Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Delay-Dependent H_∞ Control for 2-D State-Delayed Systems ⋆

Dan Peng * **Xinping Guan** * **Chengnian Long** *

* *Institute of Electrical Engineering, Yanshan University, Qinhuangdao, 066004, P.R. China*

Abstract: Considering a class of two-dimensional (2-D) local state-space (LSS) Fornasini-Marchesini (FM) second model with delays in the states, this paper studies delay-dependent H_∞ control problem. First, we propose delay-dependent bounded real lemma. Then a dynamic output feedback controller is developed, which assures that the closed-loop system is asymptotically stable and has H_∞ performance γ in terms of linear matrix inequalities' (LMIs') feasibility. Furthermore, the minimum H_∞ performance γ can be obtained by solving a linear convex optimization problem. A numerical example demonstrates the effectiveness of our results.

1. INTRODUCTION

Along with the increasing development of modern industry and civil economy, people need to deal with more and more multivariable systems and multidimensional signals, most of which are expressed as 2-D discrete-system models (see, Roesser [1975]). In the real world, many systems and process dynamics are affected by delays. The existence of delays is frequently a source of instability and poor performance. Much work has been reported on the problem of the stability of standard, often termed 1-D in the m-D systems literature, linear systems with delays (see, Niculescu [2001]). Current efforts to achieve robust stability for 1-D time-delay systems are mainly delay-dependent approach (see, Fridman [2002], Xu et al. [2002], Wu et al. [2004], Xu and Lam [2005], He et al. [2004], and Jiang and Han [2006]), which include information on the size of delay and is less conservative than delay-independent one especially when the size of a delay is small. Recently, Wu et al. [2004] and He et al. [2004] devised a new method that used free weighting matrices to express the relationships between the terms in the Leibniz-Newton formula for robust stability of 1-D systems.

The need for 2-D stability and stabilization problems is motivated by practical relevance of 2D discrete linear systems with delays (see, Rogers and Owens [1992], Galkowski et al. [2003]). When controlling a real plant, it is also desirable to design a control system which is not only stable, but also guarantees an adequate level of performance, such as H_∞ control and guaranteed cost control. Most results for the 2-D problems focus on systems without delays, though for specific stability and H_∞ control of 2-D state-delayed systems were considered in Paszke et al. [2004] and Paszke et al. [2003], respectively. To the best of our knowledge, any other work has not been done using delay-dependent approach except for reference Paszke et al. [2006] considering the delay-dependent stability problem of 2-D time-delay systems.

⋆ The work was supported in part by the NSFC Grants 60174010, 60404022, NSFC for distinguished Yong Scholars under Grant 60525303, and the Key Scientific Research Project of the Education Ministry under Grant 204014.

This paper studies the delay-dependent H_∞ control problem for 2-D state-delayed systems. First, delay-dependent bounded real lemma is proposed. Then, a 2-D output feedback controller is designed to guarantee H_∞ disturbance attenuation γ through the solvability of LMIs. Furthermore, a corresponding optimization problem is proposed to minimize H_∞ disturbance attenuation γ. Finally, a numerical example is given to show that our results are effective.

2. DELAY-DEPENDENT BOUNDED REAL LEMMA

Consider the well known 2-D FM LSS second model proposed in Fornasini and Marchesini [1978] with state delays in each of the two independent directions of information propagation

$$\begin{aligned} x(i+1,j+1) &= A_1x(i+1,j) + A_2x(i,j+1) \\ &\quad + A_{1d}x(i+1,j-d_1) \\ &\quad + A_{2d}x(i-d_2,j+1) \\ &\quad + B_1\omega(i+1,j) + B_2\omega(i,j+1) \quad (1) \\ z(i,j) &= Cx(i,j) + D\omega(i,j) \quad (2) \end{aligned}$$

where $x(i,j) \in R^n$ is the state input, $\omega(i,j) \in R^m$ is the noise disturbance and bounded which belongs to l_2, $z(i,j) \in R^p$ is the control output and $i,j \in Z^+$. $A_k, A_{kd}, B_k(k = 1,2), C$ and D are constant matrices with appropriate dimensions. Here, d_1 and d_2 are constant positive scalars representing delays along vertical direction and horizontal direction, respectively.

The boundary conditions are assumed as

$$\begin{aligned} &\left\{x(i,j) = \varphi_{ij}\right\}, \forall i \geq 0; j = -d_1, -d_1+1, \cdots, 0, \\ &\left\{x(i,j) = \psi_{ij}\right\}, \forall j \geq 0; i = -d_2, -d_2+1, \cdots, 0, \\ &\varphi_{00} = \psi_{00}. \quad (3) \end{aligned}$$

The H_∞ performance measure for 2-D system (1)-(2) with zero boundary conditions ($\varphi_{i0} = \psi_{oj} = 0$) is defined as follows.

Definition 1. (Paszke et al. [2003]) 2-D state-delayed system (1)-(2) with zero boundary conditions is said to have H_∞ disturbance attenuation γ if it is asymptotically stable and has H_∞ performance γ, i.e.

$$\|z\|_2 < \gamma \|\omega\|_2 \tag{4}$$

where

$$z = \left[\, z^T(i+1,j) \;\; z^T(i,j+1) \,\right]^T$$
$$\omega = \left[\, \omega^T(i+1,j) \;\; \omega^T(i,j+1) \,\right]^T$$

with the l_2-norms defined by

$$\|z\|_2^2 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\|z(i+1,j)\|_2^2 + \|z(i,j+1)\|_2^2\right)$$
$$\|\omega\|_2^2 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\|\omega(i+1,j)\|_2^2 + \|\omega(i,j+1)\|_2^2\right).$$

For presentation convenience, we denote

$$A = [\, A_1 \;\; A_2 \,],\, A_d = [\, A_{1d} \;\; A_{2d} \,]$$
$$B = [\, B_1 \;\; B_2 \,],\, C_d = diag\{C, C\},\, D_d = diag\{D, D\}.$$

The following theorem presents delay-dependent bounded real lemma of 2-D system (1)-(2).

Theorem 1. 2-D state-delayed system (1)-(2) with the boundary conditions (3) has delay-dependent H_∞ disturbance attenuation γ for any delay d_k satisfying $0 \le d_k \le d_k^*(k=1,2)$ and $d^* = \max\{d_1^*, d_2^*\}$ if there exist matrices $P>0, Q>0, R_k>0, S_k>0, Y_{kl}, W_{kl}, M_{kl}(k,l=1,2), X_{l_1 l_2}>0, X_{l_1 2}(l_1=1,4,6; l_2=1,3)$ and $X_{l_3 l_4}(l_3=2,3,5; l_4=1,2,3,4)$ such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ \Phi_{12}^T & -\gamma^2 I & \Phi_{23} & L_{1d}^T & \Phi_{25} \\ \Phi_{13}^T & \Phi_{23}^T & -P & 0 & 0 \\ \Phi_{14}^T & L_{1d} & 0 & -I & 0 \\ \Phi_{15}^T & \Phi_{25}^T & 0 & 0 & -d^*S \end{bmatrix} < 0 \tag{5}$$

$$\Psi = \begin{bmatrix} X_1 & X_2 & X_3 & Y \\ X_2^T & X_4 & X_5 & W \\ X_3^T & X_5^T & X_6 & M \\ Y^T & W^T & M^T & S \end{bmatrix} \ge 0 \tag{6}$$

where

$$\Phi_{11} = \begin{bmatrix} \overline{Y}_1 & Y_{12}+Y_{21}^T & -Y_{11}+W_{11}^T & -Y_{12}+W_{21}^T \\ Y_{21}+Y_{12}^T & \overline{Y}_2 & -Y_{21}+W_{12}^T & -Y_{22}+W_{22}^T \\ -Y_{11}^T+W_{11} & -Y_{21}^T+W_{12} & \overline{W}_1 & -W_{12}-W_{21}^T \\ -Y_{12}^T+W_{21} & -Y_{22}^T+W_{22} & -W_{21}-W_{12}^T & \overline{W}_2 \end{bmatrix} + d^*\begin{bmatrix} X_1 & X_2 \\ X_2^T & X_4 \end{bmatrix}$$

$$\overline{Y}_1 = Y_{11}+Y_{11}^T - Q + R_1,\, \overline{Y}_2 = Y_{22}+Y_{22}^T - P + Q + R_2$$
$$\overline{W}_1 = -W_{11}-W_{11}^T - R_1,\, \overline{W}_2 = -W_{22}-W_{22}^T - R_2$$

$$\Phi_{12} = \begin{bmatrix} M_{11}^T & M_{21}^T \\ M_{12}^T & M_{22}^T \\ -M_{11}^T & -M_{21}^T \\ -M_{12}^T & -M_{22}^T \end{bmatrix} + d^*\begin{bmatrix} X_3 \\ X_5 \end{bmatrix}$$

$$\Phi_{13} = \begin{bmatrix} A_1^T P \\ A_2^T P \\ A_{1d}^T P \\ A_{2d}^T P \end{bmatrix},\, \Phi_{15} = \begin{bmatrix} \overline{S}_1 & d^* A_1^T S_2 \\ d^* A_2^T S_1 & \overline{S}_2 \\ d^* A_{1d}^T S_1 & d^* A_{1d}^T S_2 \\ d^* A_{2d}^T S_1 & d^* A_{2d}^T S_2 \end{bmatrix}$$

$$\Phi_{14} = \begin{bmatrix} L_d^T \\ 0_{2n\times 2p} \end{bmatrix},\, \Phi_{23} = \begin{bmatrix} B_1^T P \\ B_2^T P \end{bmatrix}$$

$$\Phi_{25} = \begin{bmatrix} d^* B_1^T S_1 & d^* B_1^T S_2 \\ d^* B_2^T S_1 & d^* B_2^T S_2 \end{bmatrix},\, X_{l_1} = \begin{bmatrix} X_{l_1 1} & X_{l_1 2} \\ X_{l_1 2}^T & X_{l_1 3} \end{bmatrix}$$

$$X_{l_3} = \begin{bmatrix} X_{l_3 1} & X_{l_3 2} \\ X_{l_3 3} & X_{l_3 4} \end{bmatrix},\, Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix},\, M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$\overline{S}_1 = d^*(A_1 - I)^T S_1,\, \overline{S}_2 = d^*(A_2 - I)^T S_2$$
$$S = diag\{S_1, S_2\}.$$

Proof. First, it will be shown that LMIs (5)-(6) assure the asymptotic stability of system (1) with $\omega(i,j)=0$.

Since LMIs (5)-(6) imply that

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{13} & \Phi_{15} \\ \Phi_{13}^T & -P & 0 \\ \Phi_{15}^T & 0 & -d^*S \end{bmatrix} < 0 \tag{7}$$

$$\Psi_1 = \begin{bmatrix} X_1 & X_2 & Y \\ X_2^T & X_4 & W \\ Y^T & W^T & S \end{bmatrix} \ge 0 \tag{8}$$

we only need to verify the system (1) ($\omega(i,j)=0$) is asymptotically stable if LMIs (7)-(8) hold.

Denote

$$x_{\xi,\eta} = x(i+\xi, j+\eta)$$

$$\begin{aligned} V_{11}(i,j) &= x_{1,1}^T P x_{1,1} + \sum_{l=-d_1}^{-1} x_{1,l+1}^T R_1 x_{1,l+1} \\ &+ \sum_{l=-d_2}^{-1} x_{l+1,1}^T R_2 x_{l+1,1} \\ &+ \sum_{\theta=-d_1+1}^{0}\sum_{l=-1+\theta}^{-1} \overline{y}_{1,l+1}^T S_1 \overline{y}_{1,l+1} \\ &+ \sum_{\theta=-d_2+1}^{0}\sum_{l=-1+\theta}^{-1} \overline{y}_{l+1,1}^T S_2 \overline{y}_{l+1,1} \\ V_{d1}(i,j) &= x_{1,0}^T Q x_{1,0} + \sum_{l=-d_1}^{-1} x_{1,l}^T R_1 x_{1,l} \\ &+ \sum_{\theta=-d_1+1}^{0}\sum_{l=-1+\theta}^{-1} \overline{y}_{1,l}^T S_1 \overline{y}_{1,l} \\ V_{d2}(i,j) &= x_{0,1}^T (P-Q) x_{0,1} + \sum_{l=-d_2}^{-1} x_{l,1}^T R_2 x_{l,1} \\ &+ \sum_{\theta=-d_2+1}^{0}\sum_{l=-1+\theta}^{-1} \overline{y}_{l,1}^T S_2 \overline{y}_{l,1} \end{aligned} \tag{9}$$

with

$$\overline{y}_{1,l} = x_{1,l+1} - x_{1,l},\, \overline{y}_{l,1} = x_{l+1,1} - x_{l,1}$$

and $P>0, Q>0, R_k>0$ and $S_k>0 (k=1,2)$ are to be determined.

Due to

$$x_{1,-d_1} = x_{1,0} - \sum_{l=-d_1}^{-1} \overline{y}_{1,l},\, x_{-d_2,1} = x_{0,1} - \sum_{l=-d_2}^{-1} \overline{y}_{l,1} \tag{10}$$

the following equation

$$\alpha = 2\left(x^T Y + x_d^T W\right)\left(x - x_d - \sum_{l=-d}^{-1} \Delta x\right) = 0 \qquad (11)$$

is true for any matrices Y and W, where

$$x = \left[\, x_{1,0}^T \;\; x_{0,1}^T \,\right]^T, x_d = \left[\, x_{1,-d_1}^T \;\; x_{-d_2,1}^T \,\right]^T$$
$$\Delta x = \left[\, \overline{y}_{1,l}^T \;\; \overline{y}_{l,1}^T \,\right]^T, d = \min\{d_1, d_2\}.$$

On the other hand, for any semi-positive definite matrix $X = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_4 \end{bmatrix} \geq 0$, the following equation holds:

$$\beta = d\xi^T X \xi - \sum_{l=-d}^{-1} \xi^T X \xi = 0 \qquad (12)$$

where $\xi = \left[\, x^T, \; x_d^T \,\right]^T$.

Now, for system (1) ($\omega(i,j) = 0$), define $\Delta V(i,j)$ as

$$\begin{aligned}\Delta V(i,j) &= V_{11}(i,j) - V_{d1}(i,j) - V_{d2}(i,j) \\ &= V_{11}(i,j) - V_{d1}(i,j) - V_{d2}(i,j) + \alpha + \beta \\ &\leq \xi^T \Theta \xi - \sum_{l=-d}^{-1} \zeta^T \Psi_1 \zeta \qquad (13)\end{aligned}$$

where $\zeta = \left[\, x^T \;\; x_d^T \;\; \triangle x^T \,\right]^T$ and

$$\Theta = \begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2^T & \Theta_3 \end{bmatrix}, S = diag\{S_1, S_2\}$$

$$\Theta_1 = A^T P A + Y + Y^T - \overline{Q} + R + dX_1 + \sum_{k=1}^{2} d_k \overline{A}_k^T S_k \overline{A}_k$$

$$\Theta_2 = A^T P A_d - Y + W^T + dX_2 + \sum_{k=1}^{2} d_k \overline{A}_k^T S_k A_d$$

$$\Theta_3 = A_d^T P A_d - W - W^T - R + dX_4 + \sum_{k=1}^{2} d_k A_d^T S_k A_d$$

$$\overline{Q} = diag\{Q, P - Q\}, R = diag\{R_1, R_2\}$$
$$\overline{A}_1 = [\, A_1 - I \;\; A_2 \,], \overline{A}_2 = [\, A_1 \;\; A_2 - I \,]$$

Due to $\Psi_1 \geq 0$ is assured by LMI (8), and applying Schur complement shows that LMIs (7) implies $\Theta < 0$, then $\Delta V(i,j) < 0$ for any $\xi \neq 0$. So, the system (1) with $\omega(i,j) = 0$ is asymptotically stable if LMIs (5)-(6) are feasible.

Next, we shall prove $\|z\|_2 < \gamma \|\omega\|_2$ under zero-initial conditions for any nonzero $\omega(i,j) \in l_2\{[0,\infty),[0,\infty)\}$. To this end, we introduce

$$J = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(z^T z - \gamma^2 \omega^T \omega\right) \qquad (14)$$

In view of the stability of the system and the zero-initial condition, we have that for any nonzero $\omega(i,j) \in l_2\{[0,\infty),[0,\infty)\}$

$$J \leq \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[z^T z - \gamma^2 \omega^T \omega + \Delta V(i,j)\right] \qquad (15)$$

Similar to Equations (11) and (12), the following two equations

$$\alpha_1 = 2\left(x^T Y + x_d^T W + \omega^T M\right)\left(x - x_d - \sum_{l=-d}^{-1} \Delta x\right) = 0 \qquad (16)$$

and

$$\beta_1 = d\xi_1^T X_1 \xi_1 - \sum_{l=-d}^{-1} \xi_1^T X_1 \xi_1 = 0 \qquad (17)$$

hold for any free weighting matrices Y, W, M and any semi-positive definite matrix $X_1 = \begin{bmatrix} X_1 & X_2 & X_3 \\ X_2^T & X_4 & X_5 \\ X_3^T & X_5^T & X_6 \end{bmatrix} \geq 0$, where $\xi_1 = \left[\, x^T, \; x_d^T, \; \omega^T \,\right]^T$.

In the same way, we can compute

$$\Delta V(i,j) = V_{11}(i,j) - V_{d1}(i,j) - V_{d2}(i,j) + \alpha_1 + \beta_1$$

then it follows that

$$J \leq \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\xi_1^T \overline{\Theta} \xi_1 - \sum_{l=-d}^{-1} \zeta_1^T \Psi \zeta_1\right]$$

where $\zeta_1 = \left[\, x^T, \; x_d^T, \; \omega^T, \; \triangle x^T \,\right]^T$ and

$$\overline{\Theta} = \begin{bmatrix} \Theta_1 + C_d^T C_d & \Theta_2 & \Theta_4 \\ \Theta_2^T & \Theta_3 & \Theta_5 \\ \Theta_4^T & \Theta_5^T & \Theta_6 \end{bmatrix}$$

$$\Theta_4 = A^T P B + C_d^T D_d + M^T + dX_3 + \sum_{k=1}^{2} d_k \overline{A}_k^T S_k B$$

$$\Theta_5 = A_d^T P B - M^T + dX_5 + \sum_{k=1}^{2} d_k A_d^T S_k B$$

$$\Theta_6 = B^T P B + D_d^T D_d - \gamma^2 I + dX_6 + \sum_{k=1}^{2} d_k B^T S_k B$$

Due to $\Psi \geq 0$ is assured by LMI (6), and LMI (5) implies $\overline{\Theta} < 0$, so $J < 0$ is guaranteed by LMIs (5)-(6) for any $\xi_1 \neq 0$.

Summarizing the above two points demonstrates that 2-D system (1)-(2) has H_∞ disturbance attenuation γ if LMIs (5)-(6) are true. This completes the proof.

Remark 1. Due to $V_{11}(i,j), V_{d1}(i,j)$ and $V_{d2}(i,j)$ involve two new variables $\overline{y}_{1,l}$ and $\overline{y}_{l,1}$ expressed by the changes of system states $x_{1,l}$ and $x_{l,1}$, so Equation (10) holds. Similar to Leibniz-Newton formula for 1-D time-delay systems in Jiang and Han [2006], the free weighting matrices Y, W and M are used in (11) and (16) to express the relationship among the terms x, x_d, and $\sum_{l=-d}^{-1} \Delta x$ and they can easily be determined by solving LMIs (5)-(6). This method avoids the conservatism that results from any system transformation.

Remark 2. Theorem 1 provides a sufficient H_∞ performance criteria for 2-D state-delayed system (1)-(2). Now, we will show that although this condition is dependent on the size of delays, by a certain choice of matrices, it also implies an extension of previous delay-independent one. Choosing the following matrices

$$X = \frac{\varepsilon I_{(4n+2m)\times(4n+2m)}}{d^*}, Y = W = 0_{2n\times 2n}$$

$$M = 0_{2m\times 2n}, S = \frac{\varepsilon I_{2n\times 2n}}{d^*}$$

for some sufficiently small positive scalar ε; LMIs (5)-(6) imply the well-known delay-independent H_∞ performance condition (see, for instance, Theorem 5 in Paszke et al. [2003]). This implies that Theorem 1 is powerful in the sense that it provides sufficient conditions for both the delay-dependent and the delay-independent cases.

3. DELAY-DEPENDENT H_∞ CONTROL VIA DYNAMIC OUTPUT FEEDBACK

In this section, we design a dynamic output feedback controller such that the closed-loop system has H_∞ disturbance attenuation γ.

Now, we consider 2-D state-delayed system with control input and state delays as

$$\begin{aligned} x(i+1,j+1) &= A_1x(i+1,j) + A_2x(i,j+1) \\ &\quad + A_{1d}x(i+1,j-d_1) \\ &\quad + A_{2d}x(i-d_2,j+1) \\ &\quad + B_{11}u(i+1,j) + B_{12}u(i,j+1) \\ &\quad + B_{21}\omega(i+1,j) + B_{22}\omega(i,j+1) \quad (18) \\ y(i,j) &= C_1x(i,j) + C_2u(i,j) \quad (19) \\ z(i,j) &= D_1x(i,j) + D_2u(i,j) + D_3\omega(i,j) \quad (20) \end{aligned}$$

where $u(i,j) \in R^p$ is the control input and $y(i,j) \in R^q$ is the measurable output, respectively. $B_{kl}, C_k, D_k (k,l = 1,2)$ and D_3 are constant matrices with appropriate dimensions. The boundary conditions are also of the form (3). Without less of generality, we assume $C_2 = 0$.

Introduce the following dynamic output feedback controller

$$\begin{aligned} \widehat{x}(i+1,j+1) &= A_{c1}\widehat{x}(i+1,j) + A_{c2}\widehat{x}(i,j+1) \\ &\quad + A_{c1d}\widehat{x}(i+1,j-d_1) \\ &\quad + A_{c2d}\widehat{x}(i-d_2,j+1) \\ &\quad + B_{c1}y(i+1,j) + B_{c2}y(i,j+1) \quad (21) \\ u(i,j) &= C_c\widehat{x}(i,j) + D_cy(i,j) \quad (22) \end{aligned}$$

where $\widehat{x}(i,j) \in R^{n_c}$. Then the closed-loop system by substituting the controller (21)-(22) to 2-D system (18)-(20) is represented as

$$\begin{aligned} \overline{x}(i+1,j+1) &= \overline{A}_1\overline{x}(i+1,j) + \overline{A}_2\overline{x}(i,j+1) \\ &\quad + \overline{A}_{1d}\overline{x}(i+1,j-d_1) \\ &\quad + \overline{A}_{2d}\overline{x}(i-d_2,j+1) \\ &\quad + \overline{B}_1\omega(i+1,j) + \overline{B}_2\omega(i,j+1) \quad (23) \\ z(i,j) &= \overline{D}\overline{x}(i,j) + D_3\omega(i,j) \quad (24) \end{aligned}$$

where $\overline{x}(i,j) = \begin{bmatrix} x^T(i,j) & \widehat{x}^T(i,j) \end{bmatrix}^T$ and

$$\overline{A}_k = \begin{bmatrix} A_k + B_{1k}D_cC_1 & B_{1k}C_c \\ B_{ck}C_1 & A_{ck} \end{bmatrix} \overline{A}_{kd} = \begin{bmatrix} A_{kd} & 0 \\ 0 & A_{ckd} \end{bmatrix}$$
$$\overline{B}_k = \begin{bmatrix} B_{2k}^T & 0 \end{bmatrix}^T (k=1,2), \overline{D} = \begin{bmatrix} D_1 + D_2D_cC_1 & D_2C_c \end{bmatrix} \quad (25)$$

Accordingly, the boundary conditions are assumed as:

$$\begin{aligned} &\overline{x}(i,j) = \{\varphi_{i,j}^T, 0\}^T, \forall i \geq 0, j = -d_1, -d_1+1, \cdots, 0; \\ &\overline{x}(i,j) = \{\psi_{i,j}^T, 0\}^T, \forall j \geq 0, i = -d_2, -d_2+1, \cdots, 0, \\ &\quad \varphi_{0,0} = \psi_{0,0}. \quad (26) \end{aligned}$$

The following Theorem 2 realizes delay-dependent H_∞ control for 2-D state-delayed system (18)-(20) through controller (21)-(22), which make the closed-loop system (23)-(24) asymptotically stable and $\|z\|_2 < \gamma \|\omega\|_2$.

Theorem 2. Given scalars t, t_1 and t_2. 2-D state-delayed system (18)-(20) with the boundary conditions (3) has a delay-dependent H_∞ disturbance attenuation γ under the action of the controller (21)-(22) for any delay d_k satisfying $0 \leq d_k \leq d_k^* (k=1,2)$ and $d^* = \max\{d_1^*, d_2^*\}$ if there exist matrices $X > 0, Y > 0, \widetilde{Q} > 0, \widetilde{R}_k > 0, D_c, Z, \overline{Z}_k, \widetilde{Z}_k, \widehat{Z}_k, \widetilde{Y}_{kl}, \widetilde{W}_{kl}, \widetilde{M}_{kl} (k,l = 1,2), \widetilde{X}_{l_1,l_2} > 0, \widetilde{X}_{l_1,2} (l_1 = 1,4,6; l_2 = 1,3)$ and $\widetilde{X}_{l_3,l_4} (l_3 = 2,3,5; l_4 = 1,2,3,4)$ such that the following LMIs hold:

$$\widetilde{\Phi} = \begin{bmatrix} \widetilde{\Gamma}_1 & \widetilde{\Gamma}_2 \\ \widetilde{\Gamma}_2^T & \widetilde{\Gamma}_3 \end{bmatrix} < 0 \quad (27)$$

$$\widetilde{\Psi} = \begin{bmatrix} \widetilde{X}_1 & \widetilde{X}_2 & \widetilde{X}_3 & \widetilde{Y} \\ \widetilde{X}_2^T & \widetilde{X}_4 & \widetilde{X}_5 & \widetilde{W} \\ \widetilde{X}_3^T & \widetilde{X}_5^T & \widetilde{X}_6 & \widetilde{M} \\ \widetilde{Y}^T & \widetilde{W}^T & \widetilde{M}^T & \widetilde{S} \end{bmatrix} \geq 0 \quad (28)$$

where

$$\widetilde{\Gamma}_1 = \begin{bmatrix} \widetilde{\Gamma}_{11} & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_{22} \end{bmatrix}$$

$$\widetilde{\Gamma}_{11} = \begin{bmatrix} \overrightarrow{Y}_{11} & \widetilde{Y}_{12} + \widetilde{Y}_{21}^T & -\widetilde{Y}_{11} + \widetilde{W}_{11}^T & -\widetilde{Y}_{12} + \widetilde{W}_{21}^T \\ \widetilde{Y}_{21} + \widetilde{Y}_{12}^T & \overrightarrow{Y}_{22} & -\widetilde{Y}_{21} + \widetilde{W}_{12}^T & -\widetilde{Y}_{22} + \widetilde{W}_{22}^T \\ -\widetilde{Y}_{11}^T + \widetilde{W}_{11} & -\widetilde{Y}_{21}^T + \widetilde{W}_{12} & \overrightarrow{W}_{11} & -\widetilde{W}_{12} - \widetilde{W}_{21}^T \\ -\widetilde{Y}_{12}^T + \widetilde{W}_{21} & -\widetilde{Y}_{22}^T + \widetilde{W}_{22} & -\widetilde{W}_{21} - \widetilde{W}_{12}^T & \overrightarrow{W}_{22} \end{bmatrix} + d^* \begin{bmatrix} \widetilde{X}_1 & \widetilde{X}_2 \\ \widetilde{X}_2^T & \widetilde{X}_4 \end{bmatrix}$$

$$\overrightarrow{Y}_{11} = \widetilde{Y}_{11} + \widetilde{Y}_{11}^T - \widetilde{Q} + \widetilde{R}_1$$
$$\overrightarrow{Y}_{22} = \widetilde{Y}_{22} + \widetilde{Y}_{22}^T - tJ_P + \widetilde{Q} + \widetilde{R}_2$$
$$\overrightarrow{W}_{11} = -\widetilde{W}_{11} - \widetilde{W}_{11}^T - \widetilde{R}_1, \overrightarrow{W}_{22} = -\widetilde{W}_{22} - \widetilde{W}_{22}^T - \widetilde{R}_2$$

$$\Gamma_{12} = \begin{bmatrix} \widetilde{M}_{11}^T & \widetilde{M}_{21}^T \\ \widetilde{M}_{12}^T & \widetilde{M}_{22}^T \\ -\widetilde{M}_{11}^T & -\widetilde{M}_{21}^T \\ -\widetilde{M}_{12}^T & -\widetilde{M}_{22}^T \end{bmatrix} + d^* \begin{bmatrix} \widetilde{X}_3 \\ \widetilde{X}_5 \end{bmatrix}$$

$$\Gamma_{22} = \begin{bmatrix} -\gamma^2 I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + d^* \widetilde{X}_6$$

$$\widetilde{\Gamma}_2 = \begin{bmatrix} tJ_{A_1}^T & J_C^T & 0 & \widetilde{J}_{A_1} & d^*t_2J_{A_1}^T \\ tJ_{A_2}^T & 0 & J_C^T & d^*t_1J_{A_2}^T & \widetilde{J}_{A_2} \\ tJ_{A_{1d}}^T & 0 & 0 & d^*t_1J_{A_{1d}}^T & d^*t_2J_{A_{1d}}^T \\ tJ_{A_{2d}}^T & 0 & 0 & d^*t_1J_{A_{2d}}^T & d^*t_2J_{A_{2d}}^T \\ tJ_{B_1}^T & D_3^T & 0 & d^*t_1J_{B_1}^T & d^*t_2J_{B_1}^T \\ tJ_{B_2}^T & 0 & D_3^T & d^*t_1J_{B_2}^T & d^*t_2J_{B_2}^T \end{bmatrix}$$

$$\widetilde{J}_{A_1} = d^*t_1(J_{A_1}^T - J_P), \widetilde{J}_{A_2} = d^*t_2(J_{A_2}^T - J_P)$$

$$J_{A_{kd}} = \begin{bmatrix} XA_{kd} & \widetilde{Z}_k \\ A_{kd} & A_{kd}Y \end{bmatrix}, J_{B_k} = \begin{bmatrix} XB_{2k} \\ B_{2k} \end{bmatrix}$$

$$J_{A_k} = \begin{bmatrix} XA_k + \overline{Z}_kC_1 & \widehat{Z}_k \\ A_k + B_{1k}D_cC_1 & A_kY + B_{1k}Z \end{bmatrix} (k=1,2)$$

$$J_C = [\, D_1 + D_2 D_c C_1 \;\; D_1 Y + D_2 Z \,]$$
$$\widetilde{\Gamma}_3 = diag\{-tJ_P, -I, -I, -d^* t_1 J_P, -d^* t_2 J_P\}$$
$$\widetilde{S} = diag\{t_1 J_P, t_2 J_P\}$$
$$J_P = \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \widetilde{X}_{l_1} = \begin{bmatrix} \widetilde{X}_{l_1 1} & \widetilde{X}_{l_1 2} \\ \widetilde{X}_{l_1 2}^T & \widetilde{X}_{l_1 3} \end{bmatrix}$$
$$X_{l_3} = \begin{bmatrix} \widetilde{X}_{l_3 1} & \widetilde{X}_{l_3 2} \\ \widetilde{X}_{l_3 3} & \widetilde{X}_{l_3 4} \end{bmatrix}, \widetilde{Y} = \begin{bmatrix} \widetilde{Y}_{11} & \widetilde{Y}_{12} \\ \widetilde{Y}_{21} & \widetilde{Y}_{22} \end{bmatrix}$$
$$\widetilde{W} = \begin{bmatrix} \widetilde{W}_{11} & \widetilde{W}_{12} \\ \widetilde{W}_{21} & \widetilde{W}_{22} \end{bmatrix}, \widetilde{M} = \begin{bmatrix} \widetilde{M}_{11} & \widetilde{M}_{12} \\ \widetilde{M}_{21} & \widetilde{M}_{22} \end{bmatrix}$$

Furthermore, the delay-dependent control system matrices $A_{ck}, A_{ckd}, B_{ck}(k = 1, 2), C_c$ and D_c can be derived as

$$\begin{aligned} A_{ck} &= (\widehat{P}_{12})^{-1}(\widehat{Z}_k - XA_kY - \overline{Z}_k C_1 Y \\ &\quad - XB_{1k} C_c P_{12}^T)(P_{12}^T)^{-1} \\ A_{ckd} &= (\widehat{P}_{12})^{-1}(\widetilde{Z}_k - XA_{kd}Y)(P_{12}^T)^{-1} \\ B_{ck} &= (\widehat{P}_{12})^{-1}(\overline{Z}_k - XB_{1k}D_c) \\ C_c &= (Z - D_c C_1 Y)(P_{12}^T)^{-1}(k = 1, 2) \end{aligned} \tag{29}$$

i.e. the delay-dependent H_∞ control problem for system (18)-(20) is solved.

Proof. Applying Theorem 1 to the closed-loop system (23)-(24), then combining with variable substituting method, Schur complement and congruence transformation, completes the proof. ■

Remark 3. The results of Theorems 2 apply the tuning parameters t, t_1 and t_2. The question arises how to find the optimal combination of these parameters. The optimal value of them can be found by the approach proposed in Fridman [2002]. Moreover, in the proof of Theorem 3, if we don't introduce the parameters t, t_1 and t_2, the LMIs obtained are bilinear. We must add some limitations on the matrices to solve the new LMIs. So, the choices of parameters t, t_1 and t_2 have assured a smaller filter performance and reduced the conservatism of the result.

Remark 4. Theorem 2 provides an approach to solve the delay-dependent H_∞ control problem for 2-D state-delayed system (18)-(20). If we choose the following matrices

$$t = 1, t_1 = t_2 = \varepsilon, \widetilde{Y} = \widetilde{W} = 0_{2n\times 2n}$$
$$\widetilde{M} = 0_{2m\times 2n}, \widetilde{X} = \frac{\varepsilon I_{(4n+2m)\times(4n+2m)}}{d^*}$$

for some sufficiently small positive scalar ε, LMIs (27)-(28) imply a sufficient condition of delay-independent H_∞ control problem for 2-D state-delayed system (18)-(20):

$$\begin{bmatrix} -J_P + J_Q + \sum_{k=1}^{2} J_{Q_k} & 0 & J_{A_1}^T & 0 & 0 & J_D^T & 0 & 0 & 0 \\ 0 & -J_Q & J_{A_2}^T & 0 & 0 & 0 & J_D^T & 0 & 0 \\ J_{A_1} & J_{A_2} & -J_P & J_{A_{1d}} & J_{A_{2d}} & 0 & 0 & J_{B_1} & J_{B_2} \\ 0 & 0 & J_{A_{1d}}^T & -J_{Q_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{A_{2d}}^T & 0 & -J_{Q_2} & 0 & 0 & 0 & 0 \\ J_D & 0 & 0 & 0 & 0 & -I & 0 & D_3 & 0 \\ 0 & J_D & 0 & 0 & 0 & 0 & -I & 0 & D_3 \\ 0 & 0 & J_{B_1}^T & 0 & 0 & D_3^T & 0 & -\gamma^2 I & 0 \\ 0 & 0 & J_{B_2}^T & 0 & 0 & 0 & D_3^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{30}$$

In Theorem 2, γ is regarded as given. However, (27) and (28) are still LMIs when γ is also a variable. Thus, it is possible to formulate the following convex optimization problem to find a delay-dependent controller assuring a smallest H_∞ norm bound γ for 2-D system (18)-(20).

Problem 1.

minimize δ *subject to*

$$X > 0, \widetilde{Y} > 0, \widetilde{Q} > 0, \widetilde{R}_k > 0 \widetilde{X}_{l_1,l_2} > 0$$

(27)

(28)

applying mincx in Matlab Toolbox for given state delays d_1 *and* d_2*, and matrices* $\widetilde{Y}_{kl}, \widetilde{W}_{kl}, \widetilde{M}_{kl}(k,\ l = 1, 2), \widetilde{X}_{l_1,2}(l_1 = 1, 4, 6)$ *and* $\widetilde{X}_{l_3,l_4}(l_3 = 2, 3, 5; l_4 = 1, 2, 3, 4)$*, where* $\gamma = \sqrt{\delta}$, we can minimize H_∞ norm bound γ and obtain the corresponding H_∞ optimal controller by (29).

4. NUMERICAL EXAMPLE

Now, we will prove the usefull and effectiveness of the delay-dependent H_∞ controller design for a stationary random field in image processing proposed in *Problem 1.*

It is known that the stationary random field can be modeled as the following 2-D system considered in Katayama and Kosaka [1979]:

$$\begin{aligned} \eta(i+1, j+1) &= a_1\eta(i+1, j) + a_2\eta(i, j+1) \\ &\quad - a_1 a_2 \eta(i, j) + \omega(i, j) \end{aligned} \tag{31}$$

where $\eta(i, j)$ is the state of the random field at spacial coordinate $(i, j), a_1^2 < 1$ and $a_1^2 < 1$ as a_1 and a_2 are, respectively, the horizontal and vertical correlations of the random field.

Now, we consider the influence of time delays to system (31) and introduce two terms $\eta(i + 1, j - d_1)$ and $\eta(i - d_2, j + 1)$ in (31) following that

$$\begin{aligned} \eta(i+1, j+1) &= a_1\eta(i+1, j) + a_2\eta(i, j+1) \\ &\quad + a_3\eta(i+1, j-d_1) \\ &\quad + a_4\eta(i-d_2, j+1) \\ &\quad - a_1 a_2 \eta(i, j) + \omega(i, j) \end{aligned} \tag{32}$$

where $a_3^2 < 1$ and $a_4^2 < 1$ as a_3 and a_4 are also, respectively, the horizontal and vertical correlations of the random field, and $\omega(i, j)$ is the measurement noise.

Denote $x^T(i, j) = \left[\, \eta^T(i, j+1) - a_2\eta^T(i, j) \;\; \eta^T(i, j) \,\right]$, and assume that the measurement output is given by $y(i, j) = [\, 3 \;\; 1 \,]\, x(i, j)$. The signal to be estimated is $z(i, j) = 0.5\eta(i, j) + 0.4u(i, j) + 0.7\omega(i, j)$.

It is easy to know that the 2-D system can be converted to the 2-D FM LSS model (18)-(20) with

$$A_1 = \begin{bmatrix} 0 & 0 \\ 1 & a_1 \end{bmatrix}, A_2 = \begin{bmatrix} a_2 & 0 \\ 0 & 0 \end{bmatrix}, A_{1d} = \begin{bmatrix} a_3 & a_1 a_3 \\ 0 & 0 \end{bmatrix}$$
$$A_{2d} = \begin{bmatrix} a_4 & a_1 a_4 \\ 0 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$$
$$B_{21} = 0, B_{22} = [\,1 \;\, 0\,]^T \qquad (33)$$

Let $a_1 = 0.2, a_2 = 0.3, a_3 = 0.15, a_4 = 0.03$, by solving *Problem 1* when assuming $d_1 = 1, d_2 = 2$ and $t_1 = 0.01, t_2 = 0.101$ and $t_3 = 0.11$, we can obtain the minimum H_∞ norm bound $\gamma_{opt} = 0.7000019$ and the system matrices of H_∞ controller (21)-(22) as

$$A_{c1} = \begin{bmatrix} -0.5664 & 4.6682 \times 10^{12} \\ 0.0000 & 5799.2 \end{bmatrix}$$
$$A_{c2} = \begin{bmatrix} -0.0698 & 1.7436 \times 10^{12} \\ 0.0000 & 8823.4 \end{bmatrix}$$
$$A_{c1d} = \begin{bmatrix} -0.0000 & 13488 \\ 0.0000 & 0.1855 \end{bmatrix}, A_{c2d} = \begin{bmatrix} 0.0000 & 5392.1 \\ 0.0000 & 0.0348 \end{bmatrix}$$
$$B_{c1} = [\,618606 \;\; -0.003732\,]^T$$
$$B_{c2} = [\,-1121904 \;\; -0.005677\,]^T$$
$$C_c = [\,-0.0000 \;\; 364202.3068\,], D_c = -0.2343 \qquad (34)$$

Moreover, Figure 1 gives the maximum singular values plot of the transfer function of the closed-loop system by substituting (34) to (33) over $0 \le \omega_1 \le 2\pi, 0 \le \omega_2 \le 2\pi$. In the figure, the griddings denote the obtained H_∞ disturbance attenuations and its maximum value is 0.699988, which is below 0.7000019.

5. CONCLUSION

This paper studies the delay-dependent H_∞ control problem of 2-D state-delayed systems described by FM LSS model. First, the delay-dependent bounded real lemma has been derived through introducing free weighting matrices. By a certain choice of the free weighting matrices, it also implies an extension of delay-independent result. Then, we design a dynamic output feedback controller to solve the H_∞ control problem in terms of LMIs. Furthermore, an optimization problem is proposed to solve a minimum upper bound of H_∞ disturbance attenuation γ. Finally, a numerical example examine the effectiveness of our results.

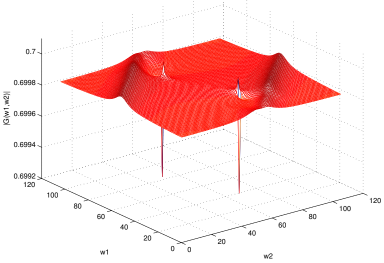

Fig. 1. The delay-dependent optimal frequency response of closed-loop system by substituting (34) to (33).

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