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Operator based Nonlinear Control Design for a Water Level Process System

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Abstract: In this paper, a method of nonlinear control systems design for a water level process is proposed. This design method uses operator based robust right coprime factorization for the nonlinear process system, as a result, robust stability of the nonlinear process system is guaranteed. For the obtained robust stable process system, an operator based process tracking controller is also designed to realize the desired output tracking performance and to eliminate the disturbance of the process system input. A simulation result obtained to a water level process control system is given to show the effectiveness of the proposed method.

1. INTRODUCTION

Recently, with development of industry and increase in kinds of product, practical processes have become complex. In general, the well-known process system includes reactors, heat exchangers, tanks, compressors etc. The control variables are temperature, pressure, water level, position, and reaction speed etc. The process model can be derived based on physical understanding of process behaviour and system estimation techniques. However, most of the above processes exhibit nonlinear performances, as a result, it is difficult to control in the same way as for an ideal linearized process. Thus, increasingly there is interest in developing methods that deal directly with it (Lee [1993]).

Control of nonlinear system has been studied in many reports. The operator theory is one of the nonlinear control system techniques, and it is based on nonlinear Lipschitz operators from a normed linear space to another normed linear space (Figueiredo and Chen [1993]). Based on this theory, nonlinear tracking control system design scheme (Deng et al. [2004]) has been studied by using robust right coprime factorization (Chen and Han [1998], Deng et al. [2006]). The merit of this method is that the process signal does not affect the process output error signal. That is, the nonlinear operator tracking system has no relation with the process output error signal. However, the effect of disturbance in the process input was not considered (Deng et al. [2008]). In this paper, we design a new tracking controller, the designed tracking controller can eliminate the effect of the disturbance and can realize the prefect tracking performance. A simulation result is given to show effectiveness.

The outline of this paper is as follows. In Section 2, several definitions of operator, right coprime factorization and bounded input and bounded output stability are given. Modeling of the process system for water level control is shown in Section 3. In Section 4, robust tracking control system using the operator theoretic approach is designed. Simulation result confirms the effectiveness of the proposed method in Section 5.

2. MATHEMATICAL PRELIMINARIES

For using operator based right coprime factorization to the process, several definitions of operator and bounded input bounded output stability (Chen and Han [1998], Figueiredo and Chen [1993], Paice et al. [1992], Vidyasagar [1985], Deng et al. [2006]) are reviewed.

Let U and Y be linear spaces over the field of real numbers, and let U^* and Y^* be normed subspaces, called the stable subspaces, of U and Y, respectively. An operator $Q : U \to Y$ is said to be bounded input bounded output (BIBO) stable or simply, stable if $Q(U^*) \subseteq Y^*$.

Let $\mathcal{S}(U, Y)$ be the set of stable operators from U and Y. Then $\mathcal{S}(U, Y)$ contains a subset defined by

$$\mathcal{U}(U, Y) = \{M : M \in \mathcal{S}(U, Y)\} \tag{1}$$

where M is invertible with $M^{-1} \in \mathcal{S}(U, Y)$. Elements of $\mathcal{U}(U, Y)$ are called unimodular operators.

In the following, well-posedness and stability of nonlinear feedback systems are described, and only those systems which are well-posed shall be considered. Consider the problem of stabilizing a nonlinear continuous time process P by a controller K, where the system is with real input spaces of continuous functions with continuous first derivative. For convenience, the feedback control system is denoted as $\{P, K\}$.

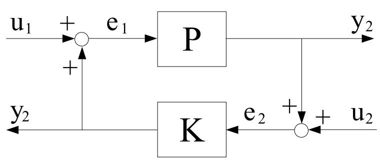

Fig. 1. The feedback system $\{P, K\}$

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The system $\{P, K\}$ which is shown in Fig. 1 is well-posed if the closed-loop system input-output operator from $[u_1\ u_2]^T$ to $[e_1\ e_2]^T$, namely

$$\begin{bmatrix} I & -K \\ -P & I \end{bmatrix}^{-1} \tag{2}$$

exists. Then the system $\{P, K\}$ is said to be internally stable if and only if for all bounded-inputs u_1, u_2 the outputs y_1, y_2 and e_1, e_2 are bounded. This is equivalent to

$$\begin{bmatrix} I & -K \\ -P & I \end{bmatrix}^{-1} \tag{3}$$

being BIBO stable.

The given process operator $P : U \to Y$ is said to have a right factorization (N, D) if and only if for any unbounded input $w \in W$, $N(w)$ or $D(w)$ is unbounded, where N and D are stable operators from the quasi-state space W to the input space U and output spaces Y.

$$P = ND^{-1}, \begin{array}{l} N : W \to Y \\ D : W \to U \end{array} \tag{4}$$

Let N and D be a right factorization of P. The factorization is said to have a right coprime factorization, if there exist two stable operators $S : Y \to U$ and $R : U \to U$, where R is invertible, satisfying the Bezout identity

$$SN + RD = M, \text{ for some } M \in \mathcal{U}(W, U) \tag{5}$$

where $\mathcal{U}(W, U)$ is the set of unimodular operator.

The Bezout identity is often used in the following equation for simplicity:

$$SN + RD = I, \ I\text{: identity operator} \tag{6}$$

The following lemmas of a right coprime factorization are employed (Paice et al. [1992]), rcf denotes right coprime factorization.

Lemma 1. Given $\{P, K\}$, and $P = ND^{-1}$ and $K = SR^{-1}$, the rcf's of the process and controller, respectively, then $\{P, K\}$ is well-posed if and only if

$$\begin{bmatrix} D & -S \\ -N & R \end{bmatrix}^{-1} \tag{7}$$

exists and is internally stable if and only if

$$\begin{bmatrix} D & -S \\ -N & R \end{bmatrix}^{-1} \tag{8}$$

is BIBO stable.

Proof. The proofs are given in Appendix A.

Hence the stability and well-posedness of the system depend on the existence and stability of operator $\begin{bmatrix} D & -S \\ -N & R \end{bmatrix}^{-1}$.

Lemma 2. Suppose $P = ND^{-1}$ and $K = SR^{-1}$, such that the operators D, N, S, R are BIBO stable. Then these are rcfs for P and K if they satisfy (8).

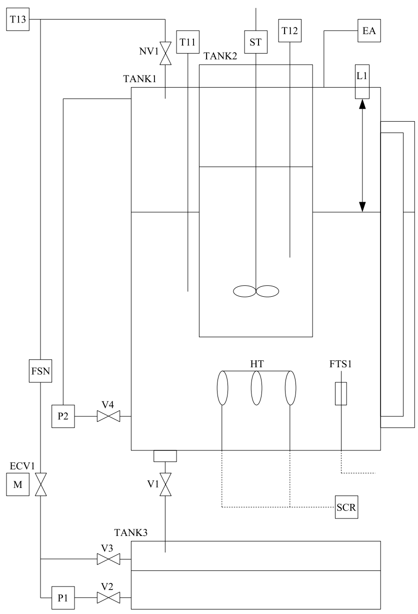

Fig. 2. The water level process system

Proof. The proofs are given in Appendix B.

Lemma 3. Suppose that Lemma 1 and Lemma 2 are satisfied. Then the system is overall stable if and only if the operator M is a unimodular operator, namely, $M \in \mathcal{U}(W, U)$.

Proof. Based on Lemma 1 and Lemma 2, the sufficiency and necessity can be proofed. Here the process of proofs is omitted.

3. MODELING OF THE PROCESS SYSTEM FOR WATER LEVEL CONTROL

In this section, the process control system is introduced. Top part of the system has two tanks and bottom part of this system has a tank. The outside tank of top part is called TANK1, and the inside one is called TANK2. The bottom part tank is called TANK3 which stores water from TANK1.

The diagram of the process control system is shown in Fig. 2. TANK1 has a supersonic wave sensor and this sensor can measure water level. An inflow mouth on TANK1, and volume of water can be measured. Water is carried from TANK3 to TANK1. We can control the volume of water by using the valve. In addition, water is outflow from bottom of TANK1 to TANK3 through a drain pipe.

Parameters of the system in Fig. 2 are given as follows.

Parameters		
D_1:	Diameter of Tank1	$[m]$
D_2:	Diameter of Tank2	$[m]$
d:	Diameter of outlet	$[m]$
h_s:	Level of Tank2 from Tank1	$[m]$
g:	Gravity acceleration	$[m/s^2]$
P_a:	Pressure in tank	$[hPa]$
ρ:	Density of water	$[kg/m^3]$
h:	Water level in Tank1	$[m]$
q_0:	Inflow	$[l/min]$
q:	Outflow	$[l/min]$

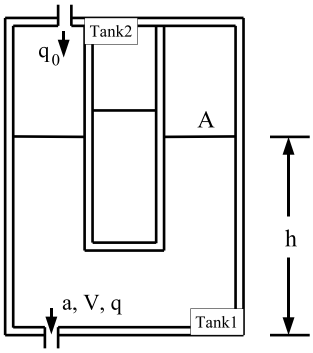

Fig. 3. Model of the water level process

In this paper, we consider that water level changes from 30[cm] to 72[cm] because a sensor can work by this range. In this range, the tank could not occur super heating or spillover. Then, sectional square A of Tank1 and square a of drain pipe is given as follows.

$$A = \frac{{D_1}^2\pi}{4} - \frac{{D_2}^2\pi}{4} \tag{9}$$

$$a = \frac{d^2\pi}{4} \tag{10}$$

In the following, we will derive a mathematical model of the water level process using the parameters from Fig. 3. Considering mass balance of the tank, the following equation is obtained.

$$A\dot{h}(t) = q_0 - aV \tag{11}$$

Based on Bernoulli theorem, the relationship between water level and outflow is obtained. This theorem is shown in the following equation.

$$\frac{v^2}{2g} + z + \frac{p}{\rho g} = const \tag{12}$$

If we consider that velocity of varying water level is $\frac{q_0}{A}$ and velocity of outflow is V, the following equation is obtained.

$$\frac{{q_0}^2}{2gA^2} + \frac{P_a}{\rho g} + h = \frac{V^2}{2g} + \frac{P_a + \Delta P_a}{\rho g} \tag{13}$$

We assume that $\Delta P_a = 0$, model of water level process is presented by the following equation.

$$\dot{h} = \frac{q_0}{A} - \frac{a}{A}\sqrt{\frac{{q_0}^2}{A^2} + 2gh} \tag{14}$$

Considering that a is much less than A in our water level process equipment, we have that

$$\frac{a}{A} \simeq 0 \tag{15}$$

Therefore, in this paper model of water level process is approximately derived as follows.

$$\dot{h} = \frac{q_0}{A} - \frac{a}{A}\sqrt{2gh} \tag{16}$$

where, the considered process is nonlinear.

The objective is to design an operator based nonlinear controller which makes process output h track the desired reference, where the control input is q_0.

4. NONLINEAR CONTROL SYSTEM DESIGN FOR THE PROCESS

Nonlinear tracking control using operator theoretic approach (Deng et al. [2004]) is used to design control system for the water level process system. The design produces are shown as follows.

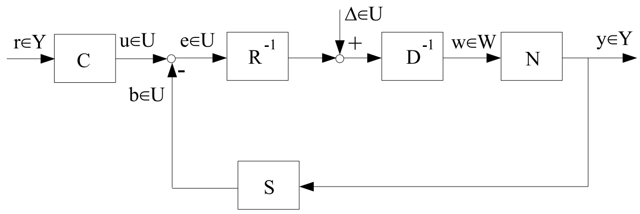

Fig. 4. Tracking control system

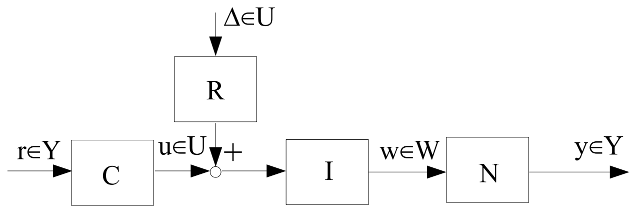

Fig. 5. Equivalent tracking control system of Fig. 4

Fig. 4 is the water level tracking control system of the process. If we consider a plant P, $P = ND^{-1}$ by right coprime factorization. Then N and D are obtained. In addition, N and D are satisfying the following Bezout identity.

$$SN + RD = I \tag{17}$$

In the real design, we need to design S and R such that Bezout identity (17) is satisfied. C is called tracking operator (Deng et al. [2004]).

The detailed design produce for the water level process system (16) is given as follows. According to the design method, we consider that q_0 is plant input u, h is plant output y and a function about u to y is plant $P(u(t))$. That is, a function $P^{-1}(y(t)) : y \to u$ can be described as follows.

$$P^{-1}(y,t) = u(t) = A\dot{y}(t) + a\sqrt{2gy(t)} \tag{18}$$

From (18), $N(w(t))$ and $D(w(t))$ are selected as

$$N(w(t)) = y(t) = \frac{w(t)^2}{2ga^2} \tag{19}$$

$$D(w(t)) = u(t) = \frac{A}{2ga^2}\frac{\mathrm{d}}{\mathrm{d}t}\{w(t)^2\} + w(t) \tag{20}$$

Design of controllers S and R is presented as follows. According to N and D, S and R must be designed to satisfy Bezout identity (17). Based on operator based right coprime factorization approach, two controllers S and R are given as

$$R = I(u(t)) \tag{21}$$

$$S = -A\dot{y}(t) \tag{22}$$

The water level process system should be stabilized by these operator based controllers. For making this system track the desired output, a stable operator based tracking controller (Deng et al. [2004]) was designed to satisfy $NM(r(t)) = I(r(t))$. In practical sytem, the disturbance $\Delta(t)$ usually exists and affects the tracking performance. According to the framework of the system which is shown in Fig. 4, signal w between D^{-1} and N can be obtained (Deng and Inoue [2006]). That is, the output of operator $R(\Delta(t))$ can be obtained (see Fig. 5), where u is known. Using this signal, the tracking controller with function of eliminating the disturbance is designed to satisfy the following condition.

$$N(SN + RD)^{-1}(C(r(t)) + R(\Delta(t))) = r(t) \tag{23}$$

where $SN + RD = I$, and the equivalent framework is shown in Fig. 5.

Remark 1: In the design of the controller S in (22), we use a derivative feedback. In general, it is inconvenient in control engineering. However, in the real control, a supersonic wave sensor for measuring the water level was used (Deng et al. [2008]), a desired control performance can be obtained.

5. SIMULATION RESULT

In this section, water level control simulation is conducted for the process system. The initialized water level is about 33cm, the reference input of r is 36cm and the total simulation time is 600 seconds. The other used parameters are shown as follows.

$A =$	$69.4 \times 10^{-3}[m^2]$	$a =$	$0.113 \times 10^{-3}[m^2]$
$D_1 =$	$0.3185[m]$	$d =$	$0.012[m]$
$D_2 =$	$0.1143[m]$	$g =$	$9.8[m/s^2]$
$P_0 =$	$1013[hPa]$	$P_a =$	$1013[hPa]$
$h_s =$	$0.2[m]$		

We assume that water inflow is limited in $[0, 4.5][l/m]$ and control input is limited in $[0, 20]$[mA]. A sine wave $1.042 \times 10^{-5} \sin \frac{\pi}{2} t$ regarded as disturbance is added into the system. Using the proposed method, the operator based controllers are designed as follows.

$$R = I(u(t)) \tag{24}$$

$$S = -69.4 \times 10^{-3}\dot{y}(t) \tag{25}$$

$$C = 0.113 \times 10^{-3}\sqrt{2gr(t)} - \bar{\Delta} \tag{26}$$

where $\bar{\Delta}$ can be obtained depending on the observation of the signal w.

Level control simulation result is shown in Fig. 6.

Control input and water inflow are shown in Figs. 7 and 8. In them, x coordinates denote simulation time [sec], y coordinates denote control input signal [mA] and water inflow [l/min]. From Fig. 6, the desired tracking result has been obtained.

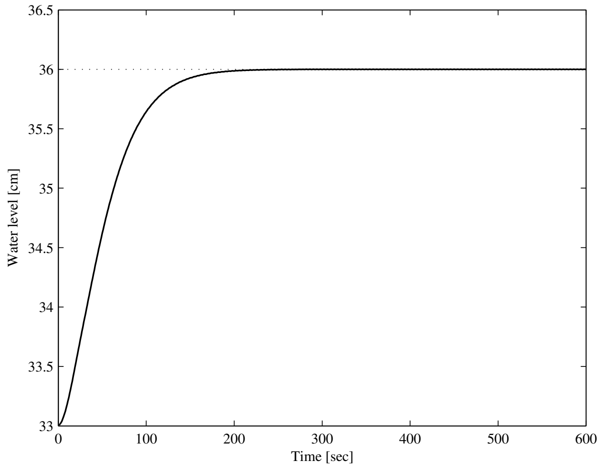

Fig. 6. Response of water level control

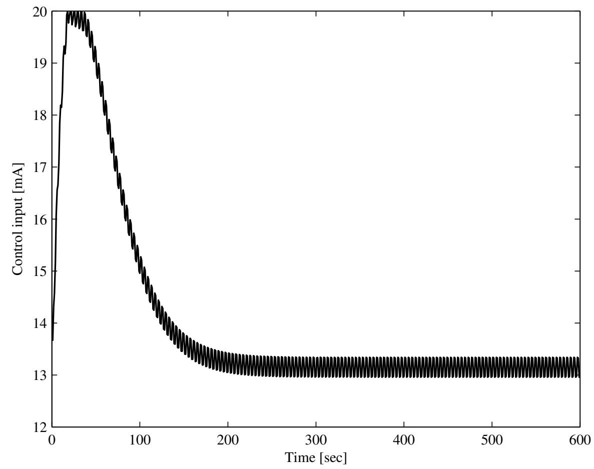

Fig. 7. Control input

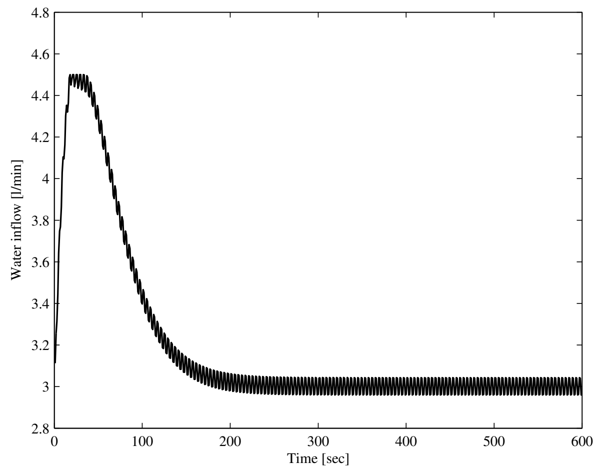

Fig. 8. Water inflow

6. CONCLUSION

A water level process system design based on operator theory is considered. The proposed method ensures the robust stability of the system and realizes the tracking performance when the disturbance appears in the process input. The effectiveness of the proposed method is confirmed through simulation.

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Appendix A

The proof of Lemma 1.

First we note that

$$\begin{aligned}\begin{bmatrix} I & -K \\ -P & I \end{bmatrix}^{-1} &= \begin{bmatrix} I & -SR^{-1} \\ -ND^{-1} & I \end{bmatrix}^{-1} \\ &= \left\{ \begin{bmatrix} D & -S \\ -N & R \end{bmatrix} \begin{bmatrix} D^{-1} & 0 \\ 0 & R^{-1} \end{bmatrix} \right\}^{-1} \quad \text{(A.1)} \\ &= \begin{bmatrix} D & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} D & -S \\ -N & R \end{bmatrix}^{-1}\end{aligned}$$

It is straightforward to see (7) holds if and only if $\{P, K\}$ is well-posed.

(Sufficiency) Suppose that (8) holds, then for a, b bounded we define c, d as follows:

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{bmatrix} D & -S \\ -N & R \end{bmatrix}^{-1} \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{(A.2)}$$

where c, d are bounded. Hence, by (A.1),

$$\begin{bmatrix} I & K \\ -P & I \end{bmatrix}^{-1} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} D & 0 \\ 0 & R \end{bmatrix}^{-1} \begin{pmatrix} c \\ d \end{pmatrix} \quad \text{(A.3)}$$

Under rcf, D and R are BIBO stable. Hence $D(c)$ and $R(d)$ are BIBO. Thus, the system inverse operator exists and it is BIBO.

(Necessity) Suppose that $\{P, K\}$ is well-posed and stable, $P = ND^{-1}$ and $K = SR^{-1}$ are stable rcf. Let

$$\begin{pmatrix} e \\ f \end{pmatrix} = \begin{bmatrix} I & -K \\ -P & I \end{bmatrix}^{-1} \begin{pmatrix} c \\ d \end{pmatrix} \quad \text{(A.4)}$$

then for all a, b bounded, we have e, f bounded. Define c, d as in (A.2), note that as a, b and e, f are bounded, the following equations hold

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} D(c) - S(d) \\ -N(c) + R(d) \end{bmatrix}^{-1} \quad \text{(A.5)}$$

$$\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} D(c) \\ R(d) \end{pmatrix} \quad \text{(A.6)}$$

As e is bounded, $D(c)$ is bounded, and since a and $D(c)$ are bounded, $S(d)$ is bounded. Similarly, as b and f are bounded, $R(d)$ and $N(c)$ are bounded. By coprimeness of ND^{-1}, since $N(c)$ and $D(c)$ are both bounded, c is bounded. Similarly, by coprimeness of SR^{-1}, d is bounded. This completes the proof.

Appendix B

The proof of Lemma 2.

Since the matrix inverse is stable we require that unbounded inputs yield unbounded inputs. Consider x an unbounded signal, and consider the action of the system as follows.

$$\begin{bmatrix} D & -S \\ -N & R \end{bmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{bmatrix} D(x) - S(0) \\ -N(x) + R(0) \end{bmatrix} \quad \text{(B.1)}$$

As x is unbounded, the output is also unbounded. Thus, we must have $D(x)$ or $N(x)$ unbounded for giving coprimeness of D, N. Considering the action of $\begin{bmatrix} D & -S \\ -N & R \end{bmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix}$ the y unbounded gives coprimeness of S, R.