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Two Improved Approaches to Fault Detection with Unknown Inputs ⋆

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Abstract: This paper addresses the fault detection problem based on H_∞ performance and L_2-gain performance. Two improved approaches compared with the recent results (Wang et al., 2005, Ding et al., 2000) are presented. One uses the inverse transfer function and state space representation to consider the H_∞ performance. The other uses dilated matrix formulation to handle different Lyapunov matrices based on L_2-gain performance. All these conditions can be efficiently solved by LMI techniques.

Keywords: Fault detection observer, Inverse transfer function, Dilated linear matrix inequalities, H_∞ performance, L_2-gain

1. INTRODUCTION

The fault detection and diagnosis (FDD) problem has received considerable attention, as modern engineering systems are large-scale and sophisticated, and safety and reliability are of practical concerns. Many fault detection approaches have been suggested in both frequency and time domains (see, e.g., the survey papers (Isermann, 2005, Venkatasubramanian et al., 2003, Patton et al., 2000) and books (Patton et al., 2000, Chiang et al., 2001, Isermann, 2006, Chen and Patton, 1999, Gertler, 1998) for a detailed classification and discussion). Among these approaches, the model-based fault detection is often used when the physical model is known. Particularly, the observer-based design approach, in which an observer plays the key role, is one of the most important and popular techniques (Isermann, 2006, Chen and Patton, 1999).

The fault detection problem commonly involves two aspects: 1) robustness to modeling errors and disturbances; and 2) sensitivity to the faults (Zhong et al., 2003). Many different performance indexes are used to measure these two aspects. For the robustness aspect, a lot of H_∞/H_2-based techniques have been introduced (see (Henry and Zolghadri, 2005a) for a survey). For the sensitivity aspect, both H_∞, H_2 and H_- norm are used. For example, in (Henry and Zolghadri, 2005a), generalized H_2 performance, as well as regional constraints on filter poles, are also included. The final formulation therein is a multi-objective optimization problem with linear matrix inequality (LMI) conditions.

The present paper is motivated by the recent works (Wang et al., 2005, Henry and Zolghadri, 2005a,b). In (Wang et al., 2005), both robustness and sensitivity are based either on H_∞ performance index or on L_2-gain index. However, their formulations actually yield bilinear matrix inequality (BMI) conditions. Furthermore, the H_∞-based design needs a pre-assigned sharp filter, which introduces additional complexity and conservatism. Besides, the LMI formulation therein for L_2-gain based design is conservative because additional constraints are made, which is not necessary. In this paper, we will attempt to overcome these obstacles using some different techniques. In fact, by using inverse transfer function and its realization, we can avoid to introduce the auxiliary dynamics, and by employing dilated LMI, we relax the design conservatism.

The rest of this paper is organized as follows. Section 2 presents the system and the design objectives. Section 3 addresses the main results. Section 4 gives several numerical examples to illustrate the advantages our approaches and Section 5 draws a conclusion.

2. PROBLEM STATEMENT

Consider the following system:

$$\dot{x} = Ax + B_1 w + B_2 f \quad (1)$$

$$y = Cx + D_1 w + D_2 f \quad (2)$$

where $x \in \mathcal{R}^n$ is the state vector, $w \in \mathcal{R}^m$ is the unknown input vector including modeling error, uncertain disturbance, process and measurement noises, $y \in \mathcal{R}^\ell$ is the measurement vector, and $f(x,t) \in \mathcal{R}^r$ is the bounded fault vector. All the matrices are properly dimensioned. We assume that the system in (1) is asymptotically stable. The fault detection observer under consideration is as follows:

$$\dot{\hat{x}}(t) = A\hat{x} + K(y - \hat{y}) \quad (3)$$

$$\hat{y} = C\hat{x} \quad (4)$$

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We define the residual vector r_{wf} as $r_{wf}(t) = y(t) - \hat{y}(t)$. We further denote:

$$r_w = r_{wf}|_{f\equiv 0}$$
$$r_f = r_{wf}|_{w\equiv 0}$$

The error system can be represented as

$$\dot{e}(t) = \tilde{A}e + \tilde{B}_1 w + \tilde{B}_2 f \quad (5)$$

$$r_{wf}(t) = Ce + D_1 w + D_2 f \quad (6)$$

where $\tilde{A} = A - KC$, $\tilde{B}_1 = B_1 - KD_1$ and $\tilde{B}_2 = B_2 - KD_2$.

The fault detection problem commonly requires that the system (5) be asymptotically stable and satisfies some performance indexes. In this paper, we only consider two kinds of such indexes: H_∞ performance index and L_2-gain performance index (Ding et al., 2000, Wang et al., 2005).

2.1 H_∞ Performance Index

The index denoted J_∞ below is widely used for the FD problems. The objective is to make J_∞ as small as possible (Ding et al., 2000, Wang et al., 2005):

$$J_\infty = \frac{\|T_{wr}(s)\|_\infty}{\|T_{fr}(s)\|_\infty}$$

where $T_{wr} = C(I - \tilde{A})^{-1}\tilde{B}_1 + D_1, T_{fr} = C(I - \tilde{A})^{-1}\tilde{B}_2 + D_2$. This objective can be cast into the following multiple objective problem:

$$\min \gamma_1 \ and \ \max \gamma_2, \ s.t. \ (8) \ and \ (9) \quad (7)$$

$$\|T_{wr}(s)\|_\infty < \gamma_1 \quad (8)$$

$$\|T_{fr}(s)\|_\infty > \gamma_2 \quad (9)$$

Inequality (8) can be easily reformulated as an LMI problem, which is due to the following bounded real lemma (Boyd et al., 1994).

Lemma 1. (Boyd et al., 1994) The system (5) is asymptotically stable and satisfies condition (8) if there exists $P_1 > 0$ such that (10) is satisfied.

$$\mathcal{A}_1^T \mathcal{P}_1 + \mathcal{P}_1 \mathcal{A}_1 + \Psi_1 < 0 \quad (10)$$

where $\mathcal{A}_1 = \begin{bmatrix} \tilde{A} & \tilde{B}_1 \\ 0 & 0 \end{bmatrix}, \Psi_1 = \begin{bmatrix} C^TC & C^TD_1 \\ D_1^TC & D_1^TD_1 - \gamma_1^2 I \end{bmatrix}, \mathcal{P}_1 = \begin{bmatrix} P_1 & 0 \\ 0 & I \end{bmatrix}$.

However, it is difficult to solve (9) efficiently, since it is not a convex problem. A well-known technique is to introduce a sharp filter/weighting function and then use the following lemma.

Lemma 2. (Henry and Zolghadri, 2005b,a, Wang et al., 2005) Given stable transfer function $W(s)$ and $T_{fr}(s)$ such that

$$\inf_{w\in R} \underline{\sigma}[W(jw)] = \delta > 0, \ \|W(s) - T_{fr}(s)\|_\infty = \varrho$$

where ϱ and δ are scalars. Then $\underline{\sigma}[T_{fr}(jw)] \geq \delta - \varrho$.

Based on Lemma 2, $\|T_{fr}(s)\|_\infty \geq \underline{\sigma}[T_{fr}(jw)] \geq \delta - \|W(s) - T_{fr}(s)\|_\infty$. Hence the condition (9) is guaranteed if the following standard H_∞ problem holds

$$\|W(s) - T_{fr}(s)\|_\infty < \delta - \gamma_2 \quad (11)$$

Note that the conservativeness of this approach lies in the facts that the inverse procedure is generally not true and no systematic method for the choice of $W(s)$ is available. Furthermore, the filter order increases as the weighting function becomes more complex. Besides, it is still difficult to obtain a less conservative LMI-based method due to the complexity of the augmented system. Hence, in (Wang et al., 2005), only a BMI-based method is proposed. In this paper, in order to avoid introducing $W(s)$ which brings conservatism, we shall propose a novel method based on inverse transformation. Instead of a sharp filter, an auxiliary matrix variable will be used to relax the conservatism.

2.2 L_2-gain Performance Index

This index is based on the L_2 norm of r_{wf}, w and f, i.e., the fault detection "unknown input signal" gain ratio $J_2 = \frac{\gamma_1}{\gamma_2}$ is made small where

$$\|r_w\|_{L_2} < \gamma_1^2 \|w\|_{L_2} + v_1(\cdot), \ w \neq 0 \quad (12)$$

$$\|r_f\|_{L_2} > \gamma_2^2 \|f\|_{L_2} + v_2(\cdot), \ f \neq 0 \quad (13)$$

where $v_1(\cdot)$ and $v_2(\cdot)$ are bounded functions related to initial conditions and satisfy $v_1(0) = v_2(0) = 0$. We assume that D_2 has full-column rank. In (Wang et al., 2005), an LMI based approach has been proposed. However, in the formulation, conservatism results from using same Lyapunov matrix for (12)-(13). In this paper, by introducing an auxiliary matrix variable, we will improve the result.

3. MAIN RESULTS

3.1 Inverse State Space and H_∞ Performance

A realization of $T_{fr}(s)$ is

$$T_{fr}(s) = \left[\begin{array}{c|c} \tilde{A} & \tilde{B}_2 \\ \hline C & D_2 \end{array}\right] \quad (14)$$

It is possible to formulate its left/right-inverse state space realization when D_2 has full column/row-rank.

Lemma 3. (Zhou et al., 1996) There exists a $T_{fr}^+(s)$ such that $T_{fr}^+(s)T_{fr}(s) = I$ (respectively, $T_{fr}(s)T_{fr}^+(s) = I$):

$$T_{fr}^+(s) = \left[\begin{array}{c|c} \tilde{A} - \tilde{B}_2 D_2^+ C & -\tilde{B}_2 D_2^+ \\ \hline D_2^+ C & D_2^+ \end{array}\right] \triangleq \left[\begin{array}{c|c} \bar{A} & \bar{B}_2 \\ \hline \bar{C} & \bar{D}_2 \end{array}\right] \quad (15)$$

where D^+ is the left inverse of D_2, i.e., $D_2^+ D_2 = I_r$ when D_2 has full column-rank (respectively, the right inverse of D_2, i.e., $D_2 D_2^+ = I_\ell$ when D_2 has full row-rank). Especially, when D_2 is invertible, we have $T_{fr}^+(s) = T_{fr}^{-1}(s)$.

Remark 4. In the case where D_2 does not have full column/row-rank, we can easily make a transformation on y or f, such that D_2 has full rank, i.e., introducing a new variable $y_1 = Ey$ such that ED_2 has full row-rank, or, $Ff_1 = f$, such that D_2F has full column-rank. Note that these transformation may only be applicable for fault detection.

It is easy to see that

$$1 = \|T_{fr}^+(s)T_{fr}(s)\|_\infty \leq \|T_{fr}^+(s)\|_\infty \|T_{fr}(s)\|_\infty$$

Hence, if

$$\|T_{fr}^{+}(s)\|_{\infty} < \frac{1}{\gamma_2} \quad (16)$$

we have $\|T_{fr}(s)\|_{\infty} > \gamma_2$, i.e., the condition (9) is satisfied. [1] So we can discuss the properties of (15) instead of considering (14).

Lemma 5. The system (15) is asymptotically stable and satisfies condition (16) if there exists $P_2 > 0$ such that (17) is satisfied.

$$\mathcal{A}_2^T \mathcal{P}_2 + \mathcal{P}_2 \mathcal{A}_2 + \Psi_2 < 0 \quad (17)$$

where $\mathcal{A}_2 = \begin{bmatrix} \bar{A} & \bar{B}_2 \\ 0 & 0 \end{bmatrix}$, $\Psi_2 = \begin{bmatrix} \bar{C}^T\bar{C} & \bar{C}^T\bar{D}_2 \\ \bar{D}_2^T\bar{C} & \bar{D}_2^T\bar{D}_2 - \frac{1}{\gamma_2^2}I \end{bmatrix}$, $\mathcal{P}_2 = \begin{bmatrix} P_2 & 0 \\ 0 & I \end{bmatrix}$.

Based on Lemmas 1 and 5, we can infer that if $P_1 > 0$ and $P_2 > 0$ exist for (10) and (17), then the system (5) is asymptotically stable and satisfies the H_∞ performance $J < \frac{\gamma_1}{\gamma_2}$. Hence we can obtain a BMI-based solution. Although there exist some algorithms (e.g.,(Kanev et al., 2004, Apkarian and Tuan, 2000)), BMI problems are generally inefficient. Hence LMI approaches are preferred. A simple result can be obtained by forcing $P_1 = P_2 = P$:

Lemma 6. The system (5) is asymptotically stable and satisfies conditions (8)-(9) if there exist matrices $P > 0$ and W such that

$$\Psi_i + \Lambda_i + \Lambda_i^T < 0,\ i = 1,2 \quad (18)$$

where

$$\Lambda_1^T = \begin{bmatrix} PA - WC & PB_1 - WD_1 \\ 0 & 0 \end{bmatrix}$$

$$\Lambda_2^T = \begin{bmatrix} PA - WC - (PB_2 - WD_2)D_2^{+}C & -(PB_2 - WD_2)D_2^{+} \\ 0 & 0 \end{bmatrix} \quad (19)$$

Furthermore, if the solution of (18) exists, the observer gain can be obtained as $K = P^{-1}W$.

In Lemma 6, we force $P_1 = P_2 = P$, which introduces additional conservatism. In the following, we will propose our less conservative algorithm based on the so-called dilated matrix technique (Xu and Xie, 2006, de Oliveira et al., 1999). Due to the space limitation, we omit the proof.

Theorem 7. The system (5) is asymptotically stable and satisfies conditions (8)-(9) if there exist matrices $P_i > 0, i = 1,2$, W and G_{ij}, $i = 1,2; j = 1,2,3$ such that

$$\begin{bmatrix} \Psi_i + \epsilon_{i1}(\hat{\Lambda}_i + \hat{\Lambda}_i^T) & \mathcal{P}_i - \epsilon_{i1}\hat{G}_i^T + \hat{\Lambda}_i \\ * & -\hat{G}_i - \hat{G}_i^T \end{bmatrix} < 0, i = 1,2 \quad (20)$$

where $\hat{G}_i = \begin{bmatrix} G_{i1} & \epsilon_{i2}G_{i1}\chi_i \\ G_{i2} & G_{i3} \end{bmatrix}$, $G_{11} = G_{21}$,

$$\hat{\Lambda}_1^T = \begin{bmatrix} G_{11}^T A - WC & G_{11}^T B_1 - WD_1 \\ \epsilon_{12}\chi_1^T(G_{11}^T A - WC) & \epsilon_{12}\chi_1^T(G_{11}^T B_i - WD_1) \end{bmatrix}$$

$$\hat{\Lambda}_2^T = \begin{bmatrix} G_{21}^T A - WC - (G_{21}^T B_2 - WD_2)D_2^{+}C & -(G_{21}^T B_2 - WD_2)D_2^{+} \\ \epsilon_{22}\chi_2^T(G_{21}^T A - WC - (G_{21}^T B_2 - WD_2)D_2^{+}C) & \epsilon_{22}\chi_2^T(G_{21}^T B_2 - WD_2)D_2^{+} \end{bmatrix} \quad (21)$$

where, χ_i are given full column-rank matrices properly dimensioned and $\epsilon_{ij}, i = 1,2, j = 1,2$ are given scalars. [2] Furthermore, if the solution of (20) exists, the observer gain can be obtained as $K = G_{11}^{-T}W$.

Remark 8. The choice of D_2^{+} is not unique. In order to obtain a small ratio of J_∞, the following optimization should be performed before the design procedure in Lemma 6:

$$\min_{D_2^{+}} \rho \text{ s.t. (22) and (23)}$$

$$\begin{bmatrix} -I & D_2^{+} \\ D_2^{+T} & -\rho \end{bmatrix} < 0 \quad (22)$$

$$D_2^{+}D_2 = I_r \text{ or } D_2D_2^{+} = I_\ell \quad (23)$$

Also, we shall note that the multi-objective problem (7) is generally difficult to solve. A possible solution is to use the so-called weighted sum method (Ehrgott, 2005).

Remark 9. In (Wang et al., 2005, Proposition 1), $W(s)$ stated in Lemma 2 is viewed as the desired transfer function (of a sharp filter) from fault f to residual r_f which T_{fr} is designed to match. A realization of $W(s) - T_{fr}(s)$ is an expanded system of the state x and auxiliary variable from $W(s)$. The main difficulties of this proposition lie in two facets. One is that it is generally hard to get an ideal realization of $W(s)$. The other is that the design procedure is actually a BMI problem. The same problem lies in (Henry and Zolghadri, 2005b). However, in our formulation, it is not necessary to find such a $W(s)$. Besides, the LMI condition is much less conservative because different Lyapunov matrices are employed.

Remark 10. (Henrion and Garulli, 2005) propose a H_∞ controller design technique for SISO linear systems based on the properties of positive polynomial matrices and LMI algorithms. This result can be also applied to the fault detection observer design using the inverse transfer functions. This kind of design method has two advantages: 1) it is suitable for fixed-order filter; 2) it is also possible to include the uncertainties into the design, such as bounded parameter uncertainties or polytope-type uncertainties. By applying the techniques introduced in (Henrion and Lasserre, 2006), this method is also applicable for MIMO systems with LMI formulations.

3.2 Dilated Matrix and L_2 Performance

Lemma 1 actually also guarantees that the performance index (12) holds (Boyd et al., 1994, Wang et al., 2005). As for the performance index (13), (Wang et al., 2005) give the following condition with the assumption that D_2 has full column-rank:

$$\tilde{\mathcal{A}}_2^T \mathcal{P}_2 + \mathcal{P}_2 \tilde{\mathcal{A}}_2 + \tilde{\Psi}_2 < 0 \quad (24)$$

[1] H_- norm cannot be treated in the same way, because it is not an induced norm.

[2] All these given parameters χ_i and ϵ_{ij} can be used to adjust the performance of fault detection observer.

where $\tilde{\Psi}_2 = -\begin{bmatrix} C^T C & C^T D_2 \\ D_2^T C & D_2^T D_2 - \gamma_2^2 I \end{bmatrix}$, $P_2 > 0$ and $\tilde{A}_2 = \begin{bmatrix} \tilde{A} & \tilde{B}_2 \\ 0 & 0 \end{bmatrix}$.

Now the fault detection problem based on L_2-gain performance can be stated as follows: If $P_i > 0$, $i = 1, 2$ exist for (10) and (24), then the system (5) is asymptotically stable and satisfies the L_2-gain performance (12)-(13).

Remark 11. In (Liu et al., 2005), a performance index named H_- index is introduced as follows: $\|G(s)\|_-^{[0,\infty]} = inf_{\omega\in[0,\infty]}\underline{\sigma}[G(jw)]$. It is easy to see that the condition (24) without requiring P_2 to be sign definite coincides with $\|G(s)\|_-^{[0,\infty]} > \gamma_2$. Hence we can easily design the observer based on the performance objective $J_{\infty/-} = \frac{\|T_{wr}\|_\infty}{\|T_{wf}\|_-}$.

It is noted that the design based on L_2-gain performance is also a BMI problem. In (Wang et al., 2005), the authors simply let $P_1 = P_2$ to obtain an LMI-based algorithm. In the following, we propose a method using the dilated matrix technique to alleviate the conservatism.

Theorem 12. The system (5) is asymptotically stable and satisfies conditions (12)-(13) if there exist matrices $P_i > 0, i = 1, 2$, W and G_{ij}, $i = 1, 2; j = 1, 2, 3$ such that

$$\begin{bmatrix} \tilde{\Psi}_i + \epsilon_{i1}(\tilde{\Lambda}_i + \tilde{\Lambda}_i^T) & \mathcal{P}_i - \epsilon_{i1}\hat{G}_i^T + \tilde{\Lambda}_i \\ * & -\hat{G}_i - \hat{G}_i^T \end{bmatrix} < 0, \ i = 1, 2 \quad (25)$$

where $\tilde{\Psi}_1 = \Psi_1$, $\tilde{\Lambda}_1 = \hat{\Lambda}_1$, $\tilde{\Psi}_2$ is given in (24),

$$\tilde{\Lambda}_2^T = \begin{bmatrix} G_{21}^T A - WC & G_{21}^T B_2 - WD_2 \\ \epsilon_{22}\chi_2^T(G_{21}^T A - WC) & \epsilon_{22}\chi_2^T(G_{21}^T B_2 - WD_2) \end{bmatrix},$$

χ_i are given full column-rank matrices properly dimensioned and $\epsilon_{ij}, i = 1, 2, j = 1, 2$ are given scalars. Furthermore, if the solution of (25) exists, the observer gain can be obtained as $K = G_{11}^{-T} W$.

Remark 13. Theorem 12 actually provides an improved LMI approach over (Wang et al., 2005, Proposition 4) in the sense of the performance index J_2, due to the fact that different Lyapunov matrices and additional free variables are appended. In fact, if we let $\epsilon_{i2} = 0$, $\epsilon_{i1} = \frac{1}{\epsilon}$, $G_{i21} = \epsilon P$, $G_{i22} = 0$, $G_{i23} = \epsilon I$ and $Q = P$ where ϵ is a sufficiently small scalar, then we can recover (Wang et al., 2005, Proposition 4).

Remark 14. Since we do not require the exact decoupling between the residual and disturbance, $r_{wf} = 0$ may not hold when $f(t) = 0$. Hence, a proper threshold (commonly denoted as J_{th}) is necessary. However, an exact threshold seems to be rather arduous to obtain, thus we propose an approximation using a cumulative sum. We assume the noise w has independent Gaussian distribution (zero mean and variance $[\sigma_1^2, ..., \sigma_\ell^2]$). In the fault-free case, if the condition of Lemma 1 is satisfied, we can actually deduce that $\|r_w\|_{L_2[0,T]} < \gamma_1^2 \|w\|_{L_2[0,T]}$ during a time window T. Let $Exp\{\|w\|_{L_2[0,T]}\} = T\sum_{i=1}^{\ell} \sigma_i^2 \triangleq \phi_w$. To obtain $\|r_w\|_{L_2[0,T]}$, we choose a sufficiently small sampling rate δ_t, such that $\|r_w\|_{L_2[0,T]} = \delta_t \sum_{i=1}^{\frac{T}{\delta_t}} r_w(t_i)^T r_w(t_i) + \epsilon \triangleq \varphi_r + \epsilon$, where ϵ is a sufficient small scalar. Now we can define the threshold as $J_{th} = \gamma_1^2 \phi_w$ and propose the following algorithm:

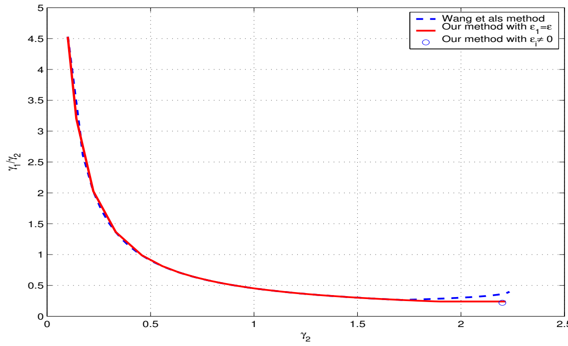

Fig. 1. Comparison of a linear system

$$\varphi_r \begin{cases} \le J_{th}, \text{ if } f = 0 \\ > J_{th}, \text{ if } f > 0 \end{cases} \quad (26)$$

However, our experience shows that this threshold does not work well if T is not long enough. Hence, we may apply the confidence level in statistic theory and choose β such that $Prob\{|w_i| < \beta\} > \rho$, where $\rho \ge 95\%$ experimentally, and redefine $\phi_w = T\sum_{i=1}^{\ell} \beta^2$. Of course, the adaptive threshold may be more useful (Shi et al., 2005).

4. NUMERICAL EXAMPLES

Example 1. Consider the following example borrowed from (Wang et al., 2005):

$$A = \begin{bmatrix} -10 & 0 & 5 & 0 \\ 0 & -5 & 0 & 2.5 \\ 0 & 0 & -2.5 & 0 \\ 0 & 5 & 0 & -3.75 \end{bmatrix}, B_1 = \begin{bmatrix} 0.8 & 0.04 \\ -2.4 & 0.08 \\ 1.6 & 0.08 \\ 0.8 & 0.08 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 4 \\ 4 \\ 8 \\ -8 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} 0.2 & 0.04 \\ 0.4 & 0.06 \end{bmatrix}, D_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

We compare the performance index J_2 using (Wang et al., 2005, Proposition 4) and Theorem 12. Instead of the multiple-objective optimization problem of searching for the sub-optimal J_2, we fix parameter γ_2 while finding minimum γ_1. Figure 1 shows the comparison result. The performance of our method actually depends on the choice of parameter $\epsilon_{ij}, i = 1, 2, j = 1, 2$. It can be easily seen from the figure that the performance of our method with $\epsilon_{21} = \epsilon_{22} = 0$ has no advantage over (Wang et al., 2005, Proposition 4) until $\gamma_2 > 1.7$. If we do a search involving all four parameters, we obtain the minimum $J_2 = 0.230$, which is superior to minimum $J_2 = 0.265$ (when $\gamma_2 = 1.712$) from (Wang et al., 2005, Proposition 4). Note that we can use the routine "fminsearch" in Matlab to search the corresponding parameters. We assume that the noise is independent Gaussian noise with zero mean and 1 variance. The fault is $f = \begin{cases} 0 & t < 5 \\ \alpha & t \ge 5 \end{cases}$. We choose a sampling time as $0.01s$ and time window $T = 0.1$. It is known that the possibilities of $|w| < \beta = 1.96, 2.24, 2.58, 3.29$ are $95\%, 97.5\%, 99\%, 99.9\%$, respectively. We may choose $0.2\beta^2$ as the approximation of ϕ_w of the thresholds.

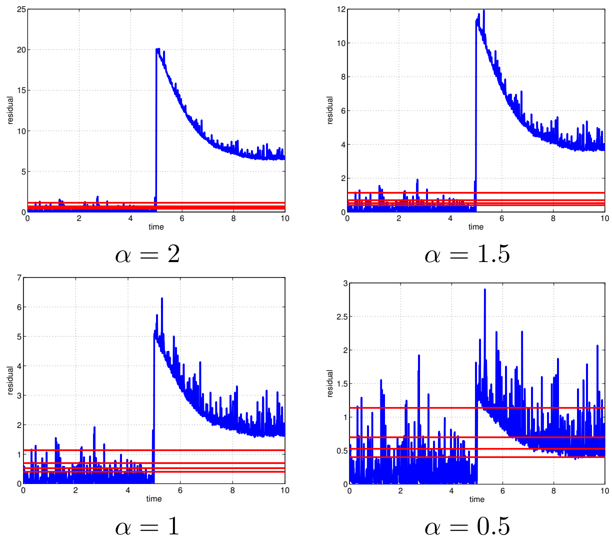

Fig. 2. Simulation results of $\|r_w\|^2$

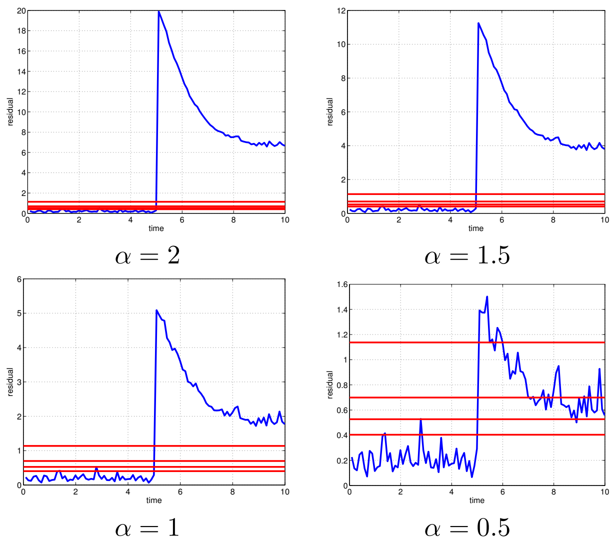

Fig. 3. Simulation results of $0.01 \sum_{i=1}^{10} r_w^T(t_i) r_w(t_i)$

When we choose $\gamma_2 = 0.2$, we have minimum $\gamma_1 = 0.4529$ based on (Wang et al., 2005, Proposition 4) or our method. A possible K is $\begin{bmatrix} 2.23138 & 0.878856 \\ -0.249055 & -5.72884 \\ 4.48168 & 1.73555 \\ -2.32028 & 3.10911 \end{bmatrix}$.

The simulation results with $\alpha = 2, 1.5, 1, 0.5$ are shown in Figures 2 and 3, where Figure 2 gives the actual value of $\|r_w\|^2$ while Figure 3 gives the value of $0.01 \sum_{i=1}^{10} r_w^T(t_i) r_w(t_i)$. It is clear that the cumulative sum performs better than actual value. Note that the value of $v(0)$ is rather big when e_0 is big. Hence it is not suitable for the threshold. That is the reason why we often assume the zero initial state conditions.

Based on (Wang et al., 2005, Proposition 4), when we choose $\gamma_2 = 1.712$, we have minimum $\gamma_1 = 0.4540$ and

$$K = \begin{bmatrix} 2.43827 & 0.817969 \\ -0.419836 & -5.65537 \\ 4.81894 & 1.63411 \\ -2.67894 & 3.2116 \end{bmatrix}$$

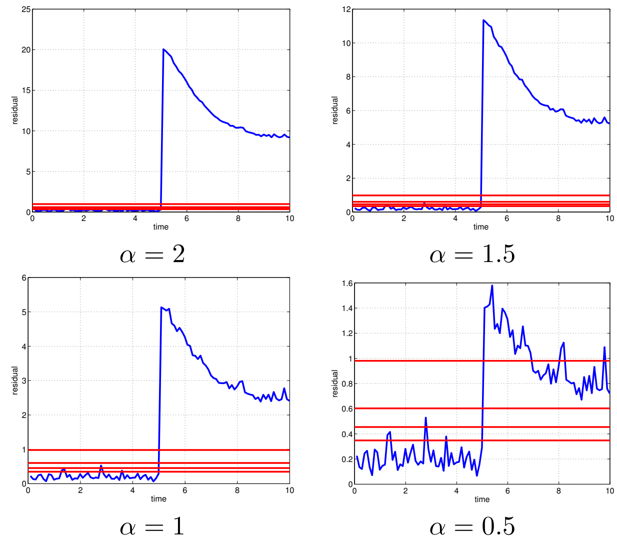

Fig. 4. Simulation results of $0.01 \sum_{i=1}^{10} r_w^T(t_i) r_w(t_i)$ based on (Wang et al., 2005, Proposition 4)

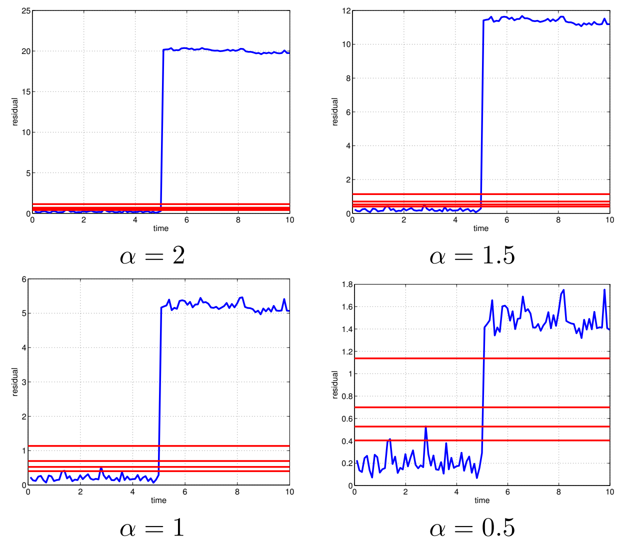

Fig. 5. Simulation results of $0.01 \sum_{i=1}^{10} r_w^T(t_i) r_w(t_i)$ of our method

where the minimum $J_2 = 0.265$ is obtained.

However, using Theorem 12 with $\epsilon_{12} = \epsilon_{22} = 0$, we can obtain the smaller $J_2 = 0.236$, if we choose $\gamma_2 = 2.21$, where we have minimum $\gamma_1 = 0.5254$ and

$$K = \begin{bmatrix} 2.4238 & 0.8532 \\ -0.7407 & -5.5031 \\ 4.8334 & 1.6776 \\ -2.4766 & 3.0464 \end{bmatrix}$$

Figures 4 and 5 show the simulation results.

Example 2. Consider the following example borrowed from (Wang et al., 2005):

$$A = \begin{bmatrix} -5.2 & 0.65 & 6.5 & 1.3 \\ -1.56 & -2.6 & 0 & 2.6 \\ -1.3 & 0 & -1.3 & 0 \\ -0.26 & 0 & 3.9 & -1.95 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.5 & 0.03 \\ -1.5 & 0.02 \\ 1 & -0.04 \\ 0.5 & 0.01 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ 2 \\ 4 \\ -4 \end{bmatrix}, C = \begin{bmatrix} -0.3 & 0.3 & 0 & 0.3 \\ 0.3 & 0 & 0.3 & 0 \end{bmatrix}$$

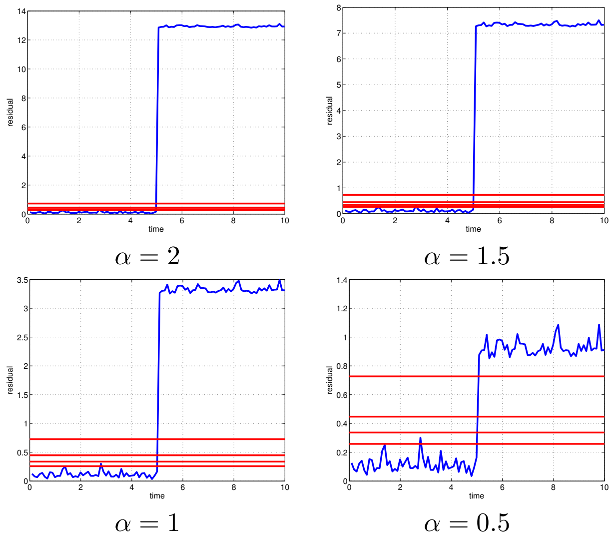

Fig. 6. Simulation results of sum of residuals

$$D_1 = \begin{bmatrix} 0.15 & 0.012 \\ 0.3 & 0.015 \end{bmatrix}, D_2 = \begin{bmatrix} 1.6 \\ -0.8 \end{bmatrix}$$

In (Wang et al., 2005), a sub-optimal solution is obtained when $\gamma_1 = 0.3400$ and $\gamma_2 = 1.785$, so that $\gamma_1/\gamma_2 = 0.1905$. However, because this example actually provides much "space" for design, using Lemma 6, we already obtain the optimal value $0.1878 = \frac{\|D_1\|}{\|D_2\|} = \frac{\gamma_1}{\gamma_2} = \frac{0.3359}{1.1788}$ with the following parameters:

$$D_2^{+} = \begin{bmatrix} 0.5 & -0.25 \end{bmatrix}, \; K = \begin{bmatrix} 1.6746 & 0.841445 \\ -0.99546 & -4.48664 \\ 3.33271 & 1.65711 \\ -1.32948 & 2.33682 \end{bmatrix}$$

Figure 6 shows the simulation results of our method with the same fault and noise setting of last example.

5. CONCLUSION

In this paper, we have discussed the fault detection observer design problem based on both H_∞ and L_2-gain performances. Several efficiently LMI-based approaches have been proposed. Compared with the results from (Wang et al., 2005), the formulations herein are more efficient and less conservative. In our continuing work, the FDI problem in finite domain frequency and of descriptor systems will be addressed.

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REFERENCES

P. Apkarian and H. D. Tuan. Robust control via concave minimization local and global algorithms. *IEEE Trans. on Automatic Control*, 45(2):299–305, 2000.

S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. Studies in applied mathematics. SIAM, 1994.

J. Chen and R. J. Patton. *Robust model-based fault diagnosis for dynamic systems*. Massachusetts: Kluwer Academic Publishers, 1999.

L. H. Chiang, E. L. Russell, and R. D. Braatz. *Fault detection and diagnosis in industrial systems*. Advanced textbooks in control and signal processing. Springer, 2001.

M. C. de Oliveira, J. Bernussou, and J. C. Geromel. A new discrete-time robust stability condition. *Systems and Control Letters*, 37:261–265, 1999.

S. X. Ding, T. Jeinsch, P. M. Frank, and E. L. Ding. A unifed approach to the optimization of fault detection systems. *International Journal of adaptive control and signal processing*, 14(7):725–745, 2000.

M. Ehrgott. *Multicriteria optimization*. Springer, Berlin Heidelberg New York, 2 edition, 2005.

J. Gertler. *Fault detection and diagnosis in engineering systems*. Marcel Dekker. Inc, New Year, USA, 1998.

D. Henrion and A. Garulli, editors. *Positive polynomials in control*, volume 312 of *LNCS*. Springer, 2005.

D. Henrion and J. B. Lasserre. Convergent relaxations of polynomial matrix inequalities and static output feedback. *IEEE Trans. on Automatic Control*, 51(2): 192–202, 2006.

D. Henry and A. Zolghadri. Design of fault diagnosis filters: A multi-objective approach. *Journal of Franklin Institute*, 342:421–446, 2005a.

D. Henry and A. Zolghadri. Design and analysis of robust residual generators for systems under feedback control. *Automatica*, 41:251–264, 2005b.

R. Isermann. Model-based fault-detection and diagnosis - status and applications. *Annual reviews in control*, 29:71–85, 2005.

R. Isermann. *Fault-diagnosis systems: an introduction from fault detection to fault tolerance*. Springer, 2006.

S. Kanev, C. Scherer, M. Verhaegen, and B. de Schutter. Robust output-feedback controller design via local BMI optimization. *Automatica*, 40:1115–1127, 2004.

J. Liu, J. Wang, and G. Yang. An LMI approach to minimum sensitivity analysis with application to fault detection. *Automatica*, 41:1995–2004, 2005.

R. J. Patton, P. M. Frank, and R. N. Clark, editors. *Issues of fault diagnosis for dynamic systems*. Springer-Verlag, London, U.K., 2000.

Z. Shi, F. Gu, B. Lennox, and A. D. Ball. The development of an adaptive threshold for model-based fault detection of a nonlinear electro-hydraulic system. *Control Engineering Practice*, 13:1357–1367, 2005.

V. Venkatasubramanian, R. Rengaswamy, S.N. Kavuri, and K. Yin. A review of process fault detection and diagnosis part III: Process history based methods. *Computers and Chemical Engineering*, 27:327–346, 2003.

H. Wang, J. Lam, S. X. Ding, and M. Zhong. Iterative linear matrix inequality algorithms for fault detection with unknown inputs. *Proc. IMechE part I: Systems and Control Engineering*, 219:161–172, 2005.

J. Xu and L. Xie. Dilated LMI characterization and a new stability criterion for polytopic uncertain systems. In *IEEE World Congress on Intelligent Control and Automation*, pages 243–247, Dalian, China, Jun 2006.

M. Zhong, S. X. Ding, J. Lam, and H. Wang. An lmi approach to design robust fault detection filter for uncertain lti systems. *Automatica*, 39:543–550, 2003.

K. Zhou, J. C. Doyle, and K. Glover. *Robust and optimal control*. Prentice Hall, 1996.