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Passive Decomposition of Multiple Nonholonomic Mechanical Systems under Motion Coordination Requirements ⋆

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Abstract: We propose a novel framework, which, under a certain geometric condition, enables us to decompose the second-order Lagrangian dynamics of the multiple nonholonomic mechanical systems into two decoupled systems according to the two most fundamental aspects of the group behaviour: shape system describing the formation aspect (i.e. group's internal shape); and locked system abstracting the maneuver aspect (i.e. group's overall motion). By controlling these decoupled locked and shape systems individually, we can then control the formation and maneuver aspects separately without any crosstalk between them. Moreover, the framework enables us to do this while respecting/utilizing the Lagrangian structure/passivity of the system's open-loop dynamics. Due to this property, we call this framework nonholonomic passive decomposition. A control design example with numerical simulation is also given to highlight some properties of the proposed framework.

Keywords: nonholonomic systems, Lagrangian dynamics, passivity, decomposition, geometry

1. INTRODUCTION

Let us consider multiple wheeled mobile robots advancing together to a target location while keeping a tight formation or a team of multiple mobile manipulators cooperatively carrying a commonly grasped object without any object-specific holding-fixture. Then, we can think of the two fundamental aspects from the group behaviour of these multiple robots: 1) *formation* aspect representing the group's internal shape (e.g. formation/grasping shape); and 2) *maneuver* aspect describing the group's overall/average motion (e.g. centroid motion of formation/grasped object). These two aspects are, in fact, universally applicable whenever we deal with multi-robot/multiagent systems.

In many applications as those mentioned above, the formation-maneuver decoupling (i.e. no crosstalk between these two aspects) and the capability to control these aspects individually and separately (yet still simultaneously) are desirable and often even imperative. For instance, in the above cooperative *fixture-less* grasping scenario, without such formation-maneuver decoupling, as we speed-up/slow-down the group's maneuver to drive the grasped object, this maneuver dynamics will then perturb the formation aspect (i.e. grasping shape), thus, may result in the (possibly dangerous) dropping of the grasped object. On the other hand, as we change the formation shape of the robots, the overall team may start drifting away due to the (energy) coupling from the formation to maneuver.

This problem - how to achieve the formation-maneuver decoupling, and, thereby, control the formation and maneuver simultaneously, separately, and precisely - has been largely remained as an open problem for the (multiple) nonholonomic mechanical systems with second-order Lagrangian dynamics, mainly due to the lack of tools to fully analyze the combined effects of the Lagrangian dynamics and the nonholonomic constraints on the formation and maneuver aspects. To our best knowledge, only the meaningful work along this line is [12], which, however, considers only the first-order drift-free kinematic nonholonomic systems, thus, can not handle with the dynamics-related effects (e.g. external force, inertial coupling). Note that, without fully considering these dynamics-related effects, we would not be able to realize many practically important applications demanding the tight formation (e.g. fixture-less cooperative grasping). This neglecting (or assuming the perfect cancellation of) the second-order Lagrangian dynamics has been, in fact, a dominant trend even for the control of a single nonholonomic mechanical system (e.g. [1, 4]). See [2, 5] for some of very few exceptions for this.

In this paper, by extending the standard passive decomposition [10, 7, 6, Lee and Li], we propose a novel framework, which, under a certain geometric condition, enables us to decompose the second-order Lagrangian dynamics of the multiple nonholonomic mechanical systems into: 1) *locked system* describing the maneuver aspect; 2) *shape system* representing the formation aspect; and 3) inertia-induced (energetically conservative) coupling between them. Then, by canceling out this coupling, we can decouple the locked and shape systems from each other (i.e. formation-maneuver decoupling is achieved). Moreover, by controlling these *decoupled* locked and shape systems individually, we can then control the formation and maneuver aspects separately without any crosstalk

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between them. These decoupled locked and shape systems, similar to their counterparts of the standard passive decomposition, individually inherit the Lagrangian-like structure/passivity from their open-loop nonholonomic mechanical systems. Thus, many powerful control techniques utilizing such passivity/Lagrangian-structure (e.g. passivity-based control) would be applicable for each of them, although control design to attain certain objectives may be quite complicated (or even impossible) here because of the nonholonomic constraints. Due to this decomposing capability and passivity preservation for the nonholonomic mechanical systems, we call this new framework *nonholonomic passive decomposition*, which may be thought of as an extension of the standard passive decomposition (i.e. formation-maneuver decomposition for the second-order unconstrained Lagrangian systems) and the work in [12] (i.e. formation-maneuver decomposition for the first-order kinematic nonholonomic systems).

The rest of this paper is organized as follows. Some preliminary materials, including the dynamics of multiple nonholonomic mechanical systems and their related geometric entities, will be discussed in Sec. 2. The standard passive decomposition will be briefly reviewed in Sec. 3 along with its shortcomings for the nonholonomic systems. The main result - nonholonomic passive decomposition - will be presented and detailed in Sec. 4, and a control design example with its numerical simulation will be given in Sec. 5. Summary and some concluding remarks on future research will be made in Sec. 6.

2. PRELIMINARY

2.1 Multiple Nonholonomic Mechanical Systems

Let us start with the dynamics of a single nonholonomic mechanical system, which consists of 1) the nonholonomic Pfaffian constraints equation:

$$A(q)\dot{q} = 0 \tag{1}$$

and 2) the Lagrange-D'Alembert equation of motion:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + A^T(q)\lambda = \tau + f \tag{2}$$

where $q, \dot{q}, \tau, f \in \Re^n$ are the configuration, velocity, control, and external force, $M, C \in \Re^{n\times n}$ are the inertia and Coriolis matrices s.t. $\dot{M} - 2C$ is skew-symmetric, $A(q) \in \Re^{p\times n}$ $(p \le n)$ defines the nonholonomic constraints,and $A^T(q)\lambda$ is the constraint force, whose magnitude is specified by the Lagrange multiplier $\lambda \in \Re^p$. We assume that these nonholonomic constraints are smooth and regular (i.e. rank A is constant). This mathematical modeling is also equally applicable to the multiple nonholonomic mechanical systems, since, by combining their individual dynamics into their (product) configuration space $\mathcal{M} \approx \Re^n$ (i.e., redefining $q := (q_1, q_2, ..., q_N)$ with q_i being i-th robot's configuration), we can obtain their group dynamics exactly in the same form as in (1)-(2) [10, 7].

Using the constraints (1) and the inertia metric $M(q)$, we can then generate four spaces at each q: 1) *constrained codistribution* $\mathcal{C}^\perp$, which is the row space of $A(q)$ determining the space of the constraint forces; 2) *unconstrained distribution* $\mathcal{D}^\top$, which is the kernel of $A(q)$ specifying the direction of $\dot{q}$ permitted by the constraints (1); 3) *constrained distribution* $\mathcal{D}^\perp$, which is the orthogonal complement of $\mathcal{D}^\top$ w.r.t. the $M(q)$-metric; and 4) *unconstrained codistribution* $\mathcal{C}^\top$, which annihilates $\mathcal{D}^\perp$. Note that $\mathcal{C}^\perp$ also annihilates $\mathcal{D}^\top$. Here, the first two are purely-kinematic (i.e. only dependent on the constraints (1)), thus, easy to compute, while the last two are inertia-dependent.

Then, at each q, the tangent space (i.e. velocity space: $T_q\mathcal{M}$) and the cotangent space (i.e. force space: $T_q^*\mathcal{M}$) respectively split s.t.

$$T_q\mathcal{M} = \mathcal{D}^\top \oplus \mathcal{D}^\perp \quad \text{and} \quad T_q^*\mathcal{M} = \mathcal{C}^\top \oplus \mathcal{C}^\perp \tag{3}$$

where $\oplus$ is the direct sum, and the velocity $\dot{q}$ and the control τ can be written as

$$\dot{q} = \underbrace{[\,\mathcal{D}_\top \;\; \mathcal{D}_\perp\,]}_{=:\mathcal{D}(q)} \begin{pmatrix} \nu \\ \xi \end{pmatrix}, \quad \tau = \underbrace{[\,\mathcal{C}_\top^T \;\; \mathcal{C}_\perp^T\,]}_{=:\mathcal{C}^T(q)} \begin{pmatrix} u \\ u_\xi \end{pmatrix} \tag{4}$$

where $\mathcal{D}_\top \in \Re^{n\times(n-p)}$, $\mathcal{D}_\perp \in \Re^{n\times p}$, $\mathcal{C}_\top \in \Re^{(n-p)\times n}$ and $\mathcal{C}_\perp \in \Re^{p\times n}$ are the matrices identifying their respective spaces. Similar also hold for f with its split coordinates being δ, δ_ξ. Since $\mathcal{D}_\perp$ describes the direction of velocity violating the constraints (1), $\xi = 0$. Also, note that the control/force in $\mathcal{C}_\top$ direction (i.e. u, δ) is fully effective not being hindered by the constraints, while those in $\mathcal{C}_\perp$ (i.e. u_ξ, δ_ξ) are completely absorbed by the constraint forces. In this paper, we assume that we can assign u arbitrarily, that is, we have full control in $\mathcal{C}^\top$ and the control under-actuation is only due to the nonholonomic constraints.

Here, from our construction, $\mathcal{C}_\perp\mathcal{D}_\top = 0$, $\mathcal{C}_\top\mathcal{D}_\perp = 0$. Also, to have the following mechanical power preservation s.t.

$$\text{power}(t) := (\tau + f)^T\dot{q} = (u + \delta)^T\nu + (u_\xi + \delta_\xi)^T\xi \tag{5}$$

we enforce $\mathcal{C}_\top\mathcal{D}_\top = I$ and $\mathcal{C}_\perp\mathcal{D}_\perp = I$. This can be achieved by simply setting $\mathcal{C} = \mathcal{D}^{-1}$, since, from $\mathcal{D}^{-1}\mathcal{D} = I$, the top $(n-p)\times n$ and the bottom $p\times n$ blocks of $\mathcal{D}^{-1}$ still identify $\mathcal{C}^\top$ and $\mathcal{C}^\perp$ respectively. Note also that, since $\xi = 0$, the last term in (5) is actually zero.

Then, using (4) with $\mathcal{D}_\top^T M\mathcal{D}_\perp = 0$ (since $\mathcal{D}^\top$ and $\mathcal{D}^\perp$ are orthogonal w.r.t. $M(q)$-metric) and $\xi = 0$, we can rewrite the dynamics (2) s.t.

$$D_\nu(q)\dot{\nu} + Q_\nu(q,\dot{q})\nu = u + \delta \tag{6}$$

$$Q_{\xi\nu}(q,\dot{q})\nu + (\mathcal{D}_\perp)^T A^T(q)\lambda = u_\xi + \delta_\xi \tag{7}$$

where $D_\nu = \mathcal{D}_\top M\mathcal{D}_\top \in \Re^{(n-p)\times(n-p)}$, and

$$\begin{bmatrix} Q_\nu & Q_{\nu\xi} \\ Q_{\xi\nu} & Q_\xi \end{bmatrix} := \mathcal{D}^T \left[M\dot{\mathcal{D}} + C\mathcal{D} \right].$$

Here, (6) is the dynamics (2) projected on $\mathcal{D}^\top$, thus no constraint force shows up there. Also, (7) is the projection on $\mathcal{D}^\perp$ where the term with $Q_{\xi\nu}$ is so called the second fundamental form [3], which quantifies the tendency of the system to deviate from $\mathcal{D}^\top$. No acceleration terms shows up in (7), since $\mathcal{D}^\top$ and $\mathcal{D}^\perp$ are orthogonal w.r.t. the $M(q)$-metric. Thus, from (7), we can directly compute the Lagrangian multiplier as a function of $q, \dot{q}, u_\xi, \delta_\xi$.

Here, since $\xi = 0$, we have

$$\kappa(t) := \tfrac{1}{2}\dot{q}^T M(q)\dot{q} = \tfrac{1}{2}\nu^T D_\nu(q)\nu.$$

It is also not so difficult to see that

$$\dot{D}_\nu - 2Q_\nu = \mathcal{D}_\top^T[\dot{M} - 2C]\mathcal{D}_\top + \dot{D}_\top^T M\mathcal{D}_\top - D_\top^T M\dot{\mathcal{D}}_\top$$

which is skew-symmetric. Combining these, we can show that both the original dynamics (1)-(2) and its projection (6)-(7) possess *(energetic) passivity property* [8]: $\forall T \ge 0$,

$$\int_0^T (\tau + f)^T\dot{q}dt = \int_0^T (u + \delta)^T\nu dt = \kappa(T) - \kappa(0). \tag{8}$$

2.2 Formation and Maneuver

For a group of multiple systems, we can think of two aspects from their group behaviour: 1) *formation* aspect - group's internal shape; and 2) *maneuver* aspect - group's overall motion. For instance, consider three wheeled mobile robots with (p_i, θ_i) as their (x, y)-position and orientation $(i = 1, 2, 3)$. Then, their (x, y)-formation shape (i.e. $(p_1 - p_2, p_2 - p_3) \in \Re^4$) and misalignment (i.e. $(\theta_1 - \theta_2, \theta_2 - \theta_3) \in \Re^2$) may represent the formation aspect, while their centroid motion (i.e. $(p_1 + p_2 + p_3)/3 \in \Re^2$) and bulk orientation (i.e. $(\theta_1+\theta_2+\theta_3)/3 \in \Re$) the maneuver aspect.

In this paper, we suppose that this formation aspect can be represented by the mapped point of a smooth function

$$h : \Re^n \to \Re^m, \quad m \leq n \tag{9}$$

which we also assume to be a submersion (i.e. its Jacobian is full-rank). Then, the level set of h, defined s.t.

$$\mathcal{H}_c := \{q \in \Re^n \mid h(q) = c, \ c \in \Re^m \ \} \tag{10}$$

is a $(n-m)$-dim. (smooth) submanifold and the collection of them forms a foliation with each submanifold being its leaf [11]. We call this map h *formation map* and its range space (identified by $\Re^m$ here) *formation manifold* $\mathcal{N}$. See Fig. 1 for an illustration, where 1) the formation aspect (i.e. shape system) is represented by the mapped point in $\mathcal{N}$; and 2) the maneuver aspect (i.e. locked system) by the trajectory moving parallel on the level set $\mathcal{H}_{h(q)}$.

In many applications, we want to decouple these formation and maneuver aspects from each other. For instance, suppose that the above three wheeled mobile robots carry a commonly grasped object by keeping a certain grasping shape (i.e. formation) among them without any holding-fixture. Then, without such a formation-maneuver decoupling, if we speed-up (or slow-down) the collectives of the robots to drive the object (i.e. maneuver control), this maneuver change can perturb the formation aspect (i.e. grasping), thus, possibly incur the dropping of the object! The standard passive decomposition[10, 7, 6, Lee and Li] enables us to achieve this formation-maneuver decoupling and, thereby, control each of them independently - unfortunately, only for the *unconstrained* mechanical systems. In this paper, we will extend this standard passive decomposition to the nonholonomic Lagrangian mechanical systems and name it *nonholonomic passive decomposition* (Sec. 4). To do this, let us first briefly summarize the standard passive decomposition and reveal some of its shortcomings for the nonholonomic mechanical systems.

3. STANDARD PASSIVE DECOMPOSITION

Consider Fig. 1. Then, at each q, for the velocity $\dot{q}$ to be parallel w.r.t. the level set $\mathcal{H}_{h(q)}$, it needs to satisfy

$$\mathcal{L}_{\dot{q}} h = \frac{\partial h}{\partial q}\dot{q} = 0$$

where $\mathcal{L}_{\dot{q}} h$ is the Lie derivative of h along $\dot{q}$. In other words, the kernel of $\partial h/\partial q \in \Re^{m\times n}$ defines the distribution (i.e. subspace of velocity) parallel to the level set. Then, similar to Sec. 2.1, using the $M(q)$-metric, we can define the following four vector spaces: 1) *normal codistribution* $\Omega^\perp$ is the row space of $\partial h/\partial q$ representing the force directions normal to the level set $\mathcal{H}_{h(q)}$; 2) *parallel distribution* $\Delta^\top$ is the kernel of $\Omega^\perp$, thus, parallel to $\mathcal{H}_{h(q)}$ and constitutes the velocity space of the maneuver aspect; 3) *normal distribution* $\Delta^\perp$ is the orthogonal complement of $\Delta_\top$ w.r.t. the $M(q)$-metric, whose image via h on $\mathcal{N}$ describes the formation aspect's evolution; and 4) *parallel codistribution* $\Omega^\top$ annihilates $\Delta^\perp$ and encodes the force directions affecting only the maneuver aspect along $\mathcal{H}_{h(q)}$. Again, the former two are purely-kinematic (i.e. dependent only on h), while the latter two are inertia-dependent.

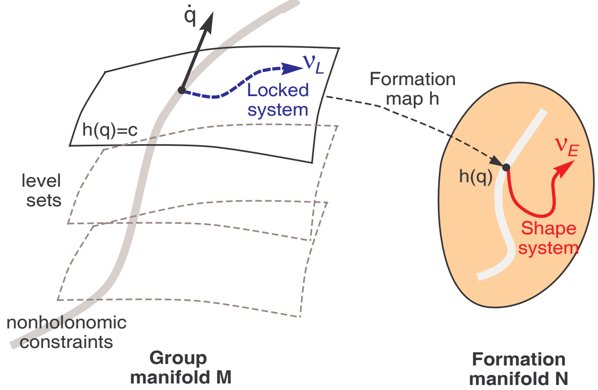

Fig. 1. Geometry of formation map and level sets.

Then, similar to (3)-(4), we have, at each q,

$$T_q\mathcal{M} = \Delta^\top \oplus \Delta^\perp, \quad T_q^*\mathcal{M} = \Omega^\top \oplus \Omega^\perp \tag{11}$$

and we can write $\dot{q}$ and τ by (similar also hold for f)

$$\dot{q} = \underbrace{\left[\, \Delta_\top \ \ \Delta_\perp \,\right]}_{=:\Delta(q)} \begin{pmatrix} v_L \\ v_E \end{pmatrix}, \quad \tau = \underbrace{\left[\, \Omega_\top^T \ \ \Omega_\perp^T \,\right]}_{=:\Omega^T(q)} \begin{pmatrix} \tau_L \\ \tau_E \end{pmatrix} \tag{12}$$

where the matrices $\Delta_\top \in \Re^{n\times(n-m)}$, $\Delta_\perp \in \Re^{n\times m}$, $\Omega_\top \in \Re^{(n-m)\times n}$ and $\Omega_\perp \in \Re^{m\times n}$ identify their respective spaces. Similar to (4), we enforce $\Omega\Delta = I$. In particular, we set $\Omega_\perp = \partial h/\partial q$ with rescaling/permutating Δ s.t.

$$\Delta = [\, \Delta_\top\alpha \ \ \Delta_\perp\beta \,] \tag{13}$$

where $\alpha = (\Omega_\top\Delta_\top)^{-1}$ and $\beta = (\frac{\partial h}{\partial q}\Delta_\perp)^{-1}$. By doing so, we can not only ensure $\Omega\Delta = I$ but also $v_E = dh/dt$ so that v_E is explicitly related to the formation aspect $h(q)$. Note that, simply setting $\Omega = \Delta^{-1}$ here as done in Sec. 2.1 does not generally guarantee $v_E = dh/dt$.

Then, using $\Delta_\top^T M \Delta_\perp = 0$, we can decompose the original dynamics (2) into: with argument omitted for brevity,

$$M_L\dot{v}_L + C_L v_L + C_{LE} v_E + \Delta_\top^T A^T\lambda = \tau_L + f_L \tag{14}$$

$$M_E\dot{v}_E + C_E v_E + C_{EL} v_L + \Delta_\perp^T A^T\lambda = \tau_E + f_E \tag{15}$$

where $M_L = \Delta_\top^T M \Delta_\top$, $M_E = \Delta_\perp^T M \Delta_\perp$, and

$$\begin{bmatrix} C_L & C_{LE} \\ C_{EL} & C_E \end{bmatrix} := \Delta^T[M\dot{\Delta} + C\Delta]. \tag{16}$$

Note that the first dynamics (14) is the projection of the original dynamics (2) onto $\Delta^\top$ (i.e. maneuver aspect), while the second (15) onto $\Delta^\perp$ (i.e. formation aspect). We call the dynamics of v_E in (15) *shape system*, since it describes the group's internal shape (i.e. formation aspect) with the explicit relation $v_E = dh/dt$. On the other hand, the dynamics of v_L in (14) we call *locked system*, since it describes the group's overall motion (i.e. maneuver aspect), especially when the formation $h(q)$ is locked so that the system's motion is confined within a single level set. Here, due to the orthogonality of $\Delta^\top$ and $\Delta^\perp$ w.r.t. the $M(q)$-metric, there is no coupling between the locked and shape systems via the acceleration channel.

Proposition 1. [9, 8] Consider the decomposed dynamics (14)-(15). Then,

(1) M_L and M_E are symmetric and positive-definite.
(2) $\dot{M}_L - 2C_L$ and $\dot{M}_E - 2C_E$ are skew-symmetric.
(3) $C_{LE} = -C_{EL}^T$.
(4) Kinetic energy and power are decomposed s.t.

$$\kappa(t) = \kappa_L(t) + \kappa_E(t)$$
$$\tau^T \dot{q} = \tau_L^T v_L + \tau_E^T v_E$$

where $\kappa_L = v_L^T M_L v_L/2$ and $\kappa_E = v_E^T M_E v_E/2$.

Proof. Probably, the only not-obvious items here would be items 2 and 3, which can be proved similarly to Sec. 2.1 by observing that the following is skew-symmetric:

$$\begin{bmatrix} \dot{M}_L - 2C_L & -2C_{LE} \\ -2C_{EL} & \dot{M}_E - 2C_E \end{bmatrix} = \frac{d[\Delta^T M \Delta]}{dt} - 2\Delta^T[M\dot{\Delta} + C\Delta]$$
$$= \Delta^T[\dot{M} - 2C]\Delta + \dot{\Delta}^T M \Delta - \Delta^T M \dot{\Delta}.$$

Thus, if there are no constraints (i.e. $A^T = 0$), 1) we can achieve the formation-maneuver decoupling by simply canceling out the coupling terms $C_{LE}v_E, C_{EL}v_L$; 2) we can control the (decoupled) locked and shape systems individually and separately without any crosstalk between them; and 3) we can utilize the Lagrangian-like structure/passivity of the locked and shape systems in designing controllers for them (e.g. passivity-based control).

Unfortunately, a direct application of this standard passive decomposition to the nonholonomic systems seems not so promising here, particularly as shown by the presence of $\Delta_\top^T A^T\lambda, \Delta_\perp^T A^T\lambda$ in (14)-(15). In addition to possibly make the control design/analysis much more complicated, these constraints terms may impose a fundamental restriction on the formation-maneuver decoupling. This is because, they may create *uncancelable* energy-coupling between the locked and shape systems. To better see this, observe the following: from (14)-(15) with Prop. 1,

$$\frac{d\kappa_L}{dt} = -v_L^T C_{LE} v_E - v_L^T \Delta_\top^T A^T \lambda + (\tau_L + f_L)^T v_L$$
$$\frac{d\kappa_E}{dt} = -v_E^T C_{EL} v_L - v_E^T \Delta_\perp^T A^T \lambda + (\tau_E + f_E)^T v_E \quad (17)$$

where, from the item 3 of Prop. 1, (1) and (12), we have

$$v_L^T C_{LE} v_E + v_E^T C_{EL} v_L = v_L^T[C_{LE} + C_{EL}^T]v_E = 0 \quad (18)$$
$$v_L^T \Delta_\top^T A^T \lambda + v_E^T \Delta_\perp^T A^T \lambda = \lambda^T A \dot{q} = 0.$$

This shows that both the Coriolis coupling terms (i.e. via C_{LE}, C_{EL}) and the constraints coupling terms (i.e. via $\Delta_\top^T A^T\lambda, \Delta_\perp^T A^T\lambda$) define (conservative) locked-shape energy coupling. However, although the former is cancelable (i.e. design $(\tau_L, \tau_E) = (C_{LE}v_E, C_{EL}v_E)$, convert to τ by (12), and project on $\mathcal{C}^\top$ by (4)), the latter is not. As long as there is such uncancelable locked-shape energy coupling, there will be no hope for us to achieve the formation-maneuver decoupling. Here, note that $\Delta_\top^T A^T\lambda, \Delta_\perp^T A^T\lambda$ are in general not individually zero (see Remark 2).

In the next section, we will show that, for the nonholonomic mechanical systems under a certain geometric condition, by extending this standard passive decomposition,

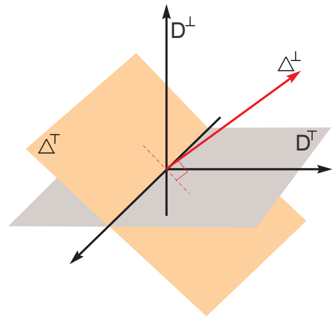

Fig. 2. Example of $\mathcal{D}^\top \neq (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp)$.

we can still achieve the locked-shape decoupling without any such uncancelable energy-coupling between them. We will call this new decomposing procedure *nonholonomic passive decomposition.*

Remark 2. In general, $v_L^T \Delta_\top^T A^T\lambda$ and $v_E^T \Delta_\perp^T A^T\lambda$ are not individually zero, although their sum is so as shown above. For instance, for a wheeled mobile robot with $h(x, y, \theta) = x$, $v_L^T \Delta_\top^T A^T\lambda = -\lambda v \,\mathrm{c}\,\theta \,\mathrm{s}\,\theta$ and $v_E^T \Delta_\perp^T A^T\lambda = \lambda v \,\mathrm{s}\,\theta \,\mathrm{c}\,\theta$, where $\lambda = (f_x \,\mathrm{s}\,\theta - f_y \,\mathrm{c}\,\theta)$, $f := (f_x, f_y)$ is the (x, y)-external force, and v is the forward-velocity of the robot.

4. NONHOLONOMIC PASSIVE DECOMPOSITION

Let us introduce the following *decomposibility condition*:

$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp) \quad (19)$$

for every q, where we assume $\mathcal{D}^\top \cap \Delta^\top \neq \emptyset$ and $\mathcal{D}^\top \cap \Delta^\perp \neq \emptyset$. Once we have this condition, as we will see below, we can still decouple the dynamics of the nonholonomic mechanical system (evolving in $\mathcal{D}^\top$) into those of $\mathcal{D}^\top \cap \Delta^\top$ (i.e. maneuver aspect) and $\mathcal{D}^\top \cap \Delta^\perp$ (i.e. formation aspect), thus, can achieve the formation-maneuver decoupling. This decomposibility condition (19) also implies the split of the dual-space $\mathcal{C}^\top$ s.t.: for all q,

$$\mathcal{C}^\top = (\mathcal{C}^\top \cap \Omega^\top) \oplus (\mathcal{C}^\top \cap \Omega^\perp)$$

since, 1) due to (19), we can split the basis of $\mathcal{D}^\top$ into $V_1 := \{e_1, ..., e_r\} \approx \mathcal{D}^\top \cap \Delta^\top$ and $V_2 := \{e_{r+1}, ..., e_{n-p}\} \approx \mathcal{D}^\top \cap \Delta^\perp$ with $\langle\langle e_i, e_j \rangle\rangle = e_i^T M(q) e_j/2 = \delta_{ij}$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise; and 2) the basis of $C^\top$, then, can also be split into $W_1 = \{d_1, ..., d_r\} \approx \mathcal{C}^\top \cap \Omega^\top$ and $W_2 = \{d_{r+1}, ..., d_{n-p}\} \approx \mathcal{C}^\top \cap \Omega^\perp$ s.t. $\langle d_i, e_j \rangle = d_i e_j = \delta_{ij}$. Here, by $A \approx B$, we mean A identifies B.

Note that this decomposibility condition (19) is not always granted (e.g. counter-example in Remark 2), although it is very tempting to believe so from (3) and (11). This is because some of the directions of $\Delta^\top$ or $\Delta^\perp$ can be cut off by the $\cap$-operation (with $\mathcal{D}^\top$), thus, with those directions missing, $\mathcal{D}^\top \cap \Delta^\top$ and $\mathcal{D}^\top \cap \Delta^\perp$ may not span the whole $\mathcal{D}^\top$-space. See Fig. 2 for an illustration. See also Remark 4 for a sufficient condition for this decomposibility condition.

Then, using the fact that $\dot{q} \in \mathcal{D}^\top$ and $\tau \in \mathcal{C}^\top$, we can write $\dot{q}$ and τ s.t.

$$\dot{q} = \underbrace{\left[\, \mathcal{D}_\top \cap \Delta_\top \;\; \mathcal{D}_\top \cap \Delta_\perp \,\right]}_{=:\mathcal{V}(q)} \begin{pmatrix} \nu_L \\ \nu_E \end{pmatrix}$$

$$\tau = \underbrace{\left[\, (\mathcal{C}_\top \cap \Omega_\top)^T \;\; (\mathcal{C}_\top \cap \Omega_\perp)^T \,\right]}_{=:\mathcal{W}^T(q)} \begin{pmatrix} u_L \\ u_E \end{pmatrix} \qquad (20)$$

where, similar to (12), each block of $\mathcal{V}(q), \mathcal{W}(q)$ identifies its corresponding vector spaces. To preserve the mechanical power (i.e. $\tau^T \dot{q} = u_L^T \nu_L + u_E^T \nu_E$), here, we also enforce $\mathcal{W}(q)\mathcal{V}(q) = I$. This can be achieved by scaling/permutating $\mathcal{V}(q)$ as done in (13).

By applying (20), we can then decompose the original nonholonomic Lagrangian dynamics (1)-(2) into:

$$D_L(q)\dot{\nu}_L + Q_L(q,\dot{q})\nu_L + Q_{LE}(q,\dot{q})\nu_E = u_L + \delta_L \qquad (21)$$

$$D_E(q)\dot{\nu}_E + Q_E(q,\dot{q})\nu_E + Q_{EL}(q,\dot{q})\nu_L = u_E + \delta_E \qquad (22)$$

where, similar to (14)-(15), diag$[D_L, D_E] := \mathcal{V}^T M \mathcal{V}$ and

$$\begin{bmatrix} Q_L & Q_{LE} \\ Q_{EL} & Q_E \end{bmatrix} := \mathcal{V}^T \left[M\dot{\mathcal{V}} + C\mathcal{V} \right]. \qquad (23)$$

Here, we call the dynamics of ν_L in (21) *unconstrained* locked system, since it is the original locked system dynamics of v_L in (14) projected to the unconstrained $\mathcal{D}^\top$. Similarly, we call the dynamics of ν_E in (22) *unconstrained* shape system. Now, we present our main result.

Theorem 3. Consider the nonholonomic mechanical system (1)-(2) with the formation map h (9). Then, if the decomposibility condition holds (19), we can decompose the system dynamics (1)-(2) into (21)-(22), where

(1) D_L and D_E are symmetric and positive-definite.
(2) $\dot{D}_L - 2Q_L$ and $\dot{D}_E - 2Q_E$ are skew-symmetric.
(3) $Q_{LE} = -Q_{EL}^T$.
(4) Kinetic energy and power are decomposed s.t.

$$\kappa(t) = \tfrac{1}{2}\nu_L^T D_L \nu_L + \tfrac{1}{2}\nu_E^T D_E \nu_E$$
$$\tau^T \dot{q} = u_L^T \nu_L + u_E^T \nu_E$$

and, furthermore, $\kappa_L = \nu_L^T D_L \nu_L/2$, $\kappa_E = \nu_E^T D_E \nu_E/2$ and $u_L^T \nu_L = \tau_L^T v_L$, $u_E^T \nu_E = \tau_E^T v_E$, where $\kappa, \kappa_L, \kappa_E$ are the kinetic energies of the total system, original locked, and shape systems, respectively.

Furthermore, by cancelling out the coupling terms $Q_{LE}\nu_L$ and $Q_{EL}\nu_E$ in (21)-(22), we can achieve the formation-maneuver decoupling.

Proof. Here, we only prove (parts of) items (2)-(4), since the rests are either easy to prove or straightforward to deduce from the given proof. First, observe that, from (23),

$$\begin{bmatrix} \dot{D}_L - 2Q_L & -2Q_{LE} \\ -2Q_{EL} & \dot{D}_E - 2Q_E \end{bmatrix} = \frac{d[\mathcal{V}^T M \mathcal{V}]}{dt} - 2\mathcal{V}^T[M\dot{\mathcal{V}} + C\mathcal{V}]$$
$$= \mathcal{V}^T[\dot{M} - 2C]\mathcal{V} + \dot{\mathcal{V}}^T M \mathcal{V} - \mathcal{V}^T M \dot{\mathcal{V}}$$

which is skew-symmetric. This proves items (2)-(3). Also, by equating (12) and (20),

$$\Delta_\top v_L = (\mathcal{D}_\top \cap \Delta_\top)\nu_L, \quad \Delta_\perp v_E = (\mathcal{D}_\top \cap \Delta_\perp)\nu_E$$

$$\Omega_\top^T \tau_L = (\mathcal{C}_\top \cap \Omega_\top)^T u_L, \quad \Omega_\perp^T \tau_E = (\mathcal{C}_\top \cap \Omega_\perp)^T u_E \qquad (24)$$

thus, we have

$$\nu_L^T (\mathcal{D}_\top \cap \Delta_\top)^T M (\mathcal{D}_\top \cap \Delta_\top)\nu_L = v_L^T \Delta_\top^T M \Delta_\top v_L$$

which proves $\nu_L^T D_L \nu_L/2 = \kappa_L$, since the left term is $\nu_L^T D_L \nu_L$, while the right term $2\kappa_L$. Using (24) with $\Omega\Delta = I$ and $\mathcal{W}\mathcal{V} = I$, we can also prove $\tau_L^T v_L = u_L^T \nu_L$, since

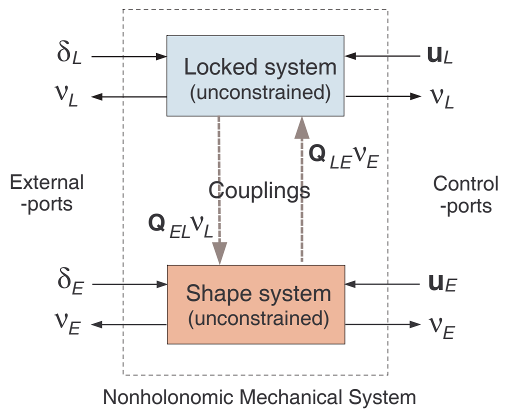

Fig. 3. Energetics of nonholonomic passive decomposition

$\tau_L^T v_L = \tau_L^T \Omega_\top \Delta_\top v_L = u_L^T (\mathcal{C}_\top \cap \Omega_\top)(\mathcal{D}_\top \cap \Delta_\top)\nu_L = u_L^T \nu_L$. Similar also holds for the unconstrained shape system.

Therefore, with the decomposibility condition (19), by cancelling out the coupling terms $Q_{LE}\nu_E, Q_{LE}\nu_E$, we can still decouple the formation and maneuver aspects from each other for the nonholonomic mechanical systems. Here, note that this decoupling control 1) requires only the measurement of (usually available) $q, \dot{q}$; and 2) cancels out the *conservative* energy coupling, that is, similar to (18), $\nu_L^T Q_{LE}\nu_E + \nu_E^T Q_{EL}\nu_L = 0$ from item (3) of Th. 3.

At this point, the following question may arise: what happened to those uncancelable constraints energy-coupling in (17)? It turns out that the decomposibility condition (19) prevents such uncancelable energy coupling via the constraints. This can be shown by: from (24) with $A \in \mathcal{C}^\perp$,

$$v_L^T \Delta_\top^T A^T = \nu_L^T (\mathcal{D}_\top \cap \Delta_\top)^T A^T = 0$$
$$v_E^T \Delta_\perp^T A^T = \nu_E^T (\mathcal{D}_\top \cap \Delta_\perp)^T A^T = 0. \qquad (25)$$

This implies that $v_L^T \Delta_\top^T A^T \lambda = 0$ and $v_E^T \Delta_\perp^T A^T \lambda = 0$ individually in (17), thus, there will be no uncancelable constraints energy-coupling. This manifests the importance of the decomposibility condition (19) for the formation-maneuver decoupling.

By using (21)-(22) with Th. 3, we can show that

$$\frac{d\kappa_L}{dt} = -\nu_L^T Q_{LE}\nu_E + (u_L + \delta_L)^T \nu_L$$
$$\frac{d\kappa_E}{dt} = -\nu_E^T Q_{EL}\nu_L + (u_E + \delta_E)^T \nu_E \qquad (26)$$

which reveals the energetic structure of (21)-(22) (or (1)-(2)) as given in Fig. 3. Note also that, here, if the couplings in (21)-(22) are canceled out (so that the terms with Q_{LE}, Q_{EL} in (26) disappear), the decoupled unconstrained locked and shape systems will individually possess passivity similar to (8). This individual passivity of the locked and shape systems would be useful in designing/analyzing controls for them.

Control design for u_L, u_E is further facilitated by the fact that $\tau_L^T v_L = u_L^T \nu_L$ and $\tau_E^T \dot{q}_E = u_E^T \nu_E$. That is, if we design the (raw) controls τ_L, τ_E (in $T_q^*\mathcal{M}$) *as if* there are no constraints, and then, project them into the (generatable) controls u_L, u_E (in $\mathcal{C}^\top$), by applying these u_L, u_E, we will still get some effects of the intended controls τ_L, τ_E, since these projected controls u_L, u_E will still generate the intended "control-power" (i.e. $\tau_L^T v_L, \tau_E^T \dot{q}_E$) in (17). Even so, due to the nonholonomic constraints, the control(-vectors) u_L, u_E here will in general produce only partial effects of the intended controls τ_L, τ_E as shown by

$$u_L = S_L(q)\tau_L, \quad u_E = S_E(q)\tau_E \tag{27}$$

where, from (24), $S_L := (\mathcal{D}_\top \cap \Delta_\top)^T(\Omega_\top)^T$ and $S_E := (\mathcal{D}_\top \cap \Delta_\perp)^T(\Omega_\perp)^T$ are "fat" matrices, showing the elimination of the intended control actions in $\mathcal{C}^\perp$.

In the next section, using these ideas, we will design a simple control for the maneuver driving with formation keeping. Before doing so, let us conclude this section by a few remarks.

Remark 4. The decomposibility condition (19) is ensured if $\Delta^\perp \in \mathcal{D}^\top$ or $\Delta^\top \in \mathcal{D}^\top$. To see this, suppose that $\Delta^\perp \in \mathcal{D}^\top$. Then, due to the tangent space split (11), the remaining space $\mathcal{D}^\top - \Delta^\perp$ is necessarily contained in $\Delta^\top$. Moreover, in this case, the original shape system (15) will become constraint-free (with $\Delta_\perp^T A^T = 0$), thus, we can control the formation aspect (i.e. $h(q)$) without being hindered by the constraints. Similar argument (e.g. constraint-free maneuver control) also holds if $\Delta^\top \in \mathcal{D}^\top$.

Remark 5. We would still be able to achieve the formation-maneuver decoupling/decomposition with the following (weaker) version of the decomposibility condition (19):

$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp) \oplus (\mathcal{D}^\top \cap \Delta^c)$$

where Δ^c is the extra-directions needed to span $\mathcal{D}^\top$. Any motion in $(\mathcal{D}^\top \cap \Delta^c)$ will, then, change the formation and maneuver aspects simultaneously, thus, break down the formation-maneuver decoupling. To overcome this, we may supplement $\mathcal{V}(q)$ (20) with $\mathcal{D}^\top \cap \Delta^c$. Then, similar to (21)-(22), we would again have the three decomposed systems (let's say D_L, D_E, D_c) projected on $\mathcal{D}^\top$, thus, by stabilizing D_c with the cancellation of the couplings among D_L, D_E, D_c, we would still be able to decouple the locked and shape systems (D_L, D_E). More details on this weaker decomposibility will be reported in future publications.

Remark 6. Note that the results presented here (e.g. formation-maneuver decoupling under the decomposibility condition (19)) are easily extended to the first-order kinematic nonholonomic systems. For instance, the motion feasibility condition in [12] is equivalent to $\mathcal{D}^\top \cap \Delta^\top \neq \emptyset$ here. This first-order kinematic model of the nonholonomic systems, however, does not allows us to address the (inertia-induced) formation-maneuver coupling (i.e. $Q_{LE}\nu_E, Q_{EL}\nu_L$ in (21)-(22)) and the external forces, both of which are of paramount importance in many applications (e.g. fixture-less cooperative grasping).

5. CONTROL DESIGN EXAMPLE: MANEUVER DRIVING WITH FORMATION KEEPING

Suppose that we want to drive the maneuver s.t. $\nu_L(t) \to \nu_L^d(t)$ (e.g. drive the centroid of the grasped object), while keeping the formation s.t. $h(q) = h_d$ (e.g. rigidly maintaining the fixture-less cooperative grasping shape). To achieve this objective, using the control design ideas given in Sec. 4, we design the controls as follows:

$$u_L = Q_{LE}\nu_E + D_L\dot{\nu}_L^d + Q_L\nu_L^d - B_L(\nu_L - \nu_L^d) \tag{28}$$

$$u_E = Q_{EL}\nu_L - B_E\nu_E - S_E[K_E(h(q) - h_d)] - \delta_E \tag{29}$$

where B_L, B_E, K_E are suitably-defined gain matrices. Here, note that the spring term in (29) is designed for τ_E of (15) *as if* there are no constraints, and then projected into u_E via (27).

Then, using (21) with its passivity property, we can easily show that, if $\delta_L = 0$, $\nu_L(t) \to \nu_L^d(t)$. Furthermore, using the passivity property of (22) with $\tau_E^T v_E = u_E^T \nu_E$ and $v_E = dh/dt$ (see Sec. 3), we have:

$$\frac{d(\kappa_E + \varphi_E)}{dt} = -\nu_E^T B_E \nu_E$$

where $\varphi_E := (h - h_d)^T K_E (h - h_d)/2$ is the spring potential. Thus, if we start with $\nu_E(0) = 0$, the formation error (as measured by φ_E) will be always less than or equal to the initial error $\varphi_E(0)$ due to the positive-definite damping dissipation $\nu_E^T B_E \nu_E$. Note also that, if we start with $h(0) = h_d$ and $\nu_E(0) = 0$, the formation control (29), even without its damping/spring terms, can still maintain $h(t) = h_d$ $\forall t \geq 0$ due to the formation-maneuver decoupling. Here, we do not include the δ_L-cancellation in (28), since, in some applications, it is desirable to perceive such external forces (e.g. teleoperation).

We apply these controls (28)-(29) to three wheeled mobile robots (with different masses and not-coinciding geometric/inertial centers). We choose $h(q) = [p_1 - p_2 - p_{12}^d; p_2 - p_3 - p_{23}^d; \theta_1 - \theta_2; \theta_2 - \theta_3] \in \Re^6$ with $h_d = 0$, where p_{12}^d, p_{23}^d are the position-offsets to make a triangle formation. We start with $h(0) = h_d$ and $\nu_E(0) = 0$. Then, the controls (28)-(29) will ensure $h(t) = h_d$ $\forall t \geq 0$ (refer the simulation results below to see this is indeed true). Thus, we assume $\theta_1 = \theta_2 = \theta_3$ in deriving the decomposition. Then, the system satisfies the decomposibility condition (19) (yet, $\Delta^\top \notin \mathcal{D}^\top$ and $\Delta^\perp \notin \mathcal{D}^\top$), and $\nu_L \in \Re^2$ describes the angular-rate/forward-velocity of the triangle formation. Simulation results are presented in Figs. 4 and 5, each of which consists of: 1) snapshots of robots' triangle formation (i.e. vertex corresponding to robot) and trajectories of robots/object; and 2) plots of formation error $||h(t)||$, maneuver error $||\nu_L - \nu_L^d||$ and the triangle's angular-rate θ_1 (with $\theta_1 = \theta_2 = \theta_3$). In Fig. 5, the three robots cooperatively carry a commonly grasped inertial/flexible object, while, in Fig. 4, no object is used.

First, note that, due to the formation-maneuver decoupling, desired formation can be maintained (i.e. $||h(t)|| = 0$) even with the ν_L^d-switchings. Moreover, with the cancellation of δ_E in (29), we can perfectly reject the disturbance on the formation from the object's inertial force (i.e. $||h(t) = 0||$ $\forall t \geq 0$). Without this δ_E-cancellation and the formation-maneuver decoupling, formation was perturbed and grasping was lost (not shown here). Note also that the object's inertia affects the triangle's motion (e.g. $||\nu_L - \nu_L^d||$ has non-zero offset in Fig. 5). This implies that, in a ν_L-teleoperation mode, humans will perceive this inertial effect (and other external forces, too).

6. SUMMARY AND FUTURE WORKS

We propose a novel nonholonomic passive decomposition, that enables us to decouple the formation and maneuver aspects of the multiple nonholonomic mechanical systems with the second-order Lagrangian dynamics and, thereby, control these two aspects individually and separately without any crosstalk between them. This is done while utilizing the Lagrangian-structure/passivity-property of the

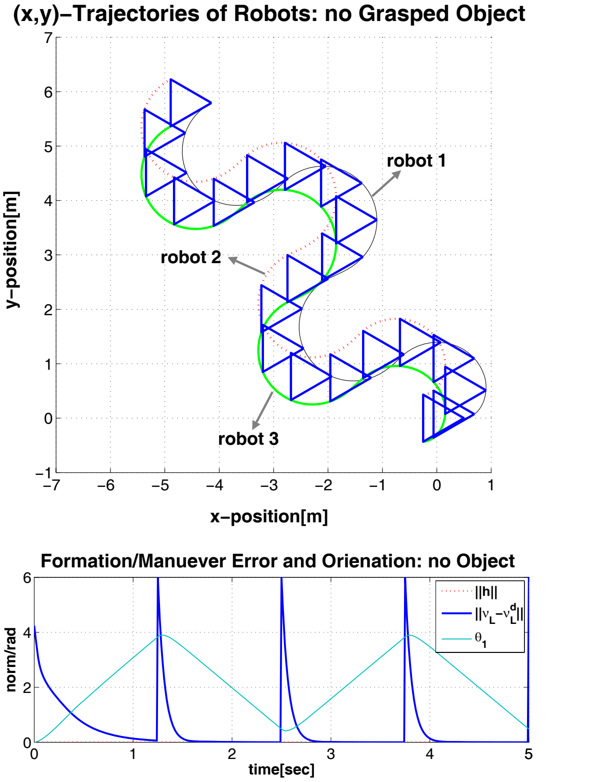

Fig. 4. Driving and formation-keeping of three wheeled mobile robots

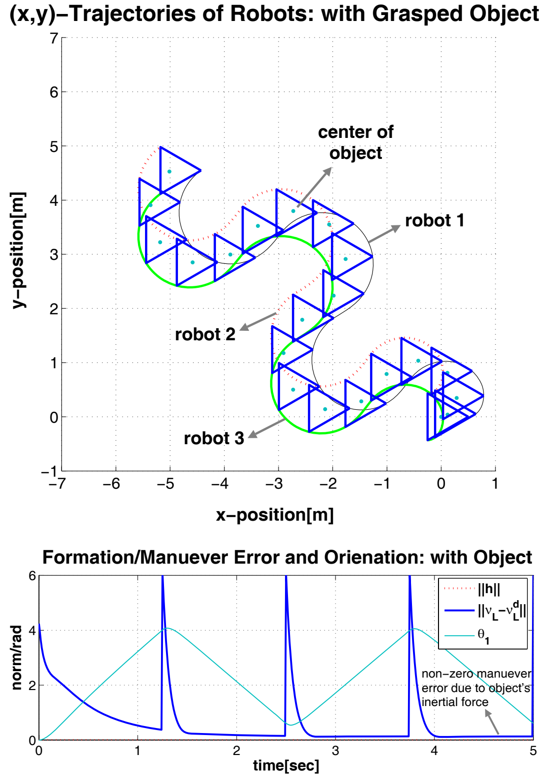

Fig. 5. Driving and formation-keeping of three wheeled mobile robots with grasped inertial/flexible object

nonholonomic mechanical system's open-loop dynamics. Some directions we will pursue for future works are as follows: 1) how to design controls for the unconstrained locked/shape systems while considering the nonholonomic constraints; 2) analyzing the geometric structure of the inherited ambient nonholonomic constraints in the unconstrained locked and shape systems; and 3) robustification and real-time numerical procedure of the decomposition for real-world complex systems (e.g. team of many mobile manipulators).

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