance criterion in Eq. (8) can be guaranteed, where $z = \begin{bmatrix} e^T & \dot{e}^T \end{bmatrix}^T$, $\bar{u} \triangleq K\sigma$ is an auxiliary control, and $W(t_0)$ is a non-negative constant depending on the initial values of e, $\dot{e}$ and $\tilde{\theta}$; (3) if $T_d(t)$ is L2-integrable, then e and $\dot{e}$ will asymptotically converge to zero. Here Λ, $R \geq 0$, $Q \geq 0$ and $\rho_{\Delta f} > 0$ are given matrices and constant, and $\tilde{\theta}$ is the parameter error given by $\tilde{\theta} \triangleq \theta - \hat{\theta}$.

Remark 1. Close observation of the proposed control law, Eq. (14), gives guidelines to improve the tracking performance by adjusting gain matrix K and the parameters ε and β. However, for a proper selection of ε and β that eliminates the effect of the uncertainties, the system tracking performance is mainly determined by the gain matrix K. Therefore, by modifying the control gain matrix K in the auxiliary control $\bar{u} = -K\sigma$, the optimal approach is to achieve the goal of disturbance attenuation using the minimal auxiliary control torque $\bar{u}$, subject to the physical capability of the actuator.

Proof: Case (1). Consider the new candidate Lyapunov function

$$V_2 = \frac{1}{2}\sigma^T J^* \sigma + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{17}$$

The time derivative of V_2 along the control law given by Eq. (14), and the update law Eq. (15), yields

$$
\begin{aligned}
\dot{V}_2 &= \sigma^T d^* + \sigma^T \left(\Lambda C^* e + \Lambda J^* \dot{e} - N^* + u^*\right) - \sigma^T \Delta f^* + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\
&\le -\sigma^T K \sigma + \|\sigma\| \left\|d^*\right\| + \varepsilon e^{-\beta t} \\
&\le -\lambda_{\min}(K) \|\sigma\|^2 + \|\sigma\| \left\|d^*\right\| + \varepsilon e^{-\beta t}
\end{aligned} \tag{18}
$$

where $\lambda_{\min}(\bullet)$ denotes the minimum eigenvalue of a matrix. Due to the bounded disturbance, if the system trajectory lies within the region $\left\{\sigma \in \mathbb{R}^3 \middle| -\lambda_{\min}(K)\|\sigma\|^2 + \|\sigma\|\left\|d^*\right\| + \varepsilon e^{-\beta t} \le 0\right\}$, then $\dot{V}_2$ is negative semi-definite. From a positive definite V_2 and $\dot{V}_2$, in the form of Eq. (18), we have $\sigma \in L_\infty$, $e \in L_\infty$, $\dot{e} \in L_\infty$, $\tilde{\theta} \in L_\infty$ and $\dot{\tilde{\theta}} \in L_\infty$. In turn, we have $\dot{\sigma} \in L_\infty$ and $\ddot{e} \in L_\infty$ from the definition of the sliding surface and assumptions 1 and 2.

Case (2). Next we prove that the performance criterion in Eq. (8) is achieved by the feasibility of the LMI problem, Eq. (16). Define the energy function

$$
W = \begin{bmatrix} e^T & \sigma \end{bmatrix} P \begin{bmatrix} e \\ \sigma \end{bmatrix} + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{19}
$$

where $P = \text{block-diag}\left[P_1, \frac{1}{2} J^*\right]$ is a symmetric positive-definite matrix. Then the time derivative of the energy function is

$$
\begin{aligned}
\dot{W} &= \begin{bmatrix} e^T & \sigma^T \end{bmatrix} \left(PA + A^T P\right) \begin{bmatrix} e \\ \sigma \end{bmatrix} \\
&+ \sigma^T \left(-\frac{\sigma \left\|\frac{1}{2} P\right\|^2 \rho_{\Delta f}^2}{\|\sigma\| \left\|\frac{1}{2} P\right\| \rho_{\Delta f} + \varepsilon e^{-\beta t}} + d^* - \Delta f^* \right)
\end{aligned} \tag{20}
$$

where the following equation is used

$$
\begin{bmatrix} \dot{e} \\ \dot{\sigma} \end{bmatrix} = A \begin{bmatrix} e \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ (J^*)^{-1} \left(\Phi(e, \dot{e}, \omega_d, \dot{\omega}_d) \tilde{\theta} - \frac{\sigma \left\|\frac{1}{2} P\right\|^2 \rho_{\Delta f}^2}{\|\sigma\| \left\|\frac{1}{2} P\right\| \rho_{\Delta f} + \varepsilon e^{-\beta t}} + d^* - \Delta f^* - C^* \sigma \right) \end{bmatrix} \tag{21}
$$

with $A = \begin{bmatrix} -\Lambda & 1 \\ 0 & -(J^*)^{-1} K \end{bmatrix}$.

From inequality (8), we have

$$
\begin{aligned}
\int_{t_0}^{T} z^T Q z dt + \int_{t_0}^{T} \bar{u}^T R \bar{u} dt &= W(t_0) - W(T) \\
&+ \int_{t_0}^{T} \left[z^T Q z + \bar{u}^T R \bar{u} + \frac{dW}{dt} \right] dt
\end{aligned} \tag{22}
$$

In view of Eqs. (21), (14) and (15), and using the definition of vector z, Eq. (22) becomes

$$
\begin{aligned}
\int_{t_0}^{T} z^T Q z dt + \int_{t_0}^{T} \bar{u}^T R \bar{u} dt &\le W(t_0) + \int_{t_0}^{T} \frac{1}{\gamma^2} \left\|d^*\right\|^2 dt \\
&+ \frac{\varepsilon}{\beta}\left(e^{-\beta t_0} - e^{-\beta T}\right) + \int_{t_0}^{T} z^T \Big\{ Q - \begin{bmatrix} \Lambda & I \end{bmatrix}^T \\
&\times \left(K - \tfrac{1}{4}\gamma^2 I\right) \begin{bmatrix} \Lambda & I \end{bmatrix} + \begin{bmatrix} \Lambda & I \end{bmatrix}^T K^T R K \begin{bmatrix} \Lambda & I \end{bmatrix} \\
&+ \begin{bmatrix} I & 0 \end{bmatrix}^T P_1 \begin{bmatrix} 0 & I \end{bmatrix} + \begin{bmatrix} 0 & I \end{bmatrix}^T P_1 \begin{bmatrix} I & 0 \end{bmatrix} \Big\} z dt
\end{aligned} \tag{23}
$$

Therefore, if the LMI problem in Eq. (22) has feasible solutions γ, K and P_1, the error achieve the required tracking performance in Eq. (8).

Case (3). In addition, if $T_d(t) \in L_2$ then $d^* \in L_2$ and Eq. (23) implies that $z \in L_2$. Since $e \in L_\infty \cap L_2$, $\dot{e} \in L_\infty \cap L_2$ and $\ddot{e} \in L_\infty$, the strongly asymptotic convergence of e and $\dot{e}$ is obtained from Barbalat's lemma. Thus the theorem is proved completely.

In above analysis, the effect of the parametric error $\tilde{\theta}$ is not considered in the robustness design. Since a poor parametric estimation will leas to an unexpected transient response, especially when the moment generated by the actuator is limited. However, it is necessary to address the attenuation of both disturbance and parametric errors. To this end, we have the following corollary.

Corollary. Consider the system given in Eq. (6) using the following control law, Eq. (24),

$$
\begin{aligned}
u^* &= -\Phi(e, \dot{e}, \omega_d, \dot{\omega}_d)\hat{\theta} - K\sigma - \frac{\sigma \left\|\frac{1}{2} P\right\|^2 \rho_{\Delta f}^2}{\|\sigma\| \left\|\frac{1}{2} P\right\| \rho_{\Delta f} + \varepsilon e^{-\beta t}} \\
&+ \frac{\gamma^2}{4} \Phi(e, \dot{e}, \omega_d, \dot{\omega}_d) \Phi^T(e, \dot{e}, \omega_d, \dot{\omega}_d) \sigma
\end{aligned} \tag{24}
$$

with the adaptation law given by Eq. (15). Then attitude tracking problem with disturbance/parametric error attenuation and control input penalty is solved if the LMI problem in Eq. (16) has a feasible solution.

Proof: The proof is straightforward and is similar to that for Theorem 1.

4. SIMULATION RESULTS

The numerical application of the proposed control schemes to the attitude control of a flexible spacecraft is presented using MATLAB/SIMULINK software. The spacecraft is characterized by a nominal main body inertia matrix (Hu and Ma, 2006). The periodic external disturbance torque, T_d, is assumed to be of the following form in the simulation

$$T_d(t) = \begin{Bmatrix} 0.3\cos(0.1t)+0.1 \\ 0.15\sin(0.1t)+0.3\cos(0.1t) \\ 0.3\sin(0.1t)+0.1 \end{Bmatrix} \quad (25)$$

All control design parameters were tuned by trial-and-error until the best control performance was achieved for the control tasks. In addition, simulations have been rendered more realistic by considering saturation on the inputs. The maximum value of the moment produced by the actuator (reaction wheel) is 0.5 Nm.

In attitude tracking, suppose that the desired attitude trajectory was selected as $q_{0d} = \cos(\Phi/2)$,

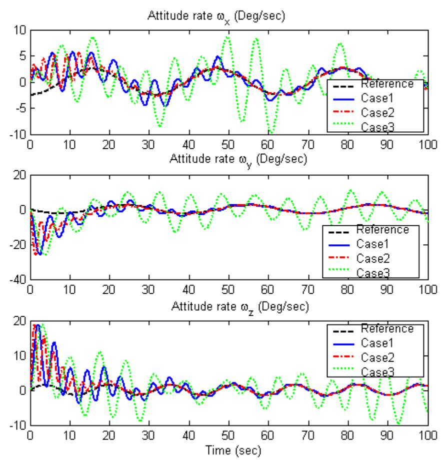
$$q_d = \begin{bmatrix}\cos(0.37t) & \sin(0.37t) & 0\end{bmatrix}^T \sin(\Phi/2) \quad (26)$$

with $\Phi = \sin(0.2t)$ and the desired velocity trajectory is

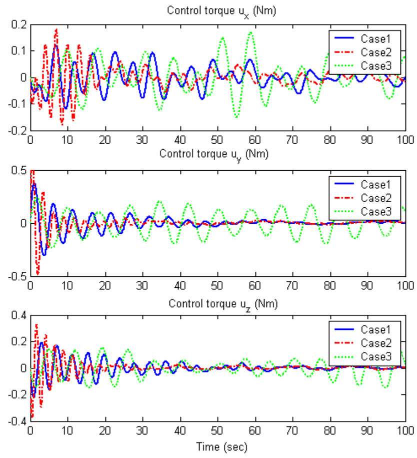
$$\omega_d = [-0.04\cos 0.2t,\ -0.04\sin 0.2t,\ 0.05\sin 0.2t + \cos 0.2t]^T \quad (27)$$

Figure 1 (solid line) gives the time responses of velocity, error quaternion, modal displacement, vibration energy defined by $E = \dot{q}^T\dot{q} + q^T Kq$, and the required control input torque given by Eq. (14). For the modified control case, Fig. 1 (dash-dot line) gives the corresponding time responses. No excessive control chattering can be observed, and also the control torque does not exceed its saturation value. This example demonstrates the effectiveness of the proposed cotnrol laws for atttiude tracking control.

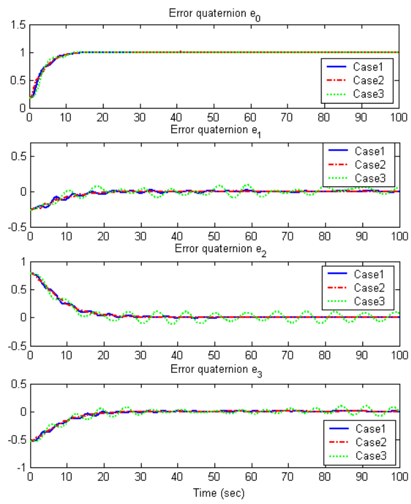
For comparison, the system is also controlled by using the traditional PD law designed using the approach of Hu and Ma (2005). The same simulation case is repeated for this TSMC, including the actuator dynamics, and the results of simulation are also shown in Fig. 1 (dotted line). For this case, the desired attitude tracking cannot be achieved, and severe oscillations are excited during tracking and can be observed, even if this designed controller is very effective when external disturbances to the system are not considered.



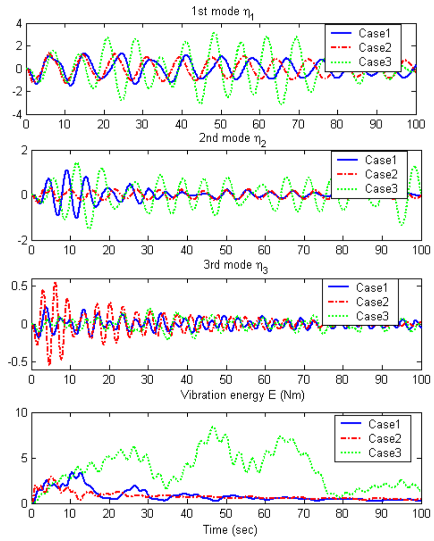
(a) Time response of the angular velocity



(b) Time response of wheel input torque



(c) Time response of the error quaternion



(d) Time response of vibration displacements and energy

Fig. 1. Attitude tracking control without vibration compensation. Case 1: Proposed sliding mode control law (solid line); Case 2: Modified sliding mode control law (dash-dot line); Case3: PD control law (dotted line).

5. CONCLUSIONS

In this paper, the attitude tracking control problem has been addressed for the electrically-driven flexible spacecraft with external disturbances by incorporating the performance criterion given by an L2-gain constraint in the controller synthesis. The design of the attitude controller was based on adaptive sliding mode control theory, and this controller can achieve arbitrary disturbance attenuation on tracking error for the external disturbances. Moreover, the developed controller does not require knowledge of the system parameters. The Lyapunov argument is also used to prove asymptotic stability when the L2-gain is less than a given small level if the linear matrix inequality (LMI) problem for control gain design is feasible. The efficiency of the proposed algorithm has been studied by the application to a three-axis stabilized flexible spacecraft system. Simulation results have demonstrated the effectiveness of the proposed algorithm.

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