

## Stress-Dependent Hysteresis Modeling and Tracking Control of Giant Magnetostrictive Actuator for Periodic Reference Input

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**Abstract:** The tracking control accuracy of giant magnetostrictive actuator (GMA) is limited due to its inherent hysteresis nonlinearity. Hysteresis characteristic of GMA is stress-dependent. This paper proposes a stress-dependent Prandtl-Ishlinskii (SDPI) hysteresis model for GMA by extending the modified Prandtl-Ishlinskii (MPI) model to account for the hysteresis of GMA at varying compressive stress. It is shown experimentally that the weights of play operators is non-sensitive to the compressive stress applied in longitudinal direction of GMA and the relationships between weights of deadzone operators and compressive stress could be modeled by quadratic functions. Then the inverse of SDPI model is established and implemented in open-loop feedforward controller for real-time tracking control for periodic reference input. Comparisons are made between feedforward controller with stress-dependent hysteresis model and stress-independent model. Experimental results show that tracking performance is noticeably improved by using SDPI model.

### 1. INTRODUCTION

Giant magnetostrictive actuator (GMA) has great potential in many applications including active vibration, micropositioning in high force regimes due to its capability for generating high force and high strain. Fig. 1 shows a schematic of magnetostrictive actuator. The displacement is provided by the magnetostrictive rod in response to the varying magnetic field generated by the surrounding solenoid. Prestressing the rod with preload spring serves to increase the performance of GMA and place the material in compression.

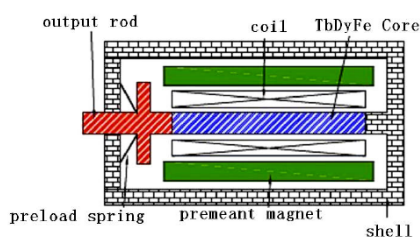


Fig. 1. Schematic of magnetostrictive actuator

A major setback of GMA is its hysteresis behavior, which give rise to undesirable inaccuracies and oscillations. A fundamental idea in coping with hysteresis is to formulate the mathematical model of hysteresis and use inverse compensation to cancel out the hysteretic effect. Hysteresis model of hysteresis can be classified into phenomenological

models models and physics-based. Preisach model is the most popular phenomenological model (Krasnosel'skii and Pokrovskii, 1989; Mayergoyz, 1991, Tan and John, 2002). Another important subclass of Preisach model is the Prandtl-Ishlinskii (PI) model, the main advantage of it is that it is simpler and its inverse can be computed analytically, which is attractive for real-time control. The PI model has odd symmetry property to the center point of the corresponding loop, which is too restrictive for real complex hysteresis actuator nonlinearities. A modified PI (MPI) model was documented to overcome the overly restriction of classical PI model by combining deadzone operators in series with classical PI model (Kuhnen, 2003). Physics-based hysteresis models of GMA include J-A model (Jile and Atherton, 1986) and Free Energy Model (Smtih etal, 2003)

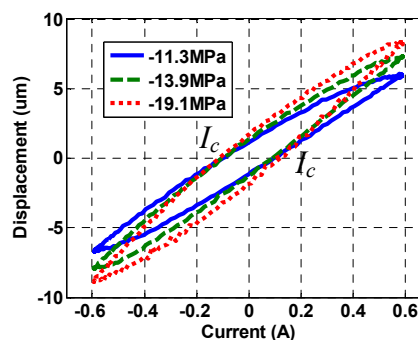


Fig. 2. Stress-dependent hysteresis of GMA

Due to strong coupling between magnetic and mechanical properties for giant magnetostrictive core, hysteresis characteristic of GMA is stress-dependent, i.e., relationship between input current and output displacement depends on the compressive stress applied in longitudinal direction of GMA, shown in Fig 2. Unfortunately, most of hysteresis models are stress-independent and little work has been done to model effect of stress on the hysteresis behavior of GMA (Kvarnsjo and Bergqvist, 1992; Yamamoto et al., 2003).

In this paper, a stress-dependent PI (SDPI) model is proposed based on the MPI model to account for hysteresis nonlinearity of GMA at varying compressive stress applied in longitudinal direction of GMA. We implement the SDPI with inverse feedforward control and compare the experimental results with the stress-independent case.

## 2. MODIFIED PRANDTL-ISHLINSKII MODEL

This section will describe the modeling of hysteresis using the modified PI operator (Kuhnen, 2003).

### 2.1 A Prandtl-Ishlinskii (PI) operator

The elementary hysteretic kernel in the PI hysteresis is a rate-independent backlash or linear-play operator  $H_r$ , shown in fig. 3, which is defined by

$$y(t) = H_r[x, y_0](t) = \max\{x(t) - r, \min\{x(t) + r, y(t_i)\}\} \quad (1)$$

for piecewise monotonous input signals with a monotonicity partition  $t_0 \leq t_1 \dots t_i \leq t \leq t_{i+1} \leq \dots \leq t_N$ , where  $x$  is the input signal and  $y$  is output signal,  $r$  is threshold value of the operator.

The initial consistency condition of (1) is given by

$$y(t_0) = \max\{x(t_0) - r, \min\{x(t_0) + r, y_0\}\} \quad (2)$$

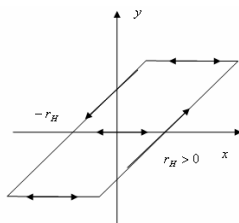


Fig. 3 Characteristic of the play operator

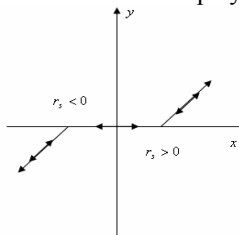


Fig. 4 Characteristic of the dead-zone operator

for the output signal at initial time  $t_0$ . The play operator depends on the independent initial value  $y_0 \in \mathfrak{R}$  of the output and is characterized by its threshold parameter  $r \in \mathfrak{R}^+$ .

Complex hysteretic nonlinearity can be modeled by the so-called threshold-discrete Prandtl-Ishlinskii hysteresis operator by the linear weighted superposition of many play operators with different threshold values. From this follows

$$y(t) = \mathbf{w}_h^T \cdot \mathbf{H}_{r_h}[x, \mathbf{y}_0](t) \quad (3)$$

with weight vector  $\mathbf{w}_h^T = [w_{h0} \dots w_{hn}]$ , the threshold vector  $\mathbf{r}_h = [r_{h0} \dots r_{hn}]^T$  with  $0 = r_{h0} < r_{h1} < \dots < r_{hn} < +\infty$ , the vector of the initial states  $\mathbf{y}_0 = [y_{00} \dots y_{0n}]^T$  and the vector of the play operator  $\mathbf{H}_r[x, \mathbf{y}_0](t) = [H_{r_{h0}}[x, y_{00}](t) \dots H_{r_{hn}}[x, y_{0n}](t)]^T$ .

### 2.2 Modified Prandtl-Ishlinskii operator

The closed loops of PI operator have an odd symmetry property to the center point of the corresponding loop. The fact that most real actuator hysteresis loops are not symmetric reduces its applicability in practice. To overcome this overly restrictive property, modified Prandtl-Ishlinskii hysteresis modeling approach has been developed by combining a so-called Prandtl-Ishlinskii superposition operator (PISO) in series with the hysteresis operator. PISO is a weighted linear superposition of one-sided dead zone operators. A dead zone operator is a non-convex, non-symmetrical, and memory-free nonlinear operator given by

$$S(x(t), r_s) = \begin{cases} \max\{x(t) - r_s, 0\}; & r_s > 0 \\ x(t) & ; r_s = 0 \\ \min\{x(t) - r_s, 0\}; & r_s < 0 \end{cases} \quad (4)$$

This elementary superposition operator is also fully characterized by a threshold parameter  $r_s \in \mathfrak{R}$  shown in fig. 4. The complex superposition operator for the approximation of more general continuous memory-free nonlinearities is given by

$$S[x](t) = \mathbf{w}_s^T \cdot \mathbf{S}_{r_s}[x](t) \quad (5)$$

with the vector of weights  $\mathbf{w}_s^T = (w_{s-l} \dots w_{s0} \dots w_{s-l})$ , the vector of thresholds  $\mathbf{r}_s^T = (r_{s-l} \dots r_{s0} \dots r_{s-l})$  with  $-\infty < r_{s-l} < \dots < r_{s0} = 0 < \dots < r_{s-l} < +\infty$  and the vector of the one-sided dead-zone operators  $\mathbf{S}_r^T = (S_{r_{s-l}} \dots S_{r_{s0}} \dots S_{r_{s-l}})$ .

The modified PI operator is thus

$$y(t) = \Gamma[L, \mathbf{y}_0](t) = \mathbf{w}_s^T \cdot \mathbf{S}_{r_s}[\mathbf{w}_h^T \cdot \mathbf{H}_{r_h}[L, \mathbf{y}_0]](t) \quad (6)$$

### 2.3 Inverse MPI operator

For MPI model, a linear inequality constraints has been given to guarantee the existence of inverse of (6) uniquely (Kuhnen, 2003). The inverse MPI operator is given by

$$\Gamma^{-1}[y](t) = \mathbf{w}_h^T \cdot \mathbf{H}_{\mathbf{r}_h} [\mathbf{w}_s^T \cdot \mathbf{S}_{\mathbf{r}_s} [y, \mathbf{y}_0]](t) \quad (7)$$

$$r_{hi}' = \sum_{j=0}^i w_{hj} (r_{hi} - r_{hj}); \quad i = 0 \dots n \quad (8)$$

$$w_{h0}' = \frac{1}{w_{h0}},$$

$$w_{hi}' = - \frac{w_{hi}}{(w_{h0} + \sum_{j=1}^i w_{hj})(w_{h0} + \sum_{j=1}^{i-1} w_{hj})}; \quad i = 1 \dots n \quad (9)$$

$$y_{0i}' = \sum_{j=0}^i w_j y_{0i} + \sum_{j=i+1}^n w_j y_{0j} \quad ; \quad i = 0 \dots n \quad (10)$$

$$r_s \in \mathfrak{R}_0^+,$$

$$r_{si}' = \sum_{j=0}^i w_{sj} (r_{si} - r_{sj}) \quad ; \quad i = 0 \dots l \quad (11)$$

$$w_{si}' = - \frac{w_{si}}{(w_{s0} + \sum_{j=1}^i w_{sj})(w_{s0} + \sum_{j=1}^{i-1} w_{sj})}; \quad i = 1 \dots l \quad (12)$$

$$r_s \in \mathfrak{R}^-,$$

$$r_{si}' = \sum_{j=i}^0 w_{sj} (r_{si} - r_{sj}) \quad ; \quad i = -l \dots 0 \quad (13)$$

$$w_{s0}' = \frac{1}{w_{s0}}$$

$$w_{si}' = - \frac{w_{si}}{(w_{s0} + \sum_{j=i}^{-1} w_{sj})(w_{s0} + \sum_{j=i+1}^{-1} w_{sj})}; \quad i = -l \dots -1 \quad (14)$$

#### 2.4 Parameter Identification

In the stress-independent MPI model formulated by (6), parameters  $\mathbf{r}_s$ ,  $\mathbf{r}_h$ ,  $\mathbf{w}_s$ ,  $\mathbf{w}_h$  should be determined and a least square scheme is proposed to identify the parameters of MPI operator (Ang et al ,2007, Zhang and Mao, 2007). We first have to measure the hysteresis  $y$  vs  $I$  experimentally. We set the threshold  $\mathbf{r}_s'$  and  $\mathbf{r}_h$ , which are usually chosen to be equal intervals. Then the weights  $\mathbf{w}_h$  and  $\mathbf{w}_s'$  are determined by performing the least squares fit of (15), and  $\mathbf{r}_s$  and  $\mathbf{w}_s$  can be calculated by (11) ~ (14).

$$\mathbf{w}_s^T \cdot \mathbf{S}_{\mathbf{r}_s} [y](t) = \mathbf{w}_h^T \cdot \mathbf{H}_{\mathbf{r}_h} [x, \mathbf{y}_0](t) \quad (15)$$

### 3. STRESS-DEPENDENT PRANDTL-ISHLINSKII (SDPI) HYSTERESIS MODEL

#### 3.1 Stress-Dependent hysteresis Model

In this section we propose an extension to MPI model to model the stress-dependent characteristics of GMA hysteresis. One of the advantage of the MPI hysteresis model is that it is purely phenomenological, which means that it is not necessary to establish the relationship between the modeling parameters and the physics mechanism of the hysteresis. In this paper, we would model the stress-dependent hysteresis with reference only to the experimental observations.

It could be found from Fig 2 that changes of the  $I_c$  at varying compressive stress is very small. By analogy with coercive force  $H_c$  of ferromagnetic material, we call  $I_c$  as 'coercive current'. It is apparent from (6) that the  $I_c$  is just determined by the weights of the play operators  $\mathbf{w}_h$  with holding values of  $\mathbf{r}_h$  and  $\mathbf{r}_s$ . It could reasonably be thought that the  $\mathbf{w}_h$  of play operators are non-sensitive to compressive stress in longitudinal direction of GMA with constant  $\mathbf{r}_h$  and  $\mathbf{r}_s$ . Hence we think that the weights of play operators is not stress-dependent and hold it, as well as the threshold value  $\mathbf{r}_s$  and  $\mathbf{r}_h$  constant while attempting to construct a relationship between the weights of the deadzone operators  $\mathbf{w}_s$  and the compressive stress applied in longitudinal direction of GMA. The SDPI hysteresis model is expressed as (16).

$$y(t) = \Gamma[I, \sigma, \mathbf{y}_0](t) \quad (16)$$

$$= \mathbf{w}_s^T(\sigma) \cdot \mathbf{S}_{\mathbf{r}_s} [\mathbf{w}_h^T \cdot \mathbf{H}_{\mathbf{r}_h} [I, \mathbf{y}_0]](t)$$

the vector of the weights  $\mathbf{w}_s^T(\sigma) = [w_{s-l}(\sigma), \dots, w_{s0}(\sigma), \dots, w_{si}(\sigma)]$ , element  $w_{si}(\sigma)$  is the  $i$ th deadzone operator weight, which is a function of compressive stress and modelled experimentally.

#### 3.2 Inverse SDPI operator

A new constraint condition is given in proposition 3.1 to guarantee the unique existence of inversion of (16).

#### Proposition 3.1

For SDPI model with  $\mathbf{r}_h \in \mathfrak{R}_+^n$ ,  $\mathbf{r}_s \in \mathfrak{R}^{2l+1}$ ,  $\mathbf{w}_h \in \mathfrak{R}^{n+1}$ ,  $\mathbf{w}_s(\sigma) : \mathfrak{R}_- \mapsto \mathfrak{R}^{2l+1}$ , suppose that there exists  $\varepsilon > 0$

$$\left\{ \begin{matrix} \mathbf{U}_H \\ \mathbf{U}_S \end{matrix} \right\} \cdot \left\{ \begin{matrix} \mathbf{w}_H \\ \mathbf{w}_S(\sigma) \end{matrix} \right\} \geq \left\{ \begin{matrix} \mathbf{u}_H \\ \mathbf{u}_S \end{matrix} \right\},$$

$$\mathbf{U}_H = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix} \in \mathfrak{R}^{(n+1) \times (n+1)}, \quad \mathbf{u}_H = \begin{bmatrix} \varepsilon \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathfrak{R}^{n+1},$$

$$\mathbf{U}_s = \begin{bmatrix} 1 & \cdots & 1 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 1 & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathfrak{R}^{2l+1 \times 2l+1}, \mathbf{u}_s = \begin{bmatrix} \varepsilon \\ \vdots \\ \varepsilon \\ \varepsilon \\ \vdots \\ \varepsilon \end{bmatrix} \in \mathfrak{R}^{2l+1}$$

then the inverse model of SDPI exists uniquely anywhere as

$$\Gamma^{-1}[y](t) = \mathbf{w}_h^T \cdot \mathbf{H}_{r_h} \cdot [\mathbf{w}_s^T(\sigma) \cdot \mathbf{S}_{r_s} [y, \mathbf{y}_0]](t) \quad (17)$$

**Proof.** The proof of it is apparent.  $\forall \sigma_s \in \mathfrak{R}_-$ , proposition 3.1 is the same as linear inequality constraints for MPI (Kuhnen, 2003) with weights vector  $\mathbf{w}_s(\sigma_s)$ , so the inverse of SDPI model exists uniquely anywhere.

The inverse model parameters of play operators are still expressed as (8) ~ (10), those of deadzone operators are expressed as

$$r_s \in \mathfrak{R}_0^+, \quad r_{si}' = \sum_{j=0}^i w_{sj}(\sigma)(r_{si} - r_{sj}) \quad ; i = 0 \dots l \quad (18)$$

$$w_{si}'(\sigma) = \frac{w_{si}(\sigma)}{(w_{s0}(\sigma) + \sum_{j=1}^i w_{sj}(\sigma))(w_{s0}(\sigma) + \sum_{j=1}^{i-1} w_{sj}(\sigma))} ; i = 1 \dots l \quad (19)$$

$$r_s \in \mathfrak{R}^-, \quad r_{si}' = \sum_{j=i}^0 w_{sj}(\sigma)(r_{si} - r_{sj}) \quad ; i = -l \dots 0 \quad (20)$$

$$w_{s0}'(\sigma) = \frac{1}{w_{s0}(\sigma)} \quad (21)$$

$$w_{si}'(\sigma) = \frac{w_{si}(\sigma)}{(w_{s0}(\sigma) + \sum_{j=i}^{-1} w_{sj}(\sigma))(w_{s0}(\sigma) + \sum_{j=i+1}^{-1} w_{sj}(\sigma))} \quad (22)$$

$$i = -l \dots -1$$

### 3.3 Parameter Identification

In the SDPI model formulated by (16), parameters  $\mathbf{r}_s$ ,  $\mathbf{r}_h$ ,  $\mathbf{w}_h$  and weights function  $\mathbf{w}_s(\sigma)$  should be determined. The following scheme is employed to identify them:

Step 1. First we measure the hysteresis  $y$  vs  $I$  with a constant compressive stress  $\sigma_0$  experimentally. In this case the compressive stress generated only by preload spring and support plate is taken as  $\sigma_0$ . From experimental data, parameters  $\mathbf{r}_s$ ,  $\mathbf{r}_h$ ,  $\mathbf{w}_s$ ,  $\mathbf{w}_h$  could be identified by method as section II described.

Step 2. A series of hysteresis  $y-I$  at differential compressive stress  $\sigma_1, \dots, \sigma_m$  are measured. Holding

parameters  $\mathbf{r}_s$ ,  $\mathbf{r}_h$ ,  $\mathbf{w}_h$  constant value, which are identified in step 1, the weights of deadzone operators  $\mathbf{w}_s^{\sigma_i T} = [w_{s-l}^{\sigma_i}, w_{s0}^{\sigma_i}, \dots, w_{s+l}^{\sigma_i}]$  at compressive stress  $\sigma_i$  could be identified by performing the least squares fit of (21).

$$y(t) = \Gamma[I, \sigma_i, \mathbf{y}_0](t) = \mathbf{w}_s^{\sigma_i T} \cdot \mathbf{S}_{r_s} [\mathbf{w}_h^T \cdot \mathbf{H}_{r_h} [x, \mathbf{y}_0]](t) \quad (21)$$

Step 3. A series of weights of deadzone operators  $\mathbf{w}_s^{\sigma_0}, \mathbf{w}_s^{\sigma_1}, \dots, \mathbf{w}_s^{\sigma_m}$  are used to model the weights function  $\mathbf{w}_s^T(\sigma) = [w_{s-l}(\sigma), w_{s0}(\sigma), \dots, w_{s+l}(\sigma)]$  satisfying proposition 3.1.

## 4. MODELING EXPERIMENTS

In this section, the proposed SDPI model is used to model stress-dependent hysteresis of GMA (manufactured by BEIHANG University, which measures  $\phi 50mm \times 200mm$  with  $\phi 7mm$  core rod). Experimental setup is shown as Fig. 5. The current input is provided by a power supply (Model GF-20) operating in current mode, which is controlled by a Pentium computer with a dSPACE DS1103 PPC Controller Board. The displacement of the actuator is measured with eddy current sensor (accuracy  $0.1\mu m$ ). It should be noticed that hysteresis of GMA is rate-dependent, which means that hysteresis characteristics is relative to rate of input current and has been well documented (Tan and John, 2004; Zhang and Mao, 2007). In this paper, we only discuss the effect of stress on the hysteresis of GMA. In order to avoid the effect of rate of input signal, the frequency of input signal used to modeling and tracking control is 3hz as hysteresis of GMA is thought as rate-independent below 5hz (Tan and John, 2004).

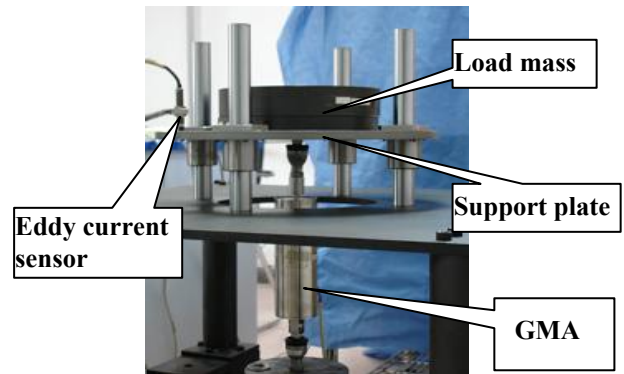


Fig. 5 Experimental setup

We let  $n=9$  (ten play operators) and  $l=3$  (seven deadzone operators) for both SDPI model and stress-independent MPI model. The responses of GMA are measured subject to sinusoidal input current with 3Hz at different compressive stress applied in longitudinal direction of GMA over range of  $-11.3 \sim -24MPa$  (from zero load mass to 50Kg load mass). We perform the SDPI parameters identification for the measured response of GMA at different compressive stress, the weights  $\mathbf{w}_s^T(\sigma) = [w_{s-l}(\sigma), w_{s0}(\sigma), \dots, w_{s+l}(\sigma)]$  are plotted against the compressive stress as shown in Fig.6 and

is modeld by Matlab polynomial curve fitting toolbox with degree of 2, which satisfy proposition 3.1.

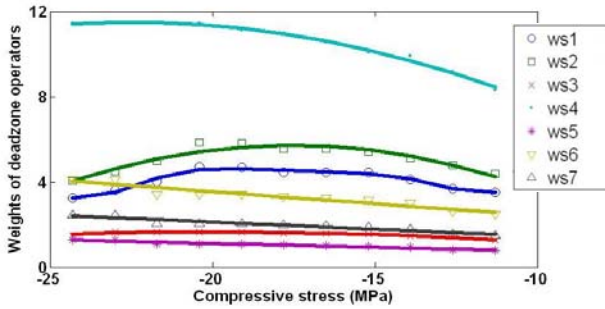


Fig. 6 Variation of weights of the deadzone operators respect to the compressive stress

Fig. 7 shows the stress-dependent hysteresis comparison between measured in experiments and simulation results. It is seen that the simulation results agree with the experimental results well.

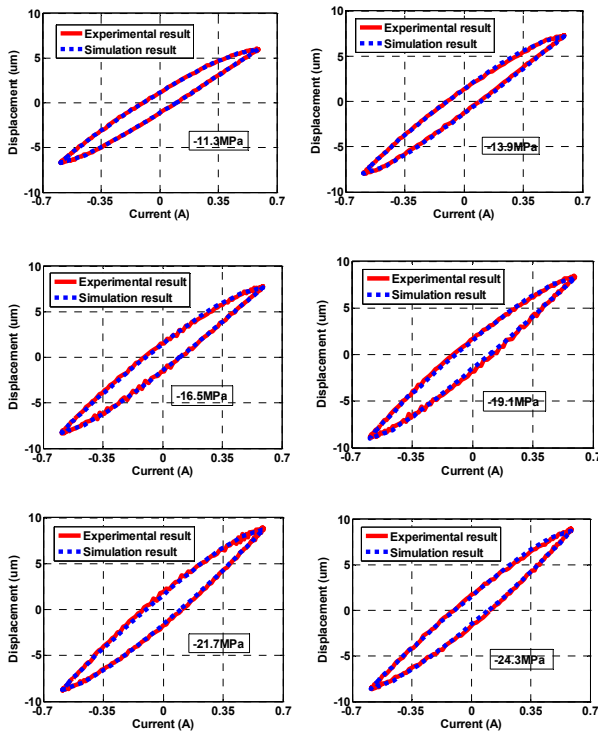


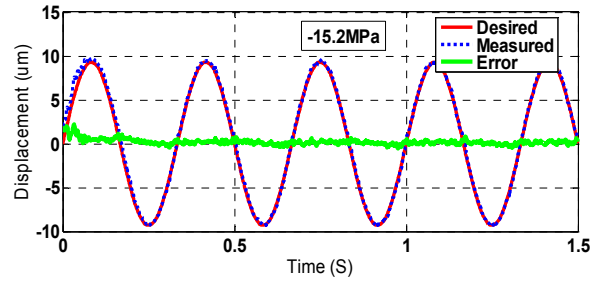
Fig. 7 Hysteresis loop of GMA at different compressive stress

### 6. TRACKING EXPERIMENTS

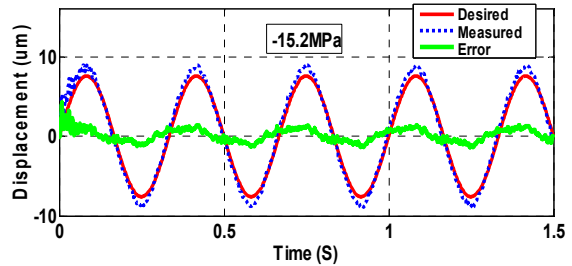
In this section tracking control experiments for periodic reference input are performed and comparisons are made between feedforward controller with inverse SDPI hysteresis model and with inverse stress-independent MPI hysteresis model in tracking a 3Hz periodic reference input.

Fig. 8 and Fig. 9 compare the experimental results of SDPI and stress-independent model tracking a 3Hz sinusoidal waveform at -15.2MPa and -19.1MPa stress respectively. Experimental results show that tracking performance is

noticeably improved by using SDPI model. The rms error in tracking periodic reference is less half that obtained using stress-independent MPI model.

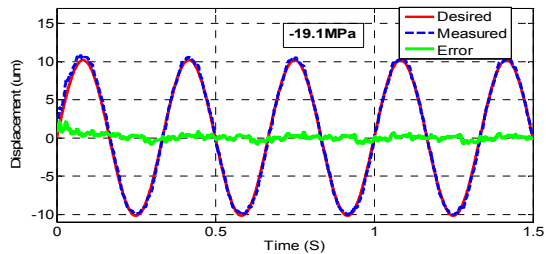


(a). stress-dependent

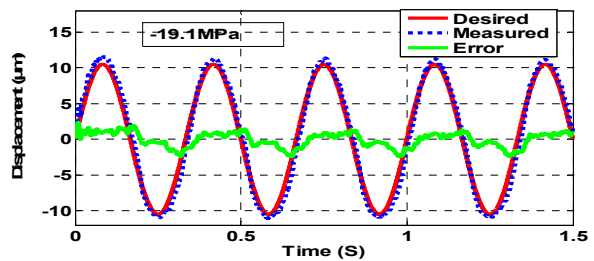


(b). stress-independent

Fig. 8. Experimental open-loop tracking results of a 3Hz sinusoid input at -15.2MPa compressive applied in longitudinal direction of GMA. (a) stress-dependent, rms tracking error is 0.3877µm. (b) stress-independent, rms tracking error is 0.8231µm .



(a). stress-dependent



(b). Stress -independent

Fig. 9. Experimental open-loop tracking results of a 3Hz sinusoid input at -19.1MPa compressive applied in longitudinal direction of GMA. (a) stress-dependent, rms tracking error 0.3942µm. (b) stress-independent, rms tracking error 1.0422µm .

## 5. CONCLUSION

We present a SDPI hysteresis model to account for the effect of the stress on the hysteresis of GMA and implement it in feedforward control scheme for motion tracking control of GMA for periodic reference input. The proposed model establishes the relationship between the weights of deadzone operators and compressive stress. Compared with stress-independent model, SDPI model yields significant better experimental results.

Some discussions of its application and future work consideration are below:

1. Nearly all hysteretic nonlinearities which occur in smart material based sensor and actuator characteristics including GMA have a non-local memory structure, a possible consequence of this complex memory structure are closed major and minor loops and intersection of branches in the same direction of the input signal (Kuhnen, 2003; Mayergoyz, I.D. (1991). The proposed SDPI model could describe the hysteretic behavior with this non-local memory structure although only major loop experiments are performed in this paper because its model structure is the same as MPI having ability to describe it (Kuhnen, 2003).

2. Although compressive stress ranging from -11.3MPa to -24.3MPa is included in the modeling experiments in this paper, the proposed SDPI is still valid for more range of compressive stress as long as changes of 'coercive current'  $I_c$  are small enough.

## 6. Acknowledgement

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