

Model-Free Adaptive Control for a Class of Nonlinear Discrete-Time Systems Based on the Partial Form Linearization¹

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Abstract: The NARMA model is an exact representation of the input-output behavior of finite-dimensional nonlinear discrete-time dynamical systems in the neighborhood of the equilibrium state. However, it is inconvenient for purposes of adaptive control due to its nonlinear dependence on the control input, even by using the neural network method. In this paper, we introduce a so called model-free adaptive control (MFAC) method, which is based on some new dynamical linearization model and concept, the partial form linearization (PFL) and the pseudo-partial derivative (PPD) of a SISO nonlinear discrete-time system. The model-free means that the controller design is only based on the I/O data of the controlled plant, no training process, no structure information and no model are needed. Rigorous analysis and extensive simulations have shown that it has BIBO stability and performs very well.

Keywords: Model free adaptive control; discrete-time nonlinear system; pseudo-partial-derivative; Stability; Partial form linearization

1. INTRODUCTION

For the adaptive control of linear systems, the theory and powerful design methods have been available and the theoretical analysis methods are well established. However, topics on nonlinear discrete-time systems, only some special nonlinearity have been considered. One of the reasons is that the well-known *Lyapunov* design technique, an extremely useful tool in continuous-time, is of little use in nonlinear discrete-time systems because the increments of the parameter estimates do not appear linearly in the increments of Lyapunov function (Elshafei, et al., 1995, Etxebarria, 1994). Moreover the methods for directly adjusting the controller parameters based on the output error for general nonlinear discrete-time systems are very few.

As we have known, the dependence on mathematical model structure of the controlled system and the un-modelled dynamics are the two main inevitable problems for the traditional adaptive control theory, therefore the design of the adaptive control system only using the I/O data of controlled plant will be of great significance both in the development and applications of control theory. Two kinds of model-free control techniques applied successfully in practice are the PID typed control technique and the adaptive control by using the neural networks method, but they both suffer some limitations themselves: The PID typed can only cope with linear time-invariant system, and the neural networks technique also has some problems which are very difficult to be overcome, such as, the need of the orders of the plant and high speed computer, the determination of numbers of nodes and hidden layers and theoretical analysis.

The MFAC is proposed by Prof. Hou Z. (1994), the design of the MFAC uses the I/O data of the controlled system only and can realize the parameter adaptive control and structure adaptive control. Under some assumptions, the convergence and stability of MFAC scheme based on tight form linearization model is proved by Hou Z., (1994, 1997, 1999), and this method is widely used in many fields, e.g. in chemical industry (Coelho et al, 2006), in sheet forming process (Liu et al., 2004), in linear motor control and in injection modelling process (Tan, et al., 1999, 2001), in PH value control (Zhang et al., 2006). At present, stability analysis of the MFAC based on partial form linearization model or full form linearization model is still an open problem. In this paper, the BIBO stability of MFAC based on the former is provided, and simulation results show the efficacy of this scheme.

The rest of the paper is organized as follows. In section 2, the problem formulation and the model transformation are presented. In section 3, the MFAC system is designed. In section 4, the BIBO stability of the MFAC system is provided. In section 5, some numerical simulation studies are given to demonstrate the efficiency and correctness for the MFAC scheme. The conclusions are drawn in the section 6.

2. PLANT DESCRIPTION AND MODEL TRANSFORMATION

Following discrete-time SISO nonlinear systems is considered:

$$y(k+1) = f(y(k), \dots, y(k-n_y), u(k), \dots, u(k-n_u)), \quad (1)$$

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where n_y, n_u are orders of output $y(k)$ and input $u(k)$ respectively, $f(\dots)$ is a nonlinear function.

Rewrite (1) as

$$y(k+1) = f(Y(k), u(k), U(k-1)), \quad (2)$$

where $Y(k) = \{y(k), y(k-1), \dots, y(k-n_y)\}$ and $U(k) = \{u(k), u(k-1), \dots, u(k-n_u)\}$, for the simplicity of notation.

It is also called NARX model. Hammerstein model, bilinear model and some other nonlinear system models can be shown to be special cases of (1) or (2).

The MFAC is based on the following assumptions made about the systems:

A1: System (1) or (2) is observable, and controllable in following meaning, that is, for some expected system output bounded signal $y^*(k+1)$, there exists a bounded control input signal in time instant k , the output $y(k+1)$ controlled by it will be equal to the set value $y^*(k+1)$.

A2: The partial derivative of with respect to control input $u(k)$ is continuous.

A3: The system (1) or (2) is generalized Lipschitz, that is, satisfying $|\Delta y(k+1)| \leq b \|\Delta U(k)\|$ for any k and $\Delta U(k)$, where $\Delta U(k) = [\Delta u(k), \dots, \Delta u(k-L+1)]^T$, and L is a positive constant, which is called as control input length constant of linearization of the discrete-time nonlinear system. b is a positive constant.

Remark 1: These assumptions of the system are reasonable and acceptable from a practical viewpoint. Assumption A1 is a basic assumption about the controlled system, controlling such a system is impossible if A1 is not satisfied. Assumption A2 is a typical condition of control system design for general nonlinear system. Assumption A3 poses a limitation on the rate of change of the system output permissible before the control law to be formulated is applicable. From the 'energy' point of view, the energy rate increasing inside a system can not go to infinite if the energy rate of change of input is in a finite altitude.

Theorem 1: For the nonlinear system (1) or (2), satisfying assumptions A1-A3, then there must exist $\Phi(k)$, called pseudo-partial-derivative (PPD) vector, when $\|\Delta U(k)\| \neq 0$ we have

$$\Delta y(k+1) = \Phi^T(k) \Delta U(k), \quad (3)$$

and $\|\Phi(k)\| \leq b$. Where $\Phi(k) = [\phi_1(k), \dots, \phi_L(k)]^T$.

Remark 2: The validity of the theorem 1 is proved by Hou Z. (1994, 1997, and 1999). The element of PPD vector is obviously a time-varying parameter even though the (1) or (2) is time-invariant system. It is clearly that $\Phi(k)$ has some relations with inputs and outputs of the system till time instant k . The theorem 1 gives that $\Phi(k)$ is a "differential" signal in some sense and bounded for any k , so we have certain reasons to say that $\Phi(k)$ is a slowly time-varying

parameter and the relation with $U(k)$ can be ignored when the magnitude of $\|\Delta U(k)\|$ and the sampling period are not too large.

Remark 3: From theorem 1 and remark 1, we know that (3) is a dynamic linear system with slowly time-varying parameter when $\Delta U(k) \neq 0$ and $\|\Delta U(k)\|$ is not too big. Therefore, besides the condition $\Delta U(k) \neq 0$ will be considered in the control system design, some free adjustable parameter should be added in the control input criterion function, which is used to keep the rate of change of control input signal not vary too large.

Remark 4: When the constant $L=1$, the partial form linearization (PFL) of nonlinear system becomes the tight form linearization (TFL), which is much easier and simpler for use and implementation than the partial form linearization here.

3. MODEL FREE ADAPTIVE CONTROL SYSTEM DESIGN

Control law algorithm: For the one-step-ahead controller (Goodwin, et al., 1984), excessive control effort may be called for to bring $y(k+1)$ to $y_r(k+1)$ in one step, particularly in the early stages of parameter tuning. The weighted one-step-ahead controller, in general, leads to steady-state tracking error. So we used the following control input criterion function

$$J(u(k)) = [(y(k+1) - y_r(k+1))^2 + \lambda(u(k) - u(k-1))^2], \quad (4)$$

where λ is a weighting constant.

Rewrite (3) as

$$y(k+1) = y(k) + \Phi^T(k) \Delta U(k), \quad (5)$$

Substituting (5) into (4) and differentiating (4) with respect to $u(k)$ and setting it be zero give the control law as follows:

$$u(k) = u(k-1) + \rho_k \phi_1(k) (y_r(k+1) - y(k)) / (\lambda + \phi_1^2(k)) - \rho_k \phi_1(k) \sum_{i=2}^L \phi_i(k) \Delta u(k-i+1) / (\lambda + \phi_1^2(k)). \quad (6)$$

Remark 5: The ρ_k in control law algorithm (6) is a step-size constant series, which is added in order to get it generality.

Remark 6: From (4) and (6), we can see that λ is not only a penalty factor on $\Delta u(k)^2$, so the substitution domain of that system (2) is substituted by system (3) can be limited in some extent, which, as a results, makes PPD vector $\Phi(k)$ not change in value too much, but also is a part of denominator in (6). This is an important parameter for this control system. Computer simulation results show that suitable choice of λ can improve the performance of the control system.

Remark 7: From control law algorithm (14), we can see that this kind of control law has an iterative form, it is different from the control law in (Goodwin, et al., 1984), and has no relations with any structural information (mathematical model, structure, orders) of controlled plant, it is designed only by I/O data of controlled system.

PPD estimation algorithm: From the remark 2, we know that the unknown parameter vector PPD is usually a slowly time-varying under some condition, so the conventional time-varying parameter estimation algorithm could be used to estimate it. Such as, the modified projection algorithm(Hou, et al., 1997, 1999), the Least-Square algorithm with a time-varying forgetting factor (Goodwin, et al., 1984), or the leakage recursive least-square algorithm (Tan, et al., 2001).

The modified Projection algorithm is:

$$\hat{\Phi}(k+1) = \hat{\Phi}(k) + \eta_k \Delta U(k) \Delta y(k+1) / (\mu + \|\Delta U(k)\|^2) - \eta_k \Delta U(k) \Delta U^T(k) \hat{\Phi}(k) / (\mu + \|\Delta U(k)\|^2), \quad (7)$$

Remark 8: The differences between the algorithm (7) and the projection algorithm (also known as NLMS algorithm) are as follows: the addition of the small constant μ to the denominator of the NLMS algorithm is only for avoiding division by zero, no practical meaning, but μ here in algorithm (7) is a weighting constant which punishes the rate of change of parameter estimate, and the methods which the NLMS and the (7) are derived are also different.

The MFAC scheme, using the parameter estimate algorithm (7), control law algorithm above, is as follows

$$u(k) = u(k-1) + \rho_k \hat{\phi}_1(k) (y_r(k+1) - y(k)) / (\lambda + \hat{\phi}_1^2(k)) - \rho_k \hat{\phi}_1(k) \sum_{i=2}^k \hat{\phi}_1(k) \Delta u(k-i+1) / (\lambda + \hat{\phi}_1^2(k)), \quad (8)$$

$$\hat{\Phi}(k+1) = \hat{\Phi}(k) + \eta_k \Delta U(k) \Delta y(k+1) / (\mu + \|\Delta U(k)\|^2) - \eta_k \Delta U(k) \Delta U^T(k) \hat{\Phi}(k) / (\mu + \|\Delta U(k)\|^2), \quad (9)$$

$$\hat{\phi}_1(k+1) = \hat{\phi}_1(1), \text{ if } |\hat{\phi}_1(k+1)| \leq \varepsilon, \quad (10)$$

where the step-size series usually can be set $\rho_k, \eta_k \in (0, 2)$, and λ, μ are two weighting constants, ε is a small positive constant, $\hat{\phi}_1(1)$ is the initial value of $\hat{\phi}_1(k)$.

Remark 9: Since the unknown parameter vector, i.e., the PPD, are time-varying, the conventional projection or least squares algorithm cannot handle this case properly, so some time-varying algorithms should be used to estimate the unknown parameters of PPD. In fact, any time-varying parameter estimate algorithm can be used to estimate the PPD.

Remark 10: The controller above has L parameters needed to be adjusted on-line, it is quite different from the traditional adaptive control system design, in which usually there are $2n$ parameters needed to be estimated on-line, where n denotes the order of the controlled system. For the case of $L=1$, the number of controller parameters needed to be adjusted on-line for SISO nonlinear system is only one, that is, the dimension of PPD vector. It can be designed much easier than that of the traditional adaptive control system.

Remark 11: In order to make the condition $\Delta U(k) \neq 0$ in theorem 1 satisfied, and meanwhile to make parameter

estimate algorithm have stronger ability to track time-varying parameter, a reset measurement of an estimation algorithm should be taken as (10).

Remark 12: The choice of the Control Input Length Constant of Linearization can usually be set 1. For a complex system, it can be set to the order sum estimated value of the plant.

Remark 13: The whole scheme has no relation with the controlled plant except the I/O data of the plant, no prior knowledge (no form of the system model, order of the system and training process are needed) are used. This is the reason why we call it be Model Free Adaptive Control. All information and assumptions about the model before are just to want to make discussion clearly.

As we have known, the designing of the controller and estimator of traditional adaptive control system depends on the structure and the orders of mathematical model of controlled plant, but the structure and the orders of controlled system are very difficult to identify, and sometimes have relations with time and ambient, so the applications of various adaptive control systems reported may be failure due to the un-modelled dynamics. The MFAC system presented in this paper only use the I/O data of controlled system, thus the un-modelled dynamics disappear, therefore it should have strong robustness.

4. STABILITY ANALISYS

Lemma 1 (Payne, 1987): Consider the time-varying difference equation

$$x(k+1) = F(k)x(k) + v(k), \quad (11)$$

where $x(t)$ and $v(t)$ are real vectors of finite dimension. Suppose that the sequence of matrixes $\{F(k)\}$ and $x(0) = x_0$ are bounded and that the free system

$$z(k+1) = F(k)z(k), k \geq 0, \quad (12)$$

is exponentially stable. Further, supposed that there exist sequences of non-negative real number $\{\gamma(k)\}$ and $\{\delta(k)\}$ and an integer $N \geq 0$, such that

$$\|v(k)\| \leq \gamma(k) \sum_{i=0}^N \|x(k-i)\| + \delta(k), \quad (13)$$

where $\|\cdot\|$ denotes the usual Euclidean norm. Under those conditions, if $\{\gamma(k)\}$ converge to zero and $\{\delta(k)\}$ are bounded, then $x(k)$ and $v(k)$ are bounded. If, in addition, $\{\delta(k)\} \rightarrow 0$, then $\{x(k)\} \rightarrow 0$ and $\{v(k)\} \rightarrow 0$.

Lemma 2 (Gerschgorin, 1931): Let $A = [a_{ij}] \in C^{n \times n}$. Assume that $\sigma_i = \sum_{j=1, j \neq i}^n |a_{ij}|$, then each eigenvalue of A is in at least one of the disk $|z - a_{ii}| \leq \sigma_i$.

Theorem 2: Under Assumptions A1, A2 and A3, Suppose the plant described by (1) is controlled by (3) and estimate $\hat{\Phi}(k)$ is

identified using the algorithm (7a), and if $y_{sp} = const$, $y_r(k+1) = \alpha^k y_{sp} + (1-\alpha^k)y(k)$, $\alpha < 1$ and λ is carefully chosen, then we have

$$\lim_{k \rightarrow \infty} y_{sp} - y(k) = 0, \{y(k)\}, \{u(k)\} \text{ are bounded sequences.}$$

Proof:

Two steps to prove this theorem. First step is to prove the estimated value of PPD is bounded. Then we prove the BIBO property.

We now prove that the estimated value of PPD is bounded.

Let $\tilde{\Phi}(k) = \hat{\Phi}(k) - \Phi(k)$, then parameter estimation algorithm becomes

$$\tilde{\Phi}(k) = \Phi(k-1) - \Phi(k) + (I - \Delta U(k-1)\Delta U(k-1)^T / (\mu + \|\Delta U(k-1)\|^2))\tilde{\Phi}(k-1). \quad (14)$$

Taking norms on both sides of (14)

$$\|\tilde{\Phi}(k)\| \leq \left\| (I - (\Delta U(k-1)\Delta U(k-1)^T) / (\mu + \|\Delta U(k-1)\|^2)) \right\| \|\tilde{\Phi}(k-1)\| + \|\Phi(k-1) - \Phi(k)\|. \quad (15)$$

There exist positive constant $d < 1$ such that following inequality holds

$$\|\tilde{\Phi}(k)\| \leq d^k \|\tilde{\Phi}(0)\| + 2D(1-d^k)/(1-d). \quad (16)$$

In view of (16), $\tilde{\Phi}(k)$ is bounded, $\Phi(k)$ is bounded, so $\hat{\Phi}(k)$ is bounded.

Based on the estimated value of $\hat{\Phi}(k)$, now we prove the tracking error, input and output of system are bounded.

Let

$$e(k) = y_{sp} - y(k). \quad (17)$$

Substituting $y_r(k+1) = \alpha^k y_{sp} + (1-\alpha^k)y(k)$ into (8) gives

$$\Delta u(k) = \frac{\rho_k \hat{\phi}_1(k)}{\lambda + \hat{\phi}_1^2(k)} [\alpha^k e(k) - \sum_{i=2}^L \hat{\phi}_1(k) \Delta u(k-i+1)]. \quad (18)$$

Set $x(k) = [\Delta u(k) \ \dots \ \Delta u(k-L+1)]^T$, $B = [1 \ 0 \ \dots \ 0]^T$,

$$A = \begin{bmatrix} \frac{\rho_k \hat{\phi}_1(k) \hat{\phi}_2(k)}{\lambda + \hat{\phi}_1^2(k)} & \frac{\rho_k \hat{\phi}_1(k) \hat{\phi}_3(k)}{\lambda + \hat{\phi}_1^2(k)} & \dots & \frac{\rho_k \hat{\phi}_1(k) \hat{\phi}_L(k)}{\lambda + \hat{\phi}_1^2(k)} & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

$$r(k) = \rho_k \hat{\phi}_1(k) / (\lambda + \hat{\phi}_1^2(k)).$$

Equation (18) can be rewrite as

$$x(k) = Ax(k-1) + \alpha^k r(k) B e(k). \quad (19)$$

Let $r_{\max} = \max_{i \in [1, k]} r(i)$ and taking norms on both sides of (19) we have

$$\|x(k)\| \leq \|A\| \|x(k-1)\| + \alpha^k r_{\max} \|e(k)\|. \quad (20)$$

From Lemma 2 and choosing λ sufficiently large, such that $\sum_{j=2}^L |\rho_k \hat{\phi}_1(k) \hat{\phi}_j(k) / (\lambda + \hat{\phi}_1^2(k))| < 1$, then the spectral radius ρ of A is less than 1.

From (20) we have

$$\|x(k)\| \leq r_{\max} \sum_{i=0}^k \rho^{k-i} \alpha^i \|e(i)\|, \quad (21)$$

Substituting (5) and (8) into (17) and let

$$C(k) = (\phi_1(k)r(k)\hat{\phi}_2(k) - \phi_2(k) \ \phi_1(k)r(k)\hat{\phi}_3(k) - \phi_3(k) \ \dots, \ \phi_1(k)r(k)\hat{\phi}_L(k) - \phi_L(k) \ 0)^T$$

we have

$$e(k+1) = (1 - \hat{\phi}_1(k)\phi_1(k) / (\lambda + \hat{\phi}_1^2(k)))e(k) + C(k)x(k-1), \quad (22)$$

where $e(k+1) = (1 - \hat{\phi}_1(k)\phi_1(k) / (\lambda + \hat{\phi}_1^2(k)))e(k)$ is exponential stable.

Taking norm on the second item of right side of (22)

$$\|C(k)x(k-1)\| \leq \|C(k)\| r_{\max} \max(\rho, \alpha)^{k-1} \left(\sum_{i=0}^{k-1} \|e(i)\| \right). \quad (23)$$

From (20) we have

$$\begin{aligned} & \left\| [u(k) \ u(k-1) \ \dots \ u(k-L+1)]^T \right\| \\ & \leq \sum_{i=0}^k \|x(i)\| \leq r_{\max} \max_{0 \leq i \leq k} \|e(i)\| / ((1-\rho)(1-\alpha)). \end{aligned} \quad (24)$$

From Lemma 1, (23) and (24), the results can be easily obtained.

5. SIMULATION STUDY

In this section, the simulation results of adaptive control of two different discrete-time SISO nonlinear systems in series by using the same controller, even the same initial values, under same conditions, are given to demonstrate the effectiveness of the model-free adaptive scheme proposed, and the correctness of the declaration that the controller can be designed without the prior knowledge about the plant controlled, these knowledge are needed for the traditional adaptive control system design, which includes the form of mathematics model of the system, the structure of the system, such as, linear, affine nonlinear or linear in parameter, etc., the order and relative degree of the system, and so on. All the models below are only used for collection of I/O data.

The initial conditions in all the following two examples are set the same, they are $u(1) = u(2) = \dots = u(5) = 0$, $y(1) = y(2) = y(3) = 0$, $y(4) = 1$, $y(5) = y(6) = 0$. The initial

value of PPD are set to be $\phi(6) = [1, 0, 0]^T$, $\varepsilon = 10^{-4}$, $L = 3$. The reset initial value of PPD is set to be 0.5.

The first example is a system that consists of a non-minimum phase nonlinear subsystem (Hou, 1994, 1999) and a linear subsystem. This system has the property that the structure, order and phase of the controlled system is time varying. The second example is a nonlinear system with a step change time-varying parameter.

Example 1:

$$y(k+1) = \begin{cases} 2.5y(k)y(k-1)/(1+y(k)^2+y(k-1)^2) \\ \quad + 0.7\sin(0.5(y(k)+y(k-1))) \\ \quad + 1.4u(k-1)+1.2u(k), k \leq 200, \\ -0.1y(k)-0.2y(k-1)-0.3y(k-2)+0.1u(k) \\ \quad + 0.02u(k-1)+0.03u(k-2), k > 200, \end{cases} \quad (25)$$

The simulation results using the MFAC scheme and the PID controller are shown in Fig.1, where PID controller is

$$u(k) = K_p[e(k) + \sum_{j=0}^k e(j)/T_i + T_d(e(k)-e(k-1))]$$

The PID tuning parameters are set to be $K_p = 0.15$, $T_i = 0.5, T_d = 0$, at which the best control performance is achieved.

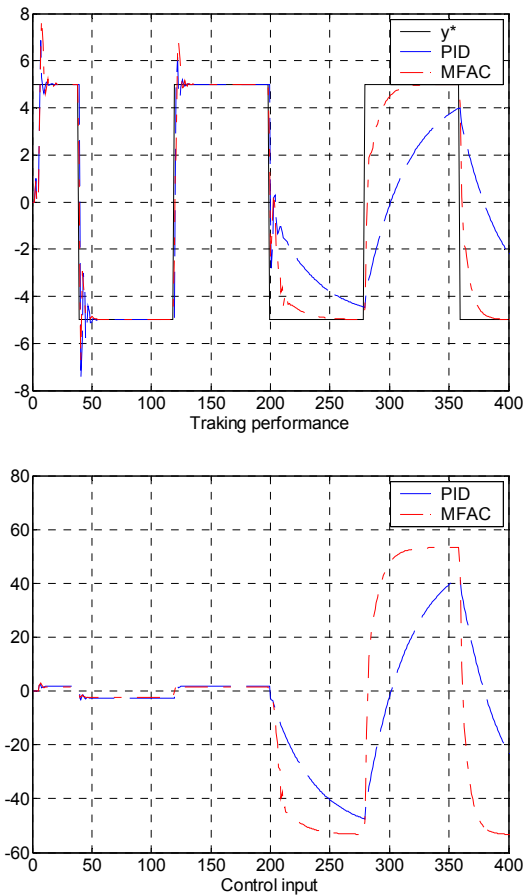


Fig.1. Comparison simulation results between MFAC and PID.

It is deserved to point out that the tuning of PID parameters must be chosen carefully, the control performance is sensitive to the choice. Careless selection of them can yields the oscillations or even unstable. But the tuning of the MFAC controller parameter λ is much easier than that of PID.

Example 2:

$$y(k+1) = y(k)y(k-1)y(k-2)u(k-1) \times (y(k)-1)/(1+y(k)^2+y(k-1)^2+y(k-2)^2) + (1+a(k))u(k)/(1+y(k)^2+y(k-1)^2+y(k-2)^2) \quad (26)$$

where $a(k)$ is a time-varying parameter.

When $a(k) = 1$, the example is controlled by using multilayer recurrent neural network with a special architecture (Jin, et al., 1994). For the reader who interested in the details please see the reference. The simulation result using the MFAC scheme is shown in fig.2.

From the performance above, we can see that the control effect is quite satisfied, which even better than that of using neural network (Jin, et al., 1994). Furthermore the computation burden is much less than using neural network and it is much easier than using neural network.

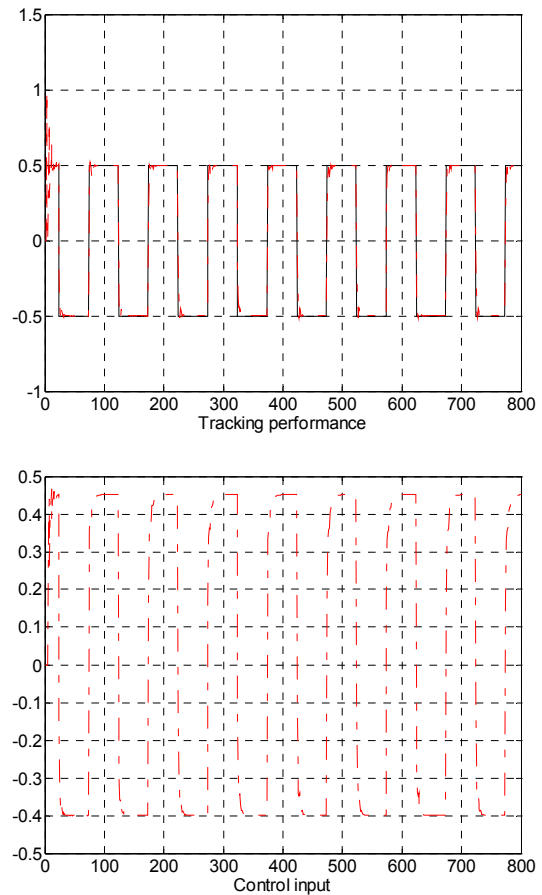


Fig.2. Simulation results using MFAC with $a(k) = 1$

When $a(k) = \begin{cases} 0 & k \leq 100 \\ 1 & 100 < k \leq 300 \\ 2 & 300 < k \leq 500 \\ 3 & 500 < k \leq 800 \end{cases}$, the simulation result using the

MFAC scheme is shown in fig.3. The abrupt changes in tracking performance are caused by the suddenly change of the time-varying parameter $a(k)$.

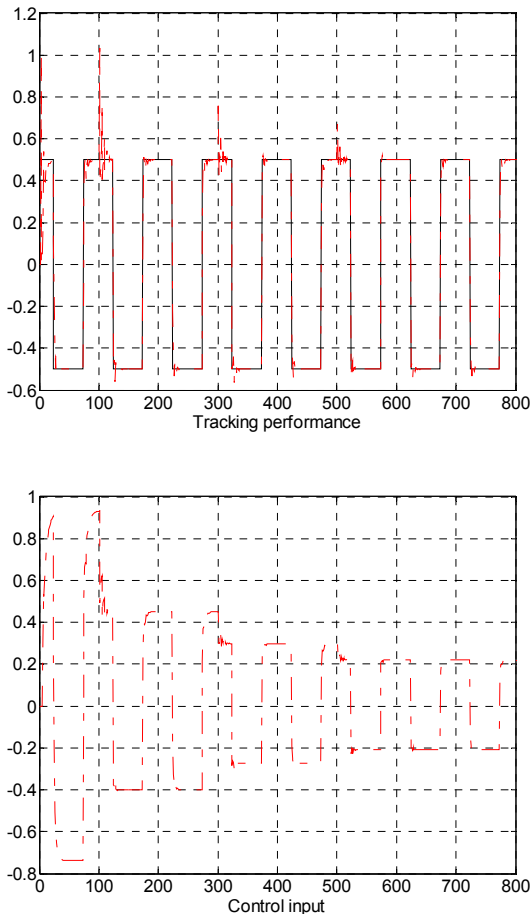


Fig.3. Simulation results using MFAC with time-varying parameter $a(k)$

6. CONCLUSIONS

In this paper, the MFAC scheme based on the partial form linearization model is introduced and its BIBO stability is provided. In contrast to other adaptive control schemes, the features of this new typed adaptive control method are as follows: First, the proposed MFAC scheme uses the I/O data of controlled system only. No mathematical model and structural information of controlled plant are needed, which implies that we can design the controller independently, and also implies that we can develop a generic controller for the industrial processes. Secondly, the MFAC mechanism does not require any external testing signals and any training process, which are necessary to the nonlinear system adaptive control by using neural networks, it implies that it is less expensive and low cost. Thirdly, the scheme proposed is simple and can be easily used and implemented, and has

minimum computational burden and strong robustness. Finally, the all results of this paper can be extended easily to the MISO and MIMO nonlinear cases.

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