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Robust State Estimation for Multi-delayed Neural Networks: An LMI Approach

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Abstract: The robust state estimation problem is studied in this paper for a class of neural networks with multiple time-varying delays and norm-bounded parameter uncertainties. The problem is to estimate the neuron states through available measured outputs such that for all admissible time-delays and parameter uncertainties, the dynamics of the estimation error is globally stable. A sufficient condition for the existence of such estimators for the multi-delayed neural networks is derived via the linear matrix inequality (LMI) approach, and a design procedure of the estimators is presented in terms of the feasible solutions to a certain LMI. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Keywords: Global stability; Linear matrix inequalities (LMIs); Neural networks; State estimation; Delays

1. INTRODUCTION

In the past decades, neural networks have received a great interest due to their wide applications in signal and image processing, artificial intelligence, system identification, industrial automation, and other areas. It is well known that time delays are likely to be present due to the finite switching speed of amplifiers and occur in the signal transmission among neurons in the electronic implementation of neural networks, which will affect the dynamics and other properties of neural networks (Baldi and Atiya, 1994). In Baldi and Atiya, 1994, the effects of delays on the dynamics and, in particular, on the oscillatory properties of simple neural networks are investigated. It is pointed out in Baldi and Atiya, 1994 that the delays in neural networks have a dramatic influence on the stability of the corresponding networks. In particular, many convergent networks become oscillatory due to the presence of delays. In recent years, the stability problem of different classes of time-delay neural networks, such as bidirectional associative neural networks, cellular neural networks, etc., has been extensively studied and a lot of stability conditions have been obtained for these neural networks, see (Arik et al., 2005; Xu et al., 2005; Huang et al., 2005; Li et al. 2005; Arik et al. 2000; Senan et al., 2005; Cao et al., 2002; Arik et al., 2002) for example. In most of these works, sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the asymptotical or exponential stability of the neural networks. Nevertheless, it is well known that the stability of a well-designed system may be destroyed by the unavoidable uncertainty due to the existence of external disturbance, modeling error and parameter fluctuation during the operation. So it is necessary to take the robust stability problem into consideration. Recently, several global and robust stability criteria for different kinds of neural networks with time-delays have been proposed (Li et al., 2004; Chen et al., 2004; Singh et al., 2005; Liao et al., 2005; Arik et al., 2003; Cao et al., 2005; Liao et al., 2004; Li et al., 2004).

On the other hand, the neuron state estimation problem becomes precursor for many applications. In large-scale neural networks, it is often the case that only partial information about the neuron states is available in the measurement outputs (Wang et al., 2005). Therefore, in order to utilize the neural networks, it is essential to estimate the neuron state through available measurement. In Wang et al., 2005, studied the state estimation problem for neural networks with time-varying delays and provided a design procedure to the desired state estimators. However, Wang et al., 2005 did not consider the parameter uncertainties and perturbations in the model of the neural networks. To the best of our knowledge, few results have been reported in literature on the robust state estimation for delayed neural networks with parameter uncertainties.

In this paper, the robust state estimation problem for neural networks with multiple time-varying delays and parameter uncertainties is studied. The system parameter uncertainties are assumed to be norm-bounded. The problem under consideration is to estimate the neuron states through available measured outputs such that for all admissible time-delays and parameter uncertainties, the dynamics of the estimation error is globally robustly stable. Using the linear matrix inequality (LMI) approach, we first derive a sufficient condition for the existence of the desired estimators for the delayed neural networks, and then show that this condition is equivalent to the feasibility of a certain LMI and the feasible solutions to this LMI are used to construct the estimators. Finally, a numerical example is presented to illustrate the effectiveness of the proposed method.

2. PROBLEM DESCRIPTION AND PREPARATION

Consider the following multi-delayed neural network with n neurons:

$$\dot{u}(t) = -Au(t) + Wg(u(t)) + \sum_{k=1}^{r} B_k g(u(t-\tau_k(t))) + V \qquad (1)$$

where $u(t) \in R^n$ is the state vector of the neural network, $A = \text{diag}\{a_1, a_2, \cdots, a_n\}$ is a diagonal matrix with positive entries $a_i > 0$. $W = (w_{ij})_{n\times n}$ and $B_k = (B_{ij}^k)_{n\times n}$ are the connection weighting matrix and the delayed connection weighting matrices, respectively. $g(u(t)) = [g_1(u_1), g_2(u_2), \cdots, g_n(u_n)]^T$ denotes the neuron activation function with $g(0) = 0$, and $V = [v_1, v_2, \cdots, v_n]^T$ is a constant external input vector. $\tau_k(t)$, $k = 1, 2, \cdots, r$ denotes the time-varying delays satisfying

$$0 \le \tau_k(t) \le \overline{\tau}_k < \infty, \; \dot{\tau}_k(t) \le \eta_k < 1, \; k = 1, 2, \cdots, r \tag{2}$$

where $\overline{\tau}_k$ and η_k are known scalar constants. Suppose that the parameter matrices of the system (1) are uncertain and of the following form:

$$\begin{aligned} A &= A_0 + \Delta A(t) = A_0 + E_a \Delta_a(t) F_a \\ W &= W_0 + \Delta W(t) = W_0 + E_w \Delta_w(t) F_w \\ B_k &= B_0^{(k)} + \Delta B^{(k)}(t) = B_0^{(k)} + E_b^{(k)} \Delta_b^{(k)}(t) F_b^{(k)} \end{aligned} \tag{3}$$

where A_0 , W_0 , and $B_0^{(k)}$ denote, respectively, the nominal matrices of A , W , and B_k . E_a , F_a , E_w , F_w , $E_b^{(k)}$, and $F_b^{(k)}$ are known constant matrices which describe the structure of the uncertainties. $\Delta_a(t)$, $\Delta_w(t)$ and $\Delta_b^{(k)}(t)$ are unknown real time-varying matrices satisfying $\Delta_a^T \Delta_a \le I$, $\Delta_w^T \Delta_w \le I$, $\Delta_b^{(k)T} \Delta_b^{(k)} \le I$, and I is the identity matrix of appropriate dimension.

When modelling a neural network, a typical assumption is that the activation functions are continuous, differentiable, monotonically increasing and bounded, such as the sigmoid-type of function. However, in many electronic circuits, the input-output functions of amplifiers may be neither monotonically increasing nor continuously differentiable. Thus, non-monotonic functions may be more appropriate to describe the neuron activation in designing and implementing an artificial neural network. As discussed in Wang et al., 2005, in this paper, we assume that the neuron activation function $g(\cdot)$ in (1) satisfies the following Lipschitz condition:

$$\|g(x) - g(y)\| \le \|G(x - y)\| \tag{4}$$

where $G \in R^{n\times n}$ is a known constant matrix. In this paper, the network measurements are assumed to be

$$y(t) = Cu(t) + f(t, u(t)) \tag{5}$$

where $y(t) \in R^m$ is the measurement output, C is a known constant matrix. $f : R \times R^n \to R^m$ is the nonlinear disturbances on the network outputs, and satisfies the following Lipschitz condition:

$$\|f(t, x) - f(t, y)\| \le \|F(x - y)\| \tag{6}$$

where the constant matrix $F \in R^{n\times n}$ is known.

In practice, it is often the case that the information about the neuron states is incomplete from the network measurements. That is, only partial information about the neuron states is available in the network measurements. On the other hand, the network measurements are subject to nonlinear disturbances. Therefore, estimating the neuron states through measured output is essential for further applications, and neuron states estimation becomes a significant research field of neural networks. The objective of this paper is to develop an efficient estimation algorithm to observe the neuron states from the available network outputs.

Consider the full-order state estimator described by

$$\begin{aligned} \dot{\hat{u}}(t) = &-A\hat{u}(t) + Wg(\hat{u}(t)) + \sum_{k=1}^{r} B_k g(\hat{u}(t - \tau_k(t))) + V \\ &+ K[y(t) - C\hat{u}(t) - f(t, \hat{u}(t))] \end{aligned} \tag{7}$$

where $\hat{u}(t)$ is the estimation of the neuron state, and $K \in R^{n\times m}$ is the estimator gain matrix to be designed. Define the error state as

$$e(t) = u(t) - \hat{u}(t) \tag{8}$$

then it follows from (1), (5), and (7) that

$$\begin{aligned} \dot{e}(t) = &(-A - KC)e(t) + W[g(u(t)) - g(\hat{u}(t))] \\ &+ \sum_{k=1}^{r} B_k [g(u(t - \tau_k(t))) - g(\hat{u}(t - \tau_k(t)))] \\ &- K[f(t, u(t)) - f(t, \hat{u}(t))] \end{aligned} \tag{9}$$

Our task is to design a state estimator for the delayed neural network described by (1) and (5), such that the error system (9) is globally robustly stable, for the nonlinear activation function $g(\cdot)$, the nonlinear disturbance $f(\cdot, \cdot)$, the time-varying delays $\tau_k(t)$ and all admissible uncertainties.

3. MAIN RESULTS

The following lemma, known as Schur Complement Lemma, will be used in establishing our main results.

Lemma 1. (Wang et.al., 2005) Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \text{, or } \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0$$

We first derive a condition for the stability of the error system (9).

Theorem 1. For a given matrix K, if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_{2k} > 0$, $\varepsilon_3 > 0$, $k = 1, 2, \cdots, r$ and a matrix $P > 0$ such that the following quadratic matrix inequality holds

$$\begin{aligned}&(-A-KC)^T P + P(-A-KC) + \varepsilon_1^{-1} PWW^T P \\ &+\varepsilon_1 G^T G + \sum_{k=1}^{r} \varepsilon_{2k}^{-1} PB_k B_k^T P \\ &+\varepsilon_3^{-1} PKK^T P + \varepsilon_3 F^T F + \sum_{k=1}^{r} \varepsilon_{2k} (1-\eta_k)^{-1} G^T G < 0 \end{aligned} \quad (10)$$

Then the error system (9) is globally asymptotically stable.

Proof: Define

$$A_k = -A - KC \quad (11)$$

$$\Psi(t) = g(u(t)) - g(\hat{u}(t)) \quad (12)$$

$$\phi(t) = f(t, u(t)) - f(t, \hat{u}(t)) \quad (13)$$

Then from (4) and (6), we have immediately that

$$\Psi^T(t)\Psi(t) = \| g(u(t)) - g(\hat{u}(t)) \|^2 \le \| Ge \|^2 = e^T G^T Ge \quad (14)$$

$$\phi^T(t)\phi(t) = \| f(t,u(t)) - f(t,\hat{u}(t)) \|^2 \le \| Fe \|^2 = e^T F^T Fe \quad (15)$$

Choose the following Lyapunov function for system (9)

$$\Phi(e(t)) = e^T(t) Pe(t) + \sum_{k=1}^{r} \int_{t-\tau_k(t)}^{t} e^T(s) Q_k e(s) ds \quad (16)$$

where

$$Q_k = \varepsilon_{2k}(1-\eta_k)^{-1} G^T G \quad (17)$$

It follows from (9) and (10) that

$$\begin{aligned}&\dot{\Phi}(e(t)) = e^T(t)(A_k^T P + PA_k)e(t) + 2e^T(t)P\,[W\Psi(t) \\ &+\sum_{k=1}^{r} B_k \Psi(t-\tau_k(t)) - K\phi(t)] \\ &+\sum_{k=1}^{r} [e^T(t) Q_k e(t) - (1-\dot{\tau}_k(t)) e^T(t-\tau_k(t)) Q_k e(t-\tau_k(t))]\end{aligned}$$

By Lemma 2.4 in Xie (1996), $\dot{\Phi}(e(t)) < 0$ is equivalent to that there exist positive scalars ε_1, ε_{2k}, ε_3, $i = 1, 2, \cdots, r$ such that the following inequality holds

$$\begin{aligned}&e^T(t)(A_k^T P + PA_k)e(t) + \varepsilon_1^{-1} e^T(t) PWW^T Pe(t) + \varepsilon_1 e^T(t) G^T Ge(t) \\ &+\sum_{k=1}^{r} \varepsilon_{2k}^{-1} e^T(t) PB_k B_k^T Pe(t) + \sum_{k=1}^{r} \varepsilon_{2k} e^T(t-\tau_k(t)) G^T Ge(t-\tau_k(t)) \\ &+\varepsilon_3^{-1} e^T(t) PKK^T Pe(t) + \varepsilon_3 e^T(t) F^T Fe(t) \\ &+\sum_{k=1}^{r} [e^T(t) Q_k e(t) - (1-\eta_k) e^T(t-\tau_k(t)) Q_k e(t-\tau_k(t))] \\ &= e^T(t)[A_k^T P + PA_k + \varepsilon_1^{-1} PWW^T P + \varepsilon_1 G^T G + \sum_{k=1}^{r} \varepsilon_{2k}^{-1} PB_k B_k^T P \\ &+\varepsilon_3^{-1} PKK^T P + \varepsilon_3 F^T F + \sum_{k=1}^{r} Q_k] e(t) < 0 \end{aligned} \quad (18)$$

Therefore, condition (10) guarantees that $\dot{\Phi}(e(t)) < 0$, thus we can conclude from Lyapunov stability theory that (10) ensures that the error system (9) is asymptotically stable. This completes the proof of Theorem 1.

Since the matrix inequality (10) contains the parameter uncertainties, it is difficult to check the truth of the inequality (10). From Theorem 1, an easily verifiable LMI-based condition for the stability of the error system (9) is presented in the following theorem.

Theorem 2. For a given matrix K, if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_6 > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{5i} > 0$, $i = 1, 2, \cdots, r$, and a matrix $P > 0$ such that the following linear matrix inequality holds

$$\begin{bmatrix} \Omega_{11} & PW_0 & PB_0^{(1)} & \cdots & PB_0^{(r)} & PK & PE_w & PE_b^{(1)} & \cdots & PE_b^{(r)} & PE_a \\ * & -\varepsilon_1 I + \varepsilon_4 F_w^T F_w & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & -\varepsilon_{21} I + \varepsilon_{51} F_b^{(1)T} F_b^{(1)} & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & (1,1) & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & -\varepsilon_3 I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & * & -\varepsilon_4 I & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & * & * & -\varepsilon_{51} I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & * & * & * & * & \cdots & -\varepsilon_{5r} I & 0 \\ * & * & * & \cdots & * & * & * & * & \cdots & * & \varepsilon_6 I \end{bmatrix} < 0 \quad (19)$$

Then the error system (9) is globally asymptotically stable, where

$$\begin{aligned}\Omega_{11} &= (-A_0 - KC)^T P + P(-A_0 - KC) + \varepsilon_1 G^T G \\ &+\varepsilon_3 F^T F + \sum_{i=1}^{r} \varepsilon_{2i} (1-\eta_i)^{-1} G^T G - \varepsilon_6 F_a^T F_a\end{aligned}$$

$$(1,1) = -\varepsilon_{2r} I + \varepsilon_{5r} F_b^{(r)T} F_b^{(r)} \quad (20)$$

Proof: By Lemma 1, it follows that inequality (10) is equivalent to

$$\Psi = \begin{bmatrix} (2,2) & P(W_0 + E_w\Delta_w F_w) & P(B_0^{(1)} + E_b^{(1)}\Delta_b^{(1)} F_b^{(1)}) & \cdots & P(B_0^{(r)} + E_b^{(r)}\Delta_b^{(r)} F_b^{(r)}) & PK \\ * & -\varepsilon_1 I & 0 & \cdots & 0 & 0 \\ * & * & -\varepsilon_{21} I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -\varepsilon_{2r} I & 0 \\ * & * & * & \cdots & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (21)$$

where

$$(2,2) = \begin{pmatrix} \bar{\Omega}_{11} - PE_a\Delta_a F_a \\ -F_a^T \Delta_a^T E_a^T P \end{pmatrix}$$

$$\bar{\Omega}_{11} = (-A_0 - KC)^T P + P(-A_0 - KC) + \varepsilon_1 G^T G + \varepsilon_3 F^T F + \sum_{i=1}^{r} \varepsilon_{2i}(1-\eta_i)^{-1} G^T G$$

Define

$$D_a = \begin{bmatrix} (PE_a)^T & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T$$
$$H_a = \begin{bmatrix} F_a & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T$$
$$D_w = \begin{bmatrix} (PE_w)^T & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T$$
$$H_w = \begin{bmatrix} 0 & F_w & 0 & \cdots & 0 & 0 \end{bmatrix}^T$$
$$D_b^{(i)} = \begin{bmatrix} (PE_b^{(i)})^T & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^T, \ i = 1, \cdots, r$$
$$H_b^{(i)} = \begin{bmatrix} 0 & 0 & \delta_1 F_b^{(1)} & \cdots & \delta_r F_b^{(r)} & 0 \end{bmatrix}^T$$

$\delta_i = 1$, and $\delta_j = 0$ for $j \neq i$, $i = 1, \cdots, r$

Since

$$\begin{aligned} \Psi &= \Xi - D_a\Delta_a H_a^T - H_a\Delta_a^T D_a^T + D_w\Delta_w H_w^T + H_w\Delta_w^T D_w^T \\ &\quad + \sum_{i=1}^{r}\left(D_b^{(i)}\Delta_b^{(i)}\left(H_b^{(i)}\right)^T + H_b^{(i)}\Delta_b^{(i)}\left(D_b^{(i)}\right)^T\right) \\ &\leq \Xi + \varepsilon_4^{-1} D_w D_w^T + \varepsilon_4 H_w H_w^T + \varepsilon_6^{-1} D_a D_a^T + \varepsilon_6 H_a H_a^T \\ &\quad + \sum_{i=1}^{r}\left(\varepsilon_{5r}^{-1} D_b^{(i)}\left(D_b^{(i)}\right)^T + \varepsilon_{5r} H_b^{(i)}\left(H_b^{(i)}\right)^T\right) = \Pi \end{aligned}$$

where

$$\Xi = \begin{bmatrix} \bar{\Omega}_{11} & PW_0 & PB_0^{(1)} & \cdots & PB_0^{(r)} & PK \\ * & -\varepsilon_1 I & 0 & \cdots & 0 & 0 \\ * & * & -\varepsilon_{21} I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -\varepsilon_{2r} I & 0 \\ * & * & * & \cdots & * & -\varepsilon_3 I \end{bmatrix}$$

$\Pi < 0$ implies the inequality (21). Applying Lemma 1 again, we conclude that the matrix inequality $\Pi < 0$ is equivalent to (19). Thus, the inequality (10) is true if the inequality (19) holds. The proof is completed.

Based on Theorem 2, a procedure will be developed as follows to determine the estimator gain K.

Theorem 3. Consider system (9), If there exist matrices $P > 0$, R and scalars $\varepsilon_1 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_6 > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{5i} > 0$, $i = 1, 2, \cdots, r$ such that the following linear matrix inequality holds

$$\begin{bmatrix} \tilde{\Omega}_{11} & PW_0 & PB_0^{(1)} & \cdots & PB_0^{(r)} & R & PE_w & PE_b^{(1)} & \cdots & PE_b^{(r)} & PE_a \\ * & -\varepsilon_1 I + \varepsilon_4 F_w^T F_w & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & (3^{(1)}, 3^{(1)}) & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & (3^{(r)}, 3^{(r)}) & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & -\varepsilon_3 I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & * & -\varepsilon_4 I & 0 & \cdots & 0 & 0 \\ * & * & * & \cdots & * & * & * & -\varepsilon_{51} I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & * & * & * & * & \cdots & -\varepsilon_{5r} I & 0 \\ * & * & * & \cdots & * & * & * & * & \cdots & * & -\varepsilon_6 I \end{bmatrix} < 0 \quad (22)$$

where

$$(3^{(i)}, 3^{(i)}) = -\varepsilon_{2i} I + \varepsilon_{5i} F_b^{(i)T} F_b^{(i)}, \ i = 1, \cdots, r$$

$$\tilde{\Omega}_{11} = -A_0^T P - PA_0 - C^T R^T - RC + \varepsilon_1 G^T G + \varepsilon_3 F^T F + \sum_{i=1}^{r} \varepsilon_{2i}(1-\eta_i)^{-1} G^T G + \varepsilon_6 F_a^T F_a$$

Then the error system (9) with the estimator gain $K = P^{-1}R$ is globally asymptotically stable.

Proof: Denoting $R = PK$ in (19), the inequality (22) is equivalent to the inequality (19). Hence, it follows from Theorem 2 that the error system (9) with the estimator gain given by (24) is globally asymptotically stable. This completes the proof of Theorem 3.

Remark 1. (22) is an LMI in the variables $\varepsilon_1 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_6 > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{5i} > 0$, $i = 1, 2, \cdots, r$, P, and R. Therefore, the robust estimation problem for a class of neural networks with multiple time-varying delays and parameter uncertainties is reduced to the feasibility problem of an LMI. The latter can be effectively solved by corresponding LMI Control Toolbox in MATLAB. Furthermore, if the LMI (22) is feasible, then a robust state estimator can be constructed in terms of the feasible solution to this LMI.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate the usefulness of the proposed results.

Example. Consider the system (1) with

$$\tau_1(t) = 0.4\sin(t),\ \tau_2(t) = 0.5\sin(t)$$

$$A = A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ W_0 = \begin{bmatrix} 2.005 & 1.005 \\ 0.005 & 0.15 \end{bmatrix},\ V = [0.5 \quad 0.5]^T$$

$$B_0^{(1)} = \begin{bmatrix} 1.525 & 0.13 \\ 0.025 & 0.125 \end{bmatrix},\ B_0^{(2)} = \begin{bmatrix} 2.025 & 0.225 \\ 0 & 0.225 \end{bmatrix}$$

$$f(t,u) = \begin{bmatrix} 0.4\cos(u_1 + u_2) \\ -0.3\cos(u_1 - u_2) \end{bmatrix},\ g(t,u) = \begin{bmatrix} 0.5\cos(u_2) \\ -0.4\cos(u_1) \end{bmatrix}$$

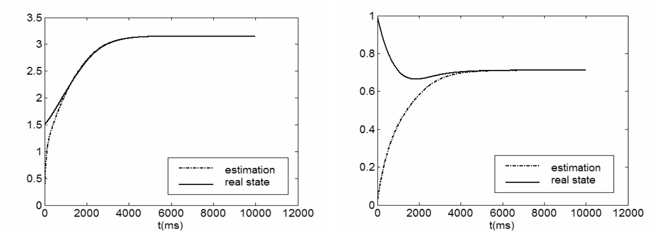
$$E_w = \begin{bmatrix} 0.005 & 0.005 \\ 0.005 & 0.05 \end{bmatrix},\ E_b^{(1)} = \begin{bmatrix} 0.025 & 0.03 \\ 0.025 & 0.025 \end{bmatrix}$$

$$E_b^{(1)} = \begin{bmatrix} 0.025 & 0.025 \\ 0 & 0.025 \end{bmatrix},\ F_w = F_b^{(1)} = F_b^{(2)} = I$$

By using the LMI toolbox in MATLAB, it follows that the LMI (22) is feasible and we obtain the following estimator gain matrix

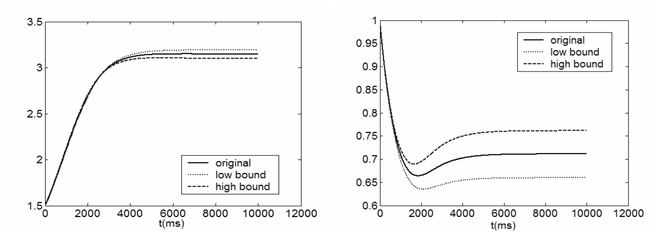
$$K = \begin{bmatrix} 21.2173 & 0.4735 \\ 0.4734 & 0.4278 \end{bmatrix}$$

For the nominal system, with initial condition $u(0) = [1.5 \quad 1]^T$ and $\hat{u}(0) = [0 \quad 0]^T$, the system state u_1 (respectively, u_2) and its estimate $\hat{u}_1$ (respectively, $\hat{u}_2$) are



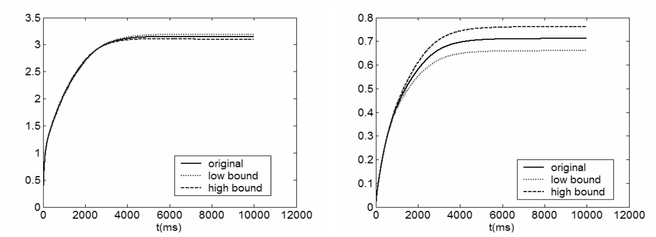
state u_1 and its estimation   state u_2 and its estimation

Fig. 1. States and state estimations of nominal system



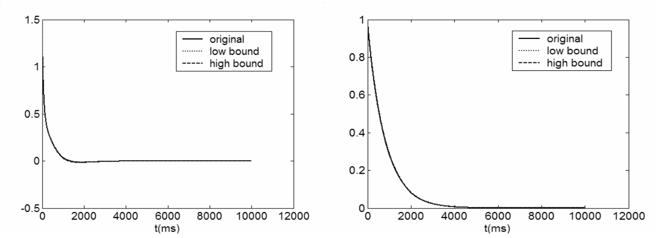
u_1 with different system parameters u_2 with different system parameters

Fig. 2. System states with different system parameters



$\hat{u}_1$ with different system parameters $\hat{u}_2$ with different system parameters

Fig. 3. Estimated states with different system parameters



e_1 with different system parameters e_2 with different system parameters

Fig. 4. Errors with different system parameters

shown in Fig. 1. The simulation results illustrate that, with the obtained estimator gain K, error states e_1, e_2 converge to zero, and the stable point of the neural network is $u = [3.148 \quad 0.711]^T$. Next, we illustrate the robust stability of the error dynamic system. Fig. 2, Fig. 3, and Fig. 4 show, respectively, the response of the system state, the estimated state, and the error signal under different system parameters. There are three curves in each figure. The solid curves represent the signals' responses under nominal parameters, and the dot curves depict the signals' responses under the minimum system parameters ($W = W_0 - E_w F_w$ for example), while the dash curves show the signals' responses under the maximal system parameters ($W = W_0 + E_w F_w$ for example). We learn from Fig, 2 and Fig. 3 that the stable points of the system states and the estimated states vary as the system parameters vary. However, it can be seen from Fig. 4 that no matter what the variations are, all the responses of the error signals converge to zero under the designed estimator, which implies that the error dynamic system is robustly stable in the presence of the uncertain parameters.

5. CONCLUSIONS

The robust state estimation problem was investigated in this paper for a class of neural networks with multiple time-varying delays and parameter uncertainties. We removed the traditional monotonicity and smoothness assumptions on the activation function, and extended the previous work to multi-delayed neural networks with norm-bounded parameter uncertainties. A linear matrix inequality (LMI) approach has been developed to solve the addressed problem, and a constructive design procedure of the robust estimator was presented in terms of the feasible solutions to a certain LMI. The effectiveness of the proposed results was finally illustrated by a numerical example.

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