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On Strongly Stabilizing Controller Synthesis for Time Delay Systems ⋆

H. Özbay *

* *Department of Electrical & Electronics Engineering, Bilkent University, Ankara, 06800 Turkey (e-mail: hitay@bilkent.edu.tr)*

Abstract: In this paper, a strongly stabilizing controller design method, proposed earlier for finite dimensional systems, is extended to a class of time delay systems. A special factorization of the plant is done first. Then, an infinite dimensional one-block $\mathcal{H}^\infty$ control problem is solved using existing techniques. The solution of this $\mathcal{H}^\infty$ control problem, together with the stable coprime factors of the plant give the stable controller stabilizing the feedback system. The method is illustrated with a numerical example. The example also shows the effects of internal and input/output time delays on the solvability of the strong stabilization problem using this approach.

Keywords: Feedback Stabilization; Stable Controllers; Time Delay; Unstable Plants.

1. INTRODUCTION

The strong stabilization problem, i.e., finding a stable controller for a given unstable plant satisfying the parity interlacing property (PIP), has been investigated over the last 20-30 years, see for example earlier papers: Ganesh and Pearson (1986); Sideris and Safanov (1985); Vidyasagar (1985); Youla et al. (1974). Very recent works, such as Cheng et al. (2007); Li and Petersen (2007); Gumussoy and Ozbay (2007), show that there is still an interest to this difficult problem. Various versions of the strong stabilization problem has been solved; but there are still many open problems in this context.

For single-input-single-output finite dimensional plants an interpolation-based procedure can be used to find a strongly stabilizing controller for a given plant satisfying the PIP (which is the necessary and sufficient condition under which this problem is solvable), Doyle et al. (1992); Vidyasagar (1985). For the multi-input-multi-output case there are some strong stabilization techniques working under certain sufficient conditions only (with or without added H_∞ and H_2 performance restrictions), see e.g. Barabanov (1996); Campos-Delgado and Zhou (2003); Choi and Chung (2001); Chou et al. (2003); Gumussoy and Ozbay (2005); Halevi (1994); Lee and Soh (2002); Petersen (2006); Saif et al. (1997); Zeren and Ozbay (1999), and their references. Among the vast literature on strong stabilization, very few works are devoted systems with time delays, see e.g. Gumussoy and Ozbay (2004, 2006); Suyama (1991) and their references. These papers either develop new techniques, or extend existing methods to cover time delay systems. For example, Gumussoy and Ozbay (2006, 2007) gave extensions of the technique used in Ganesh and Pearson (1986) to find strongly stabilizing controllers (leading to optimal or suboptimal $\mathcal{H}^\infty$ sensitivity levels) for a class of time delay systems.

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In this paper, the technique proposed in Zeren and Ozbay (2000) is extended to systems with time delays. First, a special coprime factorization of the plant is done. Then, an infinite dimensional one-block $\mathcal{H}^\infty$ control problem is solved using existing techniques such as Foias et al (1996); Iftime and Zwart (2002); Kashima et al. (2007); Meinsma et al. (2002); Mirkin (2003); Meinsma and Mirkin (2005). The solution of this $\mathcal{H}^\infty$ control problem, together with the stable coprime factors of the plant give the stable controller stabilizing the feedback system.

In Section 2 we pose the strong stabilization problem formally and define various classes of infinite dimensional plants to be considered. The design procedure of Zeren and Ozbay (2000) is applied to this problem in Section 3. This method is illustrated with a numerical example in Section 4. The example also shows the effects of internal and input/output time delays on the solvability of the strong stabilization problem using this approach. Approximation-based strong stabilization of time delay systems is discussed in Section 5, and concluding remarks are made in Section 6.

2. PROBLEM DEFINITION AND PLANT CLASSES CONSIDERED

Consider the standard feedback system formed by a controller C and a plant P, as shown in Figure 1. Suppose that the plant can be written as $P = D_p^{-1}N_p$, where $D_p, N_p \in \mathcal{H}^\infty$.

Definition. We say that the controller $C = Q \in \mathcal{H}^\infty$ is *strongly stabilizing* the plant if

$$U := D_p + N_pQ \tag{1}$$

is unimodular, i.e., $U, U^{-1} \in \mathcal{H}^\infty$.

When we deal with MIMO systems D_p, N_p and Q are appropriate size matrices whose entries are in $\mathcal{H}^\infty$. In that case U is a square matrix whose entries are in $\mathcal{H}^\infty$, and without specifying the matrix size we still write

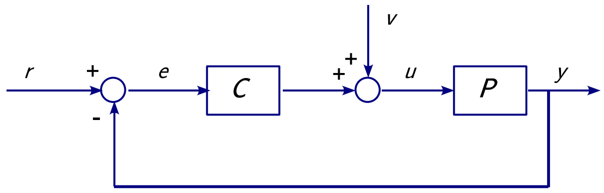

Fig. 1. Feedback Control System

$D_p, N_p, Q, U \in \mathcal{H}^\infty$. In this paper, we consider different cases where some of these matrices have entries whose transfer functions contain time delay elements in their numerators and/or denominators. Hence, these systems are infinite dimensional.

The system given below illustrates one possible class of plants which can be studied in this framework:

$$P(s) = \frac{(s-2)e^{-\tau s}}{(s+a-ke^{-hs})} \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} \\ 0 & 0 & \frac{e^{-2\tau s}}{s+4+e^{-s}} \end{bmatrix} \qquad (2)$$

where $a, h, k, \tau > 0$, with $kh < 1$ and $k > a$. The plant contains a single pole α in $\mathbb{C}_+$; it is determined as the solution of the equation

$$\alpha = ke^{-h\alpha} - a.$$

Blocking zeros of the plant in the extended right half plane are $\{2, \infty\}$. Therefore, P satisfies the PIP (equivalently, admits a strongly stabilizing controller) if and only if $\alpha < 2$. For the plant factorization we define

$$D_p(s) = \frac{(s-\alpha)}{(s+1)} I \quad \text{and} \quad N_p(s) = N_{pi}(s) N_{po}(s) N_{p1}(s)$$

where

$$N_{pi}(s) = \frac{(s-2)}{(s+2)} e^{-\tau s} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2\tau s} \end{bmatrix}$$

$$N_{po}(s) = \frac{1}{s+1} I$$

$$N_{p1}(s) = \frac{(s-\alpha)}{(s+a-ke^{-hs})} \begin{bmatrix} \frac{s+2}{s+1} & 1 & \frac{s+2}{s+3} \\ 0 & 0 & \frac{s+2}{s+4+e^{-s}} \end{bmatrix}$$

Note that the term N_{p1} admits a right inverse in the form

$$N_{p1}^\dagger(s) = \frac{(s+a-ke^{-hs})}{(s-\alpha)} \begin{bmatrix} 2\frac{s+1}{s+2} & 0 \\ -1 & -\frac{s+4+e^{-s}}{s+3} \\ 0 & \frac{s+4+e^{-s}}{s+2} \end{bmatrix}$$

which is in $\mathcal{H}^\infty$. Therefore, if we define $Q = N_{p1}^\dagger Q_1$, then U is reduced to

$$U = D_p + N_{pi} N_{po} Q_1 \qquad (3)$$

where N_{pi} is inner, and N_{po} is outer and strictly proper.

The factorizations made for the example given above can be done for a large class of MIMO plants. Thus, the strong stabilization problem reduces to finding a $Q_1 \in \mathcal{H}^\infty$ such that U defined by (3) is unimodular for the given $D_p, N_{pi}, N_{po} \in \mathcal{H}^\infty$. The problem data is such that the zeros of D_p are the poles of the plant. Blocking zeros of the plant are collected in the product $N_{pi}N_{po}$.

3. SMALL GAIN BASED STRONG STABILIZATION OF SYSTEMS WITH TIME DELAYS

By using the small gain theorem, the problem of finding a $Q_1 \in \mathcal{H}^\infty$ making U, defined by (3), unimodular can be cast as a one-block $\mathcal{H}^\infty$ problem as shown in Zeren and Ozbay (1999, 2000). The basic idea is to rewrite U as

$$U = I + (D_p - I) + N_{pi} N_{po} Q_1.$$

Let $R_p := (D_p - I)$. Then, for a given $Q_1 \in \mathcal{H}^\infty$ we have a unimodular U if

$$\|R_p + N_{pi} N_{po} Q_1\|_\infty < 1. \qquad (4)$$

In Zeren and Ozbay (2000) the problem data R_p, $N_{pi}N_{po}$ consists of finite dimensional transfer matrices. As shown in Section 2, for systems with internal and input/output delays N_{pi} may contain time delay terms.

Assumption. The plant has finitely many poles in $\mathbb{C}_+$.

Computation of these poles may require numerical techniques for stability analysis of general time delay systems, such as those developed by Engelborghs et al. (2001, 2002); Jarlebring (2006); Louisell (2004); Olgac and Sipahi (2004); Vyhlidal and Zitek (2003, 2006). Under the above assumption, we have a finite dimensional and strictly proper R_p. Recall that N_{po} is also strictly proper. Therefore, the design of Q_1 can be restricted to a finite bandwidth.

It is also interesting to note that in the finite dimensional case, when $N_{pi}N_{po}$ is low (less than two) order, then the existence of $Q_1 \in \mathcal{H}^\infty$ satisfying (4) is equivalent to PIP, i.e., the conservatism introduced by the use of the small gain theorem is lifted, see Ozbay and Gundes (2008).

The solution of (4) can be obtained for different classes of systems using different techniques. For example, when N_{pi} is an inner matrix in the form $\Lambda_1 \widehat{N}_{pi} \Lambda_2$, where Λ_i's are diagonal matrices whose non-zero entries are in the form $e^{-h_k s}$, and the entries of $\widehat{N}_{pi}$ are finite dimensional, the methods developed by Meinsma et al. (2002); Mirkin (2003); Meinsma and Mirkin (2005); Mirkin and Tadmor (2002); Nagpal and Ravi (1997); Zhong (2006) are applicable. For more general classes of inner functions other methods may be used, see e.g. Foias et al (1996); Gumussoy and Ozbay (2006); Hirata et al. (2000); Iftime and Zwart (2002); Kashima et al. (2007).

Remark. There are also several interesting classes of time delay systems with infinitely many poles in $\mathbb{C}_+$. If these systems have no input-output delays and have finitely many zeros in $\mathbb{C}_+$ then they can also be factored as $P = D_p^{-1} N_p$ where $N_p = N_{pi} N_{po} N_{p1}$, with N_{p1} is right invertible in $\mathcal{H}^\infty$, $N_{pi}N_{po}$ is finite dimensional but D_p is infinite dimensional. The $\mathcal{H}^\infty$ problem (4) appearing for these types of plants can be solved using the Nevanlinna-Pick interpolation approach, Foias and Frazho (1990); Foias et al (1996) as illustrated in Gumussoy and Ozbay (2006, 2007). In this paper we leave this class of plants aside.

In the next section, as an example, we shall study the problem (4) arising for the plant given in (2). In particular, we will discuss the effect of the time delay τ on the solvability of the problem (4).

4. EXAMPLE

Consider $R_p(s) = (\frac{s-\alpha}{s+1} - 1)\, I = -\frac{(\alpha+1)}{s+1}\, I$, and recall that

$$N_{pi}(s) = \frac{(s-2)}{(s+2)} e^{-\tau s} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2\tau s} \end{bmatrix} \qquad N_{po}(s) = \frac{1}{s+1}\, I.$$

In this example the problem data for (4) consists of diagonal matrices, so we can choose Q_1 to be diagonal, i.e.,

$$Q_1(s) = \begin{bmatrix} q_1(s) & 0 \\ 0 & q_2(s) \end{bmatrix}.$$

Thus we have two decoupled scalar $\mathcal{H}^\infty$ problems, and (4) is solvable if and only if

$$\inf_{q_2 \in \mathcal{H}^\infty} \| -\frac{(\alpha+1)}{s+1} + \frac{(s-2)}{(s+2)(s+1)} e^{-3\tau s} q_2(s) \|_\infty < 1. \quad (5)$$

Note that the dual problem of finding the optimal q_1 is easier than finding the optimal q_2 because the time delay in the second problem is three times larger than that of the first problem. The inequality (5) is satisfied if and only if

$$\gamma_o = \inf_{\widehat{q}_2 \in \mathcal{H}^\infty} \| \frac{1}{s+1} \left(1 - \frac{(s-2)}{(s+2)} e^{-3\tau s} \widehat{q}_2(s) \right) \|_\infty < \frac{1}{\alpha+1}.$$

By using the formulae given in Foias et al (1996) we can compute γ_o as the largest γ in the range $0 \le \gamma \le 1$ satisfying

$$2\tan^{-1}(\frac{\omega_\gamma}{2}) + \tan^{-1}(\omega_\gamma) + 3\tau\omega_\gamma = \pi \quad (6)$$

where $\omega_\gamma := \sqrt{\gamma^{-2} - 1}$. When $\tau = 0$ we find $\omega_\gamma = 2\sqrt{2}$ satisfies (6) i.e., $\gamma_o = 1/3$. Hence, for $\tau = 0$ the problem is solvable if and only if $\frac{1}{3} < \frac{1}{\alpha+1}$, which is equivalent to $\alpha < 2$. Also note that the plant satisfies the PIP if and only if $\alpha < 2$. In the light of the above discussion we define the maximal α for which we can find a feasible solution to (4) as α_{max}, and this quantity varies with τ. Figure 2 shows α_{max} versus τ.

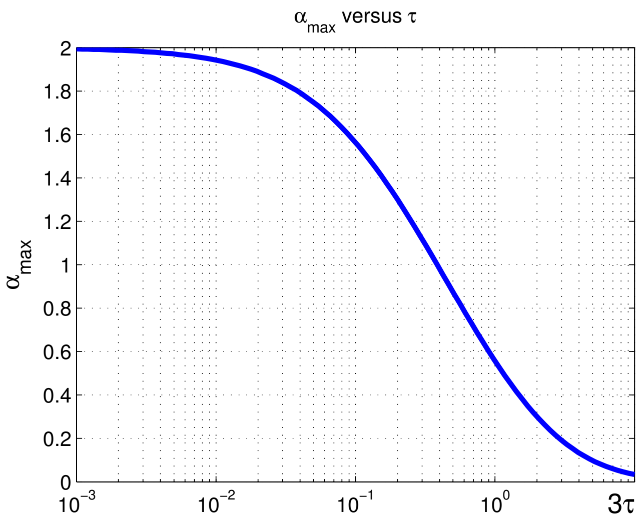

Fig. 2. α_{max} versus τ

It is interesting to note that when the delay is small, e.g. in the range $3\tau \le 0.01$, maximum allowable α for which the problem (4) is solvable is larger than 1.9, which is close to 2. Even for the non-delayed case, $\tau = 0$, a strongly stabilizing controller exists if and only if $\alpha < 2$. Thus, the effect of time delay on finding a strongly stabilizing controller using this approach is small if the delay is small, but as the delay 3τ gets larger than 0.1, its effect becomes significant.

5. APPROXIMATION-BASED STRONG STABILIZATION

Another way to find a strongly stabilizing controller for systems with time delays is to approximate the delay term appearing in N_{pi} and then design a stable controller for the corresponding finite dimensional system. We now discuss this approach briefly.

Recall that the strong stabilization problem is solvable if a $Q_1 \in \mathcal{H}^\infty$ can be found so that U defined by (3) is unimodular (i.e. invertible in $\mathcal{H}^\infty$). Let us consider the approximate problem

$$U_a = D_p + N^a_{pi} N_{po} Q^a_1 \quad (7)$$

where N^a_{pi} is a finite dimensional approximation of N_{pi} and $Q^a_1 \in \mathcal{H}^\infty$ is a finite dimensional strongly stabilizing controller (if it exists) for the "plant" $D_p^{-1} N^a_{pi} N_{po}$.

Typically Padé approximation is used in N^a_{pi} for delay terms in N_{pi}; if the order of the approximation is odd then extra blocking zeros will appear in the positive real axis. Location of these zeros with respect to the right half plane poles and other existing blocking zeros of the plant will determine whether a feasible Q^a_1 exists. When the approximation is even all new zeros are complex conjugate pairs and none of them are on the real line, in this case existence of a feasible Q^a_1 does not depend on the value of the delay. However, in either case we have to check whether this Q^a_1 strongly stabilizes the "original plant" $D_p^{-1} N_{pi} N_{po}$, i.e., check if

$$U_1 := D_p + N_{pi} N_{po} Q^a_1$$

is unimodular or not. Note that we can re-write U_1 as

$$U_1 = U_a + (N_{pi} - N^a_{pi}) N_{po} Q^a_1.$$

Suppose that there exists $Q^a_1 \in \mathcal{H}^\infty$ which makes U_a unimodular. Then, by the small gain theorem, we have a unimodular U_1 if

$$\|(N_{pi} - N^a_{pi}) N_{po} Q^a_1 U_a^{-1}\|_\infty < 1. \quad (8)$$

We now have an interesting design problem for the selection of the approximation order and construction of Q^a_1. The approximation error should be small enough to satisfy (8). More precisely, let $W_a(s)$ be a weighting function overbounding the approximation error

$$|W_a(j\omega)| > \sigma_{\max}(N_{pi}(j\omega) - N^a_{pi}(j\omega)) \quad \forall\ \omega$$

then we want

$$\|W_a N_{po} Q^a_1 U_a^{-1}\|_\infty < 1.$$

Whether we can find $Q^a_1 \in \mathcal{H}^\infty$ leading to a unimodular U_a and satisfying this inequality also depends on the right half plane pole and zero locations of $D_p^{-1} N^a_{pi} N_{po}$. If the pole-zero pattern is "close" to violating the parity interlacing property, then it is "difficult" to find $Q^a_1 \in \mathcal{H}^\infty$ satisfying (8) even if W_a is "small." An illustrative example will be given in the expanded version of the paper to be published elsewhere.

6. CONCLUSIONS

A strongly stabilizing controller design method developed earlier for finite dimensional plants is shown to work for a large class of systems with time delays. In this approach plant factorizations are done and an $\mathcal{H}^\infty$ problem is formulated. Using the special structure of the problem data in (4), (in particular, the fact that R_p is finite dimensional) all solutions $Q_1 \in \mathcal{H}^\infty$ can be obtained, see e.g. the recent work of Kashima et al. (2007). Then strongly stabilizing controllers are obtained as $C = N_{p1}^\dagger Q_1$. The approach is illustrated by an example, where the effect of input-output delay is computed explicitly. Strong stabilization using approximations of the delay terms in N_{pi} is also discussed.

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