Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Output based Observation and Control for Visual Servoing of VTOL UAV's

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Abstract: An Image-Based Visual Servo (IBVS) control strategy for stabilisation of Vertical Take-Off and Landing (VTOL) vehicles with respect to a fixed target is proposed. A visual feature based on the combination of the first order un-normalized spherical moment and the image length of the target image is considered. To avoid the lack of the measurement of the translational velocity of the camera and the thrust of the vehicle, a nonlinear observer-based visual servo controller is proposed. Local asymptotic stability of the system is proved and an estimate of the basin of attraction for the closed-loop system is provided. Simulation results are finally presented to illustrate the performance of the control algorithm.

INTRODUCTION

Vision based algorithms have been extensively developed over the last five years to control Unmanned Aerial Vehicles (UAV); posing a number of unique problems in sensing and control. A key issue is to provide robust and simple sensing and control systems that stabilize the vehicle relative to its local environment. At the core of this problem is the ability of the vehicle to measure its position and velocity relative to a local environment and to measure the orientation and orientation velocity with respect to the inertial frame. For payload and cost reasons, this must be achieved using a limited sensor suite, typically an Inertial Measurement Unit (IMU) and a camera. Using an IMU for attitude and angular velocity estimation has heavily studied in the last ten years and different algorithms have been proposed and tested (traditional linear Kalman filter techniques Jun (1999), EKF techniques Marins (2001); Rehbinder (2004) and complementary filters Tayebi (2007); Thienel (2003); Mahony (2005)). Using a camera as the primary sensor for relative position leads to a visual servo control problem. Two main approaches have been identified in visual servo-control, the first one, termed Pose-Based Visual Servo (PBVS) involves reconstruction of the target pose and leads to a Cartesian motion planning problem Hutchinson (1996). The second method, termed Image-Based Visual Servo (IBVS), aims to control the nonlinear dynamics of features in the image plane directly Espiau (1992). It has the advantage to be inherently robust to camera calibration and target modelling errors.

Such IBVS controllers have been proposed in recent works using the first order un-normalized spherical moment of a set of target points as image feature Hamel (2002); Bourquardez (2006). This vector is in bijection with the cartesian pose of the VTOL vehicle and, therefore, it can be used as the first state of the system dynamics considered. To deal with the full dynamics, the sum of visual flow of the observed target points was used in earlier work Mahony (2007) to provide a velocity estimate. The main drawback of this approach relies on the fact that the visual flow is closely associated to the inertial depth, obtained from visual flow integration, leading to a non-observable system, Le Bras (2007). To overcome this problem a new visual measurement feature is used in Le Bras (2007); it is a combination of the first order un-normalized spherical moment with the visual length of the target image. The control strategy proposed in Le Bras (2007) allows stability of the system without velocity measurements. A virtual state is introduced to overcome the lack of the linear velocity measurement while assuming perfect measurement of the thrust input. The performances are satisfying as long as good measurements of the thrust (magnitude and direction) are available.

In this paper, an observer-based controller is proposed to allow the control of the system without linear velocity and thrust measurements. Local asymptotic stability of the system is proven and an estimate of the basin of attraction for the closed-loop system is provided.

The paper is organized as follows: In section 1, the nonlinear visual dynamics presented in Le Bras (2007) is given. In section 2, a new nonlinear adaptive observer, providing specific features to control the system, is derived. Section 3, is devoted to the observer-based adaptive control; a Lyapunov control function for the positioning task is proposed and the stability analysis of the closed-loop system is discussed. Section 4 presents simulation results to illustrate the performance of the control algorithm. The final section (5) provides a short summary of conclusions.

1. PROBLEM FORMULATION

Consider the images of n marks constituting a planar target observed by a vision system mounted on the VTOL vehicle throughout the flight. The dynamics of the visual feature along with the dynamics of an idealised of VTOL vehicle in quasi-hover have been previously presented in Le Bras (2007). Assuming that the camera frame coincides with the VTOL frame and the target marks form a convex set with known orthogonal direction, these dynamics are given by (Le Bras (2007)):

978-1-1234-7890-2/08/$20.00 © 2008 IFAC

10.3182/20080706-5-KR-1001.1328

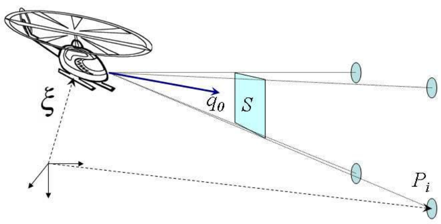

Fig. 1. Target length $L(t) = \sqrt{S}$ and centroid $q_0 = \sum \frac{R^T(P_i - \xi)}{\|P_i - \xi\|}$.

$$\dot{q}(\xi) = -\mathrm{sk}(\Omega)q(\xi) - \frac{1}{d(t)}Q_0 V \tag{1a}$$

$$\dot{V} = -\mathrm{sk}(\Omega)V + f/m \tag{1b}$$

$$\dot{R} = R\mathrm{sk}(\Omega) \tag{1c}$$

$$I\dot{\Omega} = -\mathrm{sk}(\Omega)I\Omega + \Gamma \tag{1d}$$

where (ξ, V, R, Ω) denote respectively the position, velocity, orientation matrix and the angular rate vector of the VTOL, (m, I) denote its mass and inertia matrix, $d(t)$ is the inertial depth of the camera, *i.e,* the inertial distance from the focal point to the target plane and (f, Γ) denote the force and torque applied at the vehicle on its center of mass. The force f combines thrust, lift, gravity and drag components. It is convenient to separate the gravity component from the combined aerodynamic forces and assume that the aerodynamic forces are always aligned with the thrust in the body fixed frame [1],

$$f := -Te_3 + mgR^T e_3 \tag{2}$$

where $T \in \Re$ is a scalar input representing the magnitude of external force (or thrust) applied in direction $e_3 = (0, 0, 1)'$. Control of the airframe is obtained by using the torque control $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ to align the force f as required to track the goal trajectory.

The visual feature used $q(\xi)$,

$$q = q_0 + n\ln(L) \tag{3}$$

is a combination of the un-normalized centroid vector q_0 (the sum of the spherical image points of the target marks, Hamel (2002)) and the image length L (the length of the target image on the tangent plane to the image surface parallel to the target, Le Bras (2007)), see Figure 1 .

Finally Q_0 is a known positive definite matrix. Its expression is described in Le Bras (2007).

The visual length L (Le Bras (2007)) is measurable with the available sensors and is linked with the inertial depth:

$$L(t)d(t) = \frac{1}{ma} \tag{4}$$

where $a > 0$ is an unknown parameter depending of the target size.

It has been shown in Le Bras (2007) that a pertinent method to control a VTOL through IBVS approach is to consider separately dynamics (1a-1c) in the one hand, and dynamics (1d) in the other hand.

An inner-loop controller regulates the orientation dynamics, while the first dynamics (1a-1c) are controlled by the following new control input:

$$u = \dot{f} + \mathrm{sk}(\Omega)f = (-\Omega_2 T, \Omega_1 T, -u_T) \tag{5}$$

[1] This is a reasonable assumption for the dynamics of a VTOL vehicle in quasi-stationary flight where the exogenous force is dominated by the propeller force while drag and forward thrust are negligible.

The magnitude of the thrust, is now considered as an internal state of the system thanks to the following dynamic extension:

$$\dot{T} = u_T$$

The inner-loop provides a high gain stabilization of the vehicle angular velocity based on direct measurement of the angular velocity from the IMU. The outer-loop controller uses visual features along with outputs of the partial observer. For a typical VTOL vehicle, the time-scale separation between the two loops is sufficient that the interaction terms can be ignored in the control design.

Let $b^\star$ be a stationary desired centroid vector expressed in the inertial frame and denote $q^\star = R^T b^\star$, it yields:

$$\dot{q}^\star = -\mathrm{sk}(\Omega)q^\star \tag{6}$$

Define $\delta_1 = q - q^\star$ a measurable centroid error, note that

$$\delta_1 = 0 \Leftrightarrow \frac{R^T(P_i - \xi)}{\|P_i - \xi\|} = R^T b^\star \Leftrightarrow \frac{P_i - \xi}{\|P_i - \xi\|} = b^\star \Leftrightarrow \xi = \xi^\star$$

the above relation holds because the considered function $\xi \rightarrow \frac{P_i - \xi}{\|P_i - \xi\|}$ is a diffeomorphism since $Q > 0$. Finally denote $Q = L(t)Q_0$ the Jacobian matrix, $W = amV$ the visual relative velocity and $F = af$ the relative force. The following system is considered in the sequel for the control design:

$$\dot{\delta}_1 = -\mathrm{sk}(\Omega)\delta_1 - QW \tag{7a}$$

$$\dot{W} = -\mathrm{sk}(\Omega)W + F \tag{7b}$$

$$\dot{F} = -\mathrm{sk}(\Omega)F + au \tag{7c}$$

Note that a appears as an unknown scale factor of the control input u.

2. NONLINEAR PARTIAL OBSERVER

This section presents a new partial nonlinear observer aimed to provide filtered variables allowing to control the system without measurement of both state variables (W, f). The main idea is deduced from the research works of Krstic and Kokotovic Krstic (1995) and consists in defining virtual estimates $(\hat{q}, \hat{W}, \hat{F})$ of (q, W, F) built from the sum of two filtered variables:

$$\hat{q} = q_1 + aq_2, \quad \hat{W} = W_1 + aW_2, \quad \hat{F} = F_1 + aF_2 \tag{8}$$

such that $(\hat{q}, \hat{W}, \hat{F})$ converges exponentially to (q, W, F). As the parameter a is unknown, only a partial knowledge of the estimate is given by this observer termed hereafter "*partial observer*".

Theorem 1. Consider the visual dynamics given by Equations (7a-7c), assume that the UAV evolves in a domain such that:

$$\exists \sigma > 0 \text{ such that: } Q > \sigma I_d \tag{9}$$

Consider the following dynamics of the filter:

$$\dot{q}_1 = -\mathrm{sk}(\Omega)q_1 - A_1(q - q_1) - QW_1 \tag{10a}$$

$$\dot{q}_2 = -\mathrm{sk}(\Omega)q_2 + A_1 q_2 - QW_2 \tag{10b}$$

$$\dot{W}_1 = -\mathrm{sk}(\Omega)W_1 - A_2(q - q_1) + F_1 \tag{10c}$$

$$\dot{W}_2 = -\mathrm{sk}(\Omega)W_2 + A_2 q_2 + F_2 \tag{10d}$$

$$\dot{F}_1 = -\mathrm{sk}(\Omega)F_1 - A_3(q - q_1) \tag{10e}$$

$$\dot{F}_2 = -\mathrm{sk}(\Omega)F_2 + A_3 q_2 + u \tag{10f}$$

where (A_1, A_2, A_3) are three feedback matrices defined as:

$$
\begin{aligned}
A_1 &= \frac{(k(\gamma_2^2-8)+\lambda\gamma_2\gamma_1^2)I_d-4\gamma_1 Q}{2(8-\gamma_1^2+\gamma_2^2)}\\
A_2 &= \frac{(2k\gamma_1-2\lambda\gamma_1\gamma_2)I_d+8Q}{2(8-\gamma_1^2+\gamma_2^2)}\\
A_3 &= \lambda\frac{(\lambda\gamma_1^3+k\gamma_1\gamma_2-8\lambda\gamma_1)I_d+4\gamma_2 Q}{2(8-\gamma_1^2+\gamma_2^2)}
\end{aligned} \tag{11}
$$

If the observer gains $(\gamma_1,\gamma_2,\lambda)$ are such that:

$$
\begin{gathered}
(\gamma_1,\gamma_2)\in(0;2)\times(0;2)\\
\lambda<\sigma/4\\
16\lambda-\gamma_2\gamma_1\sigma<0
\end{gathered} \tag{12}
$$

then the virtual estimates $(\hat{q},\hat{W},\hat{F})$, Eq. 8, converge exponentially to (q,W,F).

Proof the proof lies on the design of a Lyapunov function depending on observation errors

$$
\tilde{q}=q-\hat{q},\quad \tilde{W}=W-\hat{W},\quad \tilde{F}=\frac{F-\hat{F}}{\lambda} \tag{13}
$$

Differentiating Eq. 13 and recalling Eqn's 1a-1c and the filter dynamics Eqn's 10a-10f, yields:

$$
\begin{aligned}
\dot{\tilde{q}} &= -\mathrm{sk}(\Omega)\tilde{q}\ +A_1\tilde{q}-Q\tilde{W}\\
\dot{\tilde{W}} &= -\mathrm{sk}(\Omega)\tilde{W}+A_2\tilde{q}+\lambda\tilde{F}\\
\dot{\tilde{F}} &= -\mathrm{sk}(\Omega)\tilde{F}\ +\frac{1}{\lambda}A_3\tilde{q}
\end{aligned} \tag{14}
$$

Now, consider the following Lyapunov function candidate:

$$
\mathscr{L}_0=\frac{1}{2}\left(|\tilde{q}|^2+\gamma_1\tilde{q}^T\tilde{W}+|\tilde{W}|^2+|\tilde{W}|^2-\gamma_2\tilde{W}^T\tilde{F}+|\tilde{F}|^2\right) \tag{15}
$$

Introducing dynamics (14) in the time derivative of $\mathscr{L}_0$, we get:

$$
\begin{aligned}
\dot{\mathscr{L}}_0 = &\ \tilde{q}^T(A_1+\frac{\gamma_1 A_2}{2})\tilde{q}+\tilde{W}^T(2A_2-Q+\frac{\gamma_1 A_1}{2}-\frac{\gamma_2 A_3}{2\lambda})\tilde{q}\\
&+\tilde{F}^T(\frac{A_3}{\lambda}-\frac{\gamma_2 A_2}{2}+\frac{\gamma_1\lambda I_d}{2})\tilde{q}\\
&-\frac{\gamma_1}{2}\tilde{W}^TQ\tilde{W}+2\lambda\tilde{W}^T\tilde{F}-\frac{\gamma_2\lambda}{2}|\tilde{F}|^2
\end{aligned} \tag{16}
$$

Introducing the expressions of the feedback matrices (11), the time derivative of the Lyapunov function candidate becomes:

$$
\dot{\mathscr{L}}_0=-k|\tilde{q}|^2-\frac{\gamma_1}{2}\tilde{W}^TQ\tilde{W}+2\lambda\tilde{W}^T\tilde{F}-\frac{\gamma_2}{2}\lambda|\tilde{F}|^2 \tag{17}
$$

Using hypothesis (9) along with Eq. 12, one can insure that the derivative of the Lyapunov function candidate is strictly negative. Indeed, rewritten $\dot{\mathscr{L}}_0$ as follows:

$$
\begin{aligned}
\dot{\mathscr{L}}_0 \le &\ -k|\tilde{q}|^2-\frac{\gamma_1\sigma}{4}|\tilde{W}|^2-\frac{\gamma_2\lambda}{4}|\tilde{F}|^2\\
&-\left(\frac{\gamma_1\sigma}{4}|\tilde{W}|^2-2\lambda\tilde{W}^T\tilde{F}+\frac{\gamma_2\lambda}{4}|\tilde{F}|^2\right)
\end{aligned} \tag{18}
$$

The term into the brackets, is a positive definite polynomial function if:

$$
16\lambda-\gamma_2\gamma_1\sigma<0 \tag{19}
$$

which is compatible with the constraints on the observer gains (Eq. 12). The Lyapunov function derivative can then be bounded as follows

$$
\dot{\mathscr{L}}_0<-k|\tilde{q}|^2-\frac{\gamma_1\sigma}{2}|\tilde{W}|^2-\frac{\gamma_2}{2}|\tilde{F}|^2 \tag{20}
$$

and therefore, application of Lyapunov's direct method (Khalil (2002)) ensures that $(\tilde{q},\tilde{W},\tilde{F})$ converge exponentially to zero.

3. CONTROL DESIGN

The proposed control strategy stabilizes the system (7a-7c) with respect to the image features and observed data. A Lyapunov function is designed through a backstepping process.

Define a first temporary storage function T_1 as

$$
T_1=\frac{1}{2}|\delta_1|^2 \tag{21}
$$

Taking the time derivate of T_1 yields:

$$
\dot{T}_1=-\delta_1^TQW \tag{22}
$$

The following change of variable is also proposed:

$$
\varepsilon=W-k_1\delta_1 \tag{23}
$$

where k_1 is a positive control gain. Define

$$
\begin{aligned}
\varepsilon_1 &= W_1-k_1(q_1-q^\star) & \varepsilon_2 &= W_2-k_1q_2\\
\hat{\varepsilon} &= \varepsilon_1+a\varepsilon_2 & \tilde{\varepsilon} &= \varepsilon-\hat{\varepsilon}
\end{aligned} \tag{24}
$$

The derivative of T_1 can be written as:

$$
\dot{T}_1=-k_1\delta_1^TQ\delta_1-\delta_1^TQ(\varepsilon_1+a\varepsilon_2)-\delta_1^TQ\tilde{\varepsilon}
$$

Define $\rho=1/a$ and $\hat{\rho}$ an estimate of ρ, the estimation error is denoted:

$$
\tilde{\rho}=\rho-\hat{\rho} \tag{25}
$$

The dynamics of T_1 become:

$$
\dot{T}_1=-k_1\delta_1^TQ\delta_1-\delta_1^TQ\delta_2-a\tilde{\rho}\delta_1^TQ\varepsilon_1-\delta_1^TQ\tilde{\varepsilon} \tag{26}
$$

where δ_2 is a second visual error term defined as:

$$
\delta_2=\varepsilon_2+\hat{\rho}\varepsilon_1 \tag{27}
$$

Extend, now, T_1 with the adaptation error $\tilde{\rho}$ to define the first storage function S_1:

$$
S_1=T_1+a\frac{\tilde{\rho}}{2\kappa_\rho}^2 \tag{28}
$$

where κ_ρ is a positive gain and design $\hat{\rho}$ through:

$$
\dot{\hat{\rho}}=-\kappa_\rho\delta_1^TQ\varepsilon_1 \tag{29}
$$

Differentiating Eq. 28 and recalling (26) and (29), we obtain:

$$
\dot{S}_1=-k_1\delta_1^TQ\delta_1-\delta_1^TQ\delta_2-\delta_1^TQ\tilde{\varepsilon} \tag{30}
$$

The first term of S_1 acts as a stabilizer of δ_1, the second one will be compensated through a second storage function, and, as explained later in the proof, the last term will be compensated by the observer dynamics.

Define, now, a second storage function

$$
S_2=\frac{a}{2}|\delta_2|^2 \tag{31}
$$

The dynamics of δ_2 can be written as:

$$
\dot{\delta}_2=-\mathrm{sk}(\Omega)\delta_2+\Delta_2^{obs}+F_2 \tag{32}
$$

where Δ_2^{obs} is a term gathering measurable variables and observer outputs (see its expression in Appendix, Eq. 54). Define

$$
F_2^d=-\Delta_2^{obs}+Q\delta_1-k_2\delta_2 \tag{33}
$$

where k_2 is a positive control gain, and define a new error term δ_3 in the backstepping process as

$$
\delta_3=F_2-F_2^d \tag{34}
$$

we obtain:

$$
\dot{S}_2=-k_2|\delta_2|^2+\delta_2^TQ\delta_1+\delta_2^T\delta_3 \tag{35}
$$

Note that the first term of these dynamics stabilizes the error term δ_2. The second term is cancelled in the dynamics of the sum $S_1 + S_2$. Then the last term has to be compensated by the last storage function, that we define as follows:

$$S_3 = \frac{|\delta_3|^2}{2} + \frac{\tilde{a}}{2\kappa_a}^2 \quad (36)$$

where $\tilde{a} = a - \hat{a}$ is the difference between the unknown parameter a and an adaptive variable $\hat{a}$.

First, consider the dynamics of δ_3:

$$\dot{\delta}_3 = -\mathrm{sk}(\Omega)\delta_3 + \Delta_3^{obs} + M_3^{obs} W + u \quad (37)$$

where Δ_3^{obs} and M_3^{obs} are respectively a vector and a matrix designed with measured and observed data. Their expressions are given in Appendix (Eqn's 56 and (55).

Then, design the control input u as follows:

$$\begin{aligned} u = & -\delta_2 - k_3\delta_3 - \Delta_3^{obs} - M_3^{obs}(W_1 + \hat{a}W_2) \\ & -\kappa_0 M_3^{obs\,T} M_3^{obs}\delta_3 \end{aligned} \quad (38)$$

where k_3 and κ_0 are two positive gains.

Finally design the dynamics of the adaptive variable $\hat{a}$ as:

$$\dot{\hat{a}} = -\kappa_a \delta_3^T M_3^{obs} W_2 \quad (39)$$

and differentiate the storage function S_3, it yields:

$$\dot{S}_3 = -\delta_2^T\delta_3 - k_3|\delta_3|^2 + \delta_3^T M_3^{obs}\tilde{W} - \kappa_0|M_3^{obs\,T}\delta_3|^2 \quad (40)$$

The combination of results obtained through the storage functions design with the dynamics of the Lyapunov function (17) of the observer allows to prove the stability of the closed-loop system in a large basin of attraction. The following theorem presents the main contribution of this paper:

Theorem 2. Consider the system dynamics (7a-7c) along with the control input u defined by (5). Denote σ a positive parameter such that

$$Q(\xi^\star) > \sigma I_d$$

where $\xi^\star$ denotes the desired set point. Consider the following Lyapunov function candidate

$$\mathscr{L} = S_1 + S_2 + S_3 + \mu\mathscr{L}_0 \quad (41)$$

which combines the storage functions (S_1, S_2, S_3) given by (21), (31) and (36), and the Lyapunov function $\mathscr{L}_0$ of the filter (15) (μ is a positive parameter).

Design the control input u as proposed in (38) along with dynamics (39) and (29) of $\hat{a}$ and $\hat{\rho}$ respectively.

Then there exists two positive parameters (μ, Λ) such that $\mathscr{L}$ is a Lyapunov function with a definite negative derivative all initial condition such that:

$$\mathscr{L}(0) < \Lambda, \quad (42)$$

Therefore, the closed-loop trajectories converge asymptotically to the desired set point $\xi^\star$, the Jacobian matrix verifies

$$\forall t \geq 0 \quad Q(t) > \sigma I_d$$

and the observation errors $(\tilde{q}, \tilde{W}, \tilde{F})$ converge exponentially to zero.

Proof: First consider the dynamics of the storage function S_3 (40) and note that the control input u (38) depends on the term "$-\kappa_0 M_3^{obs\,T} M_3^{obs}\delta_3$", introduced by using Nonlinear Damping Assignment techniques Krstic (1995) to provide the following expression in the time derivative of S_3:

$$\delta_3^T M_3^{obs}\tilde{W} - \kappa_0|M_3^{obs\,T}\delta_3|^2$$

This expression verifies:

$$\delta_3^T M_3^{obs}\tilde{W} - \kappa_0|M_3^{obs\,T}\delta_3|^2 \leq \frac{1}{\kappa_0}|\tilde{W}|^2$$

Therefore, the proposed storage function S_3 satisfies:

$$\dot{S}_3 = -\delta_2^T\delta_3 - k_3|\delta_3|^2 + \frac{1}{\kappa_0}|\tilde{W}|^2 \quad (43)$$

Summing the three proposed storage functions (21), (31) and (36), and computing the dynamics of their sum, we obtain:

$$\begin{aligned} \frac{d}{dt}\sum S_i & \leq -k_1\delta_1^T Q\delta_1 - k_2|\delta_2|^2 - k_3|\delta_3|^2 + \delta_1^T Q\tilde{\varepsilon} + \frac{|\tilde{W}|}{\kappa_0}^2 \\ & \leq -(k_1 - \frac{1+k_1}{2\kappa_1})\delta_1^T Q\delta_1 - k_2|\delta_2|^2 - k_3|\delta_3|^2 \\ & \quad + \frac{k_1\kappa_1}{2}\tilde{q}^T Q\tilde{q} + \tilde{W}(\frac{\kappa_1}{2}Q + \frac{1}{\kappa_0}I_d)\tilde{W} \end{aligned}$$

where κ_1 is a positive gain designed such that:

$$k_1 - \frac{1+k_1}{2\kappa_1} > k_1/2 \quad (44)$$

Since the time derivative of the sum of the storage functions S_i depends on the observation error terms $(\tilde{q}, \tilde{w})$, let us extend this sum with the Lyapunov function of the observer (15).

$$\mathscr{L} = S_1 + S_2 + S_3 + \mu\mathscr{L}_0$$

The next step of the proof consists in differentiating the above function to prove that its time derivative is a definite negative function. Note, However, that the time derivative of the Lyapunov function of the observer (17) is exploitable only if the trajectory remains in the area in which the hypothesis, Eq. 9, on the jacobian matrix is ensured. This is done below by reducing the initial condition set to a basin of attraction. This illustrates a classical complexity in nonlinear observation: the observer convergence may depend from the controller developed.

First, note that

$$Q(\xi^\star) > \sigma I_d \quad \text{and} \quad \delta_1(\xi^\star) = 0$$

then, there exists $m_\delta > 0$ such that

$$\forall \xi \in R^3, \quad |\delta_1(\xi)| < m_\delta \Rightarrow Q(\xi) > \sigma I_d \quad (45)$$

Note that

$$\mathscr{L} < \frac{m_\delta^2}{2} \Rightarrow |\delta_1| < m_\delta$$

For all initial conditions such that

$$\mathscr{L}(0) < \frac{m_\delta^2}{2} \quad (46)$$

one can insure that $Q(0) > \sigma I_d$. Invoking the continuity of the solution, there exists a maximal time $t_0 > 0$ (possibly infinite) such that:

$$\forall t \in [0; t_0[, \quad Q(t) > \sigma I_d \quad (47)$$

Consequently, for any $t \in [0; t_0[$, we can introduce the observer result (17) in the dynamics of $\mathscr{L}$:

$$\begin{aligned} \dot{\mathscr{L}} \leq & -\frac{k_1}{2}\delta_1^T Q\delta_1 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - \tilde{q}^T(\mu k I_d - \frac{k_1\kappa_1}{2}Q)\tilde{q} \\ & -\tilde{W}^T\left((\frac{\mu\gamma_1\sigma}{2} - \frac{1}{\kappa_0})I_d - \frac{\kappa_1}{2}Q\right)\tilde{W} \end{aligned}$$

First, choose $\mu > \frac{2}{\kappa_0\gamma_1\sigma}$ and note that (47) implies

$$\forall t \in [0; t_0[, \quad \frac{Q}{\sigma} > -I_d$$

we also get:

$$\begin{aligned} \dot{\mathscr{L}} \leq & -\frac{k_1}{2}\delta_1^T Q\delta_1 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - (\frac{\mu k}{\sigma} - \frac{k_1\kappa_1}{2})\tilde{q}^T Q\tilde{q} \\ & -\left(\frac{\mu\gamma_1}{2} - \frac{1}{\kappa_0\sigma} - \frac{\kappa_1}{2}\right)\tilde{W}^T Q\tilde{W} \end{aligned}$$

Choosing the gain μ such that:

$$\mu > \max\left(\frac{2}{\kappa_0\gamma_1\sigma}, \frac{k_1\kappa_1\sigma}{2k}, \frac{2 + \kappa_0\kappa_1\sigma}{\kappa_0\sigma\gamma_1}\right) \quad (48)$$

the time derivative of $\mathscr{L}$ becomes a definite negative function on $[0;t_0[$ which verifies:

$$\forall t \in [0;t_0[, \quad \dot{\mathscr{L}} \le -\frac{k_1\sigma}{2}|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 \tag{49}$$

Assume now that there exists any trajectory such that its maximal time $t_0 < \infty$, the Lyapunov function decreasing in $[0;t_0]$, we get:

$$\mathscr{L}(t_0) \le \mathscr{L}(0) \Rightarrow Q(t_0) > \sigma I_d$$

Therefore, there exists $\delta t > 0$ such that the condition in (47) holds on $[0;t_0+\delta_t[$. This contradicts the existence of a maximal $t_0 < \infty$ and implies that every trajectory starting in the basin of attraction (46) is defined for all $t \ge 0$ and verifies

$$Q(t) > \sigma I_d \quad \forall t \ge 0$$

Consequently, Eq.49 holds for all time $t \ge 0$.

Therefore, Lyapunov's direct method ensures convergence of all error terms to zero and stabilization of the closed-loop system. This completes the proof.

Remark: In the above development we introduced the control variable u (5) that is fully actuated by the system inputs $(\dot{T},\Omega)$. In practice, this assignment can be achieved leaving Ω_3 free to control independently the yaw dynamics (see e.g. Le Bras (2007)).

4. SIMULATION RESULTS OF PRACTICAL STABILIZATION OF A VTOL

In this section, the control algorithm presented in Sections 2-3 is applied to an idealized model of VTOL vehicle ($m = 5kg$, helicopter type).

The simulations undertaken consider the case of tracking four target marks on a vertical plane, centered three meters above the ground level:

$$P_i' = (0, \pm 0.5, -3 \pm .5), \quad i = 1,..,4$$

The desired image of the target is chosen such that the camera set point is located three meters ago from the target plane:

$$\xi^\star = (-3, 0, -3) \tag{50}$$

To define the size of the basin of attraction (46) around the desired position we consider the simple situation in which the vehicle is in hover conditions, such that [2]:

$$\xi_0 = (-4.5, -2, -4)^T, \quad R_0 = I_d, \quad \dot{\xi}_0 = \dot{\Omega} = 0$$

This ensures that $(W, F, \Omega) \approx 0$. Choose the initial conditions of the observer such that the estimate error terms be small $(\tilde{q}, \tilde{W}, \tilde{F}) \approx 0$. To simplify the following discussion, we assume that the initial adaptation errors are small $(\tilde{a}, \tilde{\rho}) \approx 0$. This corresponds, in practice, to a knowledge of the target.

In this situation, the initial value of the Lyapunov function, giving the basin attraction, is approximated as a function of the initial visual error term ($\mathscr{L}(0) \approx \Phi(\delta_1(0))$):

$$\mathscr{L}(0) \approx \frac{a}{2}\delta_1(0)^T \left(Q + \left(\bar{k}_1 + \bar{k}_2\delta_1(0)^T Q \delta_1(0)\right) I_d\right) \delta_1(0) + \frac{a+k_1^2}{2a}|\delta_1(0)|^2 \tag{51}$$

where $\bar{k}_1 = k_1 k_2 a^{-1}$ and $\bar{k}_2 = \kappa_\rho k_1^2 a^{-2}$ are two positive constants. The control gains chosen for this example are $k_1 = k_2 = 0.1$ along with small adaptive gains, as the adaptation dynamics should be slow. Moreover given the UAV mass and the target size, we get $a = 0.2$.

[2] According to standard aeronautical conventions, height is measured down relative to the aircraft, and hence, the height of the UAV is negative with respect of the inertial frame.

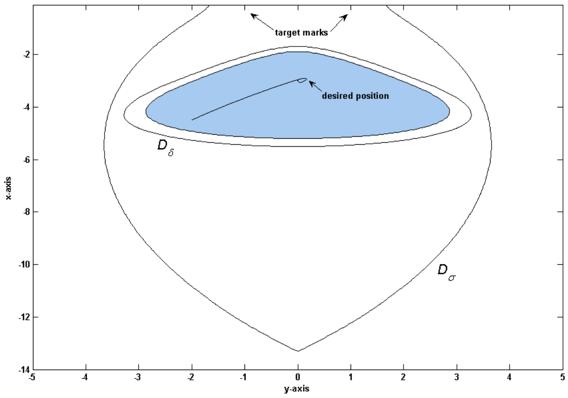

Fig. 2. Cross section in the (y, x) plane of the basin of attraction in blue, along with regions for which $Q > 0.3I_d$ ($\mathscr{D}_\sigma$) and $|\delta_1| < 2.5$ ($\mathscr{D}_\delta$). Trajectory shown is the projection onto the (y, x) plane of the ideal closed-loop response of the system starting on $(-4.5, -2, -3)$.

To evaluate the basin of attraction, we first plot the level lines of the Jacobian matrix Q and choose a minimal value σ compatible with a large basin. The corresponding level line delimitates a first domain $\mathscr{D}_\sigma$, we then consider the level lines of $|\delta_1|$ and choose a value m_δ such that the delimitated area $\mathscr{D}_\delta$ be strictly contained in $\mathscr{D}_\sigma$. Thus the basin of attraction is given by the area delimitated by the level line $\mathscr{L}(0) = \frac{m_\delta^2}{2}$ and plotted on Figure 2 along with $\sigma = 0.3$, $m_\delta = 2.5$. We notice that it contains a large basin around the desired position (about 6 meters along the y-axis and 3 along the x-axis).

On Fig.3, the performance of the control algorithm in the ideal case is shown; it clearly shows the asymptotic convergence of the error signals. The UAV achieves perfect tracking after a short transient, the initial oscillations are well damped and the set point is accurately reached.

Figure 4 shows the performance of the observer along x-axis. Both relative velocity, W, and force, F, are plotted with their estimates $(\hat{W}_x, \hat{F}_x)$. The estimations are available in simulation since the parameter a might be calculated with the target marks coordinates. This illustrates the exponential convergence of the observer.

5. CONCLUSION

In this paper, we have presented a combined observer and controller for dynamic image based visual servo control of a VTOL vehicle. The control task considered consists in stabilizing the vehicle with respect to an unknown stationary target. Visual data has been used to derive a partial observer and control design, based on an adaptive backstepping control design with nonlinear damping assignment, along with the an estimation of basin of attraction for the closed-loop system has been provided. Finally, simulation results have been presented to illustrate the performance of the observer-based control algorithm.

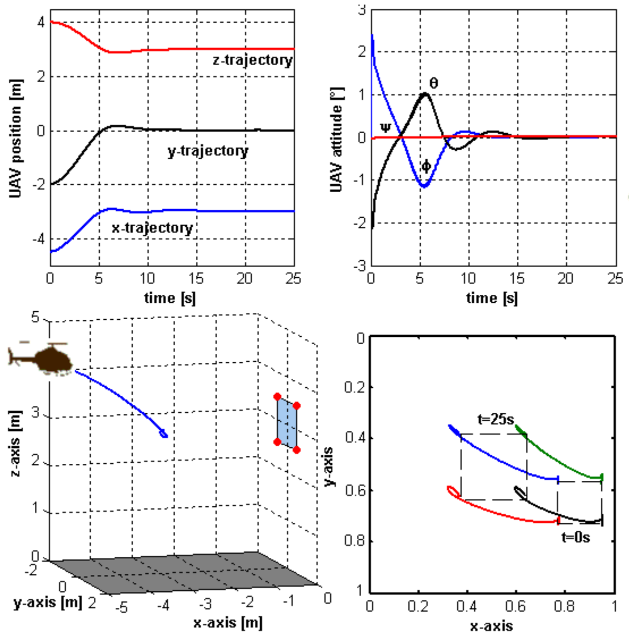

Fig. 3. Simulation results in the ideal case, upper figures show resp. the position and attitude responses. The third figure shows the 3D motion and the last one represents the evolution of the target image in the scaled camera frame.

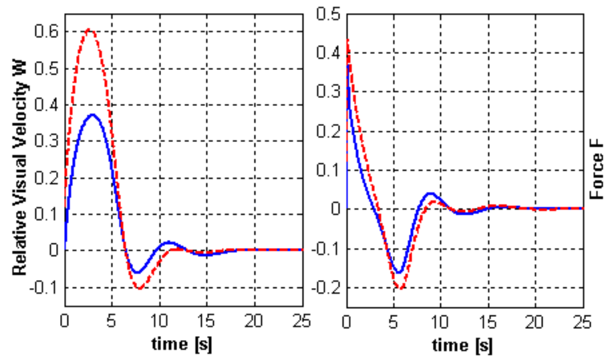

Fig. 4. Simulation results in the ideal case of the observer outputs $\hat{W}_x$ (left) and $\hat{F}_x$ (right). The real variables are in blue and the observer outputs in red dash-lines.

ACKNOWLEDGEMENT

This work was partially funded by the DGA and by ANR project SCUAV (ANR-06-ROBO-0007).

APPENDIX

The complex expressions of Δ_2^{obs}, Δ_3^{obs} and M_3^{obs} are given in this annexe, they are obtained by tedious calculations in the backstepping process. To calculate Δ_3^{obs} and M_3^{obs}, we used the following lemma giving an expression of the time derivative of the Jacobian matrix Q :

Lemma 3. Consider the following system:

$$\dot{x} = -\mathrm{sk}(\Omega)x + y \tag{52}$$

along with the jacobian matrix Q, then:

$$\frac{d}{dt}(Qx) = -\mathrm{sk}(\Omega)Qx + Qy + M_1(x)W \tag{53}$$

where $M_1(x)$ is a known matrix (see Le Bras (2007) for its detailed expression)$\diamondsuit$

Introducing

$$\alpha_2 = -\frac{2k_1\gamma_1 + 4}{8 - \gamma_1^2 - \gamma_2^2}$$

We also obtain:

$$\begin{aligned}\Delta_2^{obs} = & -\kappa_\rho(\delta_1^T Q\varepsilon_1)\varepsilon_1 + (k_1A_1 - A_2)(\hat{\rho}(q - q_1) - q_2) \\ & + k_1Q(\hat{\rho}W_1 + W_2) + \hat{\rho}F_1\end{aligned} \tag{54}$$

$$\begin{aligned}M_3^{obs} = & -\hat{\rho}(k_1A_1 - A_2)Q + Q^2 + \kappa_\rho\varepsilon_1\varepsilon_1^T\left(M_1(\delta_1) - Q^2\right) \\ & - M_1\left(\alpha_2(\hat{\rho}(q - q_1) - q_2) + k_1(\hat{\rho}W_1 + W_2) - \delta_1\right)\end{aligned} \tag{55}$$

$$\begin{aligned}\Delta_3^{obs} = & \left((k_1A_1 - A_2)A_1 - A_3 - k_1QA_2\right)(\hat{\rho}(q - q_1) - q_2) \\ & + Q(\hat{\rho}W_1 + W_2) + k_1Q(\hat{\rho}F_1 + F_2) \\ & + \kappa_\rho\delta_1^T Q\left((k_1A_1 - A_2)(q - q_1) + k_1QW_1 + F_1\right)\varepsilon_1\end{aligned} \tag{56}$$

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