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A Fuzzy Sliding Mode Controller and Its Application

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Abstract: An approach of self-learning fuzzy sliding-mode control which combines fuzzy control with the sliding-mode control, is presented for the tracking control of a class of nonlinear systems with the parameter uncertainties. The fuzzy control rules are updated through on-line learning, which make the output of fuzzy control approximate to sliding-mode equivalent control along the direction of making sliding-mode asymptotic stable. Based on Lyapunov theory, the asymptotic stability of the overall systems is proved. The proposed method is applied to some electrohydraulic servo systems, and the results of simulation show that the satisfied control precision and stability can be obtained by using proposed method for the systems.

1. INTRODUCTION

Fuzzy Control (FC) has already made great success in practical application, but FC generally relies on the experts' experience and experiments. It is difficult to analyze the stability of fuzzy control systems and obtain satisfied accuracy for some complex control objects. Sliding mode control (SMC) is a better robust control method. Once the states of controlled systems enter the sliding mode, the dynamic characteristics of overall system are determined by the designed sliding surface and independent of uncertainties. But in the practice control, the chattering phenomena in the sliding mode due to switching operating and high gain of the SMC influence the tracking accuracy and limit its application. Recently much research has been done to apply the fuzzy sliding mode control (FSMC), which includes the advantages of both FC and SMC. The References (Swiniarski R., 1990; Palm R., 1992) introduced the design means of FC into conventional SMC, which can get better consequences than FC, and eliminated the chattering of SMC to the great extent. Reference (Palm R., 1992) showed that sliding mode control with bounding layer was equivalent to FC in some meanings. Because no general stability analysis tools could be applied to fuzzy systems, reference (Kim S.W., Lee J.J., 1995) put forward a design method of FC with fuzzy sliding surfaces and Lyapunov theory could be used to testify the stability of FSMC system and the bounded properties for tracking error. Reference (Lin S.C., Chen Y.Y., 1994) presented a method of designing adaptive fuzzy sliding mode controller, the parameters in the fuzzy rule base can be updated in the terms of the adaptive algorithm and guaranteed that the states of systems can reach the prearranged sliding surfaces and slide along it. This paper proposes a control means of fuzzy sliding mode control with self-learning capability and the fuzzy logic system is employed to approximate the equivalent control of sliding mode in a compact set. Then the presented method is used to the tracking control of electrohydraulic servo system and satisfied simulation results are obtained.

2. DESCRIPTION OF PROBLEM

Consider the following system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + b(x)u + d(t), \quad y = x_1 \end{cases} \tag{1}$$

Where $x = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is system's state vectors, $b(x) > 0, f(x)$ are nonlinear scalar functions or either one is nonlinear function, and their value isn't exactly known, u is the control input of system and y is the output of system. $d(t)$ is a bounded disturbance. The control goal for the systems is that the output y can precisely track the desired output x_d when $b(x), f(x), d(t)$ contain some uncertainties.

3. DESIGN OF SELF-LEARNING FUZZY SLIDING MODE CONTROLLER

3.1 Sliding mode controller

Take tracking error as

$$e = x_d - x_1 = \left[e\,\dot{e}\cdots e^{(n-1)}\right]^T \tag{2}$$

Substituting e into (1) the error state equation of system is defined as follows:

$$e^{(n)} = x_d^{(n)} - x^{(n)} = -(f(x) + b(x)u + d(t)) + x_d^{(n)} \tag{3}$$

According to the theories of Sliding Control, choose the switching function for sliding mode as

$$s = c^T e = c_1 e + c_2 \dot{e} + \ldots + e^{(n-1)}, (c_i > 0) \quad (4)$$

According to $\dot{s}$ =0, we can get equivalent control of sliding mode

$$u_{eq} = \frac{1}{b}(-f - d + x_d^{(n)} + \sum_{i=1}^{n-1} c_i e^{(i)}) \quad (5)$$

If the accurate values of b, f, d have been known, from (5) we can directly get u_{eq}. Based on the theory of sliding mode control, control law is chosen as

$$u = u_{eq} + u_s \quad (6)$$

Where u_s is a nonlinear control, its function is to produce sliding mode. In practical control course, there often exist some uncertainty factors in b, f, d, such as the parameter uncertainties caused by nonlinear or disturbance etc., which made equivalent control not able to be gained directly. Under above situation, a fuzzy system is employed to approximate sliding mode equivalent control u_{eq}. Then (6) becomes

$$u = u_f + u_s \quad (7)$$

3.2 Design of fuzzy controller

Choose s and $\dot{s}$ as the input of fuzzy control, then the output u_f of fuzzy controller can be defined from the following m fuzzy control rules $R_1, R_2, ..., R_m$.The general form of *j*-th rule can be described as

$$R_j : If \quad s \; is \; A_{1j}(m_{1j}, \sigma_{1j}) \quad and \quad \dot{s} \; is \; A_{2j}(m_{2j}, \sigma_{2j})$$

Then $u_f^j = p_j \; (j = 1,2, \cdots m)$ (8)

Where $A_{1j}(m_{1j}, \sigma_{1j})$ and $A_{2j,}(m_{2j}, \sigma_{2j})$ are the fuzzy subsets of $A_1(m_1, \sigma_1)$ and $A_2(m_2, \sigma_2)$ respectively, $A_1(m_1, \sigma_1)$ and $A_2(m_2, \sigma_2)$ are corresponding to fuzzy input s and $\dot{s}$ respectively. Here take Gauss membership function

$$\mu_A(x) = \exp[-((x - m)/\sigma)^2] \quad (9)$$

So, fuzzy set A can be presented as $A \; (m, \sigma_x)$,

u_f^j represents the output of fuzzy controller when the *j*-th rule is satisfied.

Define *j*-th rule firing strength as

$$w_j = \min(\mu_{A1j}(s), \mu_{A2j}(\dot{s})), (j = 1 \cdots m) \quad (10)$$

Where $\mu_{A1j}(s)$ and $\mu_{A2j}(\dot{s})$ are the values of membership function corresponding to s and $\dot{s}$ in the fuzzy subsets $A_{1j}(m_{1j}, \sigma_{1j})$ and $A_{2j}(m_{2j}, \sigma_{2j})$. Then according to the weighted average defuzzification method, we can get the final output

$$u_f = \frac{\sum_{i=1}^{m} w_i p_i}{\sum_{i=1}^{m} w_i} = \frac{1}{\sum w_i} w^T p \quad (11)$$

Where $w = [w_1, \cdots, w_m]^T$, $p = [p_1, \cdots p_m]^T$, p is an unknown consequent parameter vector, it can be gotten by the following learning control method.

3.3 Design of Learning law and u_S

The destination of learning is to realize that the output parameter p_j (consequent function) is updated online, then makes the final output u_f of fuzzy control gradually approximate sliding mode equivalent control u_{eq} along the direction in which sliding mode state is asymptotically stable. So that system states can retain on the sliding surface and move along it. In order to derive the learning laws, at first, make the assumptions as follows

Assumption 1: In the formulation (1) $b(x)$ and its estimated value $\hat{b}(x)$ satisfy the following conditions

$$b(x) > 0, \text{ and } \frac{1}{\beta} \leq \frac{b(x)}{\hat{b}(x)} \leq \beta \quad (\beta > 0) \quad (12)$$

The conditions above can be satisfied for the system considered in Section 4 of this paper. In fact, equation (12) can be obtained from a number of mechantronics systems (Suolin Duan 1999).

Generally $\hat{b}(x)$ can selected as

$$\hat{b}(x) = \sqrt{b_{\max} \cdot b_{\min}} = \text{Constant} \quad (13)$$

Assumption 2: The vector set $T = [s, \dot{s}]^T$ is a compact set and $T \in R^n$, exist a group of consequent parameter vector $p = p^* = [p_1^*, p_2^*, \cdots, p_m^*]^T$, and make the following condition to be satisfied

$$\sup_{T\in R^n}\left|u_{eq}-u_f^*\right|\le\varepsilon_0,(\varepsilon_0\ge 0) \quad (14)$$

Where $u_f^*=\frac{1}{\sum w_j}w^T p^*$, and let $p-p^*=\hat{p}$.

Then

$$u_f-u_f^*=\frac{1}{\sum w_j}w^T(p-p^*)=\frac{1}{\sum w_j}w^T\hat{p} \quad (15)$$

Differentiating $s(t)$ with respect to t along the trajectory of equation (1), and using (3), (5)，(6) gives

$$\dot{s}=b(u_{eq}-u_f-u_s)$$

Thus

$$s\dot{s}=sb(u_{eq}-u_f-u_s) \quad (16)$$

Choose a Lyapunav function candidate as follows

$$V=\frac{1}{2\hat{b}}s^2+\frac{1}{2}\hat{p}^T\hat{p} \quad (17)$$

Where $\hat{b}$ is selected according to (13). Differentiating above equation (17) in two sides, then

$$\dot{V}=\frac{1}{\hat{b}}s\dot{s}+\dot{\hat{p}}^T\hat{p} \quad =\frac{1}{\hat{b}}sb(u_{eq}-u_f-u_s)+\dot{\hat{p}}^T\hat{p}$$

$$\le s\beta(u_{eq}-u^*{}_f)-s\beta(u_f-u^*{}_f)-s\beta u_s+\dot{\hat{p}}^T\hat{p}$$

Select u_s as

$$u_s=k_0 s+k_1\,\mathrm{sgn}(s) \quad (18)$$

Where $k_0>0,\ k_1>0$. From (14), (15) and (17), we have

$$\dot{V}\le s\beta\varepsilon_0-s\beta\frac{1}{\sum w_j}w^T\hat{p}-s\beta k_0 s$$
$$-k_1 s\beta\,\mathrm{sgn}(s)+\dot{\hat{p}}^T\hat{p}$$

$$\le|s|\beta\varepsilon_0-|s|\beta k_1-\beta k_0 s^2+(\dot{\hat{p}}^T$$
$$-s\beta\frac{1}{\sum w_j}w^T)\hat{p}$$

Choose

$$\left.\begin{aligned}&\dot{\hat{p}}=\dot{p}=s\beta\frac{1}{\sum w_j}w\\&k_1>\varepsilon_0\end{aligned}\right\} \quad (19)$$

Then

$$\dot{V}<-\beta k_0 s^2<0 \quad (20)$$

The result, the formula (20) indicates that the stability of SMC systems can be guaranteed when u_f, output of fuzzy control is used to approximate sliding mode equivalent control u_{eq} and sliding mode state is asymptotically stable in terms of Lyapunov theory. When $t\to\infty$, the following dynamic characteristic can be achieved

$$s=c_1 e+c_2\dot{e}+,\cdots,+e^{(n-1)}=0 \quad (21)$$

Consequently, tracking error approaches zero along sliding surface. The following theorem can be obtained.

Theorem 1: For the described system (1), the control system and sliding mode state are asymptotically stable according to the Lyapunov theory when the control law and the defined learning law given by (7), (11), (18), (19) are used, Then when $t\to\infty$, the tracking error of the system convergence to zero along sliding mode surface.

4. SIMULATION RESULTS

Consider the following electro-hydraulic position servo system described by follows equations for some structure fatigue testing machine (Suolin Duan, 1999)

$$\begin{cases}\dot{x}_1=x_2\\\dot{x}_2=x_3\\\dot{x}_3=\alpha^T x+\Delta\alpha^T x+b(x)u+F\\y=x_1\end{cases} \quad (22)$$

Where $x=[x_1x_2x_3]^T=[y_p\dot{y}_p\ddot{y}_p]^T$, y_p is output displacement of the system, $\alpha=[\alpha_1\alpha_2\alpha_3]^T$ is nominal parameter vector in the systems, and $\alpha^T=\left[-11.5\times10^5\quad-9.34\times10^7\quad-13.8\times10^3\right]$, $\|\Delta\alpha\|<7\times10^5$ is the uncertainty of the parameter vector,

u is the control input, and control gain $b(x) = 2.43\times10^7 \sqrt{1-\mathrm{sgn}(u)C_p}$ (C_p<1), is a nonlinear function, F is the bounded disturbance, and $|F| \le 1.0\times10^2$. The design procedures of the fuzzy-sliding mode controller with self-learning for above system (22) are described as follows

1) Determine the linguistic variables and universe of discourse of the fuzzy control:

Choose s and $\dot{s}$ as linguistic variables, consider their universe of discourse as [-1,1], and the sliding mode function is selected by

$$s = c_1 e + c_2 \dot{e} + \ddot{e} \tag{23}$$

Where $c_1 > 0, c_2 > 0, e = x_d - x_1$, x_d is the desired output signal.

2) Choose the linguistic values and corresponding

membership function. Every linguistic variable can individually take five linguistic values (5 fuzzy subsets). The corresponding Gauss membership functions are

$$\mu_{Ai1}(z_i) = \exp[-(\frac{z_i + 0.8}{\sigma})^2], z_i \in [-1,-0.6] \quad (24a)$$

$$\mu_{Ai2}(z_i) = \exp[-(\frac{z_i + 0.5}{\sigma})^2]\ z_i \in [-0.8,-0.2] \quad (24b)$$

$$\mu_{Ai3}(z_i) = \exp[-(\frac{z_i + 0}{\sigma})^2], z_i \in [-0.5,0.5] \quad (24c)$$

$$\mu_{Ai4}(z_i) = \exp[-(\frac{z_i - 0.5}{\sigma})^2],\ z_i \in [0.3,0.8] \quad (24d)$$

$$\mu_{Ai5}(z_i) = \exp[-(\frac{z_i - 0.8}{\sigma})^2],\ z_i \in [0.6,1] \quad (24e)$$

Where i =1, 2 is corresponding to the linguistic variables s and $\dot{s}$ respectively. Then, σ_x =0.01, the corresponding universe of discourse for the linguistic values are

$$\left\{\begin{matrix} A_{i1}(-0.8,\sigma), A_{i2}(-0.5,\sigma), A_{i3}(0,\sigma), \\ A_{i4}(0.5,\sigma), A_{i5}(0.8,\sigma) \end{matrix}\right\} \quad (i = 1,2)$$

3) Build fuzzy rules bases

To constitute fuzzy rules bases, 25 fuzzy rules are taken

R_1**:** if s is A_{11} (-0.8,σ_x) and $\dot{s}$ is A_{21} (-0.8,σ_x) Then $u_f^1 = p_1$

$\vdots$

R_{25}**:** if s is A_{15} (0.8,σ_x) and $\dot{s}$ is A_{25}(0.8,σ_x) Then $u_f^{25} = p_{25}$

4) From the fuzzy inference (Maximum and minimum Reasoning) in (10) the firing strength w_j of every rule can be determined.

5) According to (19), take **l**earning laws as

$$p_j(k) = p_j(k-1) + s\beta \frac{1}{\sum w_j} w_j$$

6) Determine the output u_f of the fuzzy controller in terms of (10), and the initial value of p_j is often taken as 0.

7) Choose u_s according to (18).

8) From (7) we can get the control input u of self-learning fuzzy sliding mode controller.

The simulation results are illustrated in Fig. 1 to Fig.4. Fig.1 and Fig.2 show the response results of tracking square wave signal with frequency 2.5Hz and 0.5Hz by using the self-learning FSMC scheme. Fig.3 and Fig.4 respectively illustrate the error curves of tracking 2.5Hz and 0.5Hz square wave. The simulation results show that the presented self-learning FSMC is of good tracking characteristics for the square wave signal with different frequency.

5. CONCLUSIONS

Based on the idea of combining fuzzy control and sliding mode control, the self-learning fuzzy sliding mode control method is presented in this paper. The proposed method is applied to an electrohydraulic servo systems, the results of simulation show that the satisfied tracking characteristics and stability can be obtained by using the proposed method for the systems to tracking the square wave signal with different frequency.

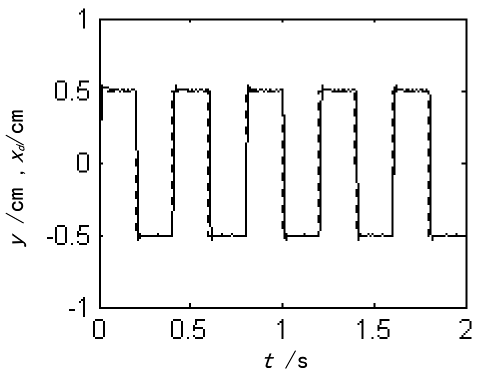

Fig. 1.Response of tracking 2.5Hz square wave signal

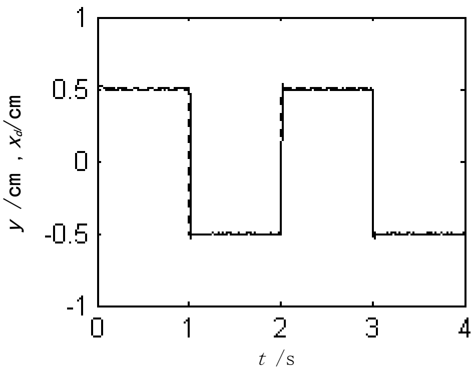

Fig.2.Response of tracking 0.5Hz square wave signal

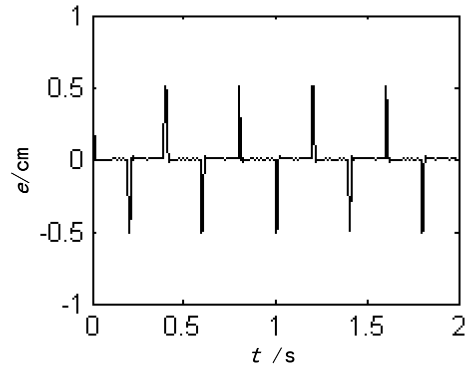

Fig.3. Error curve of tracking 2.5Hz square wave

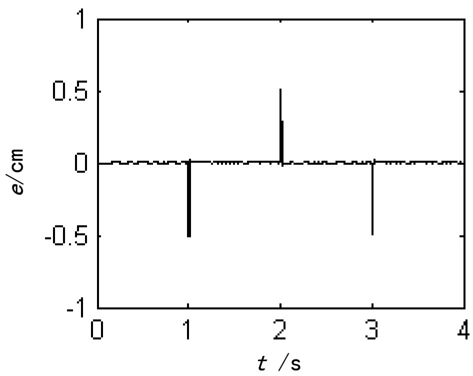

Fig.4. The error curve of tracking 0.5Hz square wave

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