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Robust Fuzzy Guaranteed Cost Controller Design via Piecewise Lyapunov Function Approach

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Abstract: This paper considers the problem of robust guaranteed cost controller design for a class of nonlinear systems subject to time-varying and norm-bounded uncertainties in both state and input matrices. The Takagi-Sugeno (T-S) fuzzy model is employed to approximate the uncertain nonlinear system. Then, two different design procedures of optimal robust guaranteed cost controller are developed by using parallel distributed compensation (PDC) scheme and piecewise Lyapunov function (PLF) approach, respectively. And it is shown that all solvability conditions for the above problem can be converted into a standard linear matrix inequality (LMI) problem. The final numerical example is given to illustrate the effectiveness of the design procedures. In addition, the results obtained by PLF method are relatively less conservative.

1. INTRODUCTION

During the last two decades, fuzzy technique has been widely and successfully used in nonlinear system modelling and control. Among various fuzzy modelling methods, the well-known Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno, 1985) is recognized as a popular and powerful tool in approximating a complex nonlinear system. As a result, the study of T-S fuzzy system has attracted ever-increasing interest of numerous investigators and various techniques have been developed for stability analysis and controller design of T-S fuzzy system (e.g. Tanaka and Sugeno, 1992; Wang *et al.*, 1996; Kim and Lee, 2000; Park *et al.*, 2001; Ding *et al.*, 2006, and references therein). In their researches, a nonlinear system is represented by a family of local linear models smoothly connected through nonlinear fuzzy membership functions. Then, a model-based fuzzy controller stabilized the T-S fuzzy model is presented in terms of linear matrix inequalities (LMIs) (Boyd *et al.*, 1994), which can be easily solved by using existing tools such as the LMI Toolbox in Matlab (Gahinet *et al.*, 1995).

Recently, the T-S fuzzy model approach has been extended to deal with the uncertain nonlinear systems. Since parameter uncertainties are frequently one of the causes of system instability and system performance degradation, different robust stability and stabilization methodologies have been proposed (e.g. Chen *et al.*, 1999; Teixeira and Zak, 1999; Lee *et al.*, 2001; Tong and Li, 2002; Lee *et al.*, 2005). However, in many practical applications, it is desirable to design a controller which not only stabilizes the system, but also achieves satisfactory performance. One solution to this problem is the so-called guaranteed cost control method (Chang and Peng, 1972). This method aims at stabilizing the system while providing an upper bound on a given quadratic performance index. Based on this idea, many significant results have been obtained (Shi *et al.*, 2003; Wu and Cai, 2004; Chen and Liu, 2005; Boukas, 2006; Chen *et al.*, 2007).

In above investigations, the stability condition of fuzzy systems is obtained from Lyapunov's direct method and the controller design is based on the parallel distributed compensation (PDC) scheme (Wang *et al.*, 1995). This method is required to find a common symmetric positive definite matrix P satisfied the Lyapunov equation for all local linear models. In many cases, it is difficult to find such a matrix, especially when the number of fuzzy rules is very large. Moreover, in a certain sense, this method may lead to a conservative result. Zhang *et al.* (2001) presented a method of stability analysis and systematic design for fuzzy control systems via the piecewise Lyapunov function (PLF) approach (Wicks *et al.*, 1994). In this approach, the single positive definite matrix will be replaced by a set of positive definite matrices. The local controller is designed for each local model separately. Then the global fuzzy controller can be composed of a set of local controllers with corresponding energy functions, and the stability of the global fuzzy system is ensured via PLF. So far, to the best of our knowledge, the robust guaranteed cost controller designed via PLF approach for uncertain fuzzy systems has not been fully investigated yet.

In this paper, attention is focused on the comparison of optimal robust guaranteed cost controller designed by PDC scheme and PLF approach, respectively. The system under consideration is approximately expressed by a set of T-S fuzzy models in the presence of norm-bounded and time-varying uncertainties. The gains of robust guaranteed cost controller are derived by the numerical solutions of a set of coupled LMIs. The last simulation results show that the robust guaranteed cost controller designed by these two methods can both stabilize the closed-loop uncertain fuzzy system and make the upper bound on the value of the given quadratic performance as small as possible. Further, the PLF approach is proved to provide a relatively less conservative result.

Throughout this paper, the following notions will be used: $\Re^n$ denotes n-dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n\times m$ real matrices. I represents identity matrix of appropriate dimension. $*$ stands for the transposed elements in the symmetric positions of a symmetric matrix, the superscript "T" denotes the transpose for vectors or matrices. Functional $Trace(\cdot)$ represents the trace of a square matrix, and $E\{\cdot\}$ denotes the mathematical expectation operator.

2. PROBLEM FORMULATION

Consider a class of uncertain nonlinear systems described by the following T-S fuzzy model with parameter uncertainties:

Plant Rule i:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip}

$$\text{THEN } \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t), \quad x(0) = x_0, \; i = 1, 2, \cdots, r. \tag{1}$$

where $x(t)\in\Re^n$ is the state vector, x_0 is the initial vector, $u(t)\in\Re^m$ is the control input vector. $z_1(t), z_2(t), \cdots, z_p(t)$ are the premise variables, which are the functions of state variables. M_{ij} $(j = 1, 2, \cdots, p)$ are fuzzy sets, r is the number of IF-THEN rules. A_i and B_i are real constant matrices with appropriate dimensions, $\Delta A_i(t)$ and $\Delta B_i(t)$ are uncertain matrices representing time-varying parameter uncertainties in the system model and are assumed to be norm bounded of the following form:

$$[\Delta A_i(t) \;\; \Delta B_i(t)] = D_i F(t)[E_{ai} \;\; E_{bi}], \; i = 1, 2, \cdots, r. \tag{2}$$

where D_i, E_{ai} and E_{bi} are real constant matrices with appropriate dimensions, which represent the structure of uncertainties. And $F(t)\in\Re^{\alpha\times\beta}$ is an unknown time-varying matrix function with Lebesgue measurable elements and satisfies

$$F^{\mathrm{T}}(t)F(t) \le I. \tag{3}$$

Using weighted average method for defuzzification, the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t)]. \tag{4}$$

where $z(t) = [z_1(t), z_2(t), \cdots, z_p(t)]$ and

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r}\mu_i(z(t))}, \; \mu_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)), \; i = 1, 2, \cdots, r.$$

in which $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in the fuzzy set M_{ij}.

In general, it is assumed that $\mu_i(z(t)) \ge 0$, $i = 1, 2, \cdots, r$ and $\sum_{i=1}^{r}\mu_i(z(t)) > 0$. Therefore, $h_i(z(t)) \ge 0$, $i = 1, 2, \cdots, r$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$.

For brevity, the functions $h_i(z(t))$ will be replaced by h_i in the subsequence.

For the uncertain fuzzy system (4), the quadratic performance cost function is considered as follows:

$$J = \int_0^{\infty}[x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)]dt. \tag{5}$$

where Q and R are given symmetric positive definite matrices.

Associated with the cost function (5), the definition of robust guaranteed cost control law is given by

Definition 1: Consider the uncertain fuzzy system (4) and the cost function (5), if there exist a control law $u^*(t)$ and a positive scalar J^* such that, for all admissible uncertainties, the closed-loop fuzzy system is robustly stable and the closed-loop value of the cost function (5) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a robust guaranteed cost control law.

The objective of this paper is to develop a procedure to design a state feedback robust guaranteed cost controller via PDC scheme and PLF approach, respectively. Meanwhile, a guaranteed cost J^* is found as small as possible such that the closed-loop uncertain fuzzy system is robustly stable and the closed-loop value of the cost function (5) satisfies $J \le J^*$ for all admissible uncertainties.

Before moving on, a Lemma is first introduced which is useful in the proof of the following sections.

Lemma 1: (Xie 1996) Given matrices Y, D, E of appropriate dimensions and with Y symmetric, then the inequality $Y + DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} < 0$ holds for all F satisfying $F^{\mathrm{T}}F \le I$ if and only if there exists a scalar $\varepsilon > 0$ such that the equality $Y + \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0$ holds.

3. ROBUST GUARANTEED COST CONTROLLER DESIGN

3.1 PDC Scheme

In this section, the concept of PDC is introduced to construct robust guaranteed cost controller. Suppose that the system state is available for feedback, then the rule for controller is illustrated as follows:

Controller Rule i:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip}

$$\text{THEN } u(t) = K_i x(t), \quad i = 1, 2, \cdots, r. \tag{6}$$

where $K_i \in \Re^{m\times n}$ are state feedback gain matrices to be determined. The overall fuzzy controller is represented by

$$u(t) = \sum_{i=1}^{r} h_i K_i x(t). \tag{7}$$

Thus, the closed-loop fuzzy system can be obtained by substituting (2) and (7) into (4).

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Lambda_{ij} x(t), \; x(0) = x_0. \tag{8}$$

where $\Lambda_{ij} = A_i + B_i K_j + D_i F(t)(E_{ai} + E_{bi} K_j)$.

In the following, a sufficient condition for the existence of state feedback robust guaranteed cost controller of the uncertain fuzzy system (4) is presented.

Theorem 1: Consider the uncertain fuzzy system (4) and the cost function (5), if there exist a symmetric positive definite matrix P and matrices K_i , $i = 1, 2, \cdots, r$, such that for all admissible uncertainties $F(t)$ satisfying equation (3), the following matrix inequalities hold.

$$Q + K_i^{\mathrm{T}} R K_i + P\Lambda_{ii} + \Lambda_{ii}^{\mathrm{T}} P < 0 \ , \ i = 1, 2, \cdots, r \ , \tag{9}$$

$$2Q + K_i^{\mathrm{T}} R K_i + K_j^{\mathrm{T}} R K_j + P(\Lambda_{ij} + \Lambda_{ji}) + (\Lambda_{ij} + \Lambda_{ji})^{\mathrm{T}} P < 0 \ , \quad i < j, \ i, j = 1, 2, \cdots, r \ . \tag{10}$$

Then, the state feedback control law (7) is a robust guaranteed cost control law and the corresponding cost upper bound satisfies

$$J \le J^* = x_0^{\mathrm{T}} P x_0 \ . \tag{11}$$

Proof: First, the Lyapunov function candidate is chosen as

$$V(x(t)) = x^{\mathrm{T}}(t) P x(t) \ .$$

where P is a symmetric positive definite matrix. Obviously, it follows from P that $V(x(t)) > 0$.

If the conditions (9) and (10) hold, then the time derivative of $V(x(t))$ along the trajectory of the closed-loop fuzzy system (8) is derived by

$$\begin{aligned}
\dot{V}(x(t)) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^{\mathrm{T}}(t)(P\Lambda_{ij} + \Lambda_{ij}^{\mathrm{T}} P)x(t) \\
&= \sum_{i=1}^{r} h_i^2 x^{\mathrm{T}}(t)(P\Lambda_{ii} + \Lambda_{ii}^{\mathrm{T}} P)x(t) \\
&\quad + \sum_{i=1}^{r}\sum_{i<j}^{r} h_i h_j x^{\mathrm{T}}(t)[P(\Lambda_{ij} + \Lambda_{ji}) + ((\Lambda_{ij} + \Lambda_{ji})^{\mathrm{T}} P]x(t) \\
&< -\sum_{i=1}^{r} h_i^2 x^{\mathrm{T}}(t)(Q + K_i^{\mathrm{T}} R K_i)x(t) \\
&\quad - \sum_{i=1}^{r}\sum_{i<j}^{r} h_i h_j x^{\mathrm{T}}(t)(2Q + K_i^{\mathrm{T}} R K_i + K_j^{\mathrm{T}} R K_j)x(t) \\
&= -\sum_{i=1}^{r} h_i x^{\mathrm{T}}(t)(Q + K_i^{\mathrm{T}} R K_i)x(t) < 0 .
\end{aligned}$$

Therefore, the closed-loop fuzzy system (8) is robustly stable. Furthermore, according to the system stability, the cost upper bound (11) can be achieved by integrating both sides of the above inequality from 0 to ∞.

$$J < V(x(0)) - V(x(\infty)) \le x_0^{\mathrm{T}} P x_0 \ .$$

Remark 1: Note that the cost upper bound (11) obtained in Theorem 1 depends on the initial condition x_0 of the uncertain fuzzy system (4). While in practice, this initial condition is difficult to be accurately determined. To overcome this disadvantage, it is assumed that the initial state x_0 is a zero mean random variable satisfying $E\{x_0 x_0^{\mathrm{T}}\} = I$. Hence, the cost upper bound (11) can be rewritten as

$$\bar{J} = E\{J\} \le E\{x_0^{\mathrm{T}} P x_0\} = Trace(P) \ . \tag{12}$$

To obtain controller gains, the conditions (9) and (10) can be transformed into a feasibility problem of a set of LMIs.

Theorem 2: Consider the uncertain fuzzy system (4) and the cost function (5), if there exist a scalar $\varepsilon > 0$, a symmetric positive definite matrix X and matrices Y_i , $i = 1, 2, \cdots, r$, such that the following LMIs are satisfied.

$$\begin{bmatrix} \Omega_{ii} & * & * & * \\ \Psi_{ii} & -\varepsilon I & * & * \\ X & 0 & -Q^{-1} & * \\ Y_i & 0 & 0 & -R^{-1} \end{bmatrix} < 0 \ , \ i = 1, 2, \cdots, r \ , \tag{13}$$

$$\begin{bmatrix} \Omega_{ij} + \Omega_{ji} & * & * & * & * & * \\ \Psi_{ij} & -\varepsilon I & * & * & * & * \\ \Psi_{ji} & 0 & -\varepsilon I & * & * & * \\ 2X & 0 & 0 & -2Q^{-1} & * & * \\ Y_i & 0 & 0 & 0 & -R^{-1} & * \\ Y_j & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0 \ , \quad i < j, \ i, j = 1, 2, \cdots, r \ . \tag{14}$$

where $\Omega_{ij} = A_i X + X A_i^{\mathrm{T}} + B_i Y_j + Y_j^{\mathrm{T}} B_i^{\mathrm{T}} + \varepsilon D_i D_i^{\mathrm{T}}$, $\Psi_{ij} = E_{ai} X + E_{bi} Y_j$. Then, the closed-loop uncertain fuzzy system (8) with controller gains $Y_i X^{-1}$, $i = 1, 2, \cdots, r$, is robustly stable. And the related cost function satisfies

$$\bar{J} \le \bar{J}^* = Trace(X^{-1}) \ . \tag{15}$$

Proof: According to Lemma 1 and Schur complement, the conditions (9) and (10) can be reduced to the LMIs (13) and (14), respectively, by denoting $X = P^{-1}$ and $Y_i = K_i X$, $i = 1, 2, \cdots, r$.

Remark 2: From the deduction of Theorem 2, it provides a parameterized representation of a set of robust guaranteed cost controllers. This important advantage can be exploited to design optimal robust guaranteed cost controller, which minimizes the upper bound of the cost function for the closed-loop uncertain fuzzy system (8). In this case, the optimization problem of robust guaranteed cost controller can be formulated as follows:

$$\begin{aligned} \min_{\varepsilon, X, Y_i} \quad & Trace(S) \\ s.t. \quad & \text{(i) (13) and (14)}, \\ & \text{(ii)} \begin{bmatrix} S & I \\ I & X \end{bmatrix} > 0 . \end{aligned} \tag{16}$$

Proof: On one hand, the condition (i) ensures the robust stability of the closed-loop uncertain fuzzy system (8). On the other hand, it follows from Schur complement that the constraint (ii) is equivalent to $S > X^{-1} > 0$. Therefore, the minimization of $Trace(S)$ implies the minimization of the cost upper bound in (15).

3.2 PLF Approach

Different from the above section, the local controller is designed for each local model separately. Then the global fuzzy controller is consisted of a set of local controllers with associated Lyapunov energy functions, and the stability of the global fuzzy system is guaranteed via the PLF approach.

Based on the fuzzy model (1), the ith state subspace is defined by

$$S_i = \{x(t) \mid h_i(x(t)) \ge h_k(x(t)), k = 1, 2, \cdots, r, k \ne i\} . \quad (17)$$

where $\{S_1, S_2, \cdots, S_r\}$ is a partition of the state space.

And the characteristic function of S_i is given as follows:

$$\eta_i = \begin{cases} 1, & x(t) \in S_i \\ 0, & x(t) \notin S_i \end{cases} \text{, and } \sum_{i=1}^{r} \eta_i = 1 . \quad (18)$$

If the fuzzy system (1) is local controllable, i.e., (A_i, B_i), $i = 1, 2, \cdots, r$, are controllable pairs, then for the ith local model, the local control law is

$$u(t) = K_i x(t) ,\ x(t) \in S_i . \quad (19)$$

Thus, the closed-loop subsystem defined on the subspace S_i is inferred as follows:

$$\dot{x}(t) = \Lambda_i x(t) ,\ x(t) \in S_i . \quad (20)$$

where $\Lambda_i = A_i + B_i K_i + D_i F(t)(E_{ai} + E_{bi} K_i)$.

Similar to the procedure of robust guaranteed cost controller design in the previous section, if the following matrix inequalities hold by selecting Lyapunov function as $V_i(x(t)) = x^{\mathrm{T}}(t) P_i x(t)$, $x(t) \in S_i$ in each subspace, then the local model (20) is robustly stable and the local control law (19) is robust guaranteed cost control law.

$$Q + K_i^{\mathrm{T}} R K_i + P_i \Lambda_i + \Lambda_i^{\mathrm{T}} P_i < 0 ,\ x(t) \in S_i . \quad (21)$$

In order to analyze the stability of the global fuzzy system constructed by

$$\dot{x}(t) = \sum_{i=1}^{r} \eta_i \Lambda_i x(t) . \quad (22)$$

the idea of PLF approach will be introduced in the following.

As all known, even if the Lyapunov functions are designed for each subspace individually, it is still needed to impose some restrictions on the control law to guarantee the stability of the systems. Accordingly, the objective of this section is to design a control law such that there exists a Lyapunov function that is non-increasing in sequential actions and also is non-increasing over each subspace.

Theorem 3: Consider the global fuzzy system (22) and the cost function (5), under the assumption that (A_i, B_i), $i = 1, 2, \cdots, r$, are local controllable pairs, if there exist a set of symmetric positive definite matrices P_i and matrices K_i defined in (19), $i = 1, 2, \cdots, r$, satisfy the matrix inequalities (21) for all admissible uncertainties $F(t)$ satisfying equation (3), then the global fuzzy system (22) is robustly stable with guaranteed cost performance. Furthermore, the corresponding cost upper bound is

$$J \le J^* = x_0^{\mathrm{T}} \overline{P} x_0 ,\ \overline{P} = \max\{P_i\} ,\ i = 1, 2, \cdots, r . \quad (23)$$

Proof: The Lyapunov function candidate is constructed as follows:

$$V(x(t)) = x^{\mathrm{T}}(t) P x(t) = x^{\mathrm{T}}(t) \left(\sum_{i=1}^{r} \eta_i P_i \right) x(t) = \sum_{i=1}^{r} \eta_i V_i(x(t)) .$$

where P_i is the solution of inequality (21) and

$$V_i(x(t)) = x^{\mathrm{T}}(t) P_i x(t) ,\ x(t) \in S_i ,\ i = 1, 2, \cdots, r . \quad (24)$$

It is easy to acquire that $V(x(t)) > 0$ from the above construction of $V(x(t))$. Due to the discontinuous property of PLF, the right and left-hand derivatives of $V(x(t))$, which define at the right and left of the discontinuous point of $V(x(t))$ respectively, are required to be ensured at the same time. Thus,

$$\begin{aligned} \dot{V}(x(t)) &= \lim_{\Delta \to 0} \frac{1}{\Delta} [V(x(t+\Delta)) - V(x(t))] \\ &= \sum_{i=1}^{r} \eta_i \left\{ \lim_{\Delta \to 0} \frac{1}{\Delta} [V_i(x(t+\Delta)) - V_i(x(t))] \right\} \\ &= \sum_{i=1}^{r} \eta_i \dot{V}_i(x(t)) . \end{aligned}$$

Then, the relations below are true from (24) and (21).

$$\dot{V}_i(x(t)) = x^{\mathrm{T}}(t)(P_i \Lambda_i + \Lambda_i^{\mathrm{T}} P_i) x(t) < x^{\mathrm{T}}(t)(-Q - K_i^{\mathrm{T}} R K_i) x(t) < 0 .$$

Thereby, $\dot{V}(x(t)) < 0$ is satisfied, i.e., the global fuzzy system (22) is robustly stable.

In addition, the cost upper bound (23) is fulfilled by integrating both sides of the above inequality from 0 to ∞.

Remark 3: For the Lyapunov function $V_i(x(t)) = x^{\mathrm{T}}(t) P_i x(t)$, $x(t) \in S_i$, the selection of state feedback gain K in the control process will be reduced to the following cases:

$$K = \begin{cases} K_i, & x(t) \in S_i , \\ K_i, & V_i(x(t)) < V_j(x(t)) \text{ and } x(t) \in S_i \cap S_j , \\ K_j, & V_i(x(t)) > V_j(x(t)) \text{ and } x(t) \in S_i \cap S_j . \end{cases} \quad (25)$$

Next, the LMI-based feasible solutions to the robust guaranteed cost controller and its convex optimization problem are presented, respectively.

Theorem 4: Consider the global fuzzy system (22) and the cost function (5), if there exist a scalar $\varepsilon > 0$, a symmetric positive definite matrix X and matrices Y_i, $i = 1, 2, \cdots, r$, satisfying the following LMIs for all admissible uncertainties $F(t)$ satisfying equation (3).

$$\begin{bmatrix} \Omega_i & * & * & * \\ E_{ai} X_i + E_{bi} Y_i & -\varepsilon I & * & * \\ X_i & 0 & -Q^{-1} & * \\ Y_i & 0 & 0 & -R^{-1} \end{bmatrix} < 0 . \quad (26)$$

where $\Omega_i = A_i X + X A_i^{\mathrm{T}} + B_i Y_i + Y_i^{\mathrm{T}} B_i^{\mathrm{T}} + \varepsilon D_i D_i^{\mathrm{T}}$. Then, the global fuzzy system (22) with controller gains $Y_i X^{-1}$, $i = 1, 2, \cdots, r$, is robustly stable. And the related cost function satisfies

$$\overline{J} \le \overline{J}^* = Trace(\overline{X}^{-1}) ,\ \overline{X}^{-1} = \max\{X_i^{-1}\} ,\ i = 1, 2, \cdots, r . \quad (27)$$

Remark 4: The optimal solution of robust guaranteed cost controller can be solved by the following optimization problem.

$$\min_{\varepsilon, X_i, Y_i} \quad Trace(S) \tag{28}$$
$$s.t. \quad \text{(i)} \quad (26),$$
$$\text{(ii)} \quad \begin{bmatrix} S & I \\ I & X_i \end{bmatrix} > 0.$$

4. NUMERICAL EXAMPLE

In this section, the above-mentioned methods will be applied to a nonlinear mass-spring-damper system (Tanaka *et al.*, 1996). The state-space model of this system can be represented as

$$\begin{aligned} \dot{x}_1(t) &= c(t)x_1(t) - 0.02x_2(t) - 0.67x_2^3(t) + u(t), \\ \dot{x}_2(t) &= x_1(t). \end{aligned} \tag{29}$$

where $c(t)$ is the uncertain term and satisfies $c(t) \in [-0.225,\ 0]$.

It is assumed that $x_1(t) \in [-1.5,\ 1.5]$ and $x_2(t) \in [-1.5,\ 1.5]$. Then two fuzzy rules in the form of (1) will be used to illustrate the system (29).

Plant Rule 1: IF $x_1(t)$ is M_1,
THEN $\dot{x}(t) = (A_1 + \Delta A_1(t))x(t) + (B_1 + \Delta B_1(t))u(t)$,
Plant Rule 2: IF $x_1(t)$ is M_2,
THEN $\dot{x}(t) = (A_2 + \Delta A_2(t))x(t) + (B_2 + \Delta B_2(t))u(t)$,

where the system parameters are given by

$$A_1 = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.1125 & -1.527 \\ 1 & 0 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$D_1 = D_2 = \begin{bmatrix} -0.1125 \\ 0 \end{bmatrix},\ E_{a1} = E_{a2} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ E_{b1} = E_{b2} = 0.$$

The membership functions as well as the weighted matrices of the cost function are chosen as follows:

$$h_1(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25},\ h_2(x_2(t)) = \frac{x_2^2(t)}{2.25}.$$
$$Q = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix},\ R = 1.$$

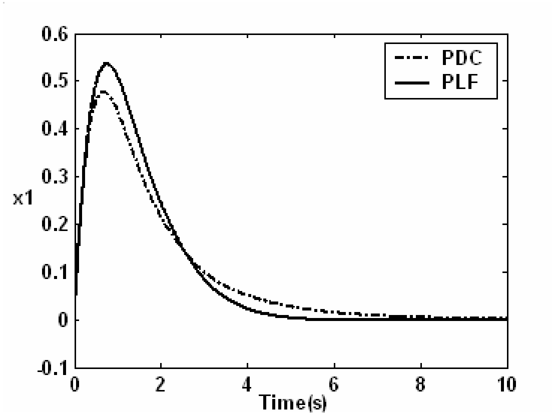
In order to be convenient for analysis and comparison between the results obtained by PDC scheme and PLF approach, respectively, the gains of robust guaranteed cost controller (RGCC) and the corresponding upper bound J of the cost function are listed in Table 1, which are achieved by solving the relevant LMIs.

From Table 1, it can be seen that the cost upper bound obtained by PLF approach are relatively less conservative than those by PDC scheme.

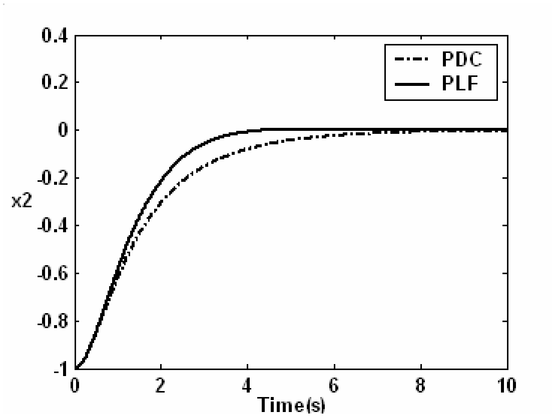
In the following, the simulation results of optimal robust guaranteed cost control will be given in Fig. 1 (a) and (b) with the initial condition $x_0 = [0 \quad -1]^{\mathrm{T}}$. To show difference between two methods discussed in this paper, the response of the identical state variable will be described in one figure. From these figures, it can be seen that the controller designed respectively by PDC scheme and PLF approach can both robustly stabilize the uncertain fuzzy system. In addition, the system controlled by PLF approach takes shorter time to converge to zero than it controlled by PDC scheme.

Table 1. The results comparison between PDC scheme and PLF approach

	RGCC Gains	J
Theorem 2	$K_1 = [-6.6124 \quad -6.2439]$, $K_2 = [-7.1243 \quad -6.4350]$.	$\bar{J}^* = 19.3626$.
Theorem 4	$K_1 = [-2.6536 \quad -1.7970]$, $K_2 = [-1.9905 \quad -0.6414]$.	$\bar{J}^* = 9.7469$.
	Optimal RGCC Gains	J
Remark 2	$K_1 = [-2.3502 \quad -1.1088]$, $K_2 = [-2.3555 \quad -1.0999]$.	$\bar{J}^* = 5.6726$.
Remark 4	$K_1 = [-1.7436 \quad -1.0178]$, $K_2 = [-1.2726 \quad -0.3039]$.	$\bar{J}^* = 3.6002$.



(a) The response of state variable x_1



(b) The response of state variable x_2

Fig. 1. The state response curves of optimal robust guaranteed cost control

5. CONCLUSIONS

In this paper, the problem of robust guaranteed cost controller design is studied for a class of uncertain nonlinear systems. Based on the T-S fuzzy model, the gains of robust guaranteed cost controller are obtained by using PDC scheme and PLF approach, respectively. In addition, a LMI-based convex optimization problem is introduced to design optimal robust guaranteed cost controller which minimizes the upper bound of the quadratic performance cost. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method and it is proved that the results achieved by PLF approach are less conservative than the ones by PDC scheme.

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