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A New Parameter-dependent Approach to Discrete-time Robust H_2 Filtering ⋆

Huijun Gao * **Xiangyu Meng** * **Tongwen Chen** **

* *Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin, 150001 China (Tel: 86-451-86402351; e-mail: hjgao@hit.edu.cn; mxydavid@yahoo.com.cn).*
** *Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4 Canada (Tel: 17804923368; e-mail: tchen@ece.ualberta.ca)*

Abstract: This paper revisits the problem of robust H_2 filtering for discrete-time systems with parameter uncertainties. Given a stable system with parameter uncertainties residing in a polytope with s vertices, the focus is on designing a robust filter such that the filtering error system is robustly asymptotically stable and has a guaranteed estimation error variance for the entire uncertainty domain. A new polynomial parameter-dependent idea is introduced to solve the robust H_2 filtering problem, which is different from the quadratic framework that entails fixed matrices for the entire uncertainty domain, or the linearly parameter-dependent framework that uses linear convex combinations of s matrices. This idea is realized by carefully selecting the structure of the matrices involved in the products with system matrices. A linear matrix inequality (LMI) condition is obtained for the existence of admissible filters, and based on this, the filter design is cast into a convex optimization problem, which can be readily solved via standard numerical software. The merit of the proposed method lies in its less conservativeness than the existing robust filter design methods, as illustrated via a numerical example.

1. INTRODUCTION

The problem of robust H_2 filtering consists of designing a linear time-invariant asymptotically stable filter that assures a prescribed bounded estimation error variance, irrespective of the uncertain parameters. In general, there are two approaches to solving this problem: the Riccati-like approach (Petersen and McFarlane [1996], Wang et al. [1999], Xie et al. [1994]) and the linear matrix inequality (LMI) approach (de Souza and Trofino [1999], Ebihara and Hagiwara [2005], Palhares et al. [2001], Shaked et al. [2001], Tuan et al. [2001]); and two kinds of parameter uncertainties have been widely used in the literature: the norm-bounded uncertainty (Petersen and McFarlane [1996], Petersen and Savkin [1999], Wang and Huang [2000], Xie and Soh [1994]) and the polytopic uncertainty (de Souza and Trofino [1999], Palhares et al. [2001], Shaked et al. [2001], Tuan et al. [2001]). In solving the robust H_2 filtering problem for uncertain systems, most of the reported results are based on quadratic Lyapunov functions, which have been largely used for robust analysis and synthesis in the past decades. Although being specially adequate for arbitrarily fast time-varying parameters, methods based on quadratic stability can produce conservative results since the same parameter independent Lyapunov function must be used for the entire uncertainty domain. One possible way to overcome this conservatism has been well recognized in considering a parameter-dependent Lyapunov function. An example of a less conservative stability condition based on parameter-dependent Lyapunov functions can be found in (de Oliveira et al. [1999]). Similar ideas have been subsequently developed to investigate the stability analysis, control and filtering synthesis problems in a few contexts (see, He et al. [2004], Shaked et al. [2001], Tuan et al. [2001], Xia and Jia [2002] and the references therein).

Solving the problem of robust filtering via parameter-dependent Lyapunov functions is an advanced research topic, whose aim is to reduce the overdesign in the quadratic framework. Results in this direction can be found in (Barbosa et al. [2005], Gao and Wang [2003, 2004], Geromel et al. [2002], Shaked et al. [2001], Tuan et al. [2001], Xie et al. [2004]). By utilizing the parameter-dependent idea, these results are generally less conservative than those in the quadratic framework, most of which have been shown either theoretically or through numerical examples. The basic idea behind these results comes from (de Oliveira et al. [1999]), that is, by introducing slack matrix variables to the well-established LMI performance conditions, the product terms between the positive definite matrices and system matrices are eliminated (these slack matrix variables are usually called multipliers). In such a way, by imposing the slack matrix variables to be fixed for the entire uncertainty domain, the positive definite matrices are relaxed to be dependent on each vertex of the polytope, which helps to achieve parameter dependence. As the additional introduced matrices are still required to be fixed for the entire uncertainty domain, the reduced conservatism is usually not significant. Another feature worth mentioning is that the parameter-dependent positive definite matrices are linearly dependent on the uncertain parameters, and thus have the same structures as that of the parameter uncertainty (for example, the λ-dependent positive definite matrix P_λ takes the form of $\sum_{i=1}^{s} \lambda_i P_i$ with s denoting the number of vertices of the polytope). One will naturally raise a question:

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whether the conservatism could be further reduced if we adopt different structures other than linear parameter dependence as described above? A possible alternative is the selection of polynomial parameter-dependent matrices (Oliveira and Peres [2006]), which to the best of the authors' knowledge has not been investigated for robust filtering problems.

In this paper, motivated by the above two aspects, we propose a structured polynomial parameter-dependent approach for robust H_2 filtering of linear uncertain systems. Given a stable system with parameter uncertainties residing in a polytope with s vertices, the focus is on designing a robust filter such that the filtering error system is robustly asymptotically stable and has a guaranteed estimation error variance for the entire uncertainty domain. The new polynomial parameter-dependent idea is introduced to solve the robust H_2 filtering problem, which is different from the quadratic framework that entails fixed matrices for the entire uncertainty domain, and the linearly parameter-dependent framework that uses linear convex combinations of s matrices. This idea is realized by carefully selecting the structure of the matrices involved in the products with system matrices. More specifically, only the $(2,1)$ and $(2,2)$ blocks of the additionally introduced slack matrix variables are required to be fixed, while the positive definite matrices and the $(1,1)$, $(1,2)$ blocks of the slack matrix variables are all relaxed to be polynomially dependent on the uncertain parameters. An LMI condition is obtained for the existence of admissible filters and based on this, the filter design is cast into a convex optimization problem, which can be readily solved via standard numerical software (Boyd et al. [1994], Gahinet et al. [1995]). If these conditions are satisfied, a desired robust filter can be readily constructed. The merit of the method presented in this paper lies in its less conservatism than the existing robust filter design methods, as shown through a numerical example.

2. PROBLEM FORMULATION

Consider a stable discrete-time system $\mathscr{S}$:

$$\mathscr{S}: \qquad \begin{aligned} x(k+1) &= A_\lambda x(k) + B_\lambda \omega(k), \\ y(k) &= C_\lambda x(k) + D_\lambda \omega(k), \\ z(k) &= L_\lambda x(k). \end{aligned} \tag{1}$$

Here $x(k) \in \mathbb{R}^n$ is the state vector; $y(k) \in \mathbb{R}^m$ is the measured output; $z(k) \in \mathbb{R}^p$ is the signal to be estimated; $\omega(k) \in \mathbb{R}^l$ is a zero-mean white noise with identity power spectrum density matrix. A_λ, B_λ, C_λ, D_λ and L_λ are appropriately dimensioned matrices. It is assumed that

$$\Omega_\lambda \triangleq (A_\lambda, B_\lambda, C_\lambda, D_\lambda, L_\lambda) \in \mathscr{R}, \tag{2}$$

where $\mathscr{R}$ is a given convex bounded polyhedral domain described by s vertices:

$$\mathscr{R} \triangleq \left\{ \Omega_\lambda \,\middle|\, \Omega_\lambda = \sum_{i=1}^{s} \lambda_i \Omega_i;\ \lambda \in \Gamma \right\},$$

with $\Omega_i \triangleq (A_i, B_i, C_i, D_i, L_i, H_i)$ denoting the vertices of the polytope, and Γ denoting the unit simplex, that is,

$$\Gamma \triangleq \left\{ (\lambda_1, \lambda_2, \ldots, \lambda_s):\ \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \geq 0 \right\}. \tag{3}$$

Here we are interested in estimating the signal $z(k)$ by a robust filter of general structure described by $\mathscr{F}$:

$$\mathscr{F}: \qquad \begin{aligned} x_F(k+1) &= A_F x_F(k) + B_F y(k), \\ z_F(k) &= C_F x_F(k), \end{aligned} \tag{4}$$

where $x_F(k) \in \mathbb{R}^n$ is the filter state vector and (A_F, B_F, C_F) are appropriately dimensioned filter matrices to be determined.

Augmenting the model of $\mathscr{S}$ to include the states of the filter, we obtain the filtering error system $\mathscr{E}$:

$$\mathscr{E}: \qquad \begin{aligned} \xi(k+1) &= \bar{A}_\lambda \xi(k) + \bar{B}_\lambda \omega(k), \\ e(k) &= \bar{C}_\lambda \xi(k), \end{aligned} \tag{5}$$

where $\xi(k) = \left[x^T(k)\ x_F^T(k) \right]^T$, $e(k) = z(k) - z_F(k)$ and

$$\bar{A}_\lambda = \begin{bmatrix} A_\lambda & \mathbf{0} \\ B_F C_\lambda & A_F \end{bmatrix}, \bar{B}_\lambda = \begin{bmatrix} B_\lambda \\ B_F D_\lambda \end{bmatrix}, \bar{C}_\lambda = \left[L_\lambda\ \ -C_F \right]. \tag{6}$$

The z transfer function of the filtering error system is given by

$$T(z,\lambda) = \bar{C}_\lambda \left[z\mathbf{I} - \bar{A}_\lambda \right]^{-1} \bar{B}_\lambda. \tag{7}$$

Then, the robust H_2 filtering problem to be addressed in this section is expressed as follows.

Problem RH2F (Robust H_2 Filtering): Given system $\mathscr{S}$ in (1) with parameter uncertainty in (2) and $\gamma > 0$, determine matrices (A_F, B_F, C_F) of filter $\mathscr{F}$ in (4), such that the filtering error system $\mathscr{E}$ in (5) is robustly asymptotically stable and satisfies

$$\max_\lambda \mathbb{E}\left\{ e^T(k) e(k) \right\} < \gamma \ \left(\text{or } \max_\lambda \|T(z,\lambda)\|_2^2 < \gamma \right). \tag{8}$$

Filters satisfying the above conditions are called robust H_2 filters.

Remark 1. The parameter uncertainties considered in this paper are assumed to be of polytopic type, entering into all the matrices of the system model. The polytopic uncertainty has been widely used in the problems of robust control and filtering for uncertain systems (see, for instance, Gao and Wang [2004], Palhares et al. [2001], Xia and Jia [2002] and the references therein), and many practical systems possess parameter uncertainties which can be either exactly modelled or overbounded by the polytopic uncertainty $\mathscr{R}$.

3. MAIN RESULTS

To solve Problem RH2F formulated in the above section, we need the following standard result (Boyd et al. [1994], Zhou et al. [1996]):
Given system $\mathscr{S}$ in (1) and filter $\mathscr{F}$ in (4), the filtering error system $\mathscr{E}$ in (5) is asymptotically stable and satisfies (8) if and only if there exist appropriately dimensioned matrix functions $P_\lambda > 0$ and $\Pi_\lambda > 0$ satisfying

$$\begin{bmatrix} -P_\lambda & P_\lambda \bar{A}_\lambda & P_\lambda \bar{B}_\lambda \\ * & -P_\lambda & \mathbf{0} \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & -P_\lambda & \bar{C}_\lambda^T \\ * & * & -\Pi_\lambda \end{bmatrix} < 0, \tag{10}$$

$$\Xi_\lambda \triangleq \operatorname{tr} \Pi_\lambda - \gamma < 0, \tag{11}$$

for all λ.

We introduce another version of the LMI-based H_2 performance (Peaucelle et al. [2000]).

Lemma 1. Given system $\mathscr{S}$ in (1) and filter $\mathscr{F}$ in (4), the filtering error system $\mathscr{E}$ in (5) is asymptotically stable and satisfies (8) if and only if there exist matrix functions $P_\lambda > 0$, $\Pi_\lambda > 0$, F_λ and W_λ satisfying (10), (11) and

$$\begin{bmatrix} -P_\lambda + \bar{A}_\lambda^T F_\lambda + F_\lambda^T \bar{A}_\lambda & -F_\lambda^T + \bar{A}_\lambda^T W_\lambda & F_\lambda^T B_\lambda \\ * & P_\lambda - W_\lambda - W_\lambda^T & W_\lambda^T B_\lambda \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (12)$$

for all λ.

Remark 2. It is worth mentioning that if we set $W_\lambda \equiv W$ and $F_\lambda \equiv F$ for the entire uncertainty domain, then by following similar procedures as in Geromel et al. [2002], Tuan et al. [2001], the robust H_2 filtering problem can be cast into an LMI-based convex optimization problem. In this way, the Lyapunov matrix P_λ is released to be dependent on the parameter λ, which is realized at the expense of setting the introduced slack matrices W_λ and F_λ to be constant for the entire uncertainty domain. In addition, it is also worth mentioning that the positive definite matrix function is selected as $P_\lambda = \sum_{i=1}^{s} \lambda_i P_i$, which is linearly dependent on the uncertain parameter λ. This approach has been shown, both theoretically and through numerical examples, to be less conservative than the filtering results in the quadratic framework where a common Lypuanov matrix is used for the entire uncertainty domain. From the above analysis, one may naturally ask: by what means can we further reduce the overdesign? In answering this question, we tentatively propose the following two ideas:

i) As W_λ and F_λ are block matrices with respect to the states of the original system and the filter respectively, can we impose part of the matrices W_λ and F_λ to be fixed, while part of them to be dependent on the uncertain parameter λ?

ii) For matrices that are relaxed to be dependent on the uncertain parameter λ, can we select other structures instead of the linearly λ-dependence?

Carefully examinations show that the conservatism can be further significantly reduced if the above two ideas are applied.

In what follows, we present a new filtering result which incorporates the above two ideas. Now, instead of setting the additionally introduced slack matrices $W_\lambda \equiv W$ and $F_\lambda \equiv F$, we select the following structure (Duan et al. [2006]):

$$W_\lambda = \begin{bmatrix} W_{1\lambda} & W_{2\lambda} \\ W_4 & W_3 \end{bmatrix}, \; F_\lambda = \begin{bmatrix} F_{1\lambda} & F_{2\lambda} \\ \alpha_1 W_4 & \alpha_2 W_3 \end{bmatrix}, \quad (13)$$

which means that the $(1,1)$ and $(1,2)$ blocks of W_λ and F_λ are assumed to be dependent on the parameter λ, while the $(2,1)$ and $(2,2)$ blocks of W_λ are assumed to be fixed and the $(2,1)$ and $(2,2)$ blocks of F_λ are related to those of W_λ with scalars α_1 and α_2. Let P_λ be partitioned as

$$P_\lambda = \begin{bmatrix} P_{1\lambda} & P_{2\lambda} \\ P_{2\lambda}^T & P_{3\lambda} \end{bmatrix}. \quad (14)$$

Without loss of generality, we assume that W_3 and W_4 are invertible. Define matrices

$$\phi \triangleq \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & W_3^{-1} W_4 \end{bmatrix}, \; \bar{P}_\lambda \triangleq \begin{bmatrix} \bar{P}_{1\lambda} & \bar{P}_{2\lambda} \\ * & \bar{P}_{3\lambda} \end{bmatrix} = \phi^T P_\lambda \phi. \quad (15)$$

Applying congruence transformations to (10) and (12) by $\mathrm{diag}\{\mathbf{I}, \phi, \mathbf{I}\}$ and $\mathrm{diag}\{\phi, \phi, \mathbf{I}\}$ respectively and considering (6), we obtain

$$\begin{bmatrix} -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -\bar{P}_{1\lambda} & -\bar{P}_{2\lambda} & L_\lambda^T \\ * & * & -\bar{P}_{3\lambda} & -W_4^T W_3^{-T} C_F^T \\ * & * & * & -\Pi_\lambda \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} \mathrm{sym}\left(\bar{\Psi}_1\right) - \bar{P}_\lambda & \bar{\Psi}_3^T - \bar{\Psi}_2^T & \bar{\Psi}_4 \\ * & \bar{P}_\lambda - \mathrm{sym}\left(\bar{\Psi}_5\right) & \bar{\Psi}_6 \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (17)$$

where

$$\bar{\Psi}_1 = \begin{bmatrix} F_{1\lambda}^T A_\lambda + \alpha_1 W_4^T B_F C_\lambda & \alpha_1 W_4^T A_F W_3^{-1} W_4 \\ W_4^T W_3^{-T} F_{2\lambda}^T A_\lambda + \alpha_2 W_4^T B_F C_\lambda & \alpha_2 W_4^T A_F W_3^{-1} W_4 \end{bmatrix},$$

$$\bar{\Psi}_2 = \begin{bmatrix} F_{1\lambda} & F_{2\lambda} W_3^{-1} W_4 \\ \alpha_1 W_4^T W_3^{-T} W_4 & \alpha_2 W_4^T W_3^{-T} W_4 \end{bmatrix},$$

$$\bar{\Psi}_3 = \begin{bmatrix} W_{1\lambda}^T A_\lambda + W_4^T B_F C_\lambda & W_4^T A_F W_3^{-1} W_4 \\ W_4^T W_3^{-T} W_{2\lambda}^T A_\lambda + W_4^T B_F C_\lambda & W_4^T A_F W_3^{-1} W_4 \end{bmatrix},$$

$$\bar{\Psi}_4 = \begin{bmatrix} F_{1\lambda}^T B_\lambda + \alpha_1 W_4^T B_F D_\lambda \\ W_4^T W_3^{-T} F_{2\lambda}^T B_\lambda + \alpha_2 W_4^T B_F D_\lambda \end{bmatrix},$$

$$\bar{\Psi}_5 = \begin{bmatrix} W_{1\lambda} & W_{2\lambda} W_3^{-1} W_4 \\ W_4^T W_3^{-T} W_4 & W_4^T W_3^{-T} W_4 \end{bmatrix},$$

$$\bar{\Psi}_6 = \begin{bmatrix} W_{1\lambda}^T B_\lambda + W_4^T B_F D_\lambda \\ W_4^T W_3^{-T} W_{2\lambda}^T B_\lambda + W_4^T B_F D_\lambda \end{bmatrix}. \quad (18)$$

Define

$$X_\lambda \triangleq F_{1\lambda}, R_\lambda \triangleq W_{1\lambda}, Y_\lambda \triangleq F_{2\lambda} W_3^{-1} W_4,$$
$$S_\lambda \triangleq W_{2\lambda} W_3^{-1} W_4, T \triangleq W_4^T W_3^{-1} W_4, \quad (19)$$

$$\begin{bmatrix} \bar{A}_F & \bar{B}_F \\ \bar{C}_F & \mathbf{0} \end{bmatrix} \triangleq \begin{bmatrix} W_4^T & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} A_F & B_F \\ C_F & \mathbf{0} \end{bmatrix} \begin{bmatrix} W_3^{-1} W_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \quad (20)$$

Substituting the above matrices into (16) and (17), we obtain

$$M_\lambda \triangleq \begin{bmatrix} -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -\bar{P}_{1\lambda} & -\bar{P}_{2\lambda} & L_\lambda^T \\ * & * & -\bar{P}_{3\lambda} & -\bar{C}_F^T \\ * & * & * & -\Pi_\lambda \end{bmatrix} < 0, \quad (21)$$

$$\Theta_\lambda \triangleq \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3^T - X_\lambda^T & \Delta_4 - \alpha_1 T & \Delta_5 \\ * & \Delta_9 & \bar{A}_F^T - Y_\lambda^T & \bar{A}_F^T - \alpha_2 T & \Delta_6 \\ * & * & \Delta_{10} & \bar{P}_{2\lambda} - S_\lambda - T & \Delta_7 \\ * & * & * & \bar{P}_{3\lambda} - \mathrm{sym}(T) & \Delta_8 \\ * & * & * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (22)$$

where

$$\Delta_1 \triangleq \mathrm{sym}\left(X_\lambda^T A_\lambda + \alpha_1 \bar{B}_F C_\lambda\right) - \bar{P}_{1\lambda},$$
$$\Delta_2 \triangleq A_\lambda^T Y_\lambda + \alpha_2 C_\lambda^T \bar{B}_F^T + \alpha_1 \bar{A}_F - \bar{P}_{2\lambda},$$
$$\Delta_3 \triangleq R_\lambda^T A_\lambda + \bar{B}_F C_\lambda, \; \Delta_4 \triangleq A_\lambda^T S_\lambda + C_\lambda^T \bar{B}_F^T,$$
$$\Delta_5 \triangleq X_\lambda^T B_\lambda + \alpha_1 \bar{B}_F D_\lambda, \; \Delta_6 \triangleq Y_\lambda^T B_\lambda + \alpha_2 \bar{B}_F D_\lambda$$
$$\Delta_7 \triangleq R_\lambda^T B_\lambda + \bar{B}_F D_\lambda, \; \Delta_8 \triangleq S_\lambda^T B_\lambda + \bar{B}_F D_\lambda,$$
$$\Delta_9 \triangleq \alpha_2 \,\mathrm{sym}\left(\bar{A}_F\right) - \bar{P}_{3\lambda}, \; \Delta_{10} \triangleq \bar{P}_{1\lambda} - \mathrm{sym}\left(R_\lambda\right).$$

Thus we have the following proposition.

Proposition 2. Given system $\mathscr{S}$ in (1), an admissible robust H_2 filter in the form of $\mathscr{F}$ in (4) exists if there exist matrices $\Pi_\lambda > 0$, $\bar{P}_\lambda = \begin{bmatrix} \bar{P}_{1\lambda} & \bar{P}_{2\lambda} \\ * & \bar{P}_{3\lambda} \end{bmatrix} > 0$, X_λ, Y_λ, R_λ, S_λ, T, $\bar{A}_F$, $\bar{B}_F$, $\bar{C}_F$ and scalars α_1, α_2 satisfying (11), (21) and (22). Moreover, under the above conditions, the matrices for an admissible robust H_2 filter in the form of (4) are given by

$$\begin{bmatrix} A_F & B_F \\ C_F & \mathbf{0} \end{bmatrix} = \begin{bmatrix} T^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \bar{A}_F & \bar{B}_F \\ \bar{C}_F & \mathbf{0} \end{bmatrix}. \tag{23}$$

Proof. (Proof of first part) Suppose there are matrices $\Pi_\lambda > 0$, $\bar{P}_\lambda > 0$, X_λ, Y_λ, R_λ, S_λ, T, $\bar{A}_F$, $\bar{B}_F$ and $\bar{C}_F$ satisfying (11), (21) and (22). Firstly, the $(4,4)$ block of (22) implies $-T-T^T<0$, which means that T is nonsingular. Thus, one can always find square and nonsingular matrices W_3 and W_4 satisfying $T = W_4^T W_3^{-1} W_4$. Now define the nonsingular matrix variable ϕ as in (15) and matrices

$$F_\lambda \triangleq \begin{bmatrix} X_\lambda & Y_\lambda W_4^{-1} W_3 \\ \alpha_1 W_4 & \alpha_2 W_3 \end{bmatrix}, \; W_\lambda \triangleq \begin{bmatrix} R_\lambda & S_\lambda W_4^{-1} W_3 \\ W_4 & W_3 \end{bmatrix},$$
$$P_\lambda \triangleq \phi^{-T} \bar{P}_\lambda \phi^{-1},$$
$$\begin{bmatrix} A_F & B_F \\ C_F & \mathbf{0} \end{bmatrix} \triangleq \begin{bmatrix} W_4^{-T} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \bar{A}_F & \bar{B}_F \\ \bar{C}_F & \mathbf{0} \end{bmatrix} \begin{bmatrix} W_4^{-1} W_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \tag{24}$$

Note that $P_\lambda > 0$. By some algebraic matrix manipulations, (21) and (22) are equivalent to

$$\begin{bmatrix} -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & -\phi^T P_\lambda \phi & \phi^T \bar{C}_\lambda^T \\ * & * & -\Pi_\lambda \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} \phi^T \Xi \phi & \phi^T \left(\bar{A}_\lambda^T W_\lambda - F_\lambda^T \right) \phi & \phi^T F_\lambda^T B_\lambda \\ * & \phi^T \left(P_\lambda - \mathrm{sym}\,(W_\lambda) \right) \phi & \phi^T W_\lambda^T B_\lambda \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \tag{26}$$

where

$$\Xi = \mathrm{sym}\left(F_\lambda^T \bar{A}_\lambda \right) - P_\lambda.$$

Performing congruence transformations to (25) by diag $\{\mathbf{I}, \phi^{-1}, \mathbf{I}\}$ and to (26) by diag $\{\phi^{-1}, \phi^{-1}, \mathbf{I}\}$ yields (10) and (12) respectively. In addition, (11) is also included in Proposition 2. Therefore, we conclude from Lemma 1 that the filter with a state-space realization (A_F, B_F, C_F) defined in (24) guarantees the filtering error system $\mathscr{E}$ in (5) to be asymptotically stable with an H_2 performance γ.

(Proof of second part) Now denote the transfer function of the filter (4) from $y(k)$ to $z_F(k)$ by $T_{z_F y}(z) = C_F (zI - A_F)^{-1} B_F$. By substituting the matrices with (24) and by considering the relationship $T = W_4^T W_3^{-1} W_4$, we have

$$\begin{aligned} T_{z_F y}(z) &= \bar{C}_F W_4^{-1} W_3 \left(z\mathbf{I} - W_4^{-T} \bar{A}_F W_4^{-1} W_3 \right)^{-1} W_4^{-T} \bar{B}_F \\ &= \bar{C}_F \left(z\mathbf{I} - T^{-1} \bar{A}_F \right)^{-1} T^{-1} \bar{B}_F. \end{aligned}$$

Therefore, an admissible filter can be given by (23), and the proof is completed. □

Remark 3. Proposition 2 tells us that not only $\bar{P}_\lambda > 0$ and $\Pi_\lambda > 0$ are allowed to be dependent on the uncertain parameter λ, but the general slack matrices X_λ, Y_λ, R_λ and S_λ are also allowed to be λ-dependent. This is different from existing results in this field, which require the slack matrices to be fixed for the entire uncertainty domain. It is worth noting that the conditions in Proposition 2 still cannot be implemented, as they are still dependent on the parameter λ, and thus are infinite-dimensional in nature. If we impose the following constraint:

$$X_\lambda \equiv X, \quad Y_\lambda \equiv Y, \quad R_\lambda \equiv R, \quad S_\lambda \equiv S,$$

and let the positive definite matrices takes the following structure (linearly dependent on the parameter λ):

$$\Pi_\lambda = \sum_{i=1}^{s} \lambda_i \Pi_i, \quad \bar{P}_\lambda = \sum_{i=1}^{s} \lambda_i \bar{P}_i,$$

then, by virtue of the inner property of convex combination, an LMI condition in terms of vertex matrices is readily obtain based on Proposition 2, which is similar to that obtained in Duan et al. [2006]. In this paper, to reduce conservatism, we assume the λ-dependent matrices in Proposition 2 (that is, $\Pi_\lambda, \bar{P}_\lambda, X_\lambda, Y_\lambda, R_\lambda, S_\lambda$) to be polynomially dependent on the parameter λ, which encompasses the linearly λ-dependence as a special case.

Now let the matrices Π_λ, $\bar{P}_\lambda$, X_λ, Y_λ, R_λ, S_λ take homogeneous forms of arbitrary degree g and depend polynomially on the uncertain parameters λ_i, $i = 1, \ldots, s$. That is,

$$\begin{aligned} \Pi_\lambda &= \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} \Pi_{k_j(g)}, \; \bar{P}_\lambda = \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} \bar{P}_{k_j(g)}, \\ X_\lambda &= \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} X_{k_j(g)}, \; R_\lambda = \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} R_{k_j(g)}, \\ Y_\lambda &= \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} Y_{k_j(g)}, \; S_\lambda = \sum_{j=1}^{J(g)} \lambda_1^{k_1} \lambda_2^{k_2} \cdots \lambda_s^{k_s} S_{k_j(g)}, \\ & [k_1 \; k_2, \cdots, k_s] = K_j(g). \end{aligned} \tag{27}$$

The notations in the above are explained as follows. Define $K(g)$ as the set of s-tuples obtained as all possible combination of $[k_1 \; k_2 \cdots k_s]$, with k_i being nonnegative integers, such that $k_1 + k_2 + \cdots + k_s = g$. $K_j(g)$ is the j-th s-tuple of $K(g)$ which is lexically ordered, $j = 1, \ldots, J(g)$. Since the number of vertices in the polytope $\mathscr{R}$ is equal to s, the number of elements in $K(g)$ is given by $J(g) = (s+g-1)!/(g!(s-1)!)$. These elements define the subscripts $k_1, k_2, \cdots, k_s$ of the constant matrices

$$\Pi_{k_1,k_2,\cdots,k_s} \triangleq \Pi_{k_j(g)}, \quad \bar{P}_{k_1,k_2,\cdots,k_s} \triangleq \bar{P}_{k_j(g)}, \quad Y_{k_1,k_2,\cdots,k_s} \triangleq Y_{k_j(g)},$$
$$X_{k_1,k_2,\cdots,k_s} \triangleq X_{k_j(g)}, \; R_{k_1,k_2,\cdots,k_s} \triangleq R_{k_j(g)}, \quad S_{k_1,k_2,\cdots,k_s} \triangleq S_{k_j(g)},$$

which are used to construct the homogeneous polynomial dependent matrices Π_λ, $\bar{P}_\lambda$, X_λ, Y_λ, R_λ, S_λ in (27).

Remark 4. Note that, when $g = 0$, we have $\Pi_\lambda = \Pi_0$, $\bar{P}_\lambda = \bar{P}_0$, $X_\lambda = X_0$, $R_\lambda = R_0$, $Y_\lambda = Y_0$, $S_\lambda = S_0$, which will lead to the standard filtering result in the quadratic framework. In addition, when $g = 1$, Π_λ, $\bar{P}_\lambda$, X_λ, R_λ, Y_λ, S_λ are linearly dependent on the parameter λ. This is why we say the polynomial λ-dependence encompasses the linear λ-dependence as a special case. It is also worth noting that since all coefficients λ_i, $i = 1, \ldots, s$, are such that $\lambda \in \Gamma$, a simple way to ensure $\Pi_\lambda > 0$ and $\bar{P}_\lambda > 0$ is to impose $\Pi_{k_j(g)} > 0$ and $\bar{P}_{K_j(g)} > 0$ for $j = 1, \cdots, J(g)$.

For each set $K(g)$, define also the set $I(g)$ with elements $I_j(g)$ given by subsets of i, $i \in \{1, 2, \ldots, s\}$, associated to the s-tuples $K_j(g)$ whose k_i's are nonzero. For each $i = 1, \ldots, s$, define the s-tuples $K_j^i(g)$ as being equal to $K_j(g)$ but with $k_i > 0$ replaced by $k_i - 1$. Note that the s-tuples $K_j^i(g)$ are defined only in the cases where the corresponding k_i is positive. Note also that, when applied to the elements of $K(g+1)$, the s-tuples $K_l^i(g+1)$ define subscripts $k_1, k_2, \cdots, k_s$ of matrices $\Pi_{k_1,k_2,\cdots,k_s}$, $\bar{P}_{k_1,k_2,\cdots,k_s}$, $X_{k_1,k_2,\cdots,k_s}$, $R_{k_1,k_2,\cdots,k_s}$, $Y_{k_1,k_2,\cdots,k_s}$, $S_{k_1,k_2,\cdots,k_s}$ associated to homogeneous polynomial parameter dependent matrices of degree g. Finally, define the scalar constant coefficients $\beta_j^i(g+1) = g!/(k_1!k_2!\ldots k_s!)$, with $[k_1, k_2, \ldots, k_s] \in K_j^i(g+1)$.

Then, we are in a position to give the main result of this paper.

Theorem 3. If there exist matrices $\bar{P}_{K_j(g)} = \begin{bmatrix} \bar{P}_{1K_j(g)} & \bar{P}_{2K_j(g)} \\ * & \bar{P}_{3K_j(g)} \end{bmatrix} > 0$, $\Pi_{K_j(g)} > 0$, $X_{K_j(g)}$, $R_{K_j(g)}$, $Y_{K_j(g)}$, $S_{K_j(g)}$, $K_j(g) \in K(g)$, $j = 1,\ldots,J(g+1)$, and scalars α_1, α_2 such that the following inequalities hold for all $K_l(g+1) \in K(g+1)$, $l = 1,\ldots,J(g+1)$:

$$\sum_{i\in I_l(g+1)} \left(\mathrm{tr}\,\Pi_{K_l^i(g+1)} - \beta_l^i(g+1)\gamma\right) < 0, \tag{28}$$

$$\sum_{i\in I_l(g+1)} \begin{bmatrix} -\beta_l^i(g+1)\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & -\bar{P}_{K_l^i(g+1)} & \beta_l^i(g+1)\bar{C}_\lambda^T \\ * & * & -\Pi_{K_l^i(g+1)} \end{bmatrix} < 0, \tag{29}$$

$$\sum_{i\in I_l(g+1)} \begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_4^T & \Lambda_7 & \Lambda_{11} \\ * & \Lambda_3 & \Lambda_5 & \Lambda_8 & \Lambda_{12} \\ * & * & \Lambda_6 & \Lambda_9 & \Lambda_{13} \\ * & * & * & \Lambda_{10} & \Lambda_{14} \\ * & * & * & * & -\beta_l^i(g+1)\mathbf{I} \end{bmatrix} < 0, \tag{30}$$

where

$$\Lambda_1 = \mathrm{sym}\left(X^T_{K_l^i(g+1)}A_i + \beta_l^i(g+1)\alpha_1\bar{B}_F C_i\right) - \bar{P}_{1K_l^i(g+1)},$$
$$\Lambda_2 = A_i^T Y_{K_l^i(g+1)} + \beta_l^i(g+1)\left(\alpha_2 C_i^T\bar{B}_F^T + \alpha_1\bar{A}_F\right) - \bar{P}_{2K_l^i(g+1)},$$
$$\Lambda_3 = \beta_l^i(g+1)\alpha_2\,\mathrm{sym}\left(\bar{A}_F\right) - \bar{P}_{3K_l^i(g+1)},$$
$$\Lambda_4 = R^T_{K_l^i(g+1)}A_i + \beta_l^i(g+1)\bar{B}_F C_i - X_{K_l^i(g+1)},$$
$$\Lambda_5 = \beta_l^i(g+1)\bar{A}_F^T - Y^T_{K_l^i(g+1)},$$
$$\Lambda_6 = \bar{P}_{1K_l^i(g+1)} - \mathrm{sym}\left(R_{K_l^i(g+1)}\right),$$
$$\Lambda_7 = A_i^T S_{K_l^i(g+1)} + \beta_l^i(g+1)C_i^T\bar{B}_F^T - \alpha_1\beta_l^i(g+1)T,$$
$$\Lambda_8 = \beta_l^i(g+1)\left(\bar{A}_F^T - \alpha_2 T\right),$$
$$\Lambda_9 = \bar{P}_{2K_l^i(g+1)} - S_{K_l^i(g+1)} - \beta_l^i(g+1)T,$$
$$\Lambda_{10} = \bar{P}_{3K_l^i(g+1)} - \beta_l^i(g+1)\,\mathrm{sym}\,(T),$$
$$\Lambda_{11} = X^T_{K_l^i(g+1)}B_i + \beta_l^i(g+1)\alpha_1\bar{B}_F D_i,$$
$$\Lambda_{12} = Y^T_{K_l^i(g+1)}B_i + \beta_l^i(g+1)\alpha_2\bar{B}_F D_i,$$
$$\Lambda_{13} = R^T_{K_l^i(g+1)}B_i + \beta_l^i(g+1)\bar{B}_F D_i,$$
$$\Lambda_{14} = S^T_{K_l^i(g+1)}B_i + \beta_l^i(g+1)\bar{B}_F D_i.$$

Then, the homogeneous polynomial matrices given by (27) assure (11), (21) and (22) for all $\lambda \in \Gamma$.

Moreover, if (28), (29), (30) are fulfilled for a given degree $\hat{g}$, then the inequalities corresponding to any degree $g > \hat{g}$ are also satisfied.

Proof. (Proof of first part) Since $\bar{P}_{1K_j(g)} > 0$, $\bar{P}_{3K_j(g)} > 0$, $\Pi_{K_j(g)} > 0$, $K_{j(g)} \in K(g)$, $j = 1,\ldots,J(g)$, we know that $\bar{P}_{1g\lambda}$, $\bar{P}_{3g\lambda}$ and $\Pi_{g\lambda}$ defined in (27) are all positive definite for all $\lambda \in \Gamma$. Now, note that Ξ_λ in (11), M_λ in (21), and Θ_λ in (22) for $(A_\lambda, B_\lambda, C_\lambda, D_\lambda, L_\lambda, H_\lambda) \in \mathscr{R}$ and $\bar{P}_{1\lambda}$, $\bar{P}_{2\lambda}$, $\bar{P}_{3\lambda}$, Π_λ, R_λ, S_λ given by (27) are homogeneous polynomial matrix equations of degree $g+1$ that can be written as

$$\Xi_\lambda = \sum_{l=1}^{J(g+1)} \lambda_1^{k_1}\lambda_2^{k_2}\cdots\lambda_s^{k_s}\left\{\sum_{i\in I_l(g+1)} (\mathrm{tr}\,\Pi_{K_l^i(g+1)} - \beta_l^i(g+1)\gamma)\right\}$$

$$M_\lambda = \sum_{l=1}^{J(g+1)} \lambda_1^{k_1}\lambda_2^{k_2}\cdots\lambda_s^{k_s}\left\{\sum_{i\in I_l(g+1)} \begin{bmatrix} -\beta_l^i(g+1)\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & -\bar{P}_{K_l^i(g+1)} & \beta_l^i(g+1)\bar{C}_\lambda^T \\ * & * & -\Pi_{K_l^i(g+1)} \end{bmatrix}\right\},$$

$$\Theta_\lambda = \sum_{l=1}^{J(g+1)} \lambda_1^{k_1}\lambda_2^{k_2}\cdots\lambda_s^{k_s}\left\{\sum_{i\in I_l(g+1)} \begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_4^T & \Lambda_7 & \Lambda_{11} \\ * & \Lambda_3 & \Lambda_5 & \Lambda_8 & \Lambda_{12} \\ * & * & \Lambda_6 & \Lambda_9 & \Lambda_{13} \\ * & * & * & \Lambda_{10} & \Lambda_{14} \\ * & * & * & * & -\beta_l^i(g+1)\mathbf{I} \end{bmatrix}\right\},$$

$$[k_1\ k_2, \cdots, k_s] = K_l(g+1).$$

Conditions (28)–(30) imposed for all l, $l = 1,\cdots,J(g+1)$ assure that $\Xi_\lambda < 0$, $M_\lambda < 0$, $\Theta_\lambda < 0$ for all $\lambda \in \Gamma$, and thus the first part is proved.

(Proof of second part) Suppose that (28)–(30) are fulfilled for a certain degree $\hat{g}$, that is, there exist $J(\hat{g})$ symmetric positive definite matrices $\bar{P}_{K_j(\hat{g})}$, $\Pi_{K_j(\hat{g})}$ and matrices $X_{K_j(\hat{g})}$, $R_{K_j(\hat{g})}$, $Y_{K_j(\hat{g})}$, $S_{K_j(\hat{g})}$, $j = 1,\cdots,J(\hat{g})$ such that $\bar{P}_\lambda$, Π_λ, X_λ, R_λ, Y_λ, S_λ defined in (27) are homogeneous polynomial parameter-dependent Lyapunov matrices assuring $\Xi_\lambda < 0$, $M_\lambda < 0$, $\Theta_\lambda < 0$. Then, the terms of the polynomial matrices $\tilde{P}_\lambda = (\lambda_1 + \cdots + \lambda_s)\bar{P}_\lambda$, $\tilde{\Pi}_\lambda = (\lambda_1 + \cdots + \lambda_s)\Pi_\lambda$, $\tilde{X}_\lambda = (\lambda_1 + \cdots + \lambda_s)X_\lambda$, $\tilde{R}_\lambda = (\lambda_1 + \cdots + \lambda_s)R_\lambda$, $\tilde{Y}_\lambda = (\lambda_1 + \cdots + \lambda_s)Y_\lambda$, $\tilde{S}_\lambda = (\lambda_1 + \cdots + \lambda_s)S_\lambda$ also satisfy the inequalities of Theorem 3 corresponding to the degree $\hat{g}+1$, which can be obtained in this case by linear combination of the inequalities of Theorem 3 for $\hat{g}$. □

Remark 5. When the tuning parameters α_1 and α_2 are given, the inequalities in Theorem 3 are LMIs. The matrices composing the homogeneous polynomial parameter-dependent matrix functions $P_\lambda, \Pi_\lambda, X_\lambda, R_\lambda, Y_\lambda$, and S_λ as well as the LMIs of (28), (29), (30) can be generated from sets $K(g)$ and $I(g)$, which can be constructed from simple routines using, for instance, a recursive code. As the degree g of the polynomial increases, the conditions become less conservative since new free variables are added to the LMIs. Although the number of LMIs is also increased, each LMI becomes easier to be fulfilled due to the extra degrees of freedom provided by the new free variables and smaller values of H_2 guaranteed costs can be obtained.

Remark 6. When the tuning parameters α_1 and α_2 are given, the minimum γ (in terms of the feasibility of (28), (29) and (30)) can be readily found by solving the following convex optimization problem:

$$\min\ \gamma$$
$$(28),\ (29),\ \text{and}\ (30)$$

over $\bar{P}_{K_j(g)} > 0, \Pi_{K_j(g)} > 0, X_{K_j(g)}, R_{K_j(g)}, Y_{K_j(g)}, S_{K_j(g)}, K_j(g) \in K(g)$, $j = 1,\ldots,J(g+1)$.

4. ILLUSTRATIVE EXAMPLE

In this section, we use a numerical example to show the less conservatism of the result developed above.

Example. Consider a discrete-time system given by

$$A = \begin{bmatrix} 0.9 & 0.1+0.06a \\ 0.01+0.05b & 0.9 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$C = [\,1\ \ 0\,],\ D = [\,0\ \ 0\ \ 1.414\,],\ L = [\,1\ \ 1\,],$$

with $|a| \leqslant 1$ and $|b| \leqslant 1$, which can be represented as a four-vertex polytopic system. This system has been considered in (Duan et al. [2006], Geromel et al. [2002], Xie et al. [2004]).

For this system, the value of minimum guaranteed H_2 cost γ^* is 44.0039 by the method in Geromel et al. [2002], 19.4682 by the method in Xie et al. [2004], 16.11 with fixed $\lambda_1 = 0$, $\lambda_2 = 0$ and 13.46 with searched $\lambda_1 = -0.89$, $\lambda_2 = -0.921$ by the method in Duan et al. [2006]. Table 1 shows the minimum guaranteed H_2 costs we obtain when using Theorem 3 for $g = 1,2$ and the associated computation time. It is clearly shown in Table 1 that the guaranteed costs obtained by our approach are much smaller than those obtained by existing results, which indicate the less conservatism of the filtering result developed above. From Table 1, we can also see that the larger the value of g, the smaller the value of γ^*.

g	$[\alpha_1, \alpha_2]$	γ^*	Evaluation Time (s)
1	[0, 0]	12.01	7.28
1	[-0.89, -0.921]	11.09	14.91
2	[0, 0]	11.52	36.20
2	[-0.89, -0.921]	10.82	80.94

Table 1. Calculation results by Theorem 3

5. CONCLUSIONS

This paper has presented a novel approach, namely structured polynomial parameter-dependent approach, to designing robust H_2 filters for linear discrete-time systems with parameter uncertainty residing in a polytope. Different from the quadratic framework and the linearly parameter-dependent framework, this paper makes the first attempt to utilize a polynomial parameter-dependent idea to solve the robust H_2 filtering problem. This idea is realized by carefully selecting the structure of the matrix involved in the products with system matrices, with easily verifiable LMI conditions obtained for the existence of desired filters. A numerical example has shown that the filter design approach presented in this paper is much less conservative than the existing robust filter design methods. The idea behind this paper may be further extended to continuous-time case and more complex systems, such as time-delay systems and two-dimensional systems.

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