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Control-oriented Sensor/Actuator Location Measures for Active Noise Control ⋆

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Abstract: In this work, a combination of measures to quantify sensor and actuator allocation according to performance, robustness and controller implementation criteria are defined. Their computation can be made with standard software, both for SISO and MIMO systems. A test is run on a simulated acoustic tube which validates the optimal measure against the best closed loop performance and lower controller order combination.

Keywords: Sensor allocation, actuator allocation, active noise control.

1. INTRODUCTION

Control system design involves six well differentiated steps, as indicated in van de Wal and de Jager (2001). One of these steps is the control structure selection, and part of it is the input/output selection. This determines the number, place, and the type of actuators and sensors. The choice of inputs and outputs affects the performance, complexity, and costs of the control system. The selection problem is combinatorial in nature and hence, quantitative measures are needed to complement the design engineer's intuition, insight and experience.

Many works has been generated in this area, particularly for flexible structure testing (Leleu et al. (2001) and chapter 7 of Gawronski (2004) and references therein) and process control (Skogestad and Postlethwaite (1996) and references therein). The definition of the sensor/actuator (S/A) location problem is somewhat different for flexible structure testing, where the $\mathcal{H}_2, \mathcal{H}_\infty$ or Hankel norms need to be maximized with the least amount of sensor and actuators (Gawronski (2004)), that in control-oriented applications. An excellent overview of the whole area and many other different applications can be found in van de Wal and de Jager (2001).

For Active Noise Control (ANC) in particular, there are several works to be cited. Katsikas et al. (1995); Ruckman and Fuller (1995) study actuator placement for active noise control and Demetriou and Fahroo (1999) focuses on the efficiency of manipulation. More recent works have been extended to uncertain model sets with dynamic uncertainty as in Pulthasthan and Pota (2006). In Leleu et al. (2001) a magnitude to measure the optimal locations based on the controllability and observability grammians: W_c and W_o respectively, is computed. In Pulthasthan and Pota (2006) another measure is added to consider the effect of model uncertainty. Nevertheless, these grammians depend on the particular state-space realization, therefore any measure derived from it could be misleading. Furthermore, the sensor and actuator location problems are treated separately, by means of two different measures, one depending on W_c, the other on W_o. This could produce situations where a good location of the sensor (good observation properties) could interact with a bad location of the actuator (poor control action) and viceversa.

From a very general point of view, in van de Wal and de Jager (2001) different S/A allocation methods have been compared, based on eight characteristics: well-founded, efficient, effective, applicable, rigorous, quantitative, controller independent and direct. The conclusions indicate that robust performance oriented measures have not been computed under a controller independent constraint. Furthermore, controller complexity (basically controller order) should be integrated with other issues, e.g. performance, and combination of S/A measures should be applied for practical purposes. In that paper, although the general control configuration in Fig. 1 was used, at the time the performance limitations for that structure where not yet available (Freudenberg et al. (2003)).

As a consequence of all previous comments, the focus of this work is in computing a practical measure for S/A allocation for ANC applications, previous to controller design. The basic characteristics of these type of applications are: a stable uncertain plant with time delays and/or right half plane (RHP) zeros, lightly damped dynamics and, as with many other applications with *fast* dynamics, the need of a low order controller for real time implementation. The measure we seek combines relevant issues concerning performance, robustness and implementation. Uncertainty is modelled as global dynamic in general without excessive conservativeness which also produces lower order controllers, as opposed to structured dynamic or parametric uncertainty which uses μ–synthesis design procedures. Other criteria are focused on robust performance with structured uncertainty (Lee et al. (1994)) but cannot compute an S/A measure previous to controller design. The general control configuration in Fig. 1 will be used and the performance limitations due to RHP zeros are based on (Freudenberg et al. (2003)). Our approach is focused in computing the optimal S/A combination achieving the best performance and controller complexity, assuming that a controller exists. This can be easily verified in

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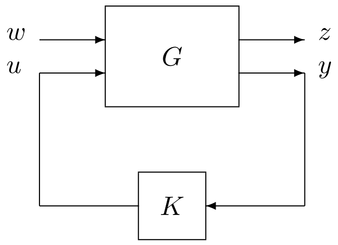

Fig. 1. Control design structure $T_{zw} = F_\ell(G, K)$.

general, e.g. for mixed sensitivity $\mathcal{H}_\infty$ control use conditions in Doyle et al. (1989). The performance weight $W_e(s)$ and its corresponding bandwidth Ω where noise attenuation is desired, is an input data for the problem .

An important part of the measure we seek considers properties of the model itself, as the *numerical* order it should have, computed by means of the Hankel Singular Values (HSV). Hence it includes simultaneously W_c and W_o, and also takes into account the controller order (proportional to the augmented model order), a key issue for real time implementation. The identification procedure is based on subspace methods (Overschee and Moor (1994); Verhaegen (1994)) which already computes the HSV and also provides a good criteria to select the best numerical model order, and can be used both for SISO (tubes, helmets) or MIMO (tubes, cavities) ANC applications. In addition, the bandwidth limitations imposed by model uncertainty and non minimum phase zeros are also taken into account. The computation of this measure can be made with standard software in both cases (SISO or MIMO). A simulated example illustrates the use of this measure in an acoustic tube used in active noise control (ANC). Finally, as mentioned in Jager and Toker (1998) it is unlikely that the methods that solve the S/A selection problem have polynomial time complexity, hence most methods are indirect in the sense that a candidate-by-candidate test should be performed. Nevertheless in this case, a controller design is not necessary to compute the S/A location measure, which reduces the time search. Therefore, based on the characteristics to evaluate S/A location methods, the one presented here is: well-founded, efficient (because it does not involve controller design, although it is not polynomial-time complexity), generally applicable (it uses the structure in Fig. 1 although it is focused to ANC), rigorous (it considers performance, robustness and implementation), quantitative, controller independent and indirect.

The paper is organized as follows: next section presents some background material and the control design setup. In section 3, the main results of this work are presented, which are illustrated by means of an ANC example in section 4. Final conclusions and future research issues are presented in section 5.

2. CONTROL PROBLEM

The configuration adopted here is the general one depicted in Fig. 1 which represents many different control problems. Here $G(s)$ is the augmented model which includes not only the nominal plant G_{yu} but also the performance and uncertainty weights, i.e.

$$G(s) = \begin{bmatrix} G_{zw}(s) & G_{zu}(s) \\ G_{yw}(s) & G_{yu}(s) \end{bmatrix}$$

where w is the vector of disturbances, z the vector of signals to be minimized, (u, y) the input and output of the system, and $F_\ell(\cdot)$ the lower linear fractional transformation operator. This setting may consider general performance and robustness constraints and applies not only to SISO but also to MIMO systems. The performance objective here is represented by the weight $W_e(s)$ that is greater than one, i.e. ($\bar{\sigma}[W_e(\jmath\omega)] > 1$) in the bandwidth Ω where noise attenuation is desired. Without loss of generality, robust performance quantified as $\min \|T_{zw}(s)\|_\infty$ could represent a typical mixed sensitivity problem. Otherwise, a better representation would be $\|T_{zw}(s, \gamma)\|_\infty < 1$, where we seek the minimum γ that weights performance as follows $\frac{1}{\gamma}W_e(s)$. A necessary pre-requisite is to assume that a controller exists for such a task, which can be verified by the existence conditions in Doyle et al. (1989) for the $\mathcal{H}_\infty$ control problem.

S/A allocation is an important part of the identification and control problem in most applications. Nominal model-based measures (Leleu et al. (2001)), or even uncertain model-based criteria (Pulthasthan and Pota (2006)) which evaluate S/A allocation are based on the controllability and observability grammians W_c and W_o. These measures depend on the state definition, hence they could inaccurately represent the physical system. Furthermore, sensor and actuator location problems are treated independently, i.e. the measures depend on both grammians separately.

To avoid this, we may use a standard state-space representation of models, which has been used for model order reduction (Moore (1981); Glover (1984)). This is the internally balanced state-space realization, which has the particular advantage that both grammians are equal and diagonal, with the (ordered) Hankel singular values in their diagonal, i.e.

$$W_c = W_o = \begin{bmatrix} \sigma_1^H & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_n^H \end{bmatrix}$$

This provides the optimal balance between controllability and observability and allows a stable and balanced model order reduction by truncation of the states corresponding to the smallest Hankel singular values. In addition, a bound on the reduction error can be obtained as a function of these values. More importantly, balanced realizations provide the minimal condition number of the observability and controllability grammians (Moore (1981)) over all possible state space realizations, i.e.

$$\min_T \max\left[\kappa(W_o), \kappa(W_c)\right] = \frac{\sigma_1^H}{\sigma_n^H}$$

where $\kappa(W) = \frac{\bar{\sigma}(W)}{\underline{\sigma}(W)}$ is the condition number. This allows a coherent distribution of the states so that the "most" (higher Hankel singular values) controllable ones are also the "most" observable ones.

As in any practical case, if accessibility is guaranteed (the states accessible from the inputs and the outputs from the states), necessary and sufficient conditions for structural state controllability and observability are guaranteed according to Morari and Stephanopoulos (1980).

A recent deep study of the performance limitations in the feedback structure adopted here (Fig. 1) has been made in Freudenberg et al. (2003), which generalizes the one in Freudenberg and Looze (1985). The limitations imposed by right half plane

(RHP) poles and zeros have been quantified. They reduce to the usual restrictions in standard feedback loops (see Freudenberg and Looze (1985)) when $\det[G] \equiv 0$ and the LFT is said to be *reducible* to a feedback loop. This is the case when the performance output is measured for feedback $z = y$ or when the control and disturbance excite the system in the same point, $w = u$. Here, due to the particular application we seek, the RHP pole limitations will not be considered.

In the general case ($\det[G] \neq 0$), the algebraic limitations on robust performance $\|T_{zw}\|_\infty$ are imposed by the RHP zeros $(\varsigma_1, \ldots, \varsigma_m)$ of G_{zu} or G_{yw} with multiplicities satisfying $m_{zw}(\varsigma) < m_{zu}(\varsigma) + m_{yw}(\varsigma)$ and are quantified as follows:

$$\|T_{zw}\|_\infty \geq \max_j |G^o_{zw}(\varsigma_j)| \stackrel{\triangle}{=} \gamma_z \tag{1}$$
$$G_{zw}(s) = G^o_{zw}(s) B_\varsigma(s)$$

where $B_\varsigma(s)$ is the Blaschke product corresponding to all RHP zeros ς_j, which absorbs them from $G_{zw}(s)$ (Corollary IV.2 of Freudenberg et al. (2003)). As a consequence, γ_z poses a lower bound on the robust performance measure γ. Usually the RHP zeros of the model G_{yu} constraint the sensitivity function, but note that here they only contribute to the performance limitation in the reducible to a feedback loop case, i.e. $\det(G) = 0$.

In the same work, a general expression of the integral constraint and a controller free lower bound are presented:

$$\int_0^\infty \log|R_{zw}|\, d\omega = \pi \left[\sum_i \mathbb{R}e(\beta_i) - \sum_j \mathbb{R}e(\alpha_j) \right] \tag{2}$$
$$\geq -\pi \sum_j \mathbb{R}e(\alpha_j) \tag{3}$$

where $R_{zw} = T_{zw}/G_{zw}$, α_j are the RHP zeros of G_{zw} not shared with T_{zw} and β_i the RHP zeros of T_{zw} not shared with G_{zw}. The negative value of the bound does not lessen the design limitation, and due to practical arguments discussed in Freudenberg et al. (2003), a reasonable lower bound to adopt is zero, i.e. the classical Bode integral. Therefore in this case there is no significant contribution to performance limitations due to analytic constraints (remember that here we consider the stable case).

Furthermore, the condition number of the model $\kappa(G)$ is an important factor in the interplay between performance and input dynamic uncertainty (clearly only for MIMO systems), as first indicated in Skogestad et al. (1988). For example in a classical loop-shaping design for weighted ($W_e(s)$) tracking error attenuation, the sufficient condition for robust performance (RP) is affected by this parameter when global (input) actuator dynamic uncertainty weighted by $W_\delta(s)$ is present (Sánchez Peña and Sznaier (1998)), i.e.

$$RP \Longleftarrow \kappa\left[G(s)\right] \bar{\sigma}\left[W_e(s)S(s)\right] + \bar{\sigma}\left[W_\delta(s)T(s)\right] < 1 \tag{4}$$

for all $s = \jmath\omega$. As a consequence, robust performance is decreased at those frequencies where $\kappa\left[G(\jmath\omega)\right] \geq 1$ is large, which may happen in practical situations, e.g. high-purity distillation plants.

In addition, general classes of model uncertainty can be represented by global dynamic multiplicative uncertainty (Doyle et al. (1992); Skogestad and Postlethwaite (1996); Sánchez Peña and Sznaier (1998)):

$$\mathcal{G} = \left\{ \tilde{G} \mid \left[I + \Delta W_\delta(s)\right] G_o(s),\ \bar{\sigma}\left[\Delta\right] < 1 \right\} \tag{5}$$
$$\bar{\sigma}\left[W_\delta(\jmath\omega)\right] \geq \bar{\sigma}\left\{ \left[\tilde{G}(\jmath\omega) - G_o(\jmath\omega)\right] G_o^{-1}(\jmath\omega) \right\}$$

This representation has the additional advantage that the uncertainty weight crossover frequency poses an upper limit for the performance bandwidth or, in general, allows robust performance only at those frequencies where $\bar{\sigma}\left[W_\delta(\jmath\omega)\right] < 1$. The limitation in the performance bandwidth can be quantified as follows:

$$\Omega_p = \left\{ \begin{array}{l} \sum_{i=1}^{n} (\omega_i^u - \omega_i^\ell),\ \forall\omega \in [\omega_1^\ell, \omega_n^u] \subseteq \Omega \text{ such that} \\ \bar{\sigma}\left\{ \left[\tilde{G}(\jmath\omega) - G_o(\jmath\omega)\right] G_o^{-1}(\jmath\omega) \right\} < 1 \end{array} \right\} \tag{6}$$

where clearly $\omega_i^u \geq \omega_i^\ell$, $\forall i = 1, \cdots, n$. This measures the relative size of the bandwidth with respect to the desired one Ω, for which robust performance should be achieved. This may well be zero if such condition is not met, i.e. $\bar{\sigma}\left\{ \left[\tilde{G}(\jmath\omega) - G_o(\jmath\omega)\right] G_o^{-1}(\jmath\omega) \right\} > 1,\ \forall\omega \in \Omega$.

Instead, the direct limitation on robust performance, as represented in equation (4) would be:

$$\ell_\delta = \min\left\{1, \left(\max \bar{\sigma}\left[W_\delta(\jmath\omega)\right],\ \omega \in \Omega\right)\right\} \tag{7}$$

As a result of the all the above restrictions, several quantifiable values can be related with performance, robustness and controller implementation:

- Model order: directly related to the controller order, e.g. in $\mathcal{H}_\infty$ optimal control, the controller has the order of the nominal model plus the robustness and performance weights. The model order can be related to the set of positive Hankel singular values.
- Right half–plane zeros limit robust performance in the general case ($\det[G] \neq 0$) as indicated in equation (1). In fact, an interpretation in Doyle et al. (1992), considers that they pose a similar performance limitation as dynamic uncertainty. Also in the case of MIMO systems, the nominal model condition number $\kappa(G_o)$ combined with actuator (input) dynamic uncertainty is also a performance limiting factor.
- Model uncertainty: in particular, if quantified as global multiplicative dynamic uncertainty, $\mathcal{G}$ poses a robust performance bandwidth limitation measured by Ω_p in (6) or directly on robust performance quantified by ℓ_δ in (7).

In this work, the criteria to define the S/A optimal location takes into account the final goal pursued by any identification and control methodology: closed loop robust performance and controller implementation. Hence, all these items should be taken into consideration when defining a measure that quantifies the S/A allocation.

The previous results are fairly general, although here we focus on disturbance attenuation of dynamically globally uncertain stable lightly damped plants with oscillatory modes, e.g. vibration, acoustic noise, flexible structures, etc. In all these cases, $T_{zw}(s)$ represents a disturbance rejection objective, the actual plant is open loop stable and it usually has RHP zeros. This last point occurs in particular when sensors and actuators are non collocated, which generate delays and non minimum phase zeros as a consequence.

3. S/A ALLOCATION MEASURE

The S/A location measure should produce higher values for higher closed loop performance. In addition, for practical implementation, the lower the controller order the better, hence the controller order decreases this measure. Both values can be calculated even before designing the controller, based only on the identified model and the uncertainty of the plant for different actuator and sensor locations. For normalization purposes it seems convenient to assign the worst and best values as 0 and 1, respectively.

To this end, and in order to have good numerical properties of the plant's model for controller design, we first produce an internally balanced realization of the nominal model at each S/A location, which will be defined as $G_{\ell_{as}}(s) \triangleq G_{yu}(s)$. For the set of all possible sensor and actuator locations $\ell_{sa} = \{\ell_s, \ell_a\}$, we define the following partial measures which quantify robustness, performance and controller implementation:

Definition 3.1. The influence of controller implementation on the optimal S/A measure can be quantified as follows:

$$\rho_o(\ell_{as}) \triangleq \left\{ i+1 \;\middle|\; \max_i \sigma_i^H \left[G_{\ell_{as}}(s)\right] > \epsilon_r \right\}^{-1} \tag{8}$$

where σ_i^H are the Hankel singular values of $G_{\ell_{as}}$ and $\epsilon_r > 0$ is a predefined (controllability/observability) safety margin [1]. The $(i+1)$ term takes into consideration the possibility of a constant model, i.e. order $i = 0$, otherwise only i should be considered in the definition.

Definition 3.2. The performance S/A measure imposed by RHP zeros in the general closed loop configuration of Fig. 1 is as follows:

$$\rho_p(\ell_{sa}) \triangleq (1+\gamma_z)^{-1} \tag{9}$$

where γ_z has been defined in (1).

Definition 3.3. The measure which relates robust performance with the limitation imposed by the model condition number is (only useful in the MIMO case combined with actuator uncertainty):

$$\rho_\kappa(\ell_{sa}) \triangleq \left\{ \frac{\max_\omega \bar{\sigma}\left[W_e(\jmath\omega)\right] \kappa\left[G_{\ell_{as}}(\jmath\omega)\right]}{\max_\omega \bar{\sigma}\left[W_e(\jmath\omega)\right]} \right\}^{-1} \tag{10}$$

Definition 3.4. The measure which relates robust performance with the limitation imposed by uncertainty could well be combined with the previous measure, both being related to equation (4):

$$\rho_\delta(\ell_{sa}) \triangleq 1 - \ell_\delta \tag{11}$$

ℓ_δ defined in (7).

Definition 3.5. The measure imposed by uncertainty on the performance bandwidth is defined as follows:

$$\rho_\Omega(\ell_{sa}) \triangleq \frac{\Omega_p}{|\Omega|} \tag{12}$$

where Ω_p is defined in (6) with the nominal model $G_o(s)$ replaced by $G_{\ell_{as}}(s)$ and $|\Omega|$ is the size of the performance bandwidth.

[1] Recall that $\sigma_n^H = 0$ or *numerically* near implies an uncontrollable and/or unobservable state space representation. Another alternative is to use the subspace identification criteria to select the model order.

Fig. 2. Acoustic noise tube experiment

All these measures contribute to determine an optimal value for the S/A location, although they quantify different aspects of robust performance and implementation issues. Probably only ρ_κ and ρ_δ are amenable to be combined, due to the fact that they are related with the same equation (4). In any case, the user could take all these issues into consideration by defining a general weighted combination of all previous values as follows:

Definition 3.6. A general control-oriented S/A measure can be defined as a convex combination of all the previous ones:

$$\rho_{as} = \max_{\ell_a,\ell_s} \sum_{\mathcal{S}} w_i\, \rho_i(\ell_{as}), \quad \mathcal{S} = \{o, p, \kappa, \delta, \Omega\}$$

As before, $(\ell_s, \ell_a) = (\ell_{sa})$ has been replaced for simplicity. The weights $w_o, w_p, w_\kappa, w_\delta, w_\Omega \in [0,1]$ with $w_o + w_p + w_\kappa + w_\delta + w_\Omega = 1$. They are constant real values which weight the relative importance of performance and controller implementation, and are supplied by the user. The weight $w_\kappa = 0$ in cases where the system is SISO or when sensor (instead of actuator uncertainty) dominates the global dynamic model set, as in $\mathcal{G}$ in equation (5). This measure will therefore be normalized in the interval $[0,1]$.

Another alternative could be to select the best S/A locations according to each measure and to combine them in order to make a pre-selection of the definite S/A location. In cases where the measures quantify different aspects of the problem, e.g. ρ_o and ρ_Ω, there is the possibility that the best S/A selection of each of them will not coincide, i.e. null intersection. In those cases, the set union would be the way to combine both selections. Instead, if the measures quantify similar aspects like the case of ρ_κ and ρ_δ, their intersection could be better to compute the best S/A selection. This will be attempted in the example presented in next section.

4. ANC APPLICATION EXAMPLE

A simulated application illustrates the usefulness of the S/A allocation measures derived previously. The model has been taken from Hong et al. (1996), but with the dimensional data of the experiment shown in Fig. 2, including an experimental validation in one of the S/A locations.

Some *a priori* specifications and information from the experimental plant have been taken into account before deciding the grid to be considered, so that the selection of the best sensor/actuator location makes sense. In this example, a grid of

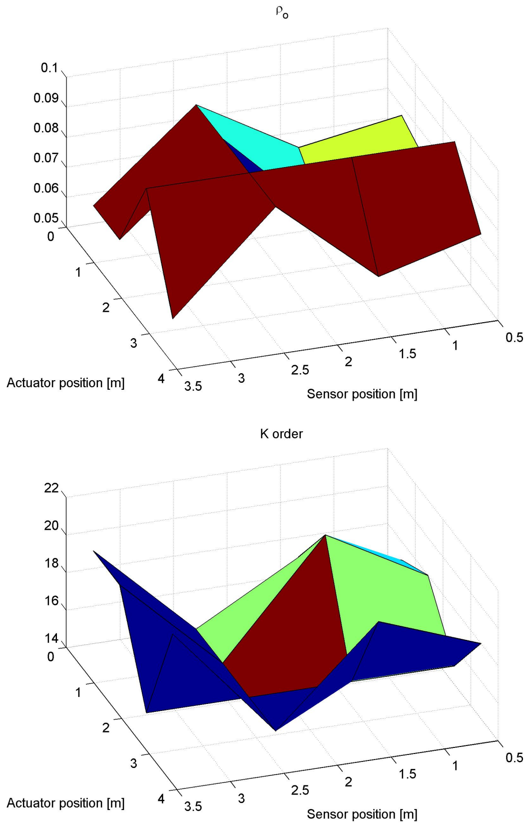

Fig. 3. Implementation issue: ρ_o (upper) and controller order (lower)

four sensor/actuator positions in the duct have been considered to simulate 16 different S/A locations.

As commented previously, the objective is to decide the best locations ***before*** the controller design, in order to reduce the analysis and design computational burden. The criteria is based on performance and implementation issues. Here, to validate the methodology, an *a posteriori* design of controllers for all test locations is presented, and their performance and order are evaluated. An important point is that the best locations detected by the measures should *include* at least, the ones produced by the controllers.

First, the practical implementation measure ρ_o which depends on the model order, is compared with the resulting controller order (which also depends on the specification weights) in Fig. 3. Due to the fact that both, performance and robustness weights used in all locations, had the same order, i.e. 2, there is a perfect coherence between the higher values of ρ_o and the lower controller orders. The higher values of ρ_o produce the following S/A selection, which corresponds with (ℓ_s, ℓ_a):

$$\mathcal{S}_{\rho_o} = \{(2.4, 0.5), (2.4, 1.46), (2.4, 3.4), (0.5, 2.4), (1.46, 2.4), (2.4, 2.4), (3.4, 2.4)\}$$

This coincides in all cases with the lower controller order, i.e. 15, obtained *a posteriori* from the design. For reasons that will be clear at the end of the example, if we expand this selection to the second higher value of ρ_o, a new location is added: $(0.5, 0.5)$. This corresponds to a controller of order 17. Both controller orders can be easily implemented for real time active noise control using the hardware at hand in our laboratory.

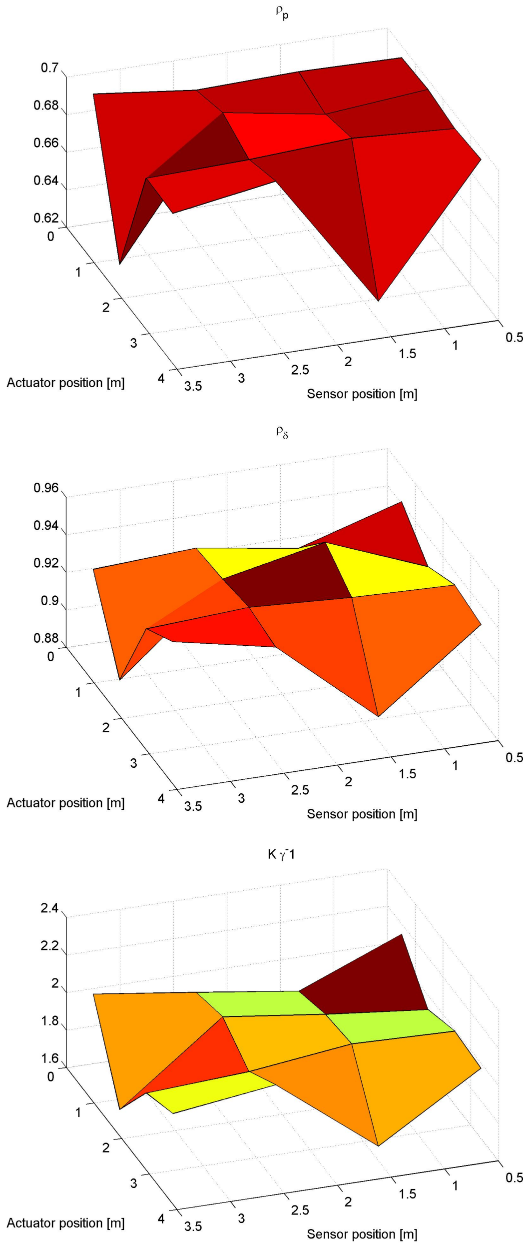

Fig. 4. Performance issues: ρ_p (upper), ρ_δ (middle) and controller performance γ^{-1} (lower)

Next, performance measures calculated *a priori* , due to model's RHP zero limitations (ρ_p) and multiplicative uncertainty (ρ_δ), are compared with the optimal γ (actually its inverse) obtained by the controller design in Fig. 4. The values of ρ_p are all very close to each other, as illustrated in the upper portion of the same figure, so little discrimination can be obtained from these values, considering possible numerical errors. Nevertheless, this has been considered as an extra information. Instead, the values of ρ_Ω which measure the practical bandwidth taken into account due to uncertainty were not used, because they had a value of one at all S/A locations ($\Omega_p \equiv \Omega$) and this imposes no limitation.

From the performance point of view, (ρ_p) and (ρ_δ) suggest several locations as the best ones, which do not intersect. Nevertheless, the union of these locations include the best positions suggested by the controller performance, measured by γ. Specifically, ρ_δ indicates three different locations:

$$\mathcal{S}_{\rho_\delta} = \{(\mathbf{0.5}, \mathbf{0.5}), (1.46, 1.46), (3.4, 3.4)\}$$

and ρ_p suggests another three:

$$\mathcal{S}_{\rho_p} = \{(\mathbf{3.4}, \mathbf{0.5}), (2.4, 1.46), (2.4, 3.4)\}$$

The best *a posteriori* performances derived from the controller design were achieved at $(\ell_s, \ell_a) = \{(\mathbf{3.4}, \mathbf{0.5}), (\mathbf{0.5}, \mathbf{0.5})\}$.

To conclude, the set of best locations obtained from performance and implementation measures do not intersect in this case, which is perfectly possible in general, due to the fact that they treat different issues. But as suggested previously, if we expand the selection through ρ_o to its second best value, the location $(0.5, 0.5)$ is added to the selection. Now it coincides with one of the best selections using the measures ρ_δ and ρ_p. In conclusion, the optimal S/A location in this case can be computed as follows:

$$(\ell_s, \ell_a)_{opt} = \left\{\mathcal{S}_{\rho_\delta} \cup \mathcal{S}_{\rho_p}\right\} \cap \left\{\mathcal{S}_{\rho_o}\right\} = (0.5, 0.5) \qquad (13)$$

Therefore the best location will have the best performance of $\gamma = 0.4673$ implemented by an $\mathcal{H}_\infty$ controller of order 17.

5. FUTURE RESEARCH

This is an ongoing research that still needs plenty of work until a precise quantitative methodology can be obtained. A future related issue is to improve this measure in order to make it less conservative for robust performance in controller free conditions. Also, exploring polynomial time computation using the ideas of checking subsets (supersets) of nonviable (viable) S/A sets as indicated in van de Wal and de Jager (2001), will be explored. The weight determination which combines all these measures into a single one could be very useful, but practical rules to determine the corresponding weights should be studied. Otherwise, the best combination of union and/or intersection of possible locations suggested by the different measures needs a deeper study. In the near future, the authors seek to validate these measures against experimental models from the tube illustrated in Fig. 2 and from a 3D cavity located in the same laboratory.

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