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Output Feedback Strict Passivity of Discrete-time Nonlinear Systems and Adaptive Control System Design ⋆

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Abstract:
In this paper, a necessary and sufficient condition for a discrete-time nonlinear system to be strictly passive is derived and OFSP (Output Feedback Strictly Passive) conditions will be established. Based on the obtained OFSP conditions, an adaptive output feedback controller design method which can solve causality problems will be proposed for a discrete-time nonlinear system.

1. INTRODUCTION

Since many practical systems contain some kind of nonlinearities, a great deal of attention has been attracted to the control of nonlinear systems. Especially, of particular interest are passivity based controller designs for the control problem on nonlinear systems (Hill and Moylan, 1998; Byrnes et al., 1991; Krstic et al., 1994; Jiang and Hill, 1998; Fradkov and Hill, 1998; Byrnes and Lin, 1994; Lin and Byrnes, 1995). Although several important results have been obtained concerning passivity based controls, most of the results however were ones for continuous-time systems (Hill and Moylan, 1998; Byrnes et al., 1991; Krstic et al., 1994; Jiang and Hill, 1998; Fradkov and Hill, 1998). Our interest here is a discrete-time passive (or strictly passive) system. For discrete-time nonlinear systems, only few passivity based controls have been investigated with respect to lossless or passive systems (Byrnes and Lin, 1994; Lin and Byrnes, 1995; Chellaboina and Haddad, 2002).

In this paper, we consider a passivity-based adaptive output feedback control for discrete-time nonlinear systems. The passivity-based control schemes can be considered one of the Lyapunov-based controls. As for the Lyapunov-based adaptive controls, several significant results have been provided for discrete-time non-linear systems(Hayakawa et al., 2004). However, the developed methods were only with state feedback forms. Unlike the former works on the Lyapunov-based adaptive control, the passivity-based adaptive control dealt with in this paper is an output feedback-based adaptive control in which only the output signal is utilized in the controller design. It is well known that one can easily design an output feedback based adaptive control for an output feedback strictly (exponentially) passive (OFSP) system (Jiang and Hill, 1998; Fradkov and Hill, 1998; Michino et al., 2003; Mizumoto et al., 2005) and the obtained control system has a strong robustness with respect to disturbances and uncertainties. The system is said to be OFSP if there exists an output feedback such that the resulting closed loop system is strictly passive. Here we investigate the OFSP property of discrete-time nonlinear systems, and consider an output feedback-based adaptive control design problem for discrete-time nonlinear systems. To this end, we first derive a discrete-time nonlinear version of Kalman-Yakubovich-Popov (KYP) Lemma for a strictly passive system. The strict passivity of the control system plays an important role in adaptive controls. The KYP-Lemma for continuous-time nonlinear systems has been interpreted (Hill and Moylan, 1998; Jiang and Hill, 1998) and the KYP-Lemma for discrete-time nonlinear systems has been investigated for lossless and passive systems (Byrnes and Lin, 1994; Lin and Byrnes, 1995). We will develop the KYP-Lemma for strictly passive discrete-time nonlinear systems in order to design an adaptive control system. After that, OFSP conditions for discrete-time nonlinear systems will be clarified, and the design of output feedback-based adaptive control will be shown. As it is well known, a passive system must have a direct feedthrough term of input, that is, a passive system must have a relative degree of 0 (Byrnes and Lin, 1994). This means that the OFSP system also has to have a direct feedthrough term of the input (i.e. relative degree of 0), possibly resulting in a causality problem in the controller design. Conditions in which one can design the adaptive controller without causality problems will be provided as strong output feedback strict passivity, and according to the obtained conditions, an adaptive output feedback controller design scheme will be shown for a discrete-time nonlinear system which does not have a relative degree of 0.

2. PREPARATION

2.1 Strictly passivity

Consider the following n-th order discrete-time SISO nonlinear system with a relative of 0.

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$$x(k+1) = f(x(k)) + g(x(k))u(k) \tag{1}$$

$$y(k) = h(x(k)) + J(x(k))u(k) \tag{2}$$

where $x(k) \in R^n$ is a state vector, $u(k)$, $y(k) \in R$ are the input and output of the system. $f(x(k)) : R^n \to R^n$, $g(x(k)) : R^n \to R^n$, $h(x(k)) : R^n \to R$ and $J(x(k)) : R^n \to R$ are smooth in $x(k)$, and we assume that $f(0) = 0$, $h(0) = 0$.

The passivity and the strict passivity of the system (1),(2) are defined as follows (Byrnes and Lin, 1994):

Definition 1. (Passivity) A system (1), (2) is said to be passive if there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$, called the storage function, such that

$$V(x(k+1)) - V(x(k)) \leq y(k)u(k) \tag{3}$$

for all $u(k) \in R$, $\forall k \geq 0$.

Definition 2. (Strict Passivity) A system (1),(2) is said to be strictly passive if there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ and a positive definite function $S(x(k)) : R^n \to R$ such that

$$V(x(k+1)) - V(x(k)) \leq y(k)u(k) - S(x(k)) \tag{4}$$

for all $u(k) \in R$, $\forall k \geq 0$.

The property of a passive or lossless system has been studied in Byrnes and Lin (1994); Lin and Byrnes (1995). Here we first investigate the strict passivity by means of the discrete-time nonlinear version of the KYP-Lemma in order to develop the adaptive controller for discrete-time nonlinear systems.

Theorem 1. A system (1),(2) is strictly passive if and only if, there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ such that

A1-1) There exist functions $l(x)$, $W(x)$ and a positive definite function $S(x)$ such that

$$V(f(x)) - V(x) = -l(x)^2 - S(x) \tag{5}$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) = h(x) - 2l(x)W(x) \tag{6}$$

$$g^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) = 2J(x) - 2W(x)^2. \tag{7}$$

A1-2) $V(f(x) + g(x)u)$ is quadratic in u.

Proof: See Appendix A.

2.2 Output feedback strict passivity

Next, we define an output feedback strict passivity for a system (1),(2).

Definition 3. (Output feedback strictly passive: OFSP) A system (1),(2) is said to be output feedback strictly passive (OFSP) if there exists an output feedback:

$$u(k) = \alpha(y(k)) + \beta(y(k))v(k) \tag{8}$$

such that the resulting closed loop system is strictly passive.

Further we define a strong output feedback strict passivity as follows:

Definition 4. (Strongly OFSP) A system (1),(2) is said to be strongly OFSP if there exists a static output feedback:

$$u(k) = -\theta^* y(k) + v(k), \theta^* > 0 \tag{9}$$

such that the resulting closed loop system from $y(k)$ to $v(k)$,

$$x(k+1) = \bar{f}(x(k)) + \bar{g}(x(k))v(k) \tag{10}$$

$$y(k) = \bar{h}(x(k)) + \bar{J}(x(k))v(k) \tag{11}$$

with

$$\bar{f}(x(k)) = f(x(k)) - \frac{\theta^*}{1+\theta^* J(x(k))} h(x(k))g(x(k)) \tag{12}$$

$$\bar{g}(x(k)) = \frac{1}{1+\theta^* J(x(k))} g(x(k)) \tag{13}$$

$$\bar{h}(x(k)) = \frac{1}{1+\theta^* J(x(k))} h(x(k)) \tag{14}$$

$$\bar{J}(x(k)) = \frac{1}{1+\theta^* J(x(k))} J(x(k)) \tag{15}$$

is strictly passive and, in addition, a transformed closed loop system with

$$\bar{v}(k) = \frac{1}{1+\theta^* J(x(k))} v(k) \tag{16}$$

as input,

$$x(k+1) = \bar{f}(x(k)) + g(x(k))\bar{v}(k) \tag{17}$$

$$y(k) = \bar{h}(x(k)) + J(x(k))\bar{v}(k) \tag{18}$$

is also strictly passive.

The sufficient conditions for a system (1),(2) to be OFSP are provided by the following theorem.

Theorem 2. A system (1),(2) is OFSP with a static output feedback (9) and a C^2 positive definite storage function if

A2-1) The system has relative degree of 0 and $J(x(k)) > 0$, $\forall x(k)$.

A2-2) The zero dynamics of the system:

$$x(k+1) = f^*(x(k)) \tag{19}$$

is stable with the following C^2 positive definite function V satisfying

a) $V(f^*(x)) - V(x) = -\zeta(x)$ (20)

with a positive definite function $\zeta(x)$.

b) $V(f^*(x) + g(x)u)$ is quadratic in u.

c) There exist positive definite matrices Γ_m, Γ_M such that

$$0 < \Gamma_m \leq \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x(k))} \leq \Gamma_M$$

A2-3) $\frac{g(x(k))}{J(x(k))}$ is bounded.

Proof: The zero dynamics of the system(1),(2) is obtained by (Byrnes and Lin, 1994)

$$x(k+1) = f^*(x(k)) = f(x(k)) - \frac{h(x(k))}{J(x(k))} g(x(k)) \tag{21}$$

Since $\bar{f}(x)$ in the closed loop system (10) can be represented from (12) and (21) by

$$\begin{aligned}\bar{f}(x) &= f(x) - \frac{\theta^*}{1+\theta^* J(x)} h(x) g(x) \\ &= f^*(x) + \tilde{J}(x) h(x) g(x) \end{aligned} \tag{22}$$

with

$$\tilde{J}(x) = \frac{1}{J(x)\,(1+\theta^* J(x))}, \tag{23}$$

from assumption A2-2), b), $V(\bar{f}(x))$ can be expressed as

$$\begin{aligned} V(\bar{f}(x)) &= V(f^*(x)) + \tilde{J}(x) h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f^*(x)} g(x) \\ &+ \frac{1}{2}\tilde{J}(x)^2 h(x)^2 g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f^*(x)} g(x). \end{aligned} \tag{24}$$

Thus we have from (20),(24) that

$$\begin{aligned} & V(\bar{f}(x)) - V(x) \\ &= -\zeta(x) + \tilde{J}(x) h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \\ &- \frac{1}{2}\tilde{J}(x)^2 h(x)^2 g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x). \end{aligned} \tag{25}$$

Now, consider a function $\bar{W}(x)$ that satisfies the following relation:

$$\bar{W}(x)^2 = \bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x). \tag{26}$$

Such function $\bar{W}(x)$ is certain to exist for a sufficiently large θ^* from assumptions A2-2),c) and A2-3). Further, consider a function $\bar{l}(x(k))$ that satisfies

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x) - 2\bar{l}(x)\bar{W}(x). \tag{27}$$

Since (27) yields that

$$\begin{aligned} \bar{l}(x)^2 \bar{W}(x)^2 &= \frac{1}{4}\left\{ \bar{h}(x)^2 - 2\bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right. \\ &+ \left.\left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right)^2 \right\}, \end{aligned} \tag{28}$$

we have from (26) and (28) that

$$\begin{aligned} & \bar{l}(x)^2 \left(\bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right) \\ &= \frac{1}{4}\left\{ \bar{h}(x)^2 - 2\bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right. \\ &+ \left.\left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right)^2 \right\}. \end{aligned} \tag{29}$$

Thus, we obtain from (29) that

$$\begin{aligned} & \bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \\ &= -2\bar{l}(x)^2 \left\{ \bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right\} \\ &+ \frac{1}{2}\left\{ \bar{h}(x)^2 + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right)^2 \right\}. \end{aligned} \tag{30}$$

Furthermore, taking the definitions of $\bar{g}(x)$ and $\bar{h}(x)$ in (13) and (14) in to account, we have from (30) that

$$\begin{aligned} & h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \\ &= -2\bar{l}(x)^2 \left\{ (1+\theta^* J(x)) J(x) - \frac{1}{2} g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x) \right\} \\ &+ \frac{1}{2}\left\{ h(x)^2 + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \right)^2 \right\}. \end{aligned} \tag{31}$$

Therefore, we obtain from (25) and (31) that

$$\begin{aligned} & V(\bar{f}(x)) - V(x) \\ &= -\zeta(x) - 2\bar{l}(x)^2 \\ &+ \frac{1}{J(x)\,(1+\theta^* J(x))} \left[\frac{1}{2}\left\{ h(x)^2 \right.\right. \\ &+ \left.\left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \right)^2 \right\} \\ &+ \left\{ \bar{l}(x)^2 - \frac{1}{2}\frac{1}{J(x)\,(1+\theta^* J(x))} h(x)^2 \right\} \\ &\times \left. g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x) \right]. \end{aligned} \tag{32}$$

Finally, we have

$$V(\bar{f}(x)) - V(x) = -\bar{l}(x)^2 - \bar{S}(x) \tag{33}$$

where

$$\begin{aligned} \bar{S}(x) &= \zeta(x) + \bar{l}(x)^2 \\ &- \frac{1}{J(x)\,(1+\theta^* J(x))} \left[\frac{1}{2}\left\{ h(x)^2 \right.\right. \\ &+ \left.\left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \right)^2 \right\} \\ &+ \left\{ \bar{l}(x)^2 - \frac{1}{2}\frac{1}{J(x)\,(1+\theta^* J(x))} h(x)^2 \right\} \\ &\times \left. g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x) \right]. \end{aligned} \tag{34}$$

$\bar{S}(x(k))$ is certain to be a positive definite function with a sufficiently large θ^*. Thus we can conclude that, for a sufficiently large θ^*, there exists a positive definite C^2 function $V(x)$ with a property that $V(f(x) + g(x)u)$ is quadratic in u, functions $\bar{W}(x(k))$, $\bar{l}(x)$ and a positive definite function $\bar{S}(x(k))$ such that

$$V(\bar{f}(x)) - V(x) = -\bar{l}(x)^2 - \bar{S}(x) \quad (35)$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x) - 2\bar{l}(x)\bar{W}(x) \quad (36)$$

$$\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = 2\bar{J}(x) - 2\bar{W}(x)^2, \quad (37)$$

that is there exists a feedback gain θ^* such that the resulting closed loop system is strictly passive. Then the system is output feedback strictly passive with a C^2 positive definite function as the storage function.

Moreover, we have the following lemma concerning the strongly OFSP conditions.

Lemma 1. Assumptions A2-1), A2-2) and A2-3) in Theorem 2 are satisfied with $J(x(k)) = d > 0$ then the system (1), (2) is strongly OFSP.

Proof: See appendix B.

3. ADAPTIVE CONTROL SYSTEM DESIGN

3.1 Problem statement

Consider the following system with $J(x) = 0$ in (1), (2):

$$x(k+1) = f(x(k)) + g(x(k))u(k) \quad (38)$$

$$y(k) = h(x(k)). \quad (39)$$

This system is not OFSP but we impose the following assumptions.

Assumption 3. (1) $g(x(k))$ is bounded for all $x(k)$.

(2) There exists a known static parallel feedforward compensator (PFC): d such that the resulting augmented system:

$$x(k+1) = f(x(k)) + g(x(k))u(k) \quad (40)$$

$$y_a(k) = y(k) + du(k) = h(x(k)) + du(k) \quad (41)$$

is rendered OFSP with a static output feedback, that is the augmented system (40), (41) satisfies the OFSP conditions in the Theorem 2.

The objective here is to design an adaptive output feedback control system under Assumption 3.

3.2 Controller design

Under Assumption 3, (2), from Theorem 2 and Lemma 1, there exists a static output feedback:

$$u^*(k) = -\theta^* y_a(k) + v(k) \quad (42)$$

for the augmented system (40), (41), such that the resulting closed loop system with the transformed signal $\bar{v}(k) = (1 + \theta^* d)^{-1} v(k)$ as the input:

$$x(k+1) = \bar{f}(x(k)) + g(x(k))\bar{v}(k) \quad (43)$$

$$y_a(k) = \bar{h}(x(k)) + d\bar{v}(k) \quad (44)$$

$$\bar{f}(x(k)) = f(x(k)) - \frac{\theta^*}{1+\theta^* d} h(x(k))g(x(k)) \quad (45)$$

$$\bar{h}(x(k)) = \frac{1}{1+\theta^* d} y(k), \quad (46)$$

is strictly passive with a C^2 positive definite storage function.

Thus, if one can design a control input by

$$u^*(k) = -\theta^* y_a(k), \quad (47)$$

then a stable control system is obtained. However for a system with uncertainties, of course, θ^* is unknown, and because of the existence a direct feedthrough term of the input, the input (47) can not be implemented due to causality problems.

To overcome these problems, we first consider the following equivalent input obtained from (41):

$$u^*(k) = -\frac{\theta^*}{1+\theta^* d} y(k) = -\tilde{\theta}^* y(k), \tilde{\theta}^* = \frac{\theta^*}{1+\theta^* d}. \quad (48)$$

Then for this ideal control input, we design the control input adaptively as follows:

$$u(k) = -\tilde{\theta}(k)y(k) \quad (49)$$

where the feedback gain $\tilde{\theta}(k)$ is adaptively adjusted by the following parameter adjusting law:

$$\tilde{\theta}(k) = \tilde{\theta}(k-1) + \gamma y_a(k)y(k), \gamma > 0. \quad (50)$$

In this case, the augmented output $y_a(k)$ can be obtained from (41) by

$$y_a(k) = \frac{\left(1 - d\tilde{\theta}(k-1)\right) y(k)}{1 + d\gamma y(k)^2} \quad (51)$$

without causality problems. It should be noted that if the controller is designed based on the input (47), then causality problems will appear.

3.3 Stability analysis

The obtained closed loop system with the input (49) is expressed by

$$x(k+1) = \tilde{f}(x(k)) + g(x(k))\Delta u(k) \quad (52)$$

$$y_a(k) = \tilde{y}(k) + d\Delta u(k), \quad (53)$$

where

$$\tilde{f}(x(k)) = f(x(k)) - \tilde{\theta}^* y(k)g(x(k)) \quad (54)$$

$$\tilde{y}(k) = \left(1 - d\tilde{\theta}^*\right) y(k) \quad (55)$$

$$\Delta u(k) = -\Delta\tilde{\theta}(k)y(k), \Delta\tilde{\theta}(k) = \tilde{\theta}(k) - \tilde{\theta}^*. \quad (56)$$

From the definition of $\tilde{\theta}^*$, we have

$$\tilde{f}(x(k)) = f(x(k)) - \frac{\theta^*}{1+\theta^* d} y(k)g(x(k)) = \bar{f}(x(k)) \quad (57)$$

$$\tilde{y}(k) = \left(1 - \frac{\theta^* d}{1+\theta^* d}\right) y(k) = \frac{1}{1+\theta^* d} y(k) = \bar{h}(x(k)). \quad (58)$$

This means that the system (52), (53) is strictly passive with C^2 positive definite storage function.

Thus, there exists a C^2 positive definite function V_1, functions $l_1(x(k))$,$W_1(x(k))$, and a positive definite function $S_1(x(k))$ such that

C1)
$$V_1(\bar{f}(x)) - V_1(x) = -l_1(x)^2 - S_1(x)$$
$$\left.\frac{\partial V_1(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) = \bar{h}(x) - 2l_1(x)W_1(x)$$
$$g^T(x)\left.\frac{\partial^2 V_1(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x) = 2d - 2W_1(x)^2$$

C2) $V_1(\bar{f}(x) + g(x)\Delta u)$ is quadratic in Δu.

Therefore, considering the difference of $V_1(x(k))$, it is easy to show that we have

$$\begin{aligned} &V_1(x(k+1)) - V_1(x(k)) \\ &\quad = y_a(k)\Delta u(k) - S_1(x(k)) \\ &\quad\quad - (l_1(x(k)) + W_1(x(k))\Delta u(k))^2. \end{aligned} \tag{59}$$

Now, consider the following positive definite function V:

$$V(k) = V_1(x(k)) + V_2(k) \tag{60}$$
$$V_2(k) = \frac{1}{2\gamma}\Delta\tilde{\theta}(k-1)^2. \tag{61}$$

Define a difference $\Delta V(k)$ as

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \Delta V_1(x(k)) + \Delta V_2(k) \end{aligned} \tag{62}$$
$$\Delta V_1(x(k)) = V_1(x(k+1)) - V_1(x(k)) \tag{63}$$
$$\Delta V_2(k) = V_2(k+1) - V_2(k). \tag{64}$$

The difference $\Delta V_2(k)$ is represented by

$$\Delta V_2(k) = \frac{1}{2\gamma}\left(\Delta\tilde{\theta}(k)^2 - \Delta\tilde{\theta}(k-1)^2\right). \tag{65}$$

Since we have from (50) that

$$\Delta\tilde{\theta}(k-1) = \Delta\tilde{\theta}(k) - \gamma y_a(k)y(k), \tag{66}$$

we obtain

$$\Delta V_2(k) = -\Delta u(k)y_a(k) - \frac{1}{2}\gamma y_a(k)^2 y(k)^2. \tag{67}$$

Consequently, the difference ΔV can be evaluated from (59) and (67) by

$$\begin{aligned} \Delta V(k) &= -S_1(x(k)) - (l_1(x(k)) + W_1(x(k))\Delta u(k))^2 \\ &\quad -\frac{1}{2}\gamma y_a(k)^2 y(k)^2 \\ &\leq -S_1(x(k)) \leq 0. \end{aligned} \tag{68}$$

From this result, we can conclude that all the signals in the control system are uniformly bounded. Further, from (68), we have $\lim_{k\to\infty} x(k) = 0$. Thus we obtain $\lim_{k\to\infty} y(k) = 0$.

Finally, we have the following theorem.

Theorem 4. Under the Assumption 3, all the signals in the resulting closed loop control system with control input in (49) are uniformly bounded, and $\lim_{k\to\infty} y(k) = 0$ is achieved.

4. CONCLUSIONS

In this paper, we considered a passivity based adaptive output feedback control design for discrete-time nonlinear systems. We first clarified a discrete-time nonlinear version of Kalman-Yakubovich-Popov (KYP) Lemma for a strictly passive system, and then investigated the OFSP property of discrete-time nonlinear systems. Furthermore, conditions in which one can design an adaptive controller without causality problems were provided as strong output feedback strict passivity, and according to the obtained conditions, an adaptive output feedback controller design scheme was shown for a discrete-time nonlinear system.

REFERENCES

C.I. Byrnes, A. Isidori, and J. C. Willems. Passivity feedback equivalence and the global stabilization of minimum phase nonlinear systems. *IEEE Trans. on Automatic Control*, 36(11):1228–1240, 1991.

C.I. Byrnes and W. Lin. Losslessness feedback equivalence and the global stabilization of discrete-time nonlinear systems. *IEEE Trans. on Automatic Control*, 39(1):83–98, 1994.

V. Chellaboina and W.M. Haddad. Stability margins of discrete-time nonlinear-non-quadratic optimal regulations. *Int. J. of Systems Science*, 33(7):577–584, 2002.

A. Fradkov and D. J. Hill. Exponential feedback passivity and stabilizability of nonlinear systems. *Automatica*, 34(6):697–703, 1998.

T. Hayakawa, W.M. Haddad, and A. Leonessa. A lyapunov-based adaptive control framework for discrete-time non-linear systems with exogenous disturbances. *Int. J. Control*, 77(3):250–263, 2004.

D. J. Hill and P. Moylan. The stability of nonlinear dissipative systems. *IEEE Trans. on Automatic Control*, 21(10):708–711, 1998.

Z.-P. Jiang and D. J. Hill. Passivity and disturbance attenuation via output feedback for uncertain nonlinear systems. *IEEE Trans. on Automatic Control*, 43(7):992–997, 1998.

M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic. Passivity and parametric robustness of a new class of adaptive systems. *Automatica*, 30(11):1703–1716, 1994.

W. Lin and C.I. Byrnes. Passivity and absolute stabilization of a class of discrete-time nonlinear systems. *Automatica*, 31(2):263–267, 1995.

R. Michino, I. Mizumoto, Z. Iwai, and M. Kumon. Robust high gain adaptive output feedback control for nonlinear systems with uncertain nonlinearities in control input term. *Int. J. of Control, Automation, and Systems*, 1(1):19–27, 2003.

I. Mizumoto, R. Michino, M. Takahashi, M. Kumon, and Z. Iwai. Adaptive output feedback control of uncertain MIMO nonlinear systems with unknown orders. *Proc. of 16th IFAC World Congress, Prague, July*, page DVD, 2005.

Appendix A. PROOF OF THEOREM 1

(Necessity): If the system (1),(2) is strictly passive, then there exist a non-negative function $V(x(k))$ and a positive definite function $S(x(k))$ such that

$$V(x(k+1)) - V(x(k)) \leq y(k)u(k) - S(x(k)) \tag{A.1}$$

Considering functions $l(x(k))$ and $W(x(k))$ to satisfy

$$\begin{aligned} &V(x(k+1)) - V(x(k)) \\ &\quad = y(k)u(k) - S(x(k)) - (l(x(k)) + W(x(k))u(k))^2, \end{aligned} \tag{A.2}$$

we have

$$V(f(x)+g(x)u)=V(x)+h(x)u+J(x)u^2-S(x)-l(x)^2 -2l(x)W(x)u-W(x)^2u^2. \quad (A.3)$$

Setting $u(k)=0$, (5) is obviously satisfied. Further, from $(A.3)$ we have

$$\frac{\partial V(f(x)+g(x)u)}{\partial u} = \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)+g(x)u} g(x) = h(x)+2J(x)u-2l(x)W(x) -2W(x)^2u, \quad (A.4)$$

$$\frac{\partial^2 V(f(x)+g(x)u)}{\partial u^2} = g^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)+g(x)u} g(x) = 2J(x)-2W(x)^2. \quad (A.5)$$

Setting $u=0$ yields (6) and (7). A1-2) is obvious.

(Sufficiency): From A1-2), $V(f(x)+g(x)u)$ can be expressed as

$$V(f(x)+g(x)u) = A(x)+B(x)u+C(x)u^2 \quad (A.6)$$

Applying the Taylor expansion formula at $u(k)=0$, we have from A1-1) that

$$A(x) = V(f(x)+g(x)u)|_{u=0} = V(f(x)) = V(x)-l(x)^2-S(x), \quad (A.7)$$

$$B(x) = \left.\frac{\partial V(f(x)+g(x)u)}{\partial u}\right|_{u=0} = \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) = h(x)-2l(x)W(x), \quad (A.8)$$

$$C(x) = \frac{1}{2}\left.\frac{\partial^2 V(f(x)+g(x)u)}{\partial u^2}\right|_{u=0} = \frac{1}{2}g^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) = J(x)-W(x)^2. \quad (A.9)$$

Thus we obtain

$$V(f(x)+g(x)u) = V(x)+h(x)u+J(x)u^2-S(x). -l(x)^2-2l(x)W(x)u-W(x)^2u^2 = V(x)+yu-S(x)-(l(x)+W(x)u))^2. \quad (A.10)$$

This yields that

$$V(x(k+1))-V(x(k)) \le y(k)u(k)-S(x(k)). \quad (A.11)$$

Finally we can conclude that the system (1),(2) with assumptions A1-1) and A1-2) is strictly passive.

Appendix B. PROOF OF LEMMA 1

Consider a system (1),(2) with $J(x(k))=d$ satisfying assumptions A2-1) to A2-3) in Theorem 2:

$$x(k+1) = f(x(k))+g(x(k))u(k) \quad (B.1)$$

$$y(k) = h(x(k))+du(k). \quad (B.2)$$

From Theorem 2, there exists a static output feedback (9) such that the resulting closed loop system:

$$x(k+1) = \bar{f}(x(k))+\bar{g}(x(k))v(k) \quad (B.3)$$

$$y(k) = \bar{h}(x(k))+\bar{d}v(k) \quad (B.4)$$

with

$$\bar{f}(x(k)) = f(x(k)) - \frac{\theta^*}{1+\theta^* d}h(x(k))g(x(k))$$

$$\bar{g}(x(k)) = \frac{1}{1+\theta^* d}g(x(k))$$

$$\bar{h}(x(k)) = \frac{1}{1+\theta^* d}h(x(k)), \bar{d} = \frac{1}{1+\theta^* d}d$$

is strictly passive with a C^2 positive definite storage function. Thus from Theorem 1, there exist a C^2 positive definite function $V(x(k))$, functions $\bar{l}(x(k))$, $\bar{W}(x(k))$ and a positive definite function $\bar{S}(x(k))$ such that

$$V(\bar{f}(x))-V(x) = -\bar{l}(x)^2-\bar{S}(x) \quad (B.5)$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x)-2\bar{l}(x)\bar{W}(x) \quad (B.6)$$

$$\bar{g}^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = 2\bar{d}-2\bar{W}(x)^2 \quad (B.7)$$

and
$V(\bar{f}(x)+\bar{g}(x)v)$ is quadratic in v. In other words, the following equality is satisfied.

$$V(x(k+1))-V(x(k)) = y(k)v(k)-\bar{S}(x(k))-\left(\bar{l}(x(k))+\bar{W}(x(k))v(k)\right)^2. \quad (B.8)$$

Considering the transformed input:

$$\bar{v}(k) = \frac{1}{1+\theta^* d}v(k), \quad (B.9)$$

$(B.8)$ can be represented by

$$V(x(k+1))-V(x(k)) = y(k)\,(1+\theta^* d)\,\bar{v}(k)-\bar{S}(x(k)) - \left(\bar{l}(x(k))+\bar{W}(x(k))\,(1+\theta^* d)\,\bar{v}(k)\right)^2. \quad (B.10)$$

Thus we have

$$\bar{V}(x(k+1))-\bar{V}(x(k)) = y(k)\bar{v}(k)-\tilde{S}(x(k)) -\left(\tilde{l}(x(k))+\tilde{W}(x(k))\bar{v}(k)\right)^2 \quad (B.11)$$

where

$$\bar{V}(x(k)) = \frac{1}{1+\theta^* d}V(x(k)) \quad (B.12)$$

$$\tilde{S}(x(k)) = \frac{1}{1+\theta^* d}\bar{S}(x(k)) \quad (B.13)$$

$$\tilde{l}(x(k)) = \frac{1}{\sqrt{1+\theta^* d}}\bar{l}(x(k)) \quad (B.14)$$

$$\tilde{W}(x(k)) = \sqrt{1+\theta^* d}\bar{W}(x(k)). \quad (B.15)$$

This means that the system with the transformed input $\bar{v}$:

$$x(k+1) = \bar{f}(x(k))+g(x(k))\bar{v}(k) \quad (B.16)$$

$$y(k) = \bar{h}(x(k))+d\bar{v}(k) \quad (B.17)$$

is strictly passive with a C^2 positive definite storage function $\bar{V}$.