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Friction Compensation with Commonly-Used PD Control and Support Vector Machine

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Abstract: This paper addresses the high-precision tracking of mechanical systems with friction effect. A simple integrated proportional-derivative (PD) scheme is proposed where a support vector machine is incorporated to deal with friction. The bounded tracking is proved with Lyapunov's direct method and the bound of tracking error can be made arbitrarily small by selecting large control gains. A major advantage of the proposed framework is that it does not use the modeling information in the controller formulation, and thus permits easy implementation in practice. Simulations performed on two single-mass servo control systems demonstrate the expected better performance of the proposed approach.

1. INTRODUCTION

It is well known that one of the major limitations to achieve high-precision performance in mechanical systems is the presence of friction, which is a nonlinear phenomenon difficult to describe analytically (Abdellatif, Grotjahn, & Heimann, 2007; Armstrong-Helouvry, Dupont, & Canudas de Wit, 1994; Canudas de Wit, Olsson, Aström, & Lischinsky, 1995). Typical errors caused by friction are steady-state errors in position regulation and tracking lags (Canudas de Wit & Lischinsky, 1997). Much effort has devoted to the development of control law for friction compensation. There are two basic strategies for friction compensation, named as model-based control and nonmodel-based control. Model-based controls such as adaptive control (Canudas de Wit & Lischinsky, 1997; Friedland & Park, 1992; Hirschorn, & Miller, 1999; Liao & Chien, 2000; Panteley, Ortega, & Gafvert, 1998) which are formulated based on the dynamic friction model and the compensation is highly dependent on the accurate of the friction model. On the other hand, nonmodel-based controls including disturbance observer techniques (Canudas de Wit & Kelly, 2007; Kempf & Kobayashi, 1999; Su, Duan, Zheng, Zhang, Chen, & Mi, 2004; Xu & Yao, 2001), nonlinear proportional-integral-derivative (PID)/PD control (Armstrong, Neevel, & Kusik, 2001; Dupont, 1994; Parra-Vega, Arimoto, Liu, Hirzinger, & Akella, 2003), learning control (Cho & Ha, 2000) and artificial neural network techniques (Du & Nair, 1999; Herrmann, Ge, & Guo, 2005; Huang, Tan, & Lee, 2002; Selmic & Lewis, 2002). Since support vector machine have an inherent capability of approximating nonlinear functions over neural network (Gunn, 1998; Vapnik, 1995), it is attractive to apply them in control systems (de Kruif & de Vries, 2001; Ong, Keerthi, Gilbert, & Zhang, 2004; Suykens, 2001; Suykens, Vandewalle, & De Moor, 2001). In particular, Wang, Li, & Bi (2004) explored support vector machine to static friction modelling for servo motion systems, and demonstrated the performance improvements of the formulated approach. A little pity is that the stability of the closed-loop system has not been thoroughly analyzed.

In this paper, a very simple integrated PD control scheme with support vector machine for friction compensation is proposed. The bounded tracking of the state errors with this simple control law is shown in agreement with Lyapunov's direct method. The control algorithm does not use the modeling information in the controller formulation, and thus, it is readily implemented. Simulations performed on two different single-mass servo motion systems demonstrate the expected better performance of the proposed approach.

2. PROBLEM STATEMENT

We consider in this paper a one-DOF mechanical system described by

$$m\ddot{x} = u - f(\dot{x}) \tag{1}$$

where m is the mass (or inertia), $x, \dot{x}, \ddot{x} \in \Re$ denotes the position (or angle), velocity, and acceleration, respectively, $f(\dot{x})$ stands for the friction force, and u denotes the torque input. We employ the dynamic model proposed in Canudas de Wit et al. (1997) to model the effect of friction force,

$$f(\dot{x}) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} \tag{2}$$

$$\dot{z} = -\sigma_0 \frac{|\dot{x}|z}{h(\dot{x})} + \dot{x} \tag{3}$$

$$h(\dot{x}) = \alpha_0 + \alpha_1 e^{-(\dot{x}/\alpha_2)^2} \tag{4}$$

where z denote the average bristle deflections of the contact forces, and σ_i, α_i, $i = 0,1,2$ are some positive parameters which are typical unknown.

As discussed in Canudas de Wit et al. (1995), it is reasonable to assume that the friction is bounded, that is,

$$|f| \le \Delta_0 + \Delta_1 |\dot{x}| \tag{5}$$

where Δ_0 and Δ_1 are two positive constants.

Let $x_d(t) \in \Re$ be any reference trajectory for the system (1) that is twice continuously differentiable, i.e., the expected trajectory-tracking speed and acceleration $\dot{x}_d, \ddot{x}_d \in \Re$ are bounded by

$$V_M = \sup\|\dot{x}_d(t)\|, \quad A_M = \sup\|\ddot{x}_d(t)\| \tag{6}$$

Define the tracking errors $e(t), \dot{e}(t) \in \Re$ as follows:

$$e = x - x_d\,, \quad \dot{e} = \dot{x} - \dot{x}_d \tag{7}$$

Assume x and $\dot{x}$ are measurable, the friction are bounded in (5), and the parameters of the mechanical systems including the friction are unknown. Our object is to design a simple tracking controller that ensures the tracking errors are bounded for all time. Furthermore, the tracking error bound can be made arbitrarily small by selecting the control gains.

3. CONTROL DEVELOPMENT

Following the idea reported in Panteley et al. (1998), we also see the friction force expressed in (2)-(4) as a disturbance to the system (1). The control law is developed in two steps, first a support vector machine is developed to model the friction and then incorporated into a simple commonly-used PD control to solve the above formulated problem. The reason behind the selection of the commonly-used PD control to realize this goal is to make the proposed scheme to a widespread field.

3.1 SVM-Based Friction Estimation

Because of the complexity and difficulty in modelling the friction, support machine (SVM) may be used to generate input-output maps using the property that a SVM can approximate any smooth function, with any desired accuracy over a compact set (Gunn, 1998; Vapnik, 1995). In fact, various learning algorithms such as neural network maybe used for this modeling. Our choice of support vector machine (SVM) is motivated by its good performances on a variety of problems and its desired properties: a theoretical error bound, an optimal solution defined by a convex quadratic programming problem, sparsity in solution representation and good generalization ability (Scholkopf & Smola, 2001; Vapnik, 1998).

For SVM, the basic idea is to map the input and output data points in a higher dimensional feature space H (reproduced kernel Hilbert space), via a nonlinear mapping ϕ, and then do linear regression in this space.

For a given training set with l samples constructed by using friction model (2)-(4)

$$D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l) | x_i, y_i \in \Re, i = 1, \ldots, l\} \tag{8}$$

where x_i and y_i denote the velocity and friction, respectively. The SVM to solve the modeling of friction can be formulated as (Vapnik, 1998)

$$\begin{aligned} &\min_{f \in F} \left(\frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{l}\left[c(\xi_i) + c(\xi_i^*)\right] \right) \\ &\textit{subject to} \quad \boldsymbol{w}\cdot\boldsymbol{x}_i + b - y_i \le \varepsilon + \xi_i \\ &\qquad\qquad\quad\;\; y_i - \boldsymbol{w}\cdot\boldsymbol{x}_i - b \le \varepsilon + \xi_i^* \\ &\qquad\qquad\quad\;\; \xi_i, \xi_i^* \ge 0,\ i = 1, \ldots, l \end{aligned} \tag{9}$$

where $C > 0$ is a parameter that determines the tradeoff between the model flatness and the training error, $c(\cdot)$ denotes the loss function, and ξ_i and ξ_i^* are slack coefficient.

The most common choice for the loss function is the following ε -insensitive loss function defined by Vapnik (1998)

$$L_\varepsilon[y, f(\boldsymbol{x})] = \begin{cases} 0, & |y - f(\boldsymbol{x})| \le \varepsilon \\ |y - f(\boldsymbol{x})| - \varepsilon, & |y - f(\boldsymbol{x})| > \varepsilon \end{cases} \tag{10}$$

where ε is referred to the expected error bound.

Following the standard approach, introducing Lagrange multiplier and kernel techniques, the resulting convex programming problem expressed in (9) is solved by its Wolfe dual formulation:

$$\begin{aligned} &\min_{\alpha_i, \alpha_i^*} \left(\frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j)k(\boldsymbol{x}_i, \boldsymbol{x}_j) - \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)y_i + \varepsilon\sum_{i=1}^{l}(\alpha_i^* + \alpha_i) \right) \\ &\textit{subject to} \quad \sum_{i=1}^{l}(\alpha_i^* - \alpha_i) = 0, \\ &\qquad\qquad\quad\; 0 \le \alpha_i, \alpha_i^* \le C,\ i = 1, \ldots, l \end{aligned} \tag{11}$$

where α_i and α_i^* denote the non-zero Lagrange multipliers, and $k(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is the kernel function. In this application, the following Spline kernel function is utilized

$$\begin{aligned} k(\boldsymbol{x}_i, \boldsymbol{x}_j) = 1 + \boldsymbol{x}_i\cdot\boldsymbol{x}_j + \frac{1}{2}(\boldsymbol{x}_i\cdot\boldsymbol{x}_j)\min(\boldsymbol{x}_i, \boldsymbol{x}_j) \\ - \frac{1}{6}\min(\boldsymbol{x}_i, \boldsymbol{x}_j)^3 \end{aligned} \tag{12}$$

Using the KKT conditions, the offset can be calculated by

$$b = \begin{cases} y_i - \sum_{j=1}^{l}(\alpha_j^* - \alpha_j)k(\boldsymbol{x}_j, \boldsymbol{x}_i) + \varepsilon & \alpha_i > 0 \\ y_i - \sum_{j=1}^{l}(\alpha_j^* - \alpha_j)k(\boldsymbol{x}_j, \boldsymbol{x}_i) - \varepsilon & \alpha_i^* > 0 \end{cases} \tag{13}$$

Finally, the output of the SVM for friction modelling can be written as

$$\hat{f}(\boldsymbol{x})=\sum_{i=1}^{l}\left(\alpha_i^*-\alpha_i\right)k(\boldsymbol{x}_i,\boldsymbol{x}_i)+\bar{b} \tag{14}$$

where $\bar{b}$ is the average value of b obtained by using (13).

The modelling error is given by

$$\left|\hat{f}(\boldsymbol{x})-f(\boldsymbol{x})\right|\le\varepsilon \tag{15}$$

3.2 Integrated Control Law

Using the SVM-based friction estimation, the following simple PD plus control law is proposed

$$u=-k_p e-k_d\dot{e}+\hat{f}(\dot{x}) \tag{16}$$

where k_p and k_d are positive proportional and derivative gains, and $\hat{f}(\dot{x})$ was defined in (14). To keep unanimity with (2), here it is written as $\hat{f}(\dot{x})$.

Substituting (16) into (1), the closed-loop error system is obtained as follows

$$\begin{aligned}&m\ddot{e}+k_p e+k_d\dot{e}=\rho\\&\rho=\hat{f}(\dot{x})-f(\dot{x})-m\ddot{x}_d\end{aligned} \tag{17}$$

Upon using (5), (6), and (15), ρ can be upper bounded by

$$|\rho|\le\varepsilon+mA_M \tag{18}$$

We are now in a position to perform a composite stability analysis for the closed-loop system given by (17) with the simple PD incorporated SVM friction compensation given by (16).

3.3 Stability Analysis

Theorem 1. Under the subsequent conditions (19) and (20), the proposed PD incorporated SVM friction compensation scheme ensures that all signals remain bounded during closed-loop operation. Furthermore, this error bound can be made arbitrarily small by selecting large control gains.

$$k_p>\max\left\{2m,\frac{\varepsilon+mA_M}{2}\right\} \tag{19}$$

$$k_d>m+\frac{\varepsilon+mA_M}{2} \tag{20}$$

Proof. The Lyapunov's direct method is employed to show the stability. To this end, we propose the following Lyapunov function candidate

$$V=\frac{1}{2}m\dot{e}^2+m\dot{e}e+\frac{1}{2}(k_p+k_d)e^2 \tag{21}$$

We first consider the following:

$$\begin{aligned}\frac{1}{4}m\dot{e}^2+m\dot{e}e+\frac{1}{2}k_p e^2&=\frac{1}{4}m(\dot{e}+2e)^2-me^2+\frac{1}{2}k_p e^2\\&\ge\frac{1}{2}(k_p-2m)e^2\end{aligned} \tag{22}$$

Substituting (22) into (21) and using (19), it follows that,

$$V\ge\frac{1}{4}m\dot{e}^2+\frac{1}{2}(k_p-2m)e^2+\frac{1}{2}k_d e^2\ge\frac{1}{4}m\dot{e}^2+\frac{1}{2}k_d e^2 \tag{23}$$

Hence, we can conclude that V is a positive definite Lyapunov function with respect to $e,\dot{e}$. Furthermore, we have the following upper bound for V

$$\lambda_1\|\eta\|^2\le V\le\lambda_2\|\eta\|^2 \tag{24}$$

with $\lambda_1=\min\left\{\frac{1}{4}m,\frac{1}{2}k_d\right\}$, $\lambda_2=\max\left\{m,\frac{1}{2}(m+k_p+k_d)\right\}$, and $\eta=[e,\dot{e}]^T$.

Differentiating V with respect to time, yields

$$\dot{V}=m\ddot{e}\dot{e}+m\ddot{e}e+m\dot{e}^2+(k_p+k_d)e\dot{e} \tag{25}$$

Substituting $m\ddot{e}$ from (17) into (25) and using (18), we have

$$\begin{aligned}\dot{V}&=(\dot{e}+e)\rho+m\dot{e}^2-k_p e^2-k_d\dot{e}^2\\&\le|(\dot{e}+e)||\rho|-k_p e^2-(k_d-m)\dot{e}^2\\&\le(\varepsilon+mA_M)(|\dot{e}|+|e|)-k_p e^2-(k_d-m)\dot{e}^2\\&=-\frac{1}{2}(\varepsilon+mA_M)\left[(|\dot{e}|-1)^2+(|e|-1)^2\right]+(\varepsilon+mA_M)\\&\qquad+\frac{1}{2}(\varepsilon+mA_M)(\dot{e}^2+e^2)-k_p e^2-(k_d-m)\dot{e}^2\\&\le-\left(k_p-\frac{\varepsilon+mA_M}{2}\right)e^2-\left[k_d-\left(m+\frac{\varepsilon+mA_M}{2}\right)\right]\dot{e}^2+(\varepsilon+mA_M)\\&\le-\lambda_3\|\eta\|^2+\lambda_4\end{aligned} \tag{26}$$

where $\lambda_3=\min\left\{k_p-\frac{\varepsilon+mA_M}{2},k_d-\left(m+\frac{\varepsilon+mA_M}{2}\right)\right\}$ and $\lambda_4=\varepsilon+mA_M$.

Upon using (24), (26) can be rewritten as

$$\dot{V}\le-\lambda_3 V+\lambda_4 \tag{27}$$

Therefore, we have

$$V(t)\le\lambda_4/\lambda_3+\left[V(0)-\lambda_4/\lambda_3\right]e^{-\lambda_3 t} \tag{28}$$

Consequently, it can be concluded that the signals e and $\dot{e}$ in the system are bounded. Furthermore, upon using (24) again, we have

$$\|\eta\|\le\sqrt{\frac{\lambda_4}{\lambda_1\lambda_3}+\frac{1}{\lambda_1}\left(V(0)-\frac{\lambda_4}{\lambda_3}\right)e^{-\lambda_3 t}} \tag{29}$$

It is clearly see that the error bound $\|\eta\|$ can be made arbitrarily small by selecting large control gains k_p and k_d. This completes the proof. □

Theorem 1 indicates that the proposed controller does not utilize the modeling information in the control law formulation, which is the simplest and would give rise to bounded tracking for mechanical system with friction effect.

4. SIMULATION RESULTS

Simulations on two servo control systems were conducted to illustrate the effectiveness of the proposed simple PD incorporated SVM friction compensation controller. The first system model can be found in (Canudas de Wit & Lischinsky, 1997). The inertial $m = 0.0022\,\mathrm{Kg}\cdot\mathrm{m}^{-2}$, and other parameters described friction are shown in Table 1.

Table 1. Nominal friction parameters for Figs. 1-6

Friction parameter	$\dot{x} > 0$	$\dot{x} < 0$	Nominal value
$\sigma_0\,(\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{rad}^{-1})$	—	—	260.0
$\sigma_1\,(\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}\cdot\mathrm{rad}^{-1})$	—	—	0.6
$\sigma_2\,(\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{rad}^{-1})$	0.0176	0.0189	0.018
$\alpha_0\,(\mathrm{N}\cdot\mathrm{m})$	0.28	0.29	0.285
$\alpha_1\,(\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}\cdot\mathrm{rad}^{-1})$	0.06	0.04	0.05
$\alpha_2\,(\mathrm{rad}\cdot\mathrm{s}^{-1})$	0.001	0.01	0.01

The sampling period was $T = 1\,\mathrm{ms}$. The reference trajectory is $x_d(t) = \sin(t)\,\mathrm{rad}$, and the initial parameters were all set as zero. The parameters of the SVM were determined as $C = 10^5$ and $\varepsilon = 10^{-4}$; and the proportional and derivative gains were chosen in accordance with stability conditions (19) and (20) as: $k_p = 5$ and $k_d = 1$. The position and velocity tracking errors with the simple PD control only are illustrated in Figs. 1 and 2, respectively. The simulation results with the proposed SVM-based PD control are shown in Figs. 3-6. It can be clearly see that, with the proposed controller, the position and velocity realize very better tracking. Notice that the favorable result is obtained with a very simple controller, which does not require any model information in the control law formulation.

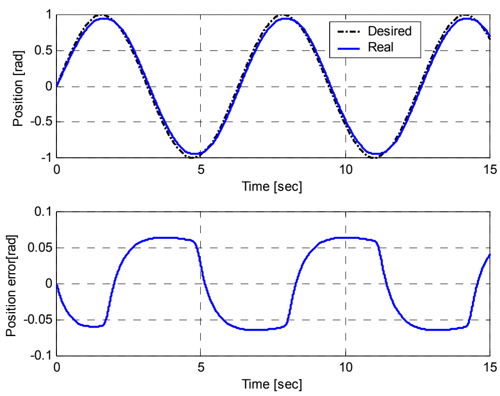

Fig. 1. Position tracking by using PD only.

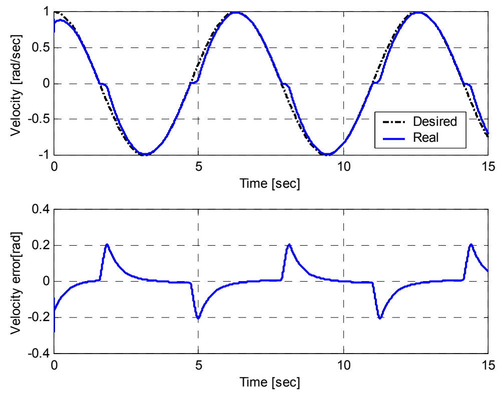

Fig. 2. Velocity tracking by using PD only.

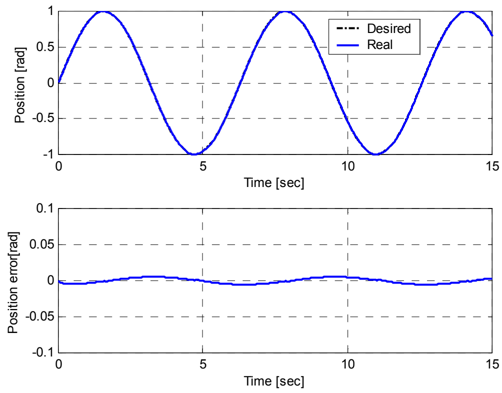

Fig. 3. Position tracking with the proposed SVM-based PD control.

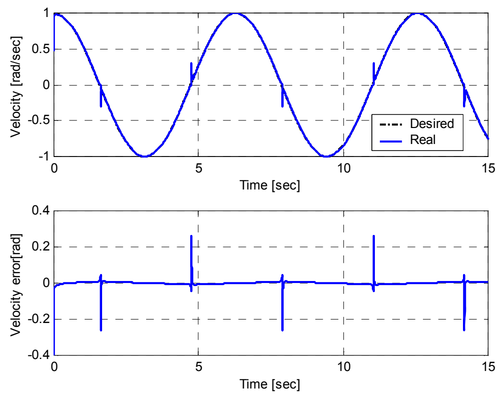

Fig. 4. Velocity tracking with the proposed SVM-based PD control.

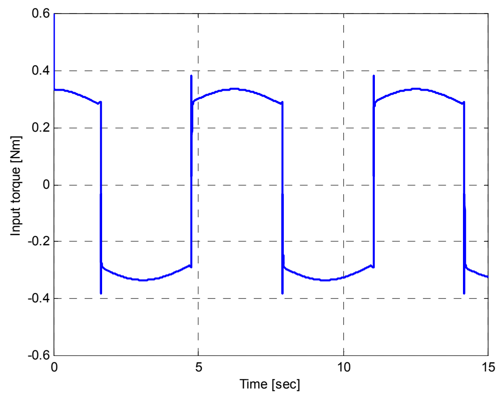

Fig. 5. Input torque of the proposed SVM-based PD control.

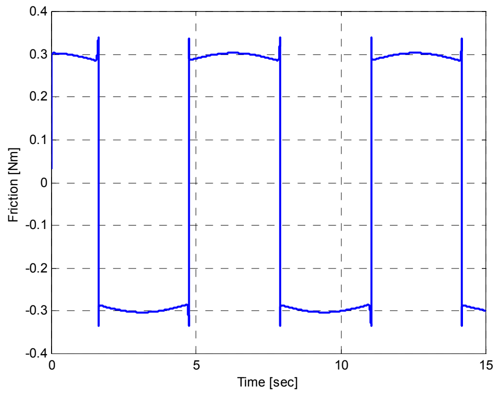

Fig. 6. Friction estimation of the SVM.

To further demonstrate the most advantage of the proposed model-free friction compensation method, simulations on another one-DOF robot system presented in (Mallon, van de Wouw, Putra, Nijmeijer, 2006) were also performed. The inertial is $m = 0.0026\,\mathrm{Kg}\cdot\mathrm{m}^{-2}$, and the friction model is

$$f(\dot{x}) = b\dot{x} + f_0(\dot{x}) \tag{30}$$

The dry friction model is expressed as a set-valued force law by the following algebraic inclusion (Mallon et al., 2006):

$$f_0(\dot{x}) = \begin{cases} g^+(\dot{x}) & \text{if } \dot{x} > 0 \\ -g^-(\dot{x}) & \text{if } \dot{x} < 0 \\ [f_s^- \quad f_s^+] & \text{if } \dot{x} = 0 \end{cases} \tag{31}$$

with $g^+(\dot{x})$ and $g^-(\dot{x})$ the Stribeck curve for positive and negative velocity, respectively. The set-valued nature of (31) at $\dot{x} = 0$ allows to model the stiction phenomena. The Stribeck curve is defined by the exponential curve (here, for $\dot{x} > 0$, indicated by the superscript "+")

$$g^+(\dot{x}) = f_c^+ + (f_s^+ - f_c^+)e^{-\left(|\dot{x}|/v^+\right)} \tag{32}$$

The friction parameters are summarized in Table 2.

Table 2. Friction parameters for another system

Friction parameter	$\dot{x} > 0$	$\dot{x} < 0$
$b\,(\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}\cdot\mathrm{rad}^{-1})$	0.0828	0.0790
$f_s\,(\mathrm{N}\cdot\mathrm{m})$	0.5735	0.5123
$f_c\,(\mathrm{N}\cdot\mathrm{m})$	0.3990	0.3887
$v\,(\mathrm{rad}\cdot\mathrm{s}^{-1})$	0.0688	0.0817

Note that the friction model presented in (30)-(32) has not only the different structure, but also the totally different parameters to that of the above mentioned first system.

The parameters of the controller including the model parameters of the SVM are kept unchanged. The simulations with PD only and proposed SVM-based PD control are shown in Figs. 7-12. It can be seen that for this totally different friction model to the learning friction model, the proposed SVM-based friction compensation PD control also obtains a better result over the conventional PD control.

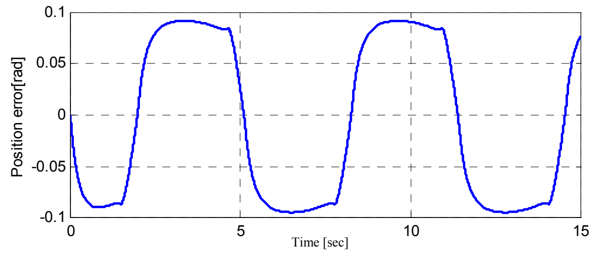

Fig. 7. Position tracking error of another system with PD only.

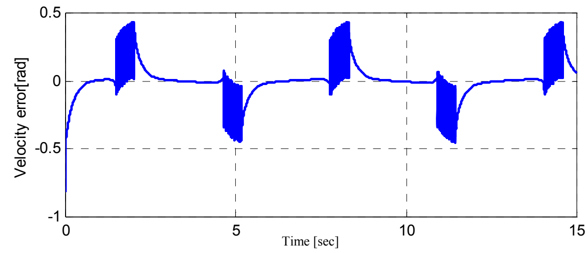

Fig. 8. Velocity tracking error of another system with PD only.

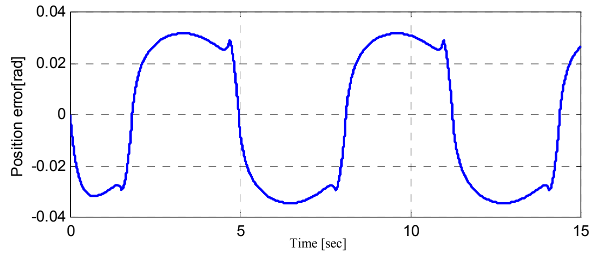

Fig. 9. Position error of another system with SVM-based PD control.

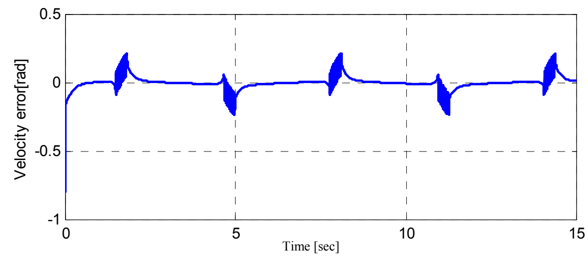

Fig. 10. Velocity error of another system with SVM-based PD control.

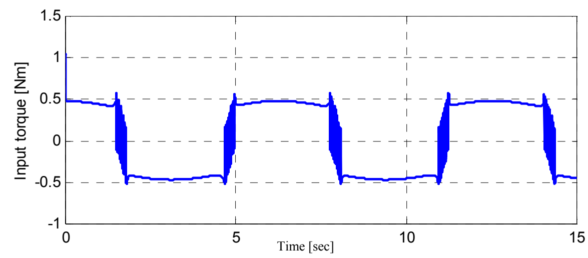

Fig. 11. Input torque of another system with SVM-based PD control.

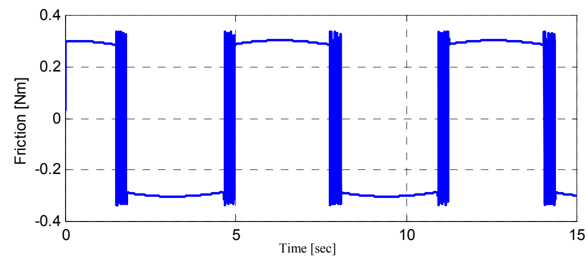

Fig. 12. Friction estimation of another system with SVM.

5. CONCLUSIONS

A very simple model-free friction compensation scheme has been proposed, by incorporating a SVM into an available commonly-used PD control. The bounded of all signals in the closed-loop system is shown with Lyapunov's direct method. The tracking error can be made arbitrarily small by selecting large control gains. The most advantage of the proposed scheme is that its formulation does not require any modelling information, and thus readily implement. Simulations demonstrate the effectiveness of the proposed approach.

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