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A Nonlinear Roll Autopilot based on 5-DOF Models of Missiles

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Abstract: This paper suggests a nonlinear control law to stabilize the rolling motion of missiles. Based on the analysis of rolling moment characteristics due to pitch/yaw control cross-coupling, we propose a conjecture that the moment is described by the bilinear form of pitch/yaw acceleration and yaw/pitch control. The conjecture fits a set of wind tunnel test data and leads to a bilinear control law. An important point is that the control law is implemented by using pitch/yaw accelerations and control signals. Simulations show that the new control law stabilizes rolling motions which cannot be stabilized by linear feedback control laws.

1. INTRODUCTION

A recent trend in advanced control systems is to increase their dynamic range as wide as possible. There is no exception in air vehicles including missiles. A conventional approach to designing missile autopilot is to separate and conquer each channel based on the assumption that cross-couplings among channels are not large. Increasing dynamic range, however, tends to make the traditional way less effective or even useless because of large cross-couplings (Arrow and Yost, 1977; Devaud, *et al.*, 2001). To make things worse, many cross-coupling components show strong nonlinearities (Cronvich, 1986).

Dynamic models integrating roll-pitch-yaw motions include kinematical, inertial, and aerodynamic cross-couplings. The kinematical couplings are composed of angular and translational velocities. The products of angular velocities consist of the inertial couplings. Aerodynamic couplings come from the asymmetric configuration of airframe with respect to the direction of flight (Cronvich, 1986). These are induced moments and control cross-couplings which are due to change in airframe attitude and unequal forces developed by control surfaces, respectively (Arrow and Yost, 1977). There are two types of control cross-couplings: 1) rolling moments due to pitch (or yaw) control surfaces, 2) pitching or yawing moments due to roll control. For typical cruciform missiles, roll motion is generally far more sensitive than pitch or yaw with respect to the same amount of asymmetric control forces, which makes controlling roll motion challenging. It even makes the problem more challenging that the rolling moment tends to be highly nonlinear.

This work focuses on a roll control problem which is influenced by control cross-couplings. Based on the fact that the source of control cross-coupling is the difference in forces generated by a pair of fins for pitch (or yaw) control, we suggest a mechanism to generate the rolling moment. We confirm that the mechanism shows a good agreement with a set of wind tunnel test data. Based on the mechanism, we come up with a nonlinear control law feeding back pitch and yaw channel information to cope with the control cross-coupling moment. 5-DOF (five-degree-of-freedom) simulations show that a control law with the nonlinear feedback works well.

This paper is organized as follows. A missile model and problem formulation are described in Section 2. Section 3 contains main results including a model of the cross-coupled rolling moment as well as the nonlinear compensator. We show some simulation results in Section 4. Section 5 describes concluding remarks.

2. PROBRAM FORMULATION

Consider a conventional cruciform missile with four tail control fins. Figure 1 shows the definition of body coordinates, relevant angles and variables .

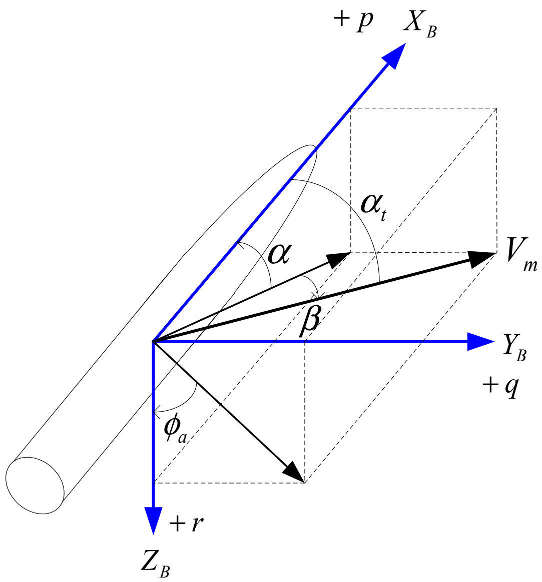

Fig. 1. Definition of the body-fixed coordinates (X_B, Y_B, Z_B), angle-of-attack α, side-slip angle β, total angle-of-attack α_t, and aerodynamic roll (or bank) angle ϕ_a, where V_m is the total velocity, and p, q, r are angular velocities about each axis.

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Under some acceptable assumptions such as a constant speed of flight, the symmetry of cruciform airframe, a rigid body and so on, a 5-DOF model which integrates all control motions including two translations and three rotations is written as follows (Brakelock, 1991; Devaud, *et al.*, 2001; Siouris, 2004).

$$\dot{\phi} = p \qquad (1)$$
$$\dot{p} = L_p p + L(\alpha, \beta, \delta_r, \delta_z, \delta_y)$$

$$\dot{\alpha} = q - \beta p + Z(\alpha, \beta, \delta_r, \delta_z, \delta_y) \qquad (2)$$
$$\dot{q} = I' pr + M_q q + M(\alpha, \beta, \delta_r, \delta_z, \delta_y)$$

$$\dot{\beta} = -r + \alpha p + Y(\alpha, \beta, \delta_r, \delta_z, \delta_y) \qquad (3)$$
$$\dot{r} = -I' pq + N_r r + N(\alpha, \beta, \delta_r, \delta_z, \delta_y)$$

In (1)-(3), ϕ is roll position, a_y, a_z are accelerations along each axis, g is the gravity, $\delta_r, \delta_z, \delta_y$ denote the control fin deflections of roll, pitch and yaw, respectively, L_p, M_q, N_r denote damping derivatives about each axis, and the other parameters are defined as follows:

$$I' = (I_{zz} - I_{xx}) / I_{yy} = (I_{yy} - I_{xx}) / I_{zz} \qquad (4)$$

$$L = QSDC_l(\alpha, \beta, \delta_r, \delta_z, \delta_y) / I_{xx} \qquad (5)$$

$$M = QSDC_m(\alpha, \beta, \delta_r, \delta_z, \delta_y) / I_{yy} \qquad (6)$$
$$N = QSDC_n(\alpha, \beta, \delta_r, \delta_z, \delta_y) / I_{zz}$$

$$Y = QSC_y(\alpha, \beta, \delta_r, \delta_z, \delta_y) / mV_m \qquad (7)$$
$$Z = QSC_z(\alpha, \beta, \delta_r, \delta_z, \delta_y) / mV_m$$

where Q is the dynamic pressure, S is the reference area, D is the reference length, C_l, C_m, C_n denote the aerodynamic moment coefficients about each axis, C_y, C_z are the aerodynamic force coefficients along each axis, m is the airframe mass, and I_{xx}, I_{yy}, I_{zz} are the moments of inertia about each axis.

The 5-DOF model (1)-(3) includes three types of couplings between roll, pitch and yaw channels. These are the kinematical couplings βp , αp , the inertial couplings pr, pq and the aerodynamic cross-couplings are hidden in (5)-(7). In order to design a roll autopilot coping with the aerodynamic cross-couplings, we need a roll system model with cross-couplings. Let us assume in this model that the pitch and yaw control loops are closed by a conventional autopilot (Zarchan, 1997) given as follows.

$$\dot{x}_z = -q + K_a a_z + K_a K_o a_{yc}^o \sin\phi - K_a K_o a_{zc}^o \cos\phi \qquad (8)$$
$$\dot{x}_y = -r - K_a a_y + K_a K_o a_{zc}^o \sin\phi + K_a K_o a_{yc}^o \cos\phi$$

$$\delta_{zc} = K_r q - K_r W_i x_z, \ \delta_{yc} = -K_r r + K_r W_i x_y \qquad (9)$$

In (8)-(9), K_o, K_a, W_i, K_r denote control gains, x_z, x_y are the states of pitch and yaw controllers, $a_{yc}{}^o, a_{zc}{}^o$ are acceleration commands given at an inertial roll reference, δ_{zc}, δ_{yc} are fin deflection commands for pitch and yaw control, respectively, and a_y, a_z are achieved accelerations along each axis and defined as

$$a_y = Y(\alpha, \beta, \delta_z, \delta_y, \delta_r) / mV_m \qquad (10)$$
$$a_z = Z(\alpha, \beta, \delta_z, \delta_y, \delta_r) / mV_m$$

For the time being, let us assume that actuators for pitch and yaw control are ideal, i.e.,

$$\delta_{zc} = \delta_z, \ \delta_{yc} = \delta_y .$$

Augmenting (1)-(3) by (8)-(9) yields a 8th-order roll system model which retains all cross-couplings between rolling, pitching and yawing motions. The augmented system can be written as

$$\dot{x} = f(x, \delta_r), \qquad (11)$$

In (11), $f \in R^8$ and the augmented state $x \in R^8$ is given as

$$x = \begin{bmatrix} \phi & p & \alpha & q & \beta & r & x_z & x_y \end{bmatrix}^T .$$

The system (11) is a 5-DOF dynamics for design and analysis of roll control laws in this work.

3. ROLL MOMENTS DUE TO PITCH/YAW CONTROL SURFACES

It is known that the rolling moment is contributed by many factors such as the pitch and yaw incidence angles and control surfaces as well as the roll control surface δ_r and roll rate p (Arrow and Yost, 1977; Cronvich, 1986; Devaud, *et al.*, 2001). Accounting the contributions decomposes the roll moment L as (Devaud, *et al.*, 2001)

$$L = f(\alpha_t)\sin(4\phi_a) + L^\delta(\alpha, \beta, \delta_r, \delta_z, \delta_y) \qquad (12)$$

The first term of (12) is the induced roll moment due to body incidence and the second is generated from control fins. The control moment is decomposed of two parts, i.e., one due to roll control fins and another due to pitch/yaw fins, written as

$$L^\delta = L_{\delta_r}(\alpha, \beta)\,\delta_r + L^{cc}(\alpha, \beta, \delta_z, \delta_y) \qquad (13)$$

The control cross-coupling moment L^{cc} is induced by the difference between two forces generated from a pair of fins for pitch or yaw control (Arrow and Yost, 1977; Cronvich, 1986). Let us think about the mechanism yielding the cross-coupling moment. Figure 2 depicts four cases how roll moments are induced from pitch/yaw control surfaces.

Consider the squeeze free relationship between four fins mounted on an airframe and deflections for roll, pitch and yaw control laws.

$$\delta_z = (\delta_1 - \delta_3)/2, \ \delta_y = (-\delta_2 + \delta_4)/2, \qquad (14)$$
$$\delta_r = (\delta_1 + \delta_2 + \delta_3 + \delta_4)/4$$

$$\delta_1 = \delta_r + \delta_z, \quad \delta_2 = \delta_r - \delta_y \qquad (15)$$
$$\delta_3 = \delta_r - \delta_z, \quad \delta_4 = \delta_r + \delta_z$$

The upper-left case in Figure 2 assumes that the airframe stays in a trim condition ($+\alpha_o, -\delta_z{}^o$) which is a steady manoeuvre with $-a_z{}^o$ acceleration. In this situation introducing an amount of control $+\delta_y$ to a of fins #2 and #4 generates only a lateral directional force along $-Y_B$ axis in principle. In reality, however, each of two fins #2 and #4 has a different effectiveness since a part of the fin #4 is shaded by airframe from the air flow with velocity V_m while the airframe has stayed in an attitude with angle-of-attack α_o. This asymmetry tends to make the force due to fin #2 be different from that due to fin #4. In this condition the fin #4 is less effective than the fin #2. The difference induces an amount of roll moment. According to the same steps employed for the upper-left one in Figure 2, the other diagrams are interpreted in their own conditions.

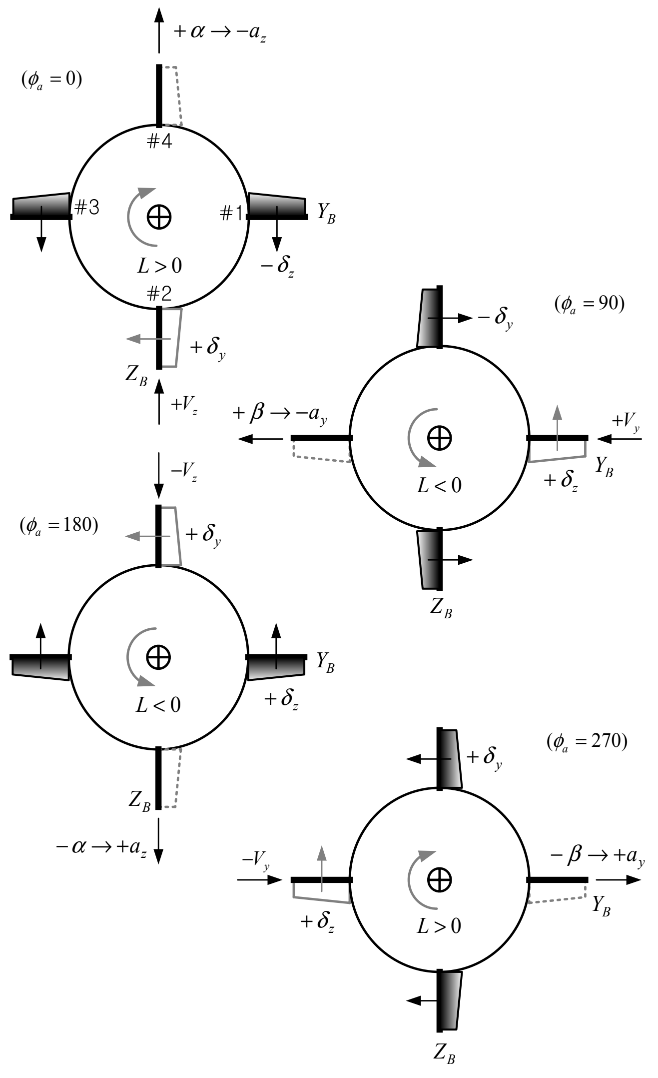

Fig. 2. Conceptual diagrams how roll moments are generated from pitch/yaw control deflections.

Now let us summarize the mechanism generating roll moments mentioned above. The forces in a trim condition which is maintained by a pair of control fins are expressed as

$$f_{\#1} = +k'\delta_1^{eff} = +k'\delta_z^o, \quad f_{\#3} = -k'\delta_3^{eff} = -k'\delta_z^o \tag{16}$$

In (16), $f_{\#1}, f_{\#3}$ denote the forces by fin #1 and fin #3, and $\delta_1^{eff}, \delta_3^{eff}$ are the effective deflection angles contributing to $f_{\#1}, f_{\#3}$, respectively, and k' is a constant. In this case $\delta_1^{eff} = \delta_3^{eff} = \delta_z^o$. The deflections for yawing in this trim state result in two different lateral directional forces. For these forces we introduce following expressions

$$f_{\#2} = +k'\delta_2^{eff} = +k'\delta_y, \quad f_{\#4} = -k'\delta_4^{eff} = -k'\delta_y \cdot (1 - \mu' |a_z^o|) \tag{17}$$

under an assumption that the effectiveness of control fin in shaded area decreases in proportion of the magnitude of trim acceleration, where $f_{\#2}, f_{\#4}$ are the forces by corresponding fins, and $\delta_2^{eff}, \delta_4^{eff}$ are the effective deflections contributing to $f_{\#2}, f_{\#4}$, respectively, and μ' is a constant. Summing the four forces in (16)-(17) gives a residual written as

$$f_r = \frac{1}{4}\sum_{i=1}^{4} f_{\#i} = k'\mu' |a_z^o| \delta_y = \mu |a_z^o| \delta_y. \tag{18}$$

In (18) f_r denotes a force which induces unwanted rolling and is due to yaw control fins. There is no difficulty to extend the expressions and steps employed for the first case to the other three cases. Table 1 summarizes the results.

The forces in the last column of Table 1 is integrated to two cases

$$f_r = \begin{cases} -\mu a_z \delta_y, & when\ \phi_a = 0, \pi \\ \mu a_y \delta_z, & when\ \phi_a = \pi/2, 3\pi/2 \end{cases}. \tag{19}$$

Four cases in Figure 2 represent specific situations in which only a pair of control fins, for pitch or yaw, contributes to the corresponding trim condition. Except the four specific cases, the velocity vector V_m always lies in between two adjacent fins. There is no objection to extend this cross-coupling mechanism yielding roll moments to general cases in which V_m lies in arbitrary direction. In these general conditions we can express the residual forces as an equation

$$f_r = -\mu a_z \delta_y + \mu a_y \delta_z \tag{20}$$

In (20), the first (or second) term of the right-hand side goes to zero only when $\phi_a = \pi/2, 3\pi/2$ (or $\phi_a = 0, \pi$), but both terms contribute in all situations except the four specific conditions.

Table 1. Forces yielding roll moments due to the deflections of pitch/yaw control surfaces

ϕ_a	$(\alpha, \beta, \delta_z, \delta_y)_{trim}$	$(a_z, a_y)_{trim}$	f_r
0	$(\alpha_o, 0, -\delta_z^o, 0)$	$(-a_z^o, 0)$	$\mu \vert a_z^o \vert \delta_y$
π	$(-\alpha_o, 0, \delta_z^o, 0)$	$(a_z^o, 0)$	$-\mu \vert a_z^o \vert \delta_y$
$\frac{\pi}{2}$	$(0, \beta_o, 0, -\delta_y^o)$	$(0, -a_y^o)$	$-\mu \vert a_y^o \vert \delta_z$
$\frac{3\pi}{2}$	$(0, -\beta_o, 0, \delta_y^o)$	$(0, a_y^o)$	$\mu \vert a_y^o \vert \delta_z$

Comparing (20) with L^{cc} in (13) suggests a new expression

$$L^{cc}(\alpha,\beta,\delta_y,\delta_z) = -k\,a_z\delta_y + k\,a_y\delta_z \quad (21)$$

with a constant k relating the forces in (19) to the corresponding moments. Finally (12) is expressed as

$$L = L_{\delta_r}(\alpha,\beta)\delta_r + L_{\delta_y}(a_z)\delta_y + L_{\delta_z}(a_y)\delta_z + f(\alpha_t)\sin(4\phi_a) \quad (22)$$

In (22) the dimensional derivatives are defined by

$$L_{\delta_y}(a_z) = -k\,a_z\,,\ L_{\delta_z}(a_y) = k\,a_y \quad (23)$$

4. ROLL CONTROL SYSTEM DESIGN

In order to design a roll control system coping with cross-couplings as well as nonlinear dynamics, let us define relevant steps.

4.1 Review of Pitch/Yaw Control System

Consider the pitch/yaw control in the aspect of roll control system. Aerodynamic forces and moments in (5) can be approximated as follows.

$$\begin{aligned} Y &= Y_\alpha(\alpha,\beta)\alpha + Y_\beta(\alpha,\beta)\beta + Y_{\delta_y}(\alpha,\beta)\delta_y \\ Z &= Z_\alpha(\alpha,\beta)\alpha + Z_\beta(\alpha,\beta)\beta + Z_{\delta_z}(\alpha,\beta)\delta_z \\ M &= M_\alpha(\alpha,\beta)\alpha + M_\beta(\alpha,\beta)\beta + M_{\delta_z}(\alpha,\beta)\delta_z \\ N &= N_\alpha(\alpha,\beta)\alpha + N_\beta(\alpha,\beta)\beta + N_{\delta_y}(\alpha,\beta)\delta_y \end{aligned} \quad (24)$$

In (24) we neglected control cross-couplings between pitch and yaw as well as with roll.

Applying (22)-(24) to (1)-(3) and (10), and using (8)-(9) gives a new expression of (11) written as

$$\dot{x} = f(x) + g(x)\delta_r\,, \quad (25)$$

where $f \in R^8, g \in R^8$.

4.2 Feedback Linearization to Compensate Roll Moments due to Pitch/yaw Control Fins

Based on (22), we come up with an auxiliary control variable δ_x defined by

$$\delta_x = \delta_r + \delta_r^{cc}\,, \quad (26)$$

$$\delta_r^{cc} = \frac{L_{\delta_y}(a_z)\delta_y + L_{\delta_z}(a_y)\delta_z}{L_{\delta_r}(\alpha,\beta)} + \frac{f(\alpha_t)\sin(4\phi_a)}{L_{\delta_r}(\alpha,\beta)}\,. \quad (27)$$

Applying (26) and (27) to (22) and letting δ_x as

$$\delta_x = -K_p p - K_\phi\phi + K_\phi\phi_{ref} \quad (28)$$

simplifies the closed-loop dynamics of roll control system to

$$\ddot{\phi} = (L_p - K_p L_{\delta_r}(\alpha,\beta))\dot{\phi} + K_\phi L_{\delta_r}(\alpha,\beta)(\phi_{ref} - \phi)\,, \quad (29)$$

where ϕ_{ref} is the reference roll position and K_p, K_ϕ are control gains. The control component δ_r^{cc} in (26)compensating the cross-coupling roll moments can be implemented to

$$\delta_r^{cc} = \frac{\hat{k}(a_z\delta_y - a_y\delta_z) + \hat{f}(\alpha_t)\sin(4\phi_a)}{\hat{L}_{\delta_r}(\alpha,\beta)} \quad (30)$$

if the estimates $\hat{k}(a_z\delta_y - a_y\delta_z)$, $\hat{L}_{\delta_r}(\alpha,\beta)$, $\hat{f}(\alpha_t)$ and the bank ϕ_a are available.

4.3 Considerations on Implementation

Let us focus on (30). The estimate $\hat{L}_{\delta_r}(\alpha,\beta)$ is a function of missile incidence angles. It is however known that the derivative can be approximated as a constant with tolerable errors for many conventional missiles. As an example, we show a trend in Figure 3 which comes from a set of wind tunnel test data of an air defence missile with four tail control fins. In case of the airframe in Figure 3, the estimation errors are less than about 15% even though the control derivatives are approximated as a constant without angle-of-attacks as well as bank angles.

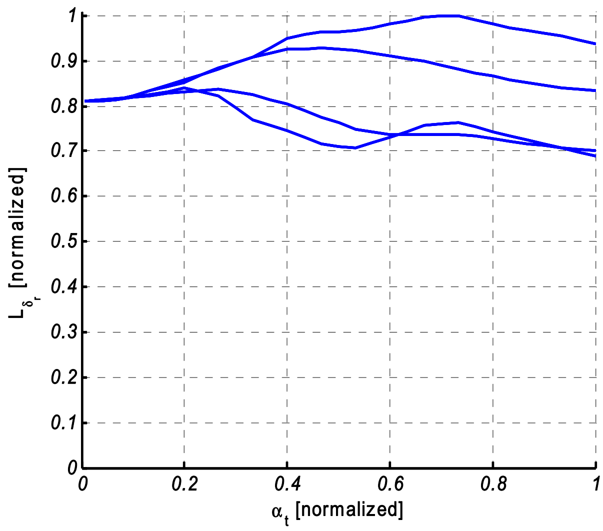

Fig. 3. An example of roll control derivatives versus the total angle-of-attack, where each line denotes a different bank angle, i.e., 0, 22.5, 45, or 90 degree

According to (23), the derivatives $L_{\delta_y}, L_{\delta_z}$ are linear with respect to a_z, a_y, respectively. With the data used in Figure 3, we depict the control coupling derivatives versus pitch/yaw accelerations in Figure 4. These figures confirm that the suggestions in (23) are acceptable. The two plots in Figure 4 show that $\hat{k}$ in (30) is approximated as a constant when the missile velocity and altitude are given.

On the other hand, the induced roll moment, the last term of (22), is a function of the total angle-of-attack and the bank angle, i.e., $f(\alpha_t)\sin(4\phi_a)$. Since the function has a period $\pi/2$, the estimation error of ϕ_a is very sensitive to the compensation error of the control cross-coupling moments. If the delay in real-time estimation is not negligible and the roll rate of vehicle is large, the effectiveness of the control law may be reduced dramatically. We, therefore, suggest a simplified compensator

$$\delta_r^{cc} = -\frac{\hat{k}}{\hat{L}_{\delta_r}}(a_z\delta_y - a_y\delta_z) \quad (31)$$

as a practical version, where $\hat{L}_{\delta_r}$ is a constant at a given velocity and altitude. The control law (31) is implemented by using pitch/yaw accelerations and controls which are all available. In a practical point view, using the control commands δ_{yc}, δ_{zc} in place of the control achievements δ_y, δ_z in (31), i.e.,

$$\delta_r^{cc} = -\frac{\hat{k}}{\hat{L}_{\delta_r}}(a_z\delta_{yc} - a_y\delta_{zc}), \qquad (32)$$

is able to show better performance and stability since the commands advance the achievements in phase. Finally employing (28) and (32) to (26) yields a new roll control law

$$\delta_r = -K_p p - K_\phi \phi + K_\phi \phi_{ref} + \frac{\hat{k}}{\hat{L}_{\delta_r}}(a_z\delta_y - a_y\delta_z). \qquad (33)$$

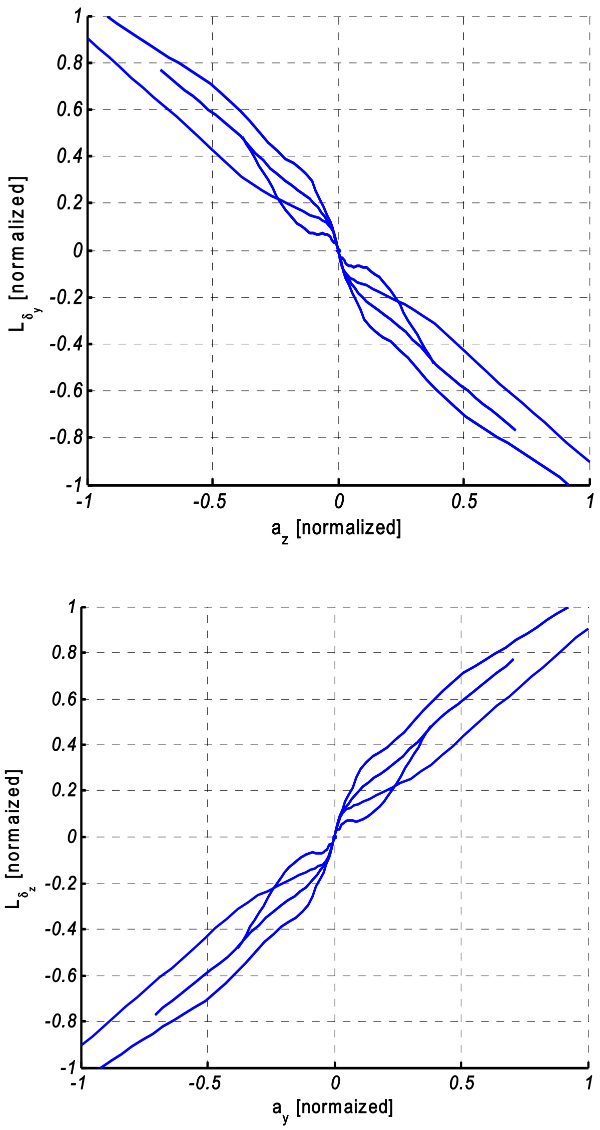

Fig. 4. An example of the roll control-coupling derivatives versus the total angle-of-attack, where each line denotes the different bank angle, i.e., 0, 22.5, 45, or 90 degree

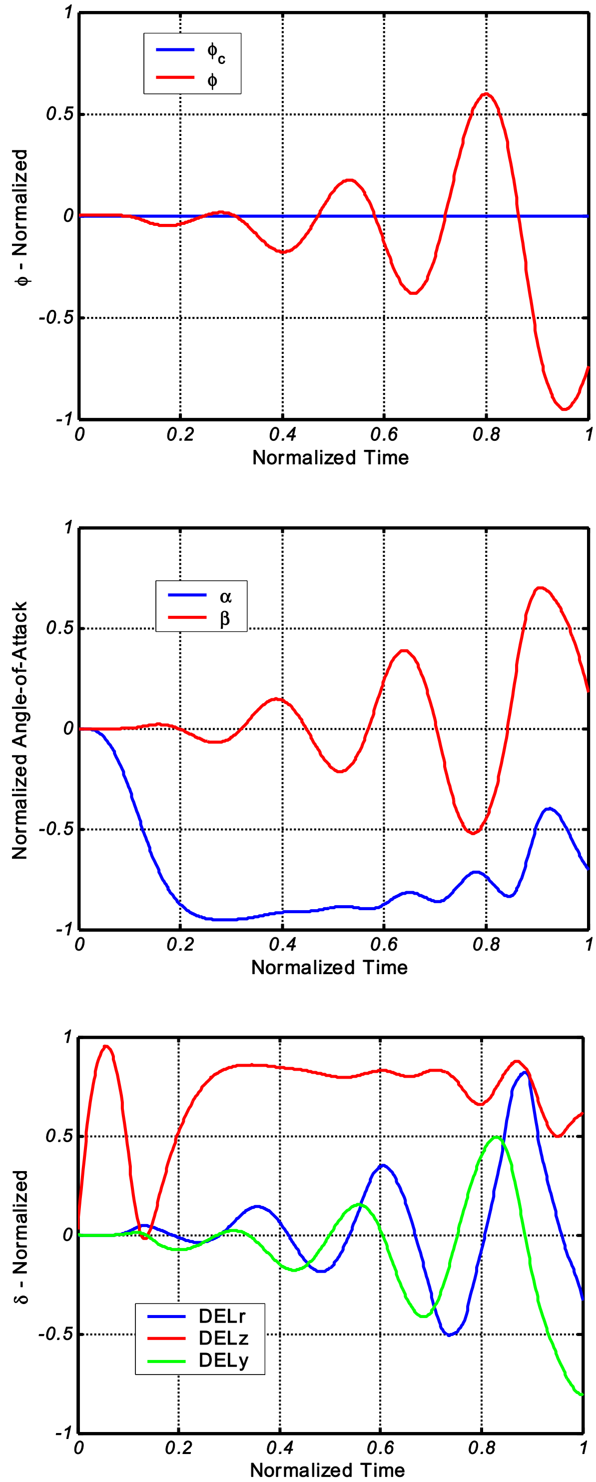

Fig. 5. 5-DOF simulation results of a given control system without the compensator

4.4 Discussions on Stability

Employing (22) with (33) for (1) yields the closed-loop dynamics of roll control system

$$\ddot{\phi} = -a_1\dot{\phi} - a_2\phi + a_2\phi_{ref} + d^{cc} \qquad (34)$$

$$d^{cc} = \tilde{k}(a_z\delta_{yc} - a_y\delta_{zc}) + f(\alpha_t)\sin(4\phi_a) \qquad (35)$$

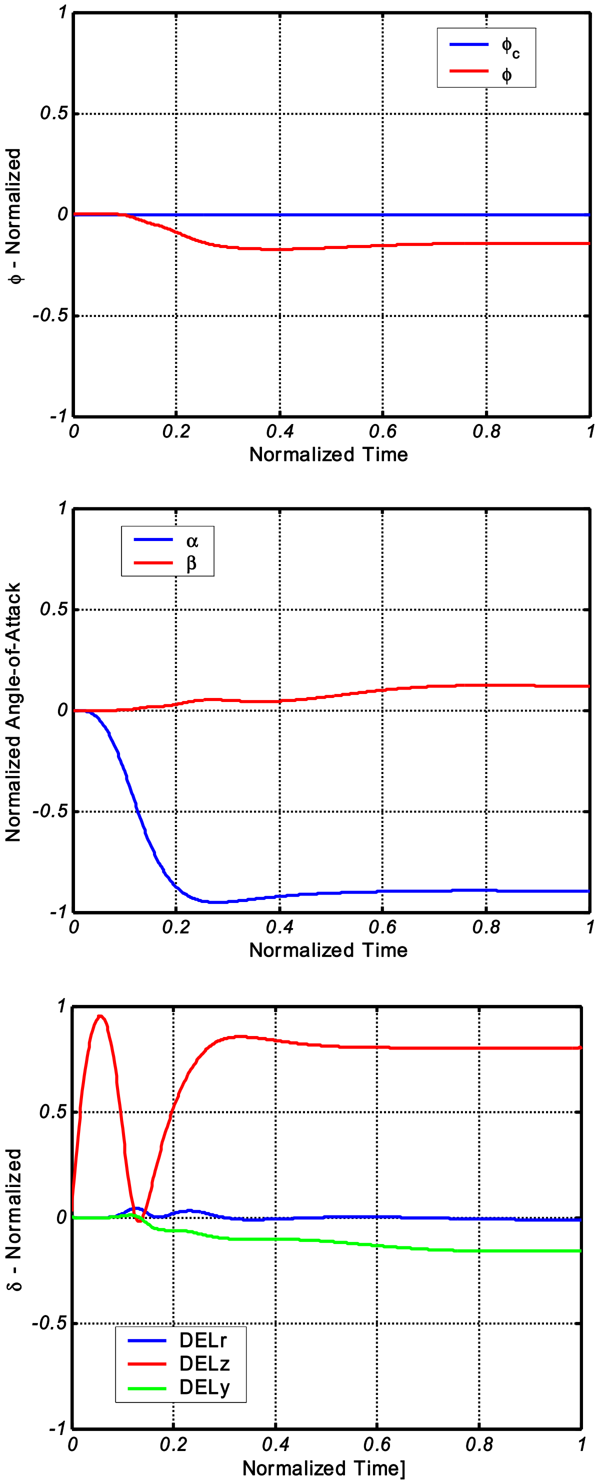

Fig. 6. 5-DOF simulation results of a given control system with the compensator

with $a_1 = L_p - K_p L_{\delta_r}(\alpha,\beta)$, $a_2 = K_\phi L_{\delta_r}(\alpha,\beta)$, and

$$\tilde{k} = k - \hat{k}\, L_{\delta_r}(\alpha,\beta) / \hat{L}_{\delta_r} . \qquad (36)$$

The closed-loop system (34) can be stabilized by the control gains K_p, K_ϕ provided that the disturbance $d^{cc}(\alpha,\beta,q,r,x_z,x_y)$ is bounded by the pitch and yaw control law (9). Another approach is to linearize the whole closed-loop system (25) with (33) at each operating condition and study the stability of each linear system. For the example in Section 5 we evaluate the stability by the second approach.

5. 5-DOF NONLINEAR SIMULATIONS

For the new roll control law (33), we simply choose $\hat{L}_{\delta_r} = L_{\delta_r}(0,0)$ by using the data in Figure 3. And the estimate of control coupling coefficient, $\hat{k}$, is evaluated by a least-squares fitting with the data in Figure 4. Given the velocity and altitude, these two parameters are constants. Control gains K_p, K_ϕ in (33) and K_r, W_i, K_a, K_o in (8)-(9) are designed for an aerodynamics data set which is including the data in Figure 3 and Figure 4. Stability of the nonlinear control system is evaluated by the linearization method on many operating conditions interested in.

Consider the 5-DOF model (25) with control laws (8)-(9) and (33). The 5-DOF model takes care of all modes of a missile except a translational motion along X_B axis under a mild assumption that the velocity and altitude are given and fixed. To this system we introduce a large acceleration command along along Z_B axis only. We show two simulation results in Figure 5 and Figure 6 for a conventional roll control law with $\hat{k} = 0$ and the new roll control, respectively. In these figures, all variables are normalized by their maximum values for an obvious reason. For easy and fair comparison, we use the same scale in Figure 5 and 6. The conventional roll control in Figure 5 shows an unstable response. It is possible to stabilize the unstable mode of low frequency by increasing the control gains K_p, K_ϕ. But large control gains tend to decrease stability at high frequency due to actuators with limited bandwidth. On the other hand, the new roll control law in Figure 6 shows an improvement. An observation on these results confirms that the proposed control law with bilinear feedback of pitch/yaw channel information has a good capability to improve stability as well as performance.

6. CONCLUDING REMARKS

In this work we introduce an efficient and simple nonlinear control law to stabilize a kind of unwanted roll moments. Based on the analysis of the process that the roll moment is generated by pitch and/or yaw control cross-couplings, we propose a conjecture that the moment is described by two bilinear terms composed of the pitch acceleration and yaw control and the yaw acceleration and pitch control. This conjecture fits a set of wind tunnel test data and leads to a bilinear control law.

An important point is that the control law is implemented by using pitch and yaw accelerations and control commands which are usually provided by conventional pitch/yaw control laws. Through some simulations based on a 5-DOF nonlinear model and a full set of aerodynamic data, we show that the new control law stabilizes rolling motions which cannot be stabilized by the conventional linear control law to feed only the roll channel information back. The new roll control law, moreover, shows relatively low sensitivity to errors in aerodynamics and incidence angles. The robustness of the new control law will be analyzed in further study.

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