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Neural network adaptive robust tracking control for uncertain robotic systems with delays ⋆

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Abstract: In this paper, the problem of the robust tracking for a class of uncertain robotic systems with delays is investigated. The uncertainty is nonlinear time-varying and does not require a matching condition. A reference model with the desired amplitude and phase properties is given to construct and error model. A neural network system is used to approximate an unknown controlled system from the strategic manipulation of the model following tracking errors. Based on the Lyapunov method and the linear matrix inequality (LMI) approach, several sufficient conditions, which guarantee the state variables of the closed loop system to converge, globally, uniformly and exponentially, to a ball in the state space with any pre-specified convergence rate, are derived. Numerical examples are given to illustrate the proposed method.

1. INTRODUCTION

For the past several years, there have been a lot of interests in applying artificial neural networks (NNs) to solve the problems of identification and control of complex nonlinear systems by exploiting the nonlinear mapping abilities of the NN. At the same time, extensive investigations have been carried out design NN controllers for robot manipulators (Tseng [2001], Kim [2000]). In addition to the nonlinearities and uncertainties, robotic systems with unknown delayed states are often encountered in practice. However, to the best of our knowledge, there is little result about the robust intelligent control scheme for the uncertain robotic systems with delays by now. In this paper, we consider the problem of robust stabilization for a class of uncertain robotic systems with multiple delayed state perturbations. Both the adaptive robust control scheme with known parameters, and the robust neural control method for unknown parameters of robotic systems, are proposed simultaneously. A reference model with the desired amplitude and phase properties is given to construct an error model. An NN system is used to represent the unknown controlled system with the desired accuracy to any degree from the strategic manipulation of the model following tracking errors. The stability and robustness properties of the proposed control scheme are established in the Lyapunov theory framework and LMI approach. The results demonstrate the feasibility of the proposed control scheme, which can guarantee parameter estimation convergence and stability robustness of the closed-loop system.

Our main contributions are as follows. First, the plants under our consideration are of multiple delays occurring in the state variables. Second, our design procedure is based upon a group of linear matrix inequalities (LMIs), whose numerical solutions can be effectively obtained by LMI toolbox in Matlab and, what is more, through which there is no need to fix a prior some parameters. Finally, the performance indices of the closed loop systems have a clear relationship with the design parameters in our controllers.

2. PRELIMINARIES

In this section, dynamical models of robotic manipulators with uncertainties will be presented in detail. According to Lagrange theory (Yi [1997]), dynamical equations of robotic manipulators with n serial links incorporating external disturbances can be expressed as

$$\begin{aligned} M(q)\ddot{q}(t) + C^{'}(q,\dot{q})\dot{q}(t) + G^{'}(q) + F^{'}(q,\dot{q}) \\ = \tau(t) + D^{'}(t), \qquad (1) \end{aligned}$$

where $\ddot{q}$, $\dot{q}$, $q \in R^n$ are vectors of joint accelerations, velocities, and positions, respectively. $M(q) \in R^{n\times n}$ is a symmetric positive definite inertia matrix; $C^{'}(q,\dot{q})q \in R^n$ being a vector of centripetal and Coriolis forces. $G^{'}(q) \in R^n$ denotes gravity vectors and $F^{'}(q,\dot{q}) \in R^n$ includes friction terms and unmodeled dynamics; $D^{'}(t) \in R^n$ is the external disturbances; $\tau(t) \in R^n$ represents torque vectors exerting on joints. It is assumed that vectors $\dot{q}$, q are measurable. The following are the properties of the robotic dynamics:

Proposition 1. The inertia matrix $M(q)$ is symmetric positive definite for every q. Both $M(q)$ and $M^{-1}(q)$ are uniformly bounded.

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Proposition 2. The matrix $\dot{M}(q) - 2C^{'}(q,\dot{q})$ is skew-symmetric for suitable representation of $C^{'}(q,\dot{q})$.

Proposition 3. $C^{'}(q,\dot{q})$ is bounded in q and linear in $\dot{q}$.

From Propositions 1-3, the equation of the manipulator can be written as

$$\begin{aligned}\ddot{q}(t) + C(q,\dot{q})\dot{q}(t) + G(q) + F(q,\dot{q})\\ = \bar{B}(q)\tau(t) + D(t),\end{aligned} \tag{2}$$

where $C(q,\dot{q}) = M^{-1}(q)C^{'}(q,\dot{q})$, $G(q) = M^{-1}(q)G^{'}(q)$, $F(q,\dot{q}) = M^{-1}(q)F^{'}(q,\dot{q})$, $\bar{B}(q) = M^{-1}(q)$ and $D(t) = M^{-1}(q)D^{'}(t)$. To simplify the notation the argument t is in many cases dropped out. Since each link transmitting the energy or the moment to the following links will have some delayed behavior due to inertia effect, the states with delayed uncertainties are unavoidable and should be included in the dynamical system. Therefore, (2) can be written as

$$\begin{aligned}&\ddot{q}(t) + C(q,\dot{q})\dot{q}(t) + G(q) + F(q,\dot{q})\\ &\qquad = \bar{B}(q)\tau(t) + \sum_{j=1}^{r} d_j(t)q(t-h_j) + D(t),\\ &q(t) = \phi(t), \quad t \in [-h, 0],\end{aligned} \tag{3}$$

where $\phi(t)$ is a continuous vector-valued initial function, $h = \max\{h_j,\ j = 1, 2, \cdots, r\}$, and $d_j(t)$, $j = 1, 2, \cdots, r$, are nonlinear time-varying continuous functions which represent the gains of the delayed state uncertainties for the system. In our control scheme, we divide the $d_j(t)$ into two parts: the known constant $\bar{d}_j$ and unknown function of time $\bar{d}_j(t)$, namely,

$$d_j(t) = \bar{d}_j + \bar{d}_j(t), \quad j = 1, 2, \cdots, r. \tag{4}$$

The reference model for the plant to follow is a linear time invariant stable system with a piecewise continuous and uniformly bounded input r_m, and the output q_m, related by

$$\ddot{q}_m + M_1\dot{q}_m + M_0 q_m = r_m, \tag{5}$$

where M_1 and M_0 are selected properly such that q_m has the desired response of the plant. Let $e = q - q_m$ denote the tracking error. In order to design a stable adaptive controller with robust tracking performance, we first select a set of parameter matrices F_1, F_0 such that the error matrix polynomial $\ddot{e} + F_1\dot{e} + F_0 e$ is a Hurwitz polynomial. Then, define z as

$$z = \ddot{q}_m - F_1\dot{e} - F_0 e. \tag{6}$$

Adding $-z$ and subtracting $C(q,\dot{q})q(t)$ and $G(q)$ from both sides of (6), we have

$$\begin{aligned}\ddot{q} - z = \bar{B}\tau - z - C(q,\dot{q})\dot{q}(t) - G(q)\\ + \sum_{j=1}^{r}(\bar{d}_j + \bar{d}_j(t))q(t-h_j) + D(t),\end{aligned} \tag{7}$$

Substituting (6) into (7), we obtain

$$\begin{aligned}\ddot{e} + F_1\dot{e} + F_0 e = \bar{B}\tau - z - C(q,\dot{q})\dot{q}(t) - G(q)\\ + \sum_{j=1}^{r}(\bar{d}_j + \bar{d}_j(t))q(t-h_j) + D(t),\end{aligned} \tag{8}$$

Therefore, the error dynamics in (8) can be written as

$$\begin{aligned}\dot{\bar{e}} = \Phi\bar{e} + B\Big(\bar{B}\tau - z - C(q,\dot{q})\dot{q}(t) - G(q)\Big)\\ + \sum_{j=1}^{r}(\bar{d}_j + \bar{d}_j(t))\bar{I}\bar{q}(t-h_j) + BD(t),\end{aligned} \tag{9}$$

where

$$\bar{q} = \begin{bmatrix}\dot{q}\\ q\end{bmatrix}_{(2n)\times 1}, \quad \bar{e} = \begin{bmatrix}\dot{e}^{\mathrm{T}}\\ e^{\mathrm{T}}\end{bmatrix}_{(2n)\times 1},$$

$$\bar{I} = \begin{bmatrix}0_{(n)} & I_{(n)}\\ 0_{(n)} & 0_{(n)}\end{bmatrix}_{(2n)\times(2n)}, \quad B = \begin{bmatrix}I_{(n)}\\ 0_{(n)}\end{bmatrix}_{(2n)\times n},$$

$$\Phi = \begin{bmatrix}-F_0 & -F_1\\ I_{(n)} & 0_{(n)}\end{bmatrix}_{(2n)\times(2n)},$$

in which $0_{(n)}$ is an $n \times n$ zero matrix. For ease of illustration, set $u(t) = \bar{B}\tau - z - C(q,\dot{q})\dot{q}(t) - G(q)$, then (9) can be rewritten as

$$\dot{\bar{e}} = \Phi\bar{e} + Bu(t) + \sum_{j=1}^{r}(\bar{d}_j + \bar{d}_j(t))\bar{I}\bar{q}(t-h_j) + BD(t), \tag{10}$$

Assumption 1. The unknown matrix-value functions of time $\bar{d}_i(t)\bar{I}$ are assumed to be of the form

$$\bar{d}_i(t)\bar{I} = B\psi_i(t), \quad i = 1, 2, \cdots, r. \tag{11}$$

Suppose that $\psi_i(t)$, $i = 0, 1, \cdots, r$ and $D(t)$ are Lebesgue measurable and norm-bounded, i.e.,

$$\|\psi_i(t)\| \le \sqrt{\rho_i}, \quad \|D(t)\| \le \sqrt{\rho_w}, \tag{12}$$

where ρ_i, $i = 1, 2, \cdots, r$, and ρ_w are positive constants.

Assumption 2. The desired trajectories q_m and desired velocity $\dot{q}_m$ are continuous and bounded known functions of time, without loss of generality, let $\|\bar{q}_m(t)\| \le \mathcal{L}$, where $\bar{q}_m(t) = \begin{bmatrix}\dot{q}_m(t)\\ q_m(t)\end{bmatrix}_{(2n)\times 1}$.

Definition 1. Define a ball $\mathcal{B}(r) := \{\bar{e} \in R^n : \|\bar{e}\| \le r\}$. The uncertain system (9) is said to be globally uniformly exponentially convergent to the ball $\mathcal{B}(r)$ at a rate $\sigma > 0$ if for any given positive number δ, there exists a positive number $\Gamma = \Gamma(\delta)$ such that

$$\begin{aligned}\|\bar{e}(t_0;\varphi)(t)\| \le r + \Gamma\exp(-\sigma(t-t_0)),\\ \forall t \ge t_0, \quad \forall \varphi \in C([-\tau, 0], R^n), \quad \|\varphi\|_c \le \delta.\end{aligned} \tag{13}$$

3. CONTROLLER DESIGN AND STABILITY ANALYSIS

3.1 Adaptive robust control scheme with known parameters

It is first assumed that the robot dynamical system (1) is known a priori. Now we consider the problem of adaptive robust controller design for system (1). Fig. 1 shows the structure of this control strategy.

Theorem 1. Consider the robotic system (1) with given $M(q)$, $C^{'}(q,\dot{q})$ and $G^{'}(q)$. Suppose that Assumption 1, 2 holds. If there exist a positive definite matrix P, positive numbers μ, ξ_0, ξ_i , ε_i, $i = 1, \cdots, r$, such that the following matrix inequalities

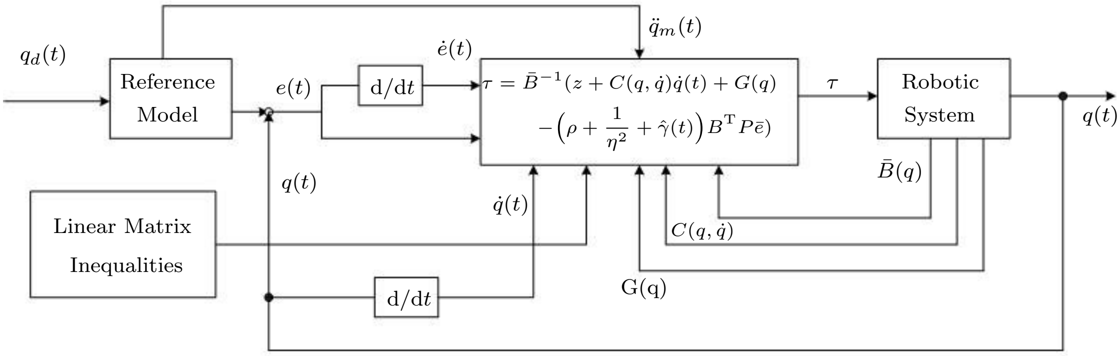

Fig. 1. Schematic diagram of adaptive robust control scheme.

$$\begin{bmatrix} P\Phi + \Phi^{\mathrm{T}}P + 2\mu P & \bar{d}_1 P\bar{I} & \bar{d}_2 P\bar{I} & \cdots & \bar{d}_r P\bar{I} \\ \bar{d}_1 \bar{I}^{\mathrm{T}}P & -\varepsilon_1 I & 0 & \cdots & 0 \\ \bar{d}_2 \bar{I}^{\mathrm{T}}P & 0 & -\varepsilon_2 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{d}_r \bar{I}^{\mathrm{T}}P & 0 & 0 & \cdots & -\varepsilon_r I \end{bmatrix} < 0, \quad (14)$$

$$\mu P - \sum_{i=1}^{r}(\varepsilon_i + \xi_i)I > 0 \quad (15)$$

hold, then the adaptive controller

$$\tau = \bar{B}\Big(z + C(q,\dot{q})\dot{q}(t) + G(q) - \Big(\rho + \frac{1}{\eta^2} + \hat{\gamma}(t)\Big)B^{\mathrm{T}}P\bar{e}\Big), \quad (16)$$

$$\dot{\hat{\gamma}}(t) = \frac{1}{\lambda}\bar{e}^{\mathrm{T}}PBB^{\mathrm{T}}P\bar{e} - \bar{\mu}\hat{\gamma}(t), \quad (17)$$

where ρ, λ, and $\bar{\mu}$ are design parameters satisfying

$$\rho > 0, \quad \lambda > 0, \quad \bar{\mu} \geq \mu, \quad (18)$$

will make the closed loop system (9), (16), (17) globally uniformly exponentially convergent to the ball $\mathcal{B}(r)$ at a rate $\sigma/2$, where r and σ are defined by

$$r = \sqrt{r_1 + r_2}, \quad (19)$$

$$\sigma = 2\mu - \frac{2}{\lambda_m(P)}\sum_{i=1}^{r}(\varepsilon_i + \xi_i)e^{\sigma\tau_{\max}}, \quad (20)$$

$$\gamma = -\rho + \frac{1}{2}\Big(\zeta + \sum_{i=1}^{r}\zeta^{-1}\rho_i\Big), \quad (21)$$

with

$$r_1 = \frac{\dfrac{\lambda\bar{\mu}^2\gamma^2}{\sum_{i=1}^{r}(\varepsilon_i + \xi_i)} + \dfrac{\zeta^{-1}\rho_w}{\lambda_m(P)}}{2\mu - \dfrac{2}{\lambda_m(P)}\sum_{i=1}^{r}(\varepsilon_i + \xi_i)},$$

$$r_2 = \frac{\rho^2\left(\sum_{i=1}^{r}\sqrt{\bar{d}_i + \rho_i\|B\bar{I}^{-1}\|}\right)^2 \mathcal{L}^2\bar{I}^{\mathrm{T}}(B^{+})^{\mathrm{T}}B^{+}\bar{I}}{2\mu\lambda_m(P) - 2\sum_{i=1}^{r}(\varepsilon_i + \xi_i)},$$

$\tau_{\max} = \max\{\tau_i, i = 1, 2, \cdots, r\}$, and ζ is a positive number which can be freely chosen.

Corollary 1. Consider the robotic system (1) with given $M(q)$, $C^{'}(q,\dot{q})$ and $G^{'}(q)$. Suppose that Assumption 1, 2 holds and the uncertainty bounds ρ_w and ρ_i, $i = 1, \cdots, r$, are known. If there exist a positive definite matrix P, positive numbers μ, ξ_0, ξ_i , ε_i, $i = 1, \cdots, r$, such that the following matrix inequalities

$$\begin{bmatrix} P\Phi + \Phi^{\mathrm{T}}P + 2\mu P & \bar{d}_1 P\bar{I} & \bar{d}_2 P\bar{I} & \cdots & \bar{d}_r P\bar{I} \\ \bar{d}_1 \bar{I}^{\mathrm{T}}P & -\varepsilon_1 I & 0 & \cdots & 0 \\ \bar{d}_2 \bar{I}^{\mathrm{T}}P & 0 & -\varepsilon_2 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{d}_r \bar{I}^{\mathrm{T}}P & 0 & 0 & \cdots & -\varepsilon_r I \end{bmatrix} < 0, \quad (22)$$

$$\mu P - \sum_{i=1}^{r}(\varepsilon_i + \xi_i)I > 0 \quad (23)$$

hold, then the adaptive controller

$$\tau = \bar{B}\Big(z + C(q,\dot{q})\dot{q}(t) + G(q) - \frac{1}{\eta^2}B^{\mathrm{T}}P\bar{e} - \big[\kappa^{-1}\rho_w + \sum_{i=1}^{r}\xi_i^{-1}\rho_i\big]B^{\mathrm{T}}P\bar{e}\Big), \quad (24)$$

where $\kappa > 0$, $\eta > 0$ are design parameters, will make the closed loop system (9), (24) globally uniformly exponentially convergent to the ball $\mathcal{B}(r)$ at a rate $\sigma/2$, where r and σ are defined by

$$r = \sqrt{\frac{\kappa}{2\mu\lambda_m(P) - \sum_{i=1}^{r}(\varepsilon_i + \xi_i)}}, \quad (25)$$

$$\sigma = 2\mu - \frac{1}{\lambda_m(P)}\sum_{i=1}^{r}(\varepsilon_i + \xi_i)e^{\sigma\tau_{\max}}, \quad (26)$$

$$\tau_{\max} = \max\{\tau_i, i = 1, 2, \cdots, r\}. \quad (27)$$

3.2 Adaptive neural robust control scheme with unknown parameters

In order to apply the NN system with adaptation capability, we first parameterize the dynamical system (2) as follows:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ a_{2n+1} & a_{2n+2} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{2n^2-2n+1} & a_{2n^2-2n+2} & \cdots & a_{2n^2-n} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$+\begin{bmatrix} a_{n+1} & a_{n+2} & \cdots & a_{2n} \\ a_{3n+1} & a_{3n+2} & \cdots & a_{4n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{2n^2-2n+1} & a_{2n^2-2n+2} & \cdots & a_{2n^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \hat{B} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$
$$+\sum_{j=1}^{r} (\bar{d}_j + \bar{d}_j(t)) q(t-h_j) + D(t), \quad (28)$$

where $\hat{B} = \dfrac{1}{\sqrt{c_1 c_2}} I_{(n)}$ for which $I_{(n)}$ is an $n \times n$ identity matrix. Let $\mathbf{a} = [a_1\ a_2 \cdots\ a_n\ a_{n+1} \cdots\ a_{2n}\ a_{2n+1} \cdots a_{3n}\ a_{3n+1} \cdots a_{2n^2-n+1}\ a_{2n^2-n+2} \cdots a_{2n^2}]^{\mathrm{T}}_{(2n^2)\times 1}$ be the unknown plant parameter vector.

Rewrite (28) in the following form:

$$\ddot{q} + V\mathbf{a} = \hat{B}\tau + \sum_{j=1}^{r} (\bar{d}_j + \bar{d}_j(t)) q(t-h_j) + D(t), \quad (29)$$

where

$$\begin{bmatrix} \bar{q}^{\mathrm{T}} & 0 & \cdots & 0 \\ 0 & \bar{q}^{\mathrm{T}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{q}^{\mathrm{T}} \end{bmatrix}_{n\times(2n^2)}, \quad (30)$$

Therefore, from (9), the error dynamics can be written as

$$\dot{\bar{e}} = \Phi\bar{e} + B(\bar{B}\tau - z - V\mathbf{a}) + \sum_{j=1}^{r} (\bar{d}_j + \bar{d}_j(t)) \bar{I}\bar{q}(t-h_j) + BD(t), \quad (31)$$

where $\bar{e}$, $\bar{I}$, Φ and B are defined as (9). For ease of illustration, set $v(t) = \bar{B}\tau - z - V\mathbf{a}$, then (31) can be rewritten as

$$\dot{\bar{e}} = \Phi\bar{e} + Bv(t) + \sum_{j=1}^{r} (\bar{d}_j + \bar{d}_j(t)) \bar{I}\bar{q}(t-h_j) + BD(t), \quad (32)$$

Let $x = [x_1\ x_2 \cdots x_n\ x_{n+1} \cdots x_{2n}]^{\mathrm{T}} = [\dot{e}^{\mathrm{T}}\ e^{\mathrm{T}}]^{\mathrm{T}}$. The basic configuration of the neural network system is implemented by using massive connections among processing units. Let $x \in R^{2n}$ be the input of the neural network system. Express $\mathbf{a}(x, \mathbf{c}) = [\mathbf{a}_1(x, \mathbf{c}_1), \cdots, \mathbf{a}_{2n^2}(x, \mathbf{c}_{2n^2})]^{\mathrm{T}}$. The neural networks output $\mathbf{a}_k(x, \mathbf{c}_k)$ for $k = 1, \cdots, 2n^2$ are composed of nonlinear neurons in every hidden layer and linear neurons in the input and output layers. For simplicity of design, the adjustable weightings $\mathbf{c}_k$ for $k = 1, \cdots, 2n^2$ are put in the output layers of the following single-output neural networks (Chang [1997]):

$$\mathbf{a}_k(x, \mathbf{c}_k) = \sum_{i=1}^{p_k} \mathbf{c}_{ki} H\Big(\sum_{j=1}^{2n} w_{ij}^k x_j + m_i^k\Big) \triangleq \xi_k^{\mathrm{T}} \mathbf{c}_k, \quad (33)$$

where $\mathbf{c}_k = (\mathbf{c}_{k1}, \cdots, \mathbf{c}_{kp_k})^{\mathrm{T}}$, $\xi_k = [H(\sum_{j=1}^{2n} w_{ij}^k x_j + m_1^k), \cdots, H(\sum_{j=1}^{2n} w_{p_k j}^k x_j + m_{p_k}^k)]^{\mathrm{T}}$, and p_k is the number of hidden neurons, $k = 1, \cdots, 2n^2$. According to the multi-layer neural network approximation theorem (Hornik [1989]), H must be a non-constant, bounded and monotonically increasing continuous function. In this work, the following hyperbolic tangent function is used:

$$H(x) = \frac{e^{\sigma(x)} - e^{\sigma(x)}}{e^{\sigma(x)} + e^{\sigma(x)}},$$

where $\sigma(x)$ is a function of the augmented state x.

For convenience, the neural network system $\mathbf{a}(x_e, \mathbf{c})$ is denoted as

$$\mathbf{a}(x, \mathbf{c}) = \begin{bmatrix} \xi_1^{\mathrm{T}} \mathbf{c}_1 \\ \xi_2^{\mathrm{T}} \mathbf{c}_2 \\ \vdots \\ \xi_{2n^2}^{\mathrm{T}} \mathbf{c}_{2n^2} \end{bmatrix} = \begin{bmatrix} \xi_1^{\mathrm{T}} & 0 & \cdots & 0 \\ 0 & \xi_1^{\mathrm{T}} & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi_{2n^2}^{\mathrm{T}} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_{2n^2} \end{bmatrix}$$
$$\triangleq \Xi(x)\mathbf{c} \quad (34)$$

Because the time-varying parameters of the controlled plant represented as the parameter vector $\mathbf{a}$ are absorbed partly into the NN system, $\mathbf{a}$ can be obtained more accurately by further estimating the unknown but constant weight vector $\mathbf{c}$ according the tracking error and the coefficients of the NN system. Let $\hat{\mathbf{a}} = \Xi\hat{\mathbf{c}}$ be the estimate of $\mathbf{a}$ due to and $\tilde{\mathbf{c}} = \hat{\mathbf{c}} - \mathbf{c}$ the error vector. Then, the certainty equivalent controller of (16) can be re-defined as

$$\tau = \hat{B}^{-1}\Big(z + V\hat{\mathbf{a}} - \Big(\rho + \frac{1}{\eta^2} + \hat{\gamma}(t)\Big)B^{\mathrm{T}}P\bar{e}\Big)$$
$$= \hat{B}^{-1}\Big(z + W\hat{\mathbf{c}} - \Big(\rho + \frac{1}{\eta^2} + \hat{\gamma}(t)\Big)B^{\mathrm{T}}P\bar{e}\Big) \quad (35)$$

where $W = V\Xi$. The configuration of proposed adaptive neural robust control scheme for the robotic system is depicted in Fig. 2.

Theorem 2. Consider the robotic system (1) with uncertain nonlinear parameter matrices $M(q)$, $C'(q, \dot{q})$ and $G'(q)$. Suppose that Assumption 1, 2 holds. If there exist a positive definite matrix P, positive numbers μ, ξ_0, ξ_i , ε_i, $i = 1, \cdots, r$, such that the following matrix inequalities

$$\begin{bmatrix} P\Phi + \Phi^{\mathrm{T}}P + 2\mu P & \bar{d}_1 P\bar{I} & \bar{d}_2 P\bar{I} & \cdots & \bar{d}_r P\bar{I} \\ \bar{d}_1 \bar{I}^{\mathrm{T}} P & -\varepsilon_1 I & 0 & \cdots & 0 \\ \bar{d}_2 \bar{I}^{\mathrm{T}} P & 0 & -\varepsilon_2 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{d}_r \bar{I}^{\mathrm{T}} P & 0 & 0 & \cdots & -\varepsilon_r I \end{bmatrix} < 0, \quad (36)$$

$$\mu P - \sum_{i=1}^{r} (\varepsilon_i + \xi_i) I > 0 \quad (37)$$

hold, then the adaptive NN robust controller

$$\tau = \bar{B}\Big(z + W\hat{\mathbf{c}} - \Big(\rho + \frac{1}{\eta^2} + \hat{\gamma}(t)\Big)B^{\mathrm{T}}P\bar{e}\Big), \quad (38)$$

$$\dot{\hat{\mathbf{c}}} = \dot{\tilde{\mathbf{c}}} = \frac{\alpha}{2}\tilde{\mathbf{c}} - \frac{1}{2}P_1 W^{\mathrm{T}} W\tilde{\mathbf{c}} - P_1 \Xi^{\mathrm{T}} V^{\mathrm{T}} B^{\mathrm{T}} P\bar{e}, \quad (39)$$

$$\dot{\hat{\gamma}}(t) = \frac{1}{\lambda}\bar{e}^{\mathrm{T}} PBB^{\mathrm{T}} P\bar{e} - \bar{\mu}\hat{\gamma}(t), \quad (40)$$

where P_1 is defined as

$$P_1(t) = \Big(\int_{0^+}^{t} \exp\Big(-\int_{0^+}^{t} \alpha(\phi)\mathrm{d}\phi\Big) W^{\mathrm{T}}(s) W(s) \mathrm{d}s\Big)^{-1} \quad (41)$$

with

$$\alpha = \alpha_0 \left(1 - \frac{\|P_1\|}{k_0}\right) \quad (42)$$

for which α_0 and k_0 are positive constants. ρ, λ, η, and $\bar{\mu}$ are design parameters satisfying

$$\rho > 0, \quad \lambda > 0, \quad \eta > 0, \quad \bar{\mu} \geq \mu, \quad (43)$$

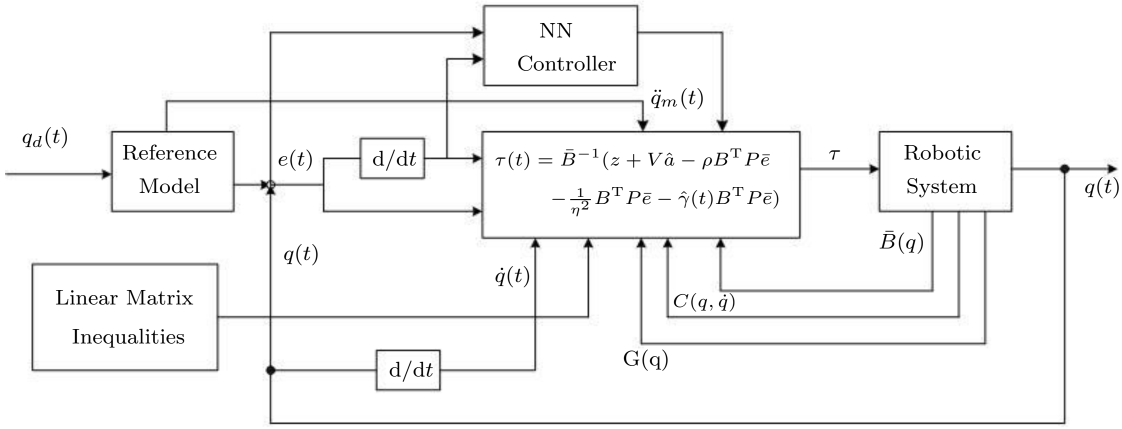

Fig. 2. Schematic diagram of adaptive neural robust control scheme.

will make the closed loop system (32), (38)-(40) globally uniformly exponentially convergent to the ball $\mathcal{B}(r)$ at a rate $\sigma/2$, where r and σ are defined by Theorem 1.

Corollary 2. Consider the robotic system (1) with uncertain nonlinear parameter matrices $M(q)$, $C^{'}(q, \dot{q})$ and $G^{'}(q)$. Suppose that Assumption 1, 2 holds and the uncertainty bounds ρ_w and ρ_i, $i = 1, \cdots, r$, are known. If there exist a positive definite matrix P, positive numbers μ, ξ_0, ξ_i , ε_i, $i = 1, \cdots, r$, such that the following matrix inequalities

$$\begin{bmatrix} P\Phi + \Phi^{\mathrm{T}} P + 2\mu P & \bar{d}_1 P\bar{I} & \bar{d}_2 P\bar{I} & \cdots & \bar{d}_r P\bar{I} \\ \bar{d}_1 \bar{I}^{\mathrm{T}} P & -\varepsilon_1 I & 0 & \cdots & 0 \\ \bar{d}_2 \bar{I}^{\mathrm{T}} P & 0 & -\varepsilon_2 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{d}_r \bar{I}^{\mathrm{T}} P & 0 & 0 & \cdots & -\varepsilon_r I \end{bmatrix} < 0, \quad (44)$$

$$\mu P - \sum_{i=1}^{r} (\varepsilon_i + \xi_i) I > 0 \quad (45)$$

hold, then the adaptive controller

$$\tau = \bar{B}\left(z + W\hat{c} - \frac{1}{\eta^2} B^{\mathrm{T}} P\bar{e} - \left[\kappa^{-1}\rho_w + \sum_{i=1}^{r} \xi_i^{-1}\rho_i\right] B^{\mathrm{T}} P\bar{e} \right), \quad (46)$$

$$\dot{\hat{c}} = \dot{\tilde{c}} = \frac{\alpha}{2}\tilde{c} - \frac{1}{2} P_1 W^{\mathrm{T}} W\tilde{c} - P_1 \Xi^{\mathrm{T}} V^{\mathrm{T}} B^{\mathrm{T}} P\bar{e}, \quad (47)$$

where P_1 is defined as (41)-(42), and $\kappa > 0$, $\eta > 0$ are design parameters, will make the closed loop system (32), (46), (47) globally uniformly exponentially convergent to the ball $\mathcal{B}(r)$ at a rate $\sigma/2$, where r and σ are defined by Corollary 1.

4. COMPUTER SIMULATION

To illustrate the adaptive neural control scheme proposed in this paper, a simulation example for gyroscopic system with single actuating input (Ferreira [1999]) is performed. The inertia of the system is concentrated in the rotor, with J as the radial moment of the inertia and I the axial moment of inertia. This system is acted upon by a single torque input $\tau(t)$ applied along the x-axis (torque axis). From Newton's law in rotational form, the equations of motion for the gyroscopic system are shown as follows:

$$J\ddot{\theta} = \tau(t) + J\dot{\beta}^2(t-h)\sin(\theta(t-h))\cos(\theta(t-h)) - E_2\dot{\beta}(t-h)\sin(\theta(t-h)) + \zeta(t), \quad (48)$$

$$J\dot{\beta}\sin^2(\theta(t)) + E_2\cos(\theta(t)) = E_1, \quad (49)$$

$$I(\dot{\psi} + \dot{\beta}(t)\cos(t)) = E_2, \quad (50)$$

where E_1, E_2 are conservation of the angular momentum constants, h is the delay time for the rate of change, β, due to the inertias of the radial and axial moments, and ζ is the bounded disturbance caused by the unbalanced effects. Without loss of generality, counterclockwise and clockwise rotations are defined as positive and negative, respectively.

There are three outputs θ, β and ψ in this gyroscopic system. From (49) and (50), we have

$$\dot{\beta} + \frac{E_2\cos(\theta) - E_1}{J\sin^2(\theta)} = 0, \quad (51)$$

$$\dot{\psi} + \frac{E_1\cos(\theta) - E_2\cos^2(\theta)}{IJ\sin^2(\theta)} = \frac{E_2}{I}. \quad (52)$$

Hence, the rate changes of β, $\dot{\beta}$, and ψ, $\dot{\psi}$, can be seen as in function of θ. From (28) and (48)-(50), since $n = 2$ for θ and $n = 1$ for ψ, we can express the plant model as follows:

$$\ddot{\theta} + a_1\dot{\theta} + a_2\theta + a_3\psi = \tau + d(\theta(t-h), \dot{\beta}(t-h)) + \zeta, \quad (53)$$

where a_i, $i = 1, 2, 3$, are unknown parameters to be estimated.

A gyroscope having the following parameters is chosen for this simulation: the inertias are taken given by $I = 2$ and $J = 1$, the delay time by $h = 0.02s$, and the angular momentum constants by $E_1 = E_2 = 1$, since (49) and (50) satisfy the conservation laws of the angular momentum. Let the reference model be specified as

$$\ddot{\theta}_m + 15\dot{\theta}_m + 75\theta_m + 125\psi_m = r_m, \quad (54)$$

where $r_m = 125$ for unit step tracking.

Specify $\alpha_0 = 1$, $k_0 = 1$, and from (8) and (30), we have

$$F_0 = \begin{bmatrix} 1024 & 64 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad V = \begin{bmatrix} \dot{\theta} & \theta & \psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{\theta} & \theta & \psi \end{bmatrix}. \quad (55)$$

In practice, friction and mass unbalance are inevitable for the gyroscope and will cause disturbance torques on the gimbals when the body is accelerating and rotating. Therefore, the sinusoidal disturbances 2sin(t) are used to simulate these imperfections.

Since the information on the bounds of parameter uncertainties and disturbances is unknown, let the initial conditions be given by $P_1(0)^{-1} = I$, $\sigma_0 = 0.05$, $r_0 = 0.5$, $\mu = \bar{\mu} = 1.0$, $\hat{c}(0) = [10\ 10 \cdots 10]^{\mathrm{T}}$, $\hat{\gamma}(0) = 0$, and set

$$\hat{B} = I_{(3)}, \quad B = [1\ 0\ 1]^{\mathrm{T}}, \quad V = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (56)$$

Then we have $a = 2.0406$. Substituting a into Step 2 of Algorithm 1 yields

$$P = \begin{bmatrix} 35.7820 & -23.1621 & -19.0652 \\ -23.1621 & 27.0633 & 16.7321 \\ -19.0652 & 16.7321 & 13.6162 \end{bmatrix},$$
$$\varepsilon_1 = 26.7851, \quad \xi_1 = 5.7889. \quad (57)$$

Update the estimates of the vectors $\hat{c}$, $\hat{\gamma}$ and obtain the weighting matrix P_1 from (39), (40) and (41), respectively. Now we choose $\lambda = 10^3$ and $\rho = 0.01$, then the adaptive controllers will be derived from (16) and (38), respectively, for comparison depending on different assumptions on the plant parameter matrices.

Figs. 3 and 4 show the simulation results with disturbances 2sin(t) as follows: (a) the tracking error $e_\beta = \beta - \beta_m$, (b) the tracking error $e_\theta = \theta - \theta_m$, (c) the tracking error $e_\psi = \psi - \psi_m$, (d) the control input τ. It is easy to see that in our tracking purpose with the NN system (i.e., without a priori knowledge on plant dynamics) can be achieved effectively than the case without (i.e., with a priori knowledge on plant dynamics). The results reveal that the proposed adaptive neural robust scheme indeed improves the system performances including convergence of tracking errors, the smoothness of the control inputs. It seems that the robustness of the proposed control scheme is excellent.

5. CONCLUSIONS

In this paper, an adaptive neural robust control scheme for MIMO uncertain robotic systems with time delays has been developed and the general idea that appropriate estimation of the adaptation process should provide a satisfactory basis for the control. The essential requirement for the delayed state uncertainties is that they satisfy matching conditions and are norm-bounded, but the bounds of the uncertainties are not necessarily known. As well, an NN system is used to represent the unknown controlled system. A reference model with the desired amplitude and phase properties is given to construct an error model. It is shown that, in the sense of Lyapunov-type stability, the proposed control scheme can guarantee estimation convergence and stability robustness of the closed-loop system.

As demonstrated in the illustrated example, the control scheme proposed in this paper can achieve a better model following tracking performance over that without using the NN system with adaptation weights.

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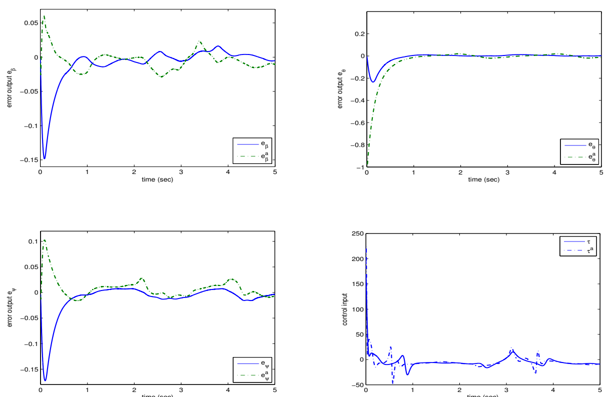

Fig. 3. Simulation results with disturbance 2sin(t) for $\rho = 0.5$, where the subscript a denotes the responses without using the NN system.

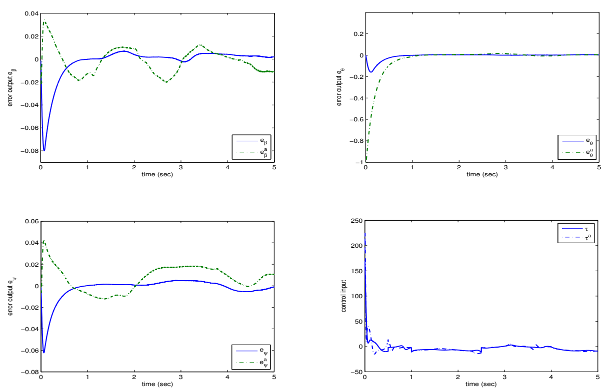

Fig. 4. Simulation results with disturbance 2sin(t) for $\rho = 0.1$, where the subscript a denotes the responses without using the NN system.