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Model-Free based Optimal Iterative Learning Control for a Class of Discrete-Time Nonlinear Systems[1]

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Abstract: A pseudo-partial-derivative based dynamic linearization method is introduced, the method can transform general discrete-time nonlinear model into discrete-time time-varying linear model. Based on this discrete-time time-varying linear model, a novel norm-optimal iterative learning control (NOILC), called model-free based norm-optimal iterative learning control (MFNOILC), is proposed for a class of discrete-time nonlinear systems. Through rigorous analysis, the convergence of the proposed algorithm is proved. The simulation results show the effectiveness of the algorithm.

Keywords: Iterative Learning control; discrete-time nonlinear system; pseudo-partial-derivative; Freeway traffic control.

1. INTRODUCTION

Iterative learning control (ILC) has been intensively studied over the past two decades (Moore, 1993; Sun et al, 1995; Chen et al, 1999; Xu et al, 2003). ILC was originally proposed in the robotics community (Arimoto, Kawamura and Miyazaki, 1984) as an intelligent teaching mechanism for robot manipulators. The basic idea of ILC is to improve the control signal for the present operation cycle by feeding back the control error in the previous cycle. Nowadays, ILC has become one of the most active research areas in control theory and applications. It is one of the most effective methodologies for repeatable control environment which deals with repeated tracking control tasks for deterministic systems. Specifically, ILC improves the transient response and tracking performance in time domain when the system executes the same motion under essentially the same initial conditions.

The formulation of ILC design problem could be divided into three categories, the first is the contraction mapping based ILC control design methods. So far, most of the ILC publications belong to this category (Chen, 1998; Kuc, Lee and Nam, 1992). This category is belong to the almost a model-free method. The second one is the energy function based ILC, which the dynamics in state space has been incorporated in ILC design (Ham, Qu, and Kaloust, 1995; Xu, and Tan, 2002). The third one is the optimization based ILC control design method, in which explicit optimization objective is introduced into the ILC control design, and the monotonic error sequence could be achieved (Amann, Owens and Rogers, 1996; Lee, Lee and Kim, 2000; Hatonen, Owens and Moore, 2004; Hatonen, 2004).

The optimization based ILC control design method is also called model-based ILC methods. The model based ILC algorithms proposed were based on the known linear time-invariant model of the controlled plant. So the algorithms become hyper-sensitive to model uncertainties and it cannot be used directly in the practice.

There are certain additional traits and requirements found in model-based norm-optimal ILC approaches. First, dynamics of almost all practical processes are intrinsically nonlinear, and the nonlinearities become exposed when the processes are operated over a wide range of conditions. For this reason, it is desirable to derive a norm-optimal ILC control algorithms that can accommodate nonlinear system models. Secondly, to model the practical plants is not an easy thing, and sometimes it is impossible in the view of cost or accuracy. Hence, it is desirable to design directly the input-output based ILC control law for the ILC control task. So we can enjoy not only extra good properties of the norm optimal ILC, but also the little requirements on the system dynamic model of the prototype of the ILC as well.

The objective of this paper is to provide a more general and comprehensive framework for quadratic criterion based ILC that is capable of addressing all the issues that mentioned to be important for the process control applications. We first introduce a dynamic linearization method that can transform general discrete-time nonlinear model into a time-varying linearized model using a concept of pseudo-partial- derivative (PPD). Based on this discrete-time linearized model, the norm-optimal ILC for discrete-time nonlinear systems is designed, and the mathematical properties are also discussed similar to the outline of Amann, Owens and Rogers (1996).

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The rest of the paper is organized as follows. In section 2, the problem formulation and the model transformation are presented. In section 3, the norm-optimal ILC is designed, and its properties are also discussed. In section 4, some numerical simulation studies are given in this section to demonstrate the efficiency and correctness for MFNOILC proposed. The conclusions are drawn in the section 5.

2. PROBLEM FORMULATION AND MODEL TRANSFORMATION

2.1 Problem formulation

The system to be controlled is described by the following SISO discrete-time nonlinear equation

$$y(t+1) = f(y(t),\cdots,y(t-n_y),u(t),\cdots,u(t-n_u)), \tag{1}$$

where $y(t),u(t)$ are the output and input at time t, $t \in \{0,1,\cdots,T-1\}$, n_y, n_u are the unknown orders, and $f(\cdot)$ is an unknown nonlinear function.

The control task is the perfect tracking in a finite interval under a repeatable control environment. The perfect tracking task implies that a trajectory must be strictly followed from very beginning of the execution. The repeatable control environment implies (1) identical target trajectory and (2) same initial condition for all trials. In more details, the control objective for ILC is to design a sequence of appropriate control inputs $u_k(t)$ such that the system output $y_k(t)$ approaches the target trajectory $y_d(t)$, $t \in \{0,1,\cdots,T-1\}$, and T > 0 is a finite number. The subscript k denotes the k-th repeated control operation period and is called k-th learning "iteration". $y_d(t)$ is invariant in iteration domain.

2.2 Model transformation

The system dynamics in the k-th iteration

$$y_k(t+1) = f(y_k(t),\cdots,y_k(t-n_y),u_k(t),\cdots,u_k(t-n_u)). \tag{2}$$

The system dynamics in the (k-1)-th iteration

$$\begin{aligned} y_{k-1}(t+1) = f(&y_{k-1}(t),\cdots,y_{k-1}(t-n_y),u_{k-1}(t),\\ &\cdots,u_{k-1}(t-n_u)). \end{aligned} \tag{3}$$

We first make two assumptions with regards to the system.

Assumptions:

A1: The partial derivative of $f(\cdot)$ with respect to control input $u_k(t)$ is continuous.

A2: The system (2) is generalized Lipschitz, that is, satisfying

$$|\Delta y_k(t+1)| \le D\|\Delta\vec{u}_k(t)\| \quad \forall t \quad and \quad \|\Delta\vec{u}_k(t)\| \ne 0, \tag{4}$$

where D is a constant, $\Delta y_k(t+1) = y_k(t+1) - y_{k-1}(t+1)$, $\Delta\vec{u}_k(t) = \vec{u}_k(t) - \vec{u}_{k-1}(t)$, $\vec{u}_k(t) = [u_k(t),u_k(t-1),\cdots,u_k(t-L+1)]^T$, and L is a positive integer known as the control input length constant.

Remark 1: These assumptions of the system are reasonable and acceptable from a practical point of view. Assumption (A1) is a typical condition for many control laws which a general nonlinear system should satisfy. Assumption (A2) poses a limitation on the rate of change of the system output permissible before the control law to be formulated is applicable.

Theorem 1: For the nonlinear systems (2) and (3), when Assumptions (A1) and (A2) hold, then for a given L, there must exist $\vec{\phi}_k(t)$, called pseudo-partial-derivative (PPD), and $\|\vec{\phi}_k(t)\| \le D$ such that if $\|\Delta\vec{u}_k(t)\| \ne 0$, the system may be described as

$$\Delta y_k(t) = \vec{\phi}_k(t)\Delta\vec{u}_k(t), \tag{5}$$

where $\vec{\phi}_k(t) = [\phi_k^1(t),\phi_k^2(t),\cdots,\phi_k^L(t)]^T$.

Proof:

Subtracting (3) from (2), and using the differential mean theorem, yields

$$\begin{aligned} y_k(t+1) = {} & y_{k-1}(t+1) + (\partial f^*/\partial u_k(t))\Delta u_k(t) \\ & + \cdots + (\partial f^*/\partial u_k(t-L+1))\Delta u_k(t-L+1) + \psi_k(t), \end{aligned} \tag{6}$$

where $\partial f^*/\partial u_k(t)$ represents the partial derivative value of f at some point in the interval $[u_k(t),u_{k-1}(t)]$, and

$$\begin{aligned} \psi_k(t) = {} & f(y_k(t),\cdots,y_k(t-n_y),u_{k-1}(t),\\ & \cdots,u_{k-1}(t-L+1),u_k(t-L),\cdots,u_k(t-n_u)) \\ & - f(y_{k-1}(t),\cdots,y_{k-1}(t-n_y),u_{k-1}(t),\\ & \cdots,u_{k-1}(t-L+1),\cdots,u_{k-1}(t-n_u)), \end{aligned} \tag{7}$$

and $\Delta u_k(t) = u_k(t) - u_{k-1}(t)$.

Rewriting (6) in a compact form, we obtain

$$\vec{y}_k = \vec{y}_{k-1} + G_k\Delta\vec{u}_k, k = 0,1,2,\cdots, \tag{8}$$

where

$\vec{y}_k = [y_k(1)\ \ y_k(2)\ \ \cdots\ \ y_k(T)]^T$, $\vec{y}_{k-1} = [y_{k-1}(1)\ \ y_{k-1}(2)\ \ \cdots\ \ y_{k-1}(T)]^T$,

$\Delta\vec{u}_k = [\Delta u_k(0)\ \ \Delta u_k(1)\ \ \cdots\ \ \Delta u_k(T-1)]^T$,

$$G_k = \begin{bmatrix} \phi_k^1(0) & & & & \\ \phi_k^2(1) & \phi_k^1(1) & & 0 & \\ \vdots & & \ddots & & \\ \phi_k^L(L-1) & & \cdots & \phi_k^1(L-1) & \\ & \ddots & & & \ddots \\ 0 & & \phi_k^L(T-1) & & \cdots & \phi_k^1(T-1) \end{bmatrix}_{T\times T}.$$

Considering the following equation with a variables $\vec{\eta}_k(t)$

$$\psi_k(t) = \vec{\eta}_k^{\,T}(t) \begin{bmatrix} \Delta u_k(t) \\ \Delta u_k(t-1) \\ \vdots \\ \Delta u_k(t-L+1) \end{bmatrix}. \quad (8)$$

Equation (8) must have at least a solution $\vec{\eta}_k(t)$ since condition $\Delta\vec{u}_k(t) \neq 0$ holds. In fact, it must have infinite solutions.
Let

$$\vec{\phi}_k(t) = \left[\partial f^* / \partial u_k(t) \ \cdots \ \partial f^* / \partial u_k(t-L+1)\right]^T + \vec{\eta}_k(t). \quad (9)$$

Replacing $\psi_k(t)$ of (7) by (9), then we have

$$\Delta y_k(t) = \vec{\phi}_k(t)\Delta\vec{u}_k(t),$$

where $\Delta\vec{u}_k(t) = [\Delta u_k(t), \Delta u_k(t-1), \cdots, \Delta u_k(t-L+1)]^T$, and $\vec{\phi}_k(t) = \left[\phi_k^1(t), \phi_k^2(t), \cdots, \phi_k^L(t)\right]^T$. The boundedness of PPD is the straightforward result from the assumption (A2) and (5).

Remark 2: Theorem 1 is an extension of the results in Hou, et al. (1994 and 1997). This theorem shows that $\vec{\phi}_k(t)$ is a differential signal in some sense along the iteration axis and bounded for any iteration number k. Furthermore, PPD $\vec{\phi}_k(t)$ is a slowly time-varying parameter along the iteration axis and its relation with $\vec{u}_k(t)$ may be ignored when $\|\Delta\vec{u}_k(t)\|$ is not too large.

Remark 3: For the time-invariant linear system

$$x(t+1) = Ax(t) + Bu(t) \qquad x(0) = x_0,$$
$$y(t) = Cx(t),$$

where $t = 0, 1, \cdots, T-1$. Obviously, the system output is $y(t) = CA^t x_0 + \sum_{i=0}^{t-1} CA^{t-1-i}Bu(i)$, then the output in the k-th iteration is

$$y_k(t+1) = CA^{t+1}x_k(0) + \sum_{i=0}^{t} CA^{t-i}Bu_k(i),$$

the output in the (k-1)-th iteration is

$$y_{k-1}(t+1) = CA^{t+1}x_{k-1}(0) + \sum_{i=0}^{t} CA^{t-i}Bu_{k-1}(i),$$

thus we have

$$\Delta y_k(t+1) = \sum_{i=0}^{t} CA^{t-i}B\Delta u_k(i), if \quad x_k(0) = x(0), \quad \forall k.$$

From the above expression, we can see that, the PPD vector $\vec{\phi}_k(t)$ is just the Markov sequence of the system if the given constant L is sufficiently large such that L=T, that is $\phi_k^1(t) = CB$, $\phi_k^2(t) = CAB$, $\cdots$, $\phi_k^T(t) = CA^{T-1}B$, this is the same description as Amann, Owens and Rogers (1996).

3. NORM-OPTIMAL ITERATIVE LEARNING CONTROL

3.1 Optimal learning control law algorithm

Let $y_d(t)$ be the desired output signal, $e_k(t) = y_d(t) - y(t)$ be the output error, and define the optimization design problem for the ILC controller as follows

$$\min_{\vec{u}_{k+1}(t)} J_{k+1}(\vec{u}_{k+1}) = \min_{\vec{u}_{k+1}(t)}(\|\vec{e}_{k+1}\|^2 + \lambda\|\vec{u}_{k+1} - \vec{u}_k\|^2), \quad (12)$$

subject to

$$\vec{e}_{k+1} = \vec{e}_k - G_{k+1}\Delta\vec{u}_{k+1}, \quad (13)$$

where $\lambda > 0$.

To simplify notions, we will hereafter use $\vec{e}_k = \vec{y}_d - \vec{y}_k$ to denote $e_k(t) = y_d(t) - y(t)$.

Inserting (13) into (12), and differentiating the objective function, and setting the differential equal to zero yields the following norm-optimal ILC control law

$$\vec{u}_{k+1} = \vec{u}_k + (\lambda I + G_{k+1}^{\ T}G_{k+1})^{-1}G_{k+1}^{\ T}\vec{e}_k. \quad (14)$$

Remark 4: The row of the matrix G_{k+1} could not be zero vector, otherwise the problem is not solvable.

Remark 5: when $L = 1$, the norm optimal ILC control law becomes

$$u_{k+1}(t) = u_k(t) + \phi_{k+1}^1(t)(y_d(t) - y_k(t))/(\lambda + \|\phi_{k+1}^1(t)\|^2), \quad \forall t \in [0, T-1], k = 1, 2, \cdots. \quad (15)$$

It is the default form for the norm-optimal ILC controller.

Since the PPD $\vec{\phi}_k(t)$ is unknown, a new parameter estimation Criterion function is used for the derivation of the estimator.

$$J(\hat{\vec{\phi}}_k(t)) = \left|\Delta y_{k-1}(t+1) - \hat{\vec{\phi}}_k(t)\Delta\vec{u}_{k-1}(t)\right|^2 + \mu\left\|\hat{\vec{\phi}}_k(t) - \hat{\vec{\phi}}_{k-1}(t)\right\|^2, \quad (16)$$

where $\mu > 0$.

By using (3), the minimization of above criterion function gives estimation algorithm

$$\hat{\vec{\phi}}_k(t) = \hat{\vec{\phi}}_{k-1}(t) + \Delta\vec{u}_{k-1}(t)(\Delta y_{k-1}(t+1) - \Delta\vec{u}_{k-1}^{\ T}(t)\hat{\vec{\phi}}_{k-1}(t))/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2), \quad (17)$$

and control law algorithm becomes

$$\vec{u}_{k+1} = \vec{u}_k + (\lambda I + \hat{G}_{k+1}^{\ T}\hat{G}_{k+1})^{-1}\hat{G}_{k+1}^{\ T}\vec{e}_k, t \in \{0, 1, \cdots T-1\}. \quad (18)$$

In order to make the condition $\|\Delta\vec{u}_k(t)\| \neq 0$ in theorem 1 satisfied, and meanwhile to make parameter estimation algorithm (17) have stronger ability to track time-varying parameter, a reset measurement of estimation algorithm should be taken

$$\hat{\vec{\phi}}_k(t) = \hat{\vec{\phi}}_0(t), \text{if } \left\|\hat{\vec{\phi}}_k(t)\right\| \le \varepsilon \text{ or } \|\Delta\vec{u}_{k-1}(t)\| \le \varepsilon, \tag{19}$$

where ε is a some small positive constant, $\hat{\vec{\phi}}_0(t)$ is the initial estimation value of $\hat{\vec{\phi}}_k(t)$.

In order to obtain the convergence and stability for the controller, another assumption about the system should be made.

A3: Estimation error of ppd $\vec{\phi}_k(t)$ is sufficiently small, such that $\left\|\hat{G}_{k+1}\hat{G}_{k+1}{}^T - G_{k+1}\hat{G}_{k+1}{}^T\right\| < \min\{\sigma_{k+1}\}, \forall t \in \{0,1,\cdots T-1\}$, and $\forall k \in \{0,1,\cdots\}$, where $\min\{\sigma_{k+1}\}$ is the smallest eigenvalue of $\hat{G}_{k+1}\hat{G}_{k+1}{}^T$.

Theorem 2 Assume A1-A3 hold, Suppose that the plant described by (2) is controlled by (18) and the estimate $\hat{\vec{\phi}}_k(t)$ is identified using (17) and (19), then

Estimate $\hat{\vec{\phi}}_k(t)$ is bounded for all $t \in \{0,1,\cdots T-1\}$ and $k \in \{0,1,\cdots\}$.

$\|\vec{e}_{k+1}\| \le \rho\|\vec{e}_k\|$, where $0 < \rho < 1$.

$\lim_{k\to\infty}\|e_k(t)\| = 0$.

Proof:

Introduce the following notation:

$$\tilde{\vec{\phi}}_k(t) = \hat{\vec{\phi}}_k(t) - \vec{\phi}_k(t). \tag{20}$$

From (5) we have

$$\Delta y_{k-1}(t+1) = \Delta\vec{u}_{k-1}{}^T(t)\vec{\phi}_{k-1}(t). \tag{21}$$

When $\|\Delta\vec{u}_{k-1}(t)\| \le \varepsilon$, from (19) we can see that $\hat{\vec{\phi}}_k(t)$ is bounded. When $\|\Delta\vec{u}_{k-1}(t)\| > \varepsilon$, subtracting $\vec{\phi}_k(t)$ from both sides of (17) yields

$$\begin{aligned}\tilde{\vec{\phi}}_k(t) &= \Delta\vec{u}_{k-1}(\Delta y_{k-1}(t) - \Delta\vec{u}_{k-1}^T(t)\hat{\vec{\phi}}_{k-1}(t))/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2) \\ &\quad + \tilde{\vec{\phi}}_{k-1}(t) + \vec{\phi}_{k-1}(t) - \vec{\phi}_k(t).\end{aligned} \tag{22}$$

Substituting (21) into (22) gives

$$\begin{aligned}\tilde{\vec{\phi}}_k(t) &= (I - \Delta\vec{u}_{k-1}(t)\Delta\vec{u}_{k-1}^T(t))\tilde{\vec{\phi}}_{k-1}(t)/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2) \\ &\quad + \vec{\phi}_{k-1}(t) - \vec{\phi}_k(t).\end{aligned} \tag{23}$$

Let

$$\Xi(t) = (I - \Delta\vec{u}_{k-1}(t)\Delta\vec{u}_{k-1}^T(t))\tilde{\vec{\phi}}_k(t)/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2). \tag{24}$$

Computing $\|\Xi(t)\|^2$ yields

$$\begin{aligned}\|\Xi(t)\|^2 &= \left\|\tilde{\vec{\phi}}_{k-1}(t)\right\|^2 + (-2 + \|\Delta\vec{u}_{k-1}(t)\|^2/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2)) \\ &\quad \times \left\|\Delta\vec{u}_{k-1}{}^T(t)\tilde{\vec{\phi}}_{k-1}(t)\right\|^2/(\mu + \|\Delta\vec{u}_{k-1}(t)\|^2).\end{aligned} \tag{25}$$

Since $\vec{\phi}_k(t)$ is an iteration dependent column vector and $\mu > 0$, we have

$$\|\Xi(t)\|^2 < \left\|\tilde{\vec{\phi}}_{k-1}(t)\right\|^2. \tag{26}$$

This implies that there exist positive constant $d < 1$ such that following inequality holds

$$\left\|\tilde{\vec{\phi}}_k(t)\right\| \le d^k \|\tilde{\vec{\phi}}_0(t)\| + 2D(1-d^k)/(1-d). \tag{27}$$

In view of (27), $\tilde{\vec{\phi}}_k(t)$ is bounded, $\vec{\phi}_k(t)$ is bounded, so all of the $\hat{\vec{\phi}}_k(t)$ is bounded.

Substituting (18) into (13)

$$\vec{e}_{k+1} = (\lambda I + \hat{G}_{k+1}\hat{G}_{k+1}{}^T)^{-1}(\lambda I + \hat{G}_{k+1}\hat{G}_{k+1}{}^T - G_{k+1}\hat{G}_{k+1}{}^T)\vec{e}_k^T. \tag{28}$$

Taking norms on both sides of (28) we have

$$\begin{aligned}\|\vec{e}_{k+1}\| &= \left\|(I - G_{k+1}(\lambda I + \hat{G}_{k+1}{}^T\hat{G}_{k+1})^{-1}\hat{G}_{k+1}{}^T)\vec{e}_k\right\| \\ &\le \frac{\lambda + \left\|\hat{G}_{k+1}\hat{G}_{k+1}{}^T - G_{k+1}\hat{G}_{k+1}{}^T\right\|}{\lambda + \min\{\sigma_{k+1}\}}\|\vec{e}_k\|.\end{aligned} \tag{29}$$

From assumption 3 and (29) we can conclude

$\|\vec{e}_{k+1}\| \le \rho\|\vec{e}_k\|$, and $\lim_{k\to\infty}\|e_k(t)\| = 0$.

Remark 6: For the time-invariant linear system

$$\begin{aligned}x(t+1) &= Ax(t) + Bu(t) \qquad x(0) = x_0, \\ y(t) &= Cx(t),\end{aligned}$$

where $t = 0,1,\cdots,T-1$.

If the given constant L is sufficiently large such that L=T, that is $\phi_k^1(t) = CB, \phi_k^2(t) = CAB, \cdots, \phi_k^T(t) = CA^{T-1}B$, and let $\lambda = 1$, then the theorem2 becomes

$\|\vec{e}_{k+1}\| \le (1/(1+\sigma^2))\|\vec{e}_k\|$, where σ^2 is the smallest eigenvalue of GG^T.

$\lim_{k\to\infty}\|e_k(t)\| = 0$.

This is the same as in Amann, Owens and Rogers (1996).

4. SIMULATION STUDY

In order to demonstrate effectiveness of the proposed ILC algorithm, two simulation examples are now presented.

Example 1: The system to be controlled is the same as in Hou (2004), the SISO nonlinear model is

$$y(t+1) = \begin{cases} y(t)/(1+y(t)^2)+u(t)^3, 0 \le t \le 50, \\ y(t)y(t-1)y(t-2)u(t-1)(y(t-2)-1)/(1+y(t-1)^2+y(t-2)^2), \\ +a(t)u(t)/(1+y(t-1)^2+y(t-2)^2), 50 \le t \le 100, \end{cases} \quad (30)$$

where $a(t) = 1+round(t/50)$ is parameter of the system.

Obviously the structure, order, and parameter of the system to be controlled are time-varying.

The control task of system is

$$y_d(t+1) = \begin{cases} 0.5\times(-1)^{(t/10)}, 0 \le t \le 30, \\ 0.5\sin(t\pi/10)+0.3\cos(t\pi/10), 30 < t \le 70. \\ 0.5\times(-1)^{(t/10)}, 70 < t \le 100, \end{cases} \quad (31)$$

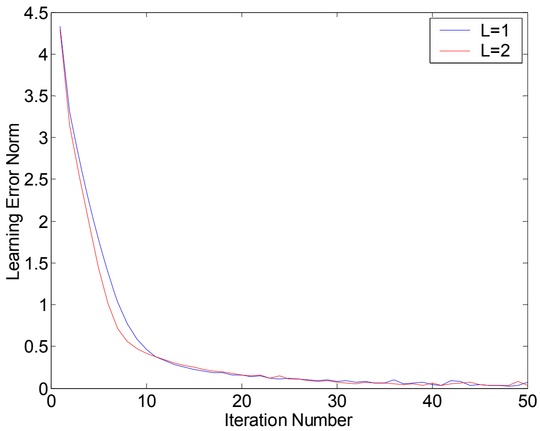

Fig.1 The norm of the error with random initial conditions

In order to illustrate the proposed MFNOILC algorithm can overcome the limitations on identical initial condition of traditional ILC, a random varying along iteration axis $y_k(0)$ is used. In the simulation we choose $L=1,2, \lambda=1.5, \mu=1$, and Fig.1 shows the tracking error with random initial conditions, where the learning error norm is a usual vector norm, e.g., $\|e_k(t)\| = \left\{\sum_{t=1}^{T} e_k(t)^2\right\}^{1/2}, \quad k=1,2\cdots$. Form fig.2 we can see that the convergence over the entire finite time interval can be guaranteed when the initial conditions are randomly varying along the iteration axis, and the tracking performance when *L*=2 is better than when *L*=1.

Example 2: Freeway traffic system

The system to be controlled is the same as in M. Parageorgiou, et al. (1990), the macroscopic traffic flow model is

$$\rho_i(t+1) = \rho_i(t) + (q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t))T/L_i, \quad (32)$$

$$q_i(t) = \rho_i(t) v_i(t), \quad (33)$$

$$\begin{aligned} v_i(t+1) = v_i(t) &+ (V(\rho_i(t)) - v_i(t))T/\tau \\ &+ v_i(t)(v_{i-1}(t) - v_i(t))T/L_i \\ &- \nu T(\rho_{i+1}(t) - \rho_i(t))/(\tau L_i(\rho_i(t)+\kappa)), \end{aligned} \quad (34)$$

$$V(\rho_i(t)) = v_{free}(1-(\rho_i(t)/\rho_{jam})^l)^m. \quad (35)$$

The definition of parameters in model and the setting of parameters are similar to M. Parageorgiou, et al. (1990), and Hou (2007).

Boundary conditions are summarized as follows

$$\rho_0(t) = q_0(t)/v_1(t),\ v_0(t) = v_1(t),\ \rho_{N+1}(t) = \rho_N(t), \quad v_{N+1}(t) = v_N(t).$$

Consider a long segment of freeway that is subdivided into 5 sections. The length of each section is 0.5 km. The desired density is $30 veh/km$ .The initial traffic volume entering section 1 is 1500 vehicles per hour. The initial density, mean speed, and other parameters used in this model are listed in Table 1.There exist an on-ramp with known traffic demands in section 3 and an off-ramp with unknown exiting traffic flow in section 4. The parameters used in this model are listed in Table 1. In order to illustrate the proposed MFNOILC algorithm can overcome the limitations on identical initial condition of traditional ILC, a random varying along iteration axis $\rho_i(0)$ is used. In the simulation we choose $L=1, \lambda=0.001, \mu=1$. Fig.2 shows the tracking error of the traffic flow density in section 3 and Fig.3 shows density tracking performance in section 3 with random initial conditions, where the learning error norm is a usual vector norm.

Table1: Parameters associated with the traffic model

v_{free}	ρ_{jam}	l	m	κ	τ
80	80	1.8	1.7	13	0.01
T	ν	$q_0(k)$	$r_i(0)$	α	
0.00417	35	1500	0	0.95	

For practical urban freeway traffic control system, form fig.2 and fig.3 we can see that the proposed ILC can overcome the limitations of traditional ILC with respect to initial condition, achieve the perfect tracking except initial point.

5. CONCLUSION

The MFNOILC scheme based on the discrete-time linearized model is proposed in this paper, and the convergence over the entire finite time interval can be guaranteed by theoretical analysis when the initial conditions are randomly varying along the iteration axis. Moreover, this paper covers the results of Owens (1996), which is the special case of the MFNOILC in LTI system. The main features of the MFNOILC scheme is that the controller design only depends on the I/O data of the dynamic system. The simulation results show effectiveness of proposed algorithm.

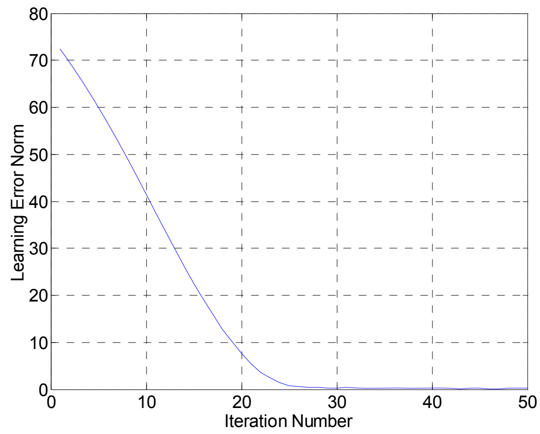

Fig.2 The norm of tracking error of traffic flow density in section 3 with random initial conditions

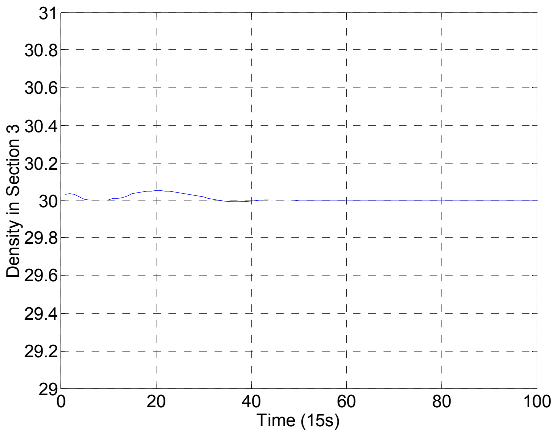

Fig.3 Traffic flow density tracking performance in section 3 with random initial conditions

REFERENCES

Arimoto, S., Kawamura, S., and Miyazaki, F. (1984). Bettering operation of robots by learning. *Journal of Robotic Systems*, **1**, (1), 123-140.

Amann, N., Owens, D. H. and Rogers, E. (1996). Iterative learning control for discrete-time systems with exponential rate of convergence. *IEE Proc.-Control Theory and Applications*, **143**, (2), 217-224.

Chen, C. J. (1998). A discrete iterative learning control for a class of nonlinear time-varying systems. *IEEE Transactions on Automatic Control*, **43**, (5), 748-752.

Chen, Y. Q. and Wen, C. Y. (1999). *Iterative Learning Control: Convergence, Robustness and Application*. Lecture Notes in Control and Information Sciences 248, Springer.

Ham, C, Qu, Z. H. and Kaloust, J. H. (1995). Nonlinear learning control for a class of nonlinear systems based on Lyapunov's direct method. *In Proceeding of 1995 IEEE American Control Conference*, 3024-3028.

Hatonen, J. J., Owens, D. H. and Moore, K. L. (2004). An algebraic approach to iterative learning control. *International Journal of Control*, **77**, (1), 45-54.

Hatonen, J. J. (2004). *Issues of algebra and optimality in iterative learning control*. PhD dissertation, University of Oulu, Finland.

Hou, Z. S. (1994). *The parameter identification, adaptive control and model-free learning adaptive control for nonlinear systems*, PhD thesis, Northeastern University, China. (in Chinese)

Hou, Z. S. and Huang, W. H. (1997). Model-free learning adaptive control of a class of SISO nonlinear systems. *Proc of Amer. Control Conf*., Albuquerque, New Mexico, 343-344.

Hou , Z . S. (1999) .*Non-Parametric Model and its Adaptive Control*. Science press, Beijing. (in Chinese)

Hou, Z. S., Xu, J. X. and Zhong, H. W. (2007). Freeway Traffic Control Using Iterative Learning Control-Based Ramp Metering and Speed Signalling. *IEEE Transaction on vehicular technology*. **56**, (2), 466-477.

Hou, Z. S., Xu, J. X. and Yan, J. W. (2007). An Iterative Learning Approach for Density Control of Freeway Traffic Flow via Ramp Metering, *Transportation Research Part C*, In Press

Moore, K. L. (1993). *Iterative Learning Control for Deterministic Systems*, Springer-Verlag, Advances in Industrial Control Series.

Kuc, T. Y. Lee, J. S. and Nam, K. (1992). An iterative learning control theory for a class of nonlinear dynamic systems. *Automatica*, **28**, (6), 1215–1221.

Lee, J. H., Lee, K. S. and Kim, W. C. (2000). Model-based iterative learning control with a quadratic criterion for time-varying linear systems. *Automatica*, **36**, (5), 641-657.

Parageorgiou, M., Blosseville, J. M. and Hadj-Salem, H. (1990). Modelling and real time control on traffic flow on the southern part of Bpulevard Peripherique in Paris. Part I: Modelling; Part II: Coordinated on-ramp metering, *Transportation Research*, **A24**, 345-370.

Sun, M. X. and Huang, B. J. (1998). *Iterative Learning control*. National Defence Industry Press, Beijing. (In Chinese)

Xu, J. X. and Tan, Y. (2003). *Linear and Nonlinear Iterative Learning Control*. Lecture Notes in Control and Information Sciences, 291, Springer.

Xu, J. X. and Tan, Y. (2002). A composite energy function based learning control approach for nonlinear systems with time varying parametric uncertainties. *IEEE Transactions on Automatic Control*, **47**, (11), 1940-1945.