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An Improved Nonlinear Speed Controller for Series DC Motors

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Abstract: The issue of speed tracking control for series DC motors is addressed. Based on the classic backstepping design technique, an improved recursive nonlinear controller is proposed to improve the transient response. In the design, two additional class $\mathcal{K}$ functions as design functions are adopted to achieve desirable varying decay rate. Application of this strategy substantially improves the transient response and convergence rate without remarkably increasing the controller "gains". Series DC motors with jumping load torques are also studied. The dynamics of such a motor with jumping load torques are modeled as a switched system. A switching controller based on the improved nonlinear design method is presented. Simulation results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

The control of a series DC motor is of great importance because of extensive engineering applications of series DC motors. Meanwhile, since the dynamics of a series DC motor can be described by a typical nonlinear system with lower triangular structure, the study of series DC motors from the control point of view also provides many challenging theoretical issues to address.

The traditional way to control a series DC motor is to adopt a linear controller which is based on the linear approximation of the original nonlinear model around a nominal operation point (Rashid (1993)). However, this linear controller is not applicable to the case when the motor operates on wide dynamical regimes. To overcome this drawback, multiple linear controllers are used to control the motor, where each linear controller is applied to a certain neighborhood of a operating point (Sira-Ramirez (1990)). As a result, a piecewise linear controller is formed. This control strategy really works in some cases and is easy to implement in engineering but may not produce satisfactory solutions in some other cases. Recent advances in nonlinear control systems makes it possible to apply nonlinear controllers. Exact feedback linearization technique was introduced to the controller design of series DC motors (Chiasson (1994); Olivier (1991)). This technique cancels all nonlinearities and transforms the nonlinear system into a linear controllable one by coordinates transformation and feedback, and thus standard design means for linear systems can be applied in the new coordinates (Krstic et al. (1995)). However, the exact feedback linearization method requires precise models and often cancel some useful nonlinearity. Backstepping is another effective nonlinear design method which preservers certain useful nonlinearity and gives the controller systematically. Also, compared with exact feedback linearization method, backstepping often has better robustness. Therefore, nonlinear control based on backstepping has attracted a lot of attention. Many results using backstepping to control DC motors are available in the literature, see, for example, Burridge et al. (2003); Wang et al. (2006).

For speed tracking control of series DC motors, nonlinear control approaches, such as backstepping and feedback linearization methods, usually give asymptotic tracking results with constant decay rates. If we want a fast transient response, large control "gains" must be applied which in many cases is impossible or means high cost. How to have a good transient response without remarkably increasing the control "gains" is a significant problem.

Since uncertainties and disturbances are unavoidable in practice, changing load torques must be taken into account when designing a controller for a series DC motor. A number of advanced robust control methods have been exploited to handle uncertainties and disturbances in motor systems. For example, a robust tracking control was given for varying parameters in Liu (2006); tracking periodic signals was addressed for uncertain parameters in Wai (2001); An adaptive backstepping method was applies to the problem of motion control (Yu et al. (2001)); A robust feedback linearization approach was presented in Bogosyan et al. (2000). On the other hand, loads can be changed abruptly, which causes jumping load torques. Most robust control methods which are effective to deal with slowly varying parameters are not applicable to control of jumping load torques. Moreover, transient responses are even more important when we control a motor with jumping load torques because there is often no sufficient time to have a transient response which is good enough before a new load torque is applied.

In this paper we study the problem of speed tracking control for series DC motors. An improved recursive nonlinear controller is proposed to improve the transient response. In the design, two additional class $\mathcal{K}$ functions as design functions are adopted to adjust the response and decay rates. Series DC motors with jumping load torques are also studied. The dynamics of such a motor can be modeled as a switched system so that the design methods for switched systems are applicable. In particular, a switching controller based on the improved design method is presented.

2. MOTOR DYNAMICS

Consider a series DC motor shown in Figure 1 below.

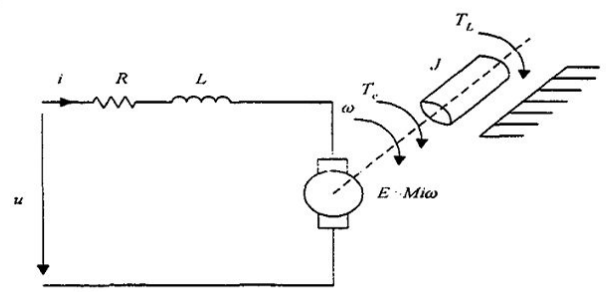

Fig. 1. Series DC motor

If we neglect magnetic saturation in field circuit, the motor can be modeled as (refer to Liu et al. (1999) for details).

$$\begin{aligned} L\frac{di}{dt} &= u - Ri - Mi\omega, \\ J\frac{d\omega}{dt} &= Mi^2 - T_L, \end{aligned} \quad (1)$$

where the physical meaning of the quantities are as follows.

i: armature current (or filed current)

u: terminal control voltage

ω: rotational speed of the motor

L: total armature and field current inductance

R: total armature and field circuit resistance

J: moment of inertia associated with both motor and the load

M: motor constant

T_L: load torque

$T_e = Mi^2$ and $E = Mi\omega$.

For notational convenience, we set

$$x_1 = \omega, x_2 = i.$$

Then, the system (1) becomes

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{M}{J}x_2^2 - \frac{T_L}{J}, \\ \frac{dx_2}{dt} &= -\frac{1}{L}(Rx_2 + Mx_1x_2) + \frac{1}{L}u. \end{aligned} \quad (2)$$

The control goal is to design a feedback controller u such that the speed of the motor x_1 tracks a constant desired speed ω_r.

3. CONTROLLER DESIGN

In this section, we develop an improved backstepping design method for the tracking control of the series DC motor. We first recall the classic backstepping design for the series DC motor.

3.1 Classic Backstepping Design

The classic backstepping design for (2) can be summarized as follows (Liu et al. (1999)).

Step 1. Let $e = x_1 - \omega_r$. Its derivative along the system is

$$\dot{e} = \dot{x}_1 = \frac{1}{J}(Mx_2^2 - T_L).$$

Define the Lyapunov function for this step by

$$V_1 = \frac{1}{2}e^2,$$

whose derivative along the system is

$$\dot{V}_1 = \frac{1}{J}e(Mx_2^2 - T_L).$$

In order to have

$$\dot{V}_1 = -k_1e^2$$

for some constant $k_1 > 0$, the virtual control x_2^2 should be

$$-\frac{J}{M}k_1e + \frac{T_L}{M}.$$

Step 2. Set

$$\begin{aligned} \eta &= M(x_2^2 + \frac{J}{M}k_1e - \frac{T_L}{M}) \\ &= Mx_2^2 + Jk_1e - T_L. \end{aligned}$$

Define the Lyapunov function for Step 2 as

$$V_2 = V_1 + \frac{1}{2}\eta^2 = \frac{1}{2}e^2 + \frac{1}{2}\eta^2.$$

Differentiating V_2 along the system yields

$$\begin{aligned} \dot{V}_2 &= -k_1e^2 + \frac{1}{J}e\eta \\ &\quad +\eta[\frac{2Mx_2}{L}(u - (Rx_2 + Mx_1x_2)) \\ &\quad +k_1\eta - k_1^2Je]. \end{aligned}$$

Choosing

$$\begin{aligned} u &= Rx_2 + Mx_1x_2 \\ &\quad +\frac{L}{2Mx_2}\left(-k_1\eta + k_1^2Je - \frac{1}{J}e - k_2\eta\right) \end{aligned}$$

with a constant $k_2 > 0$, we have

$$\dot{V}_2 \le -k_1e^2 - k_2\eta^2.$$

Therefore,

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \eta(t) = 0.$$

3.2 Improved Backstepping Design

In the classic backstepping design, the Lyapunvov function is made decreasing at a constant rate. It may take a long time to have a desired tracking error if the initial error is relatively large. If we need a shorter time to get a desired error we have to choose larger constants k_1 and k_2, which implies the increase of the control "gains". How to achieve fast transient response without remarkably increasing the controller "gains" is a basic problem in practice. A possible solution is to exploit varying k_1 and k_2 which take larger values when the error is larger and take smaller values when the error is smaller, and asymptotically tend to some constants when time goes to infinity. This is our motivation to propose an improved backstepping design. We first recall the concept of class $\mathcal{K}$ functions.

Definition (Krstic et al. (1995)). A function $\gamma(\cdot)$: $[0, \infty) \to [0, \infty)$ is called a class $\mathcal{K}$ function if it is continuous, strictly increasing and vanishes at the origin.

Now, we give the improved backstepping design.

Step 1. Let $e = x_1 - \omega_r$. Choose constants $k_1 \geq 0, k_2 \geq 0$, and smooth class $\mathcal{K}$ functions $\alpha(\cdot)$ and $\beta(\cdot)$ which are design functions determined by the required response time and the maximum of control "gains".

Define the Lyapunov function for this step by

$$V_1 = \frac{1}{2}e^2,$$

whose derivative along the system is

$$\dot{V}_1 = \frac{1}{J}e(Mx_2^2 - T_L).$$

If we choose the virtual control x_2^2 as

$$-\frac{J}{M}(k_1 + \alpha(e))e + \frac{T_L}{M},$$

we will have

$$\dot{V}_1 = -(k_1 + \alpha(e))e^2.$$

Step 2. Set

$$\begin{aligned} \eta &= M(x_2^2 + \frac{J}{M}(k_1 + \alpha(e))e - \frac{T_L}{M}) \\ &= Mx_2^2 + J(k_1 + \alpha(e))e - T_L. \end{aligned}$$

Define the Lyapunov function for Step 2 as

$$V_2 = V_1 + \frac{1}{2}\eta^2 = \frac{1}{2}e^2 + \frac{1}{2}\eta^2.$$

Differentiating V_2 along the system yields

$$\begin{aligned} \dot{V}_2 &= e\left(\frac{M}{J}x_2^2 - \frac{T_L}{J}\right) \\ &\quad + \frac{2Mx_2\eta}{L}(u - Rx_2 + Mx_1x_2) \\ &\quad + \eta J\frac{d}{dt}(k_1e + \alpha(e)e) \\ &= -(k_1 + \alpha(e))e^2 - (k_2 + \beta(e))\eta^2 \\ &\quad + \eta[\frac{1}{J}e + (k_2 + \beta(e))\eta \\ &\quad + \frac{2Mx_2}{L}(u - (Rx_2 + Mx_1x_2)] \\ &\quad + J\frac{d}{dt}(k_1e + \alpha(e)e)). \end{aligned} \tag{3}$$

Then, we design the controller

$$\begin{aligned} u &= Rx_2 + Mx_1x_2 \\ &\quad - \frac{L}{2Mx_2}[\frac{1}{J}e + (k_2 + \beta(e))\eta \\ &\quad + J\frac{d}{dt}(k_1e + \alpha(e)e)] \\ &= Rx_2 + Mx_1x_2 \\ &\quad - \frac{L}{2Mx_2}[\frac{1}{J}e + (k_2 + \beta(e))\eta \\ &\quad + (Mx_2^2 - T_L)\left(k_1 + \frac{\partial\alpha}{\partial e}e + \alpha(e)\right)]. \end{aligned} \tag{4}$$

Using the expression of η we can easily have the controller in terms of x_1, x_2 and e.

Substituting the controller (4) in to (3) results in

$$\dot{V}_2 \leq -(k_1 + \alpha(e))e^2 - (k_2 + \beta(e))\eta^2. \tag{5}$$

Since $k_1 > 0, k_2 > 0, \alpha(e) \geq 0$ and $\beta(e) \geq 0$, we have

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \eta(t) = 0,$$

which means that the objective of asymptotically speed tracking is fulfilled. Obviously, it holds that

$$\lim_{t\to\infty} \alpha(e(t)) = \lim_{t\to\infty} \beta(e(t)) = 0.$$

Therefore,

$$\lim_{t\to\infty} (k_1 + \alpha(e(t))) = k_1$$

and

$$\lim_{t\to\infty} (k_1 + \beta(e(t))) = k_2$$

which means that the decay rate of V_2 tends to a constant.

Remark 1. From (5) and in view of α and β being class $\mathcal{K}$ functions, we can see that V_2 decreases at a large rate when the tracking error is larger, which improves the transient response. On the other hand, smaller tracking error leads to smaller values of $(k_1 + \alpha)$ and $(k_2 + \beta)$, which in turn reflects smaller controller gain.

4. JUMPING LOAD TORQUES

When load changes, the corresponding load torque also changes. In this section, we will study the problem of speed tracking control for a series DC motor with jumping torques.

Suppose that we have m different load torques in use. Then the the motor has m groups of dynamics:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= \frac{M}{J}x_2^2 - \frac{T_L^j}{J}, \\ \frac{dx_2}{dt} &= -\frac{1}{L}(Rx_2 + Mx_1x_2) + \frac{1}{L}u. \end{aligned}\right. \quad j = 1, 2, \cdots, m \tag{6}$$

For each j, (6) corresponds a torque and at each time t one and only one torque is connected to the motor. Suppose that at time t_k one torque is switched off and another torque is switched on. Thus we have a switching sequence

$$\{t_0, t_1, \cdots, \cdots t_k, \cdots\},$$

which means that the i_k-th torque is switched on at t_k and switched off at t_{k+1}. Therefore, we actually have a switched system:

$$\frac{dx_1}{dt} = \frac{M}{J}x_2^2 - \frac{T_L^{\sigma(t)}}{J}, \qquad \frac{dx_2}{dt} = -\frac{1}{L}(Rx_2 + Mx_1x_2) + \frac{1}{L}u. \tag{7}$$

where

$$\sigma : [0, \infty) \to \{1, 2, \cdots m\}$$

is a switching signal which is characterized by the switching sequence, i.e.,

$$\sigma(t) = i_k \quad \text{when} \quad t \in [t_k, t_{k+1}).$$

The system resulting from (7) when $\sigma(t)$ is replaced by j is called the j-th subsystem of the switched system (7), which is exactly the j-th group dynamical equations of (6).

Now we apply the improved backstepping design method in Section 3 to design a controller u_j for the j-th subsystem of system (7)

$$\begin{aligned} u_j &= Rx_2 + Mx_1x_2 \\ &\quad -\frac{L}{2Mx_2}[\frac{1}{J}e + (k_2 + \beta(e))\eta_j \\ &\quad +(Mx_2^2 - T_L^j)\left(k_1 + \frac{\partial \alpha}{\partial e}e + \alpha(e)\right)] \end{aligned} \tag{8}$$

where

$$\eta_j = Mx_2^2 + J(k_1 + \alpha(e))e - T_L^j.$$

Let $V_2^j = \frac{1}{2}e^2 + \frac{1}{2}\eta_j^2$. Then, when $t \in [t_k, t_{k+1})$, we have

$$\dot{V}_2^j \leq -(k_1 + \alpha(e))e^2 - (k_2 + \beta(e))\eta_j^2 \tag{9}$$

which guarantees asymptotic stability of each subsystem of the switched system (7).

A switching controller is then constructed as $u = u_\sigma$.

Remark 2. In the discussion above, we construct different Lyapunov functions $V_2^j, j = 1, 2, \cdots, m$ for individual subsystem. Moreover, for different load torques we may choose different $\mathcal{K}$ functions α and β and different constants k_1 and k_2 to cope with different dynamical behavior. In this case, for the switched system (7), we have a switched Lyapunov function

$$V_2^{\sigma(t)} = \frac{1}{2}e^2 + \frac{1}{2}\eta_{\sigma(t)}^2,$$

whose derivative along the system is given by

$$\begin{aligned} \dot{V}_2^{\sigma(t)} \leq & -(k_{1\sigma(t)} + \alpha_{\sigma(t)}(e))e^2 \\ & -(k_{2\sigma(t)} + \beta_{\sigma(t)}(e))\eta_{\sigma(t)}^2. \end{aligned} \tag{10}$$

Although $V_2^{\sigma(t)}$ and $\dot{V}_2^{\sigma(t)}$ are all given in terms of e and $\eta_{\sigma(t)}$, they can be easily expressed in x_1, x_2 and e.

Remark 3. For jumping load torques, asymptotically speed tracking is impossible no matter what kind of controllers are applied. This can be seen from the dynamical equations of the motor. However, we can easily achieve acceptable tracking accurateness by properly designing feedback controllers for subsystems. This can be guaranteed by using multiple Lyapunov function principle for switched systems (Branicky (1998)). In the case that the switching law can be designed, we have more freedom to choose controllers for subsystems.

5. SIMULATIONS

To demonstrate the effectiveness of the proposed design method we have conducted two groups of simulations for the cases of non-jumping load torque and jumping load torques, respectively. In each group, we use speed and current responses to compare the proposed improved beckstepping method with the classic backstepping method.

Group 1: Non-jumping torque.

The parameters are taken from Liu et al. (1999):

$$\begin{array}{ll} R = 1\Omega, & J = 0.5kgm^2, \\ L = 0.05H, & M = 0.027H, \\ T_L = 55Nm, & \omega_r = 151.7. \end{array}$$

Figure 2 and Figure 3 show the speed and current responses respectively by the classic backstepping method with

$$k_1 = k_2 = 10.$$

Figure 4 and Figure 5 show the the speed and current responses respectively by the improved classic backstepping method with

$$k_1 = k_2 = 10$$

and

$$\alpha(e) = \beta(e) = 0.05e^2.$$

Group 2: Jumping load torques.

Parameters:

$$\begin{array}{ll} R = 1\Omega, & J = 0.5kgm^2, \\ L = 0.05H, & M = 0.027H, \\ T_L^1 = 55Nm, & T_L^2 = 110Nm \\ \omega_r = 151.7. & \end{array}$$

The switching sequence is

$$\{0, 0.5, 1, 1.5, \cdots\}$$

with the torque $T_L = 55Nm$ applied first.

Figure 6 and Figure 7 show the speed and current responses respectively by the classic backstepping method with

$$k_1 = k_2 = 10.$$

Figure 8 and Figure 9 show the speed and current responses respectively by the improved classic backstepping method with

$$k_1 = k_2 = 10$$

and

$$\alpha(e) = \beta(e) = 0.05e^2.$$

These simulation results show that in any cases, the transient responses are substantially improved with the proposed control method.

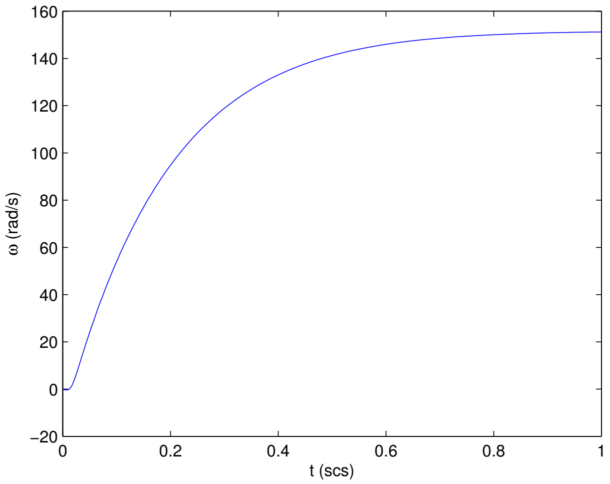

Fig. 2. Classic backstepping

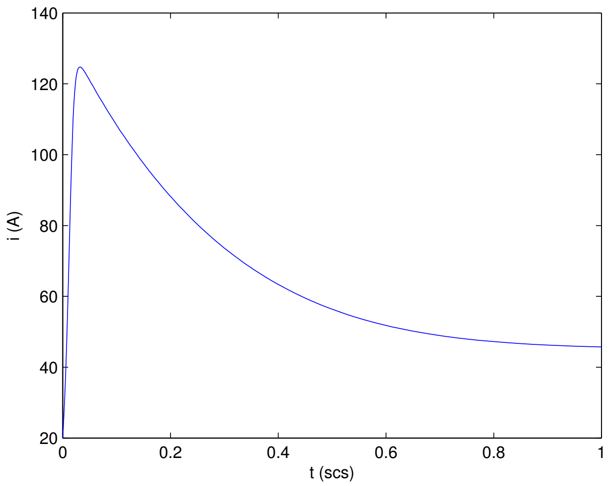

Fig. 3. Classic backstepping

6. CONCLUDING REMARKS

We have proposed an improved backstepping speed controller for series DC motors. By adopting two class $\mathcal{K}$ functions as design functions, we achieved varying decay rates which gives fast transient response without remarkably increasing the controller gains. The proposed method can be obviously extended to the case that torque and the speed of the motor are unmeasurable by using adaptive mechanism.

Jumping torques are also discussed where the improved backstepping method is more suitable. By expressing the dynamics of a series DC motor with jumping load torques as a switched system, we designed a switching controller.

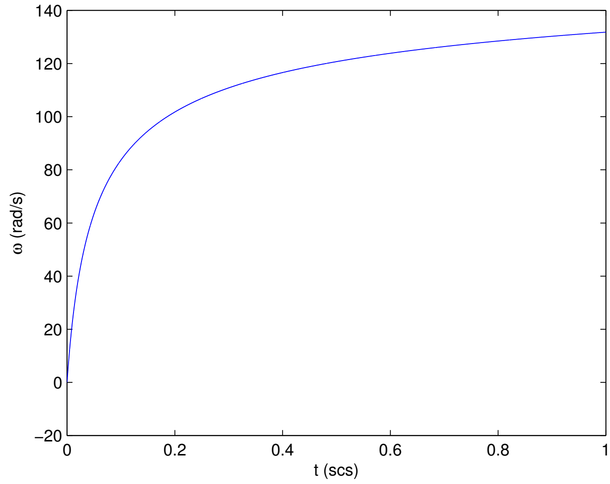

Fig. 4. Improved backstepping

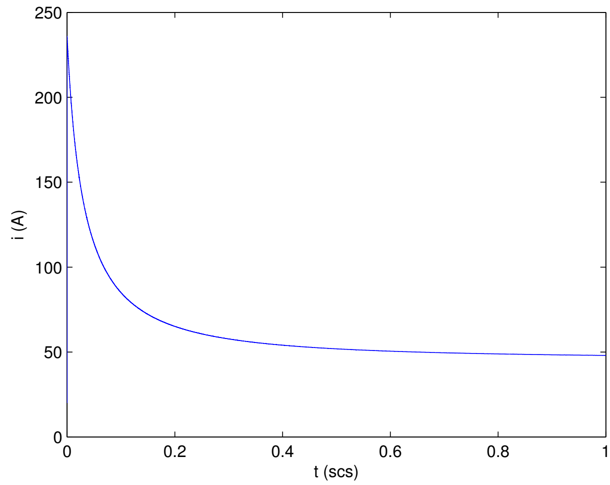

Fig. 5. Improved backstepping

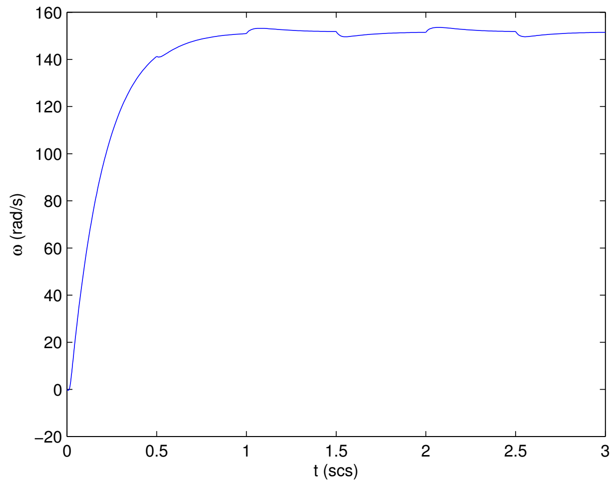

Fig. 6. Classic backstepping with jumping torques

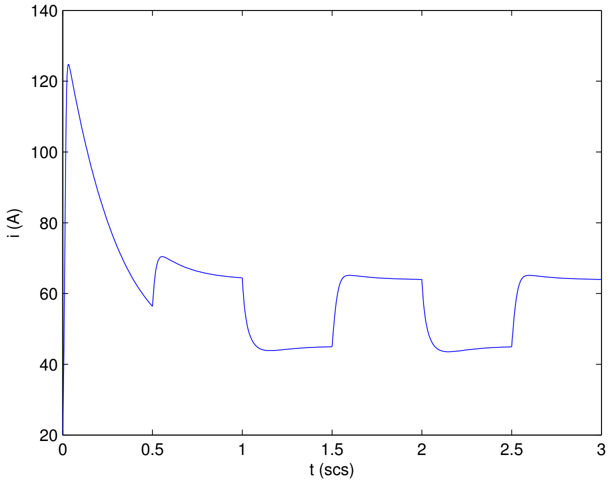

Fig. 7. Classic backstepping with jumping torques

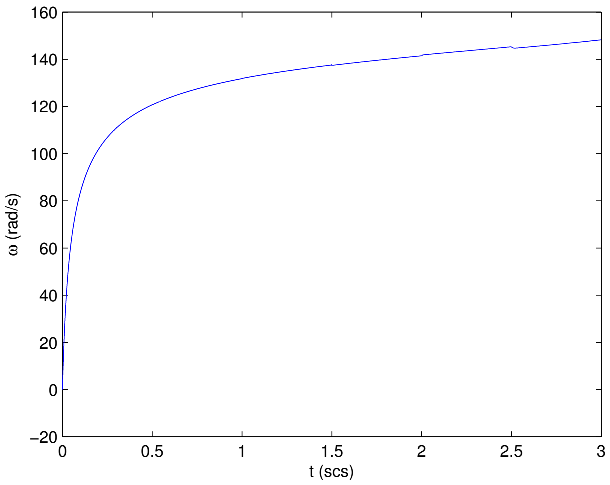

Fig. 8. Improved backstepping with jumping torques

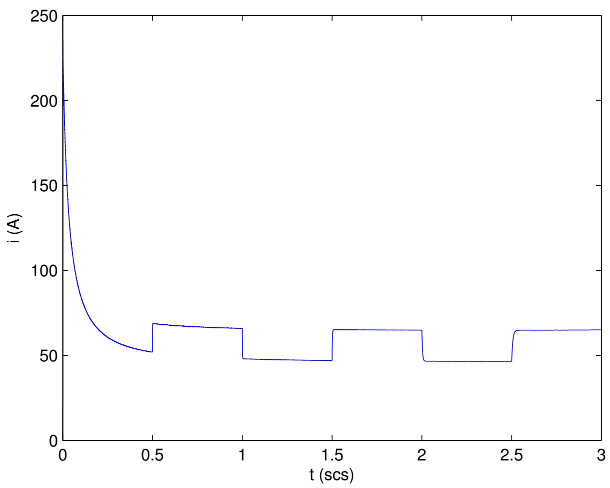

Fig. 9. Improved backstepping with jumping torques

It is worth mentioning that the transient response for each load torque is more important than that in non-jumping case because each torque is only connected to the motor for a finite time interval before it is switched off. Good response of each torque often produces good tracking effect.

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