

An LMI Design of an Observer Based Fuzzy PSS

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Abstract: Power systems are highly nonlinear systems that exhibit undesirable oscillations following disturbances. Power system stabilizers (PSS) are usually incorporated to provide auxiliary excitation signals to damp these oscillations. Our objective is to improve the PSS performance via the use of fuzzy logic and LMI techniques. A power system is viewed as a polytopic model that can be adequately represented by a Takagi-Sugeno fuzzy system. A power system stabilizer based on the parallel distributed control principle is suggested. Typically, speed measurements are used as feedback signals. Consequently, a fuzzy observer is included to estimate the unmeasured states. LMI conditions that guarantee the stability and robust pole clustering of the closed loop system are derived. Simulation results of both single-machine and multi-machine models confirm the effectiveness of the proposed algorithm.

Keywords: Intelligent control of power systems, modelling, operation and control of power systems, control system design

1. Introduction

The main problem that encounters a PSS design is that power systems frequently experience changes in operating conditions due to continuous variations in generation and load patterns, as well as changes in transmission networks DeMello and Concordia (1969). Another problem is the non-linear nature of power systems (Saadat, 1999). These two problems can be well-treated through T-S fuzzy models because such models can approximate nonlinear time varying systems (Tanaka & Wang 2001, Kang, Lee & Pusan, 1998). Dynamic T-S fuzzy models are obtained by linearization of the nonlinear plant around different operating points. Once T-S fuzzy models are created, linear control techniques are used to design a local controller for each linear model under global stability and performance conditions. LMI techniques are extensively used in multi-objective control design methods because many objectives can be expressed as convex constraints in LMI framework (Boyd, et al. 1994). Pole clustering, is used here to guarantee that all the system poles are placed in a pre-selected LMI region in the open left-half of the s-plane such that adequate damping and better time response are achieved.

Many researchers have addressed robust PSS designs using LMI techniques. Werner, Korba & Yang (2003) present the model uncertainty as a linear fractional transformation, and designed an output feedback PSS that guarantees stability for all admissible plants models, while minimizing a quadratic performance index for the nominal plant. In Rao & Sen (2000), robust pole clustering using a state feedback PSS design is studied. In Tsai et al. (2004), robust stability and selection of weighting functions that shape the open loop system are considered. Ramos et al. (2003) use a combination of an LMI technique and direct feedback linearization to achieve damping of a certain nominal plant model. In Befekadu & Erlich (2006), a robust decentralized PSS design problem is expressed as minimizing a linear objective function under LMI and bilinear matrix inequality constraints. The authors also reported the problem of designing a reduced-order decentralized H_∞ dynamic output feedback PSS based on parameter continuation method in LMI framework. A dynamic output feedback design of a PSS is proposed in Hisham (2006) for a single-machine infinite-bus system.

In this paper, the problem of a multivariable fuzzy PSS based on output feedback is addressed.

The power system model is firstly formulated as a T-S fuzzy model (IF-THEN rules). The changes that occur in a power system are well captured by active, reactive powers (P, Q) and tie line reactance (X_e). Therefore, these three variables appear in the premise parts of the IF-THEN rules. Parallel distributed control (PDC) offers a procedure to design a fuzzy stabilizer from a given T-S fuzzy model. The T-S model and the PDC are presented in Section 2. In Section 3, pole clustering is introduced to ensure that the controller can stabilize the system at different operating points. To cope with the practical case of speed feedback, a fuzzy observer is introduced in Section 4. Design validation based on nonlinear simulations of typical power systems are depicted in Section 5. Section 6 provides the conclusions.

2- T-S Model of a power system

2-1 A brief review of T-S Models and PDC

A dynamic T-S fuzzy model is described by IF-THEN rules. The consequents of the rules represent local linear input-output relations of a nonlinear system. The main feature of T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear sub-model. The overall model of the system is obtained by fuzzy blending of these linear sub-models. The i^{th} rule of the T-S fuzzy model is written as follows.

Model Rule i :

IF $z_1(t)$ is M_1^i AND ... AND $z_n(t)$ is M_n^i

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$
 $y(t) = C_i x(t)$

where $M_j^i, j = 1, 2, \dots, n$, is the j^{th} fuzzy set of the i^{th} rule and $z_1(t), \dots, z_n(t)$ are known premise variables that may be functions of state variables, external disturbances, and/or time. Let $\mu_j^i(z_j)$ be the membership function of the fuzzy set M_j^i and let

$$h_i = h_i(t) = \prod_{j=1}^n \mu_j^i(z_j)$$

Then given a pair $(z(t), u(t))$, the resulting fuzzy system is inferred as the weighted average of the local models and has a form of the following.

$$\begin{aligned} \dot{x} &= \frac{\sum_{i=1}^r h_i (A_i x + B_i u)}{\sum_{i=1}^r h_i} = \sum_{i=1}^r \alpha_i (A_i x + B_i u) \\ &= \left(\sum_{i=1}^r \alpha_i A_i \right) x + \left(\sum_{i=1}^r \alpha_i B_i \right) u = A(\alpha) x + B(\alpha) u \\ y &= C(\alpha) x \end{aligned} \quad (1)$$

where, for $i = 1, 2, \dots, r, \alpha_i = h_i / \sum_{i=1}^r h_i$. Note that

for $i = 1, 2, \dots, r, \alpha = [\alpha_1 \ \dots \ \alpha_r]^T$.

The PDC offers a procedure to design a fuzzy controller from a given T-S fuzzy model (Tanaka & Wang, 2001). To realize a PDC, a controlled plant is first represented as T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy model described in (1), the following fuzzy controller is constructed via PDC.

Model Rule i :

IF $z_1(t)$ is M_1^i AND ... AND $z_n(t)$ is M_n^i

THEN $u(t) = F_i x(t), \quad i = 1, 2, \dots, r$

The fuzzy control rules have a linear controller in the consequent parts and the overall fuzzy controller is represented by

$$u(t) = \frac{\sum_{i=1}^r h_i (F_i x)}{\sum_{i=1}^r h_i} = \left(\sum_{i=1}^r \alpha_i F_i \right) x(t) \quad (2)$$

Although the fuzzy controller (2) is constructed using local models, the feedback gains must be determined using global design conditions to guarantee global stability and control objectives. Theorems that discuss sufficient conditions of global quadratic stability for T-S fuzzy models are given in Kang, et al. (1998). Let $B_i = B, i = 1, 2, \dots, r$ (which is the case in a PSS design). The fuzzy control system (1) is globally quadratically stable via the state feedback PDC (2) if there exists a common positive definite matrix P^* such that

$$(A_i + B F_i)^T P^* + P^* (A_i + B F_i) < 0, \quad i = 1, 2, \dots, r \quad (3)$$

2-2 A T-S model for a PSS design

In this paper, the system under study comprises a single machine connected to an infinite bus through a tie line. The design model is represented by the fourth order linearized state-space model proposed by DeMello & Concordia (1969). The k -parameters (k_1, k_2, \dots, k_6) of the model depends on the loading (P, Q) and the tie line reactance (X_e). These scheduling variables (P, Q, X_e) are assumed to vary independently over the following ranges:

$$P \in [\bar{P} \ \underline{P}], \quad Q \in [\bar{Q} \ \underline{Q}] \quad \text{and} \quad X_e \in [\bar{X}_e \ \underline{X}_e].$$

These ranges are selected to encompass almost all possible operating conditions and very weak to very strong transmission networks. Therefore, the state space realization takes the following general form:

$$\begin{bmatrix} \dot{x} \\ x \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_y & D_y & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (4)$$

where, $x \in R^{4sl}$ is the state vector, u is PSS output and d is the disturbance that is represented here by variation in reference voltage signal (ΔV_{ref}). The measured output (y) is the speed deviation ($\Delta\omega$).

$B_1 = B_2 = [0 \ 0 \ 0 \ K_E/T_E]^T$, $C_y = [0 \ 1 \ 0 \ 0]$ and $D_y = 0$. K_E and T_E are the exciter's gain and time constant, respectively.

The proposed T-S model for the PSS design is given by the following eight rules.

Model Rule 1:

IF $(P \text{ is about } \bar{P})$ AND $(Q \text{ is about } \bar{Q})$ AND $(X_e \text{ is about } \bar{X}_e)$

THEN $\begin{bmatrix} \dot{x} \\ x \\ y \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & B_2 \\ C_y & D_y & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix}$

.....

Model Rule 8:

IF $(P \text{ is about } \bar{P})$ AND $(Q \text{ is about } \bar{Q})$ AND $(X_e \text{ is about } \bar{X}_e)$

THEN $\begin{bmatrix} \dot{x} \\ x \\ y \end{bmatrix} = \begin{bmatrix} A_8 & B_1 & B_2 \\ C_y & D_y & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix}$

where, A_1, A_2, \dots, A_8 are calculated at $[\bar{P}, \bar{Q}, \bar{X}_e], [\bar{P}, \bar{Q}, \bar{X}_e], \dots, [\bar{P}, \bar{Q}, \bar{X}_e]$ respectively.

The resulting fuzzy system is inferred as the weighted average of the local models and has the form

$$\begin{bmatrix} \dot{x} \\ x \\ y \end{bmatrix} = \begin{bmatrix} \left(\sum_{i=1}^8 \alpha_i A_i \right) & B_1 & B_2 \\ C_y & D_y & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (5)$$

In this paper, Z-shaped membership functions are adopted and expressed in terms of P , Q and X_e as follows.

$$L_1(\bar{P}, \bar{P}, P) = \begin{cases} 1 & , P < \bar{P} \\ 0.5 + 0.5 \cos\left(\frac{P - \bar{P}}{\bar{P} - \bar{P}} \pi\right) & , \bar{P} \leq P \leq \bar{P} \\ 0 & , P > \bar{P} \end{cases}$$

$$L_2(\bar{P}, \bar{P}, P) = 1 - L_1(\bar{P}, \bar{P}, P)$$

In a similar manner, membership functions for Q and X_e are defined.

3- Performance requirements of the PSS design

In power systems, a damping factor of at least 10% and a real part not greater than -0.5 guarantees that the low frequency oscillations, when excited, will die out in a reasonably short time (Rao & Sen, 2000). These transient response specifications can be satisfied by clustering the closed loop poles in the admissible region shown in Fig. 1. This ensures a minimum decay rate α_R and a minimum damping $\zeta_{min} = \cos(\theta/2)$. This in turn bounds the maximum overshoot and the settling time of the closed loop system. To avoid very large controller gains, the real part of the poles should be larger than $-\alpha_L$. Region 1 guarantees an upper bound on the settling time. Region 2 guarantees sufficient damping of the system. Region 3 prevents controller gains from being excessively large. The desired multi-variable PSS design must guarantee that all system roots lie in the pre-described region shown in Fig. 1. Each of the above regions is an LMI region and their intersection is also an LMI region (Chilali & Gahinet, 1999 and Chilali & Gahinet, 1996).

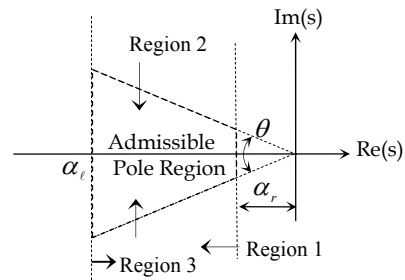


Fig. 1. A typical LMI Region.

An LMI region is any subset D of the complex plane that can be well defined as given in Chilali & Gahinet (1999)..

$$D = \left\{ s \in C: \Phi + s\Psi + \bar{s}\Psi^T < 0 \right\} \quad (6)$$

where Φ and Ψ are real matrices and Φ is a symmetric matrix. LMI constraints for different design objectives are listed below. For the analysis purpose, these inequalities will be written for a closed loop system that has the following state space realization:

$$\begin{cases} \bullet \\ x = A_{cl}x + B_{cl}d \end{cases} \quad (7)$$

The roots of system (7) lie inside an LMI region (6), if and only if there is a symmetric positive definite matrix P^* such that:

$$\Phi \otimes P^* + \Psi \otimes (P^* A_{cl}) + \Psi^T \otimes (P^* A_{cl})^T < 0 \quad (8)$$

where, \otimes denotes the Kronecker product.

4- A PSS design based on a fuzzy observer

Typically, a PSS has the speed as a feedback signal. In such case, attention is oriented towards output feedback design methods such as an observer-based design. This section presents an algorithm for state estimation of T-S fuzzy models to implement a pole-clustering observer-based stabilizer. The concept of PDC is utilized to construct a fuzzy observer as follows:

Observer Rule i :

IF $z_1(t)$ is M_1^i AND ... AND $z_n(t)$ is M_n^i
 THEN $\dot{\hat{x}}(t) = A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t))$,
 $\hat{y}(t) = C_i \hat{x}(t)$

The fuzzy observer is represented as follows.

$$\begin{aligned} \dot{\hat{x}}(t) &= \frac{\sum_{i=1}^r h_i \{A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t))\}}{\sum_{i=1}^r h_i} \\ &= \sum_{i=1}^r \alpha_i \{A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t))\} \\ \hat{y}(t) &= \sum_{i=1}^r \alpha_i C_i \hat{x}(t) \end{aligned} \quad (9)$$

If the fuzzy observer exists, the fuzzy state feedback regulator

$$u(t) = \sum_{i=1}^r \alpha_i F_i \hat{x}(t) \quad (10)$$

Combining the fuzzy observer (9) and fuzzy regulator (10) and denoting $e(t) = x(t) - \hat{x}(t)$, the augmented system is represented as follows

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \{ (A_i + B_i F_j) x(t) + B_i F_j e(t) \} \quad (11)$$

$$\dot{e}(t) = \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \{ A_i - K_i C_j \} e(t) \quad (12)$$

The closed loop system can be written as follows

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} A_i + B_i F_j & B_i F_j \\ 0 & A_i - K_i C_j \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \quad (13)$$

For the case of power systems, $B_i = B$, $C_j = C$, $i, j = 1, 2, \dots, r$ then (13) can be rewritten as follows

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \sum_{i=1}^r \alpha_i \begin{bmatrix} A_i + B F_i & B F_i \\ 0 & A_i - K_i C \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \quad (14)$$

The set of LMI conditions for pole-clustering based-observer design that is obtained by a quadratic Lyapunov function. The roots of system (14) lie inside an LMI region (6), if and only if the following LMIs hold

$$P_i, P_2 > 0 \quad (15)$$

$$\Phi \otimes P_i + \Psi \otimes (A_i P_i + B M_i) + \Psi^T \otimes (A_i P_i + B M_i)^T < 0 \quad (16)$$

$$\Phi \otimes P_2 + \Psi \otimes (P_2 A_1 - N_i C) + \Psi^T \otimes (P_2 A_1 - N_i C)^T < 0 \quad (17)$$

The feedback gains and the observer gains can then be obtained as $F_i = M_i P_i^{-1}$ and $K_i = P_2^{-1} N_i$, $i = 1, \dots, 8$, where M_i and N_i are decision variables obtained from (16)-(17).

The design steps can be summarized as follows

- i. Determine the ranges $P \in \left[\begin{smallmatrix} - & + \\ P & P \end{smallmatrix} \right]$, $Q \in \left[\begin{smallmatrix} - & + \\ Q & Q \end{smallmatrix} \right]$ and $X_e \in \left[\begin{smallmatrix} - & + \\ X_e & X_e \end{smallmatrix} \right]$ that encompass all practically operating conditions.
- ii. Define the eight subsystems by calculating A_i, A_2, \dots, A_8, B and C .
- iii. Define the membership functions according to their shapes and the ranges of P, Q and X_e .
- iv. Generate the T-S fuzzy system defined in (5).
- v. Define α_R, α_L and θ , then compute the LMI region matrices Φ and Ψ as shown in Chilali and Gahinet (1999).
- vi. To design a robust pole-placement observer-based stabilizer, solve the optimization problem (15)-(17).

The above steps are carried out off-line. The resulting gains of the controller and observer are used to implement the proposed fuzzy PSS on-line using (9)-(10).

5- Design validation

The proposed PSS algorithm is validated in this section based on two different nonlinear models. The first model is a single-machine infinite-bus model which is used to illustrate the design steps. The second model is a four-machine two-area system which is used as a bench mark problem in the literature. In applying our algorithm to the multi-machine system, each machine is considered as a single machine connected to an infinite bus. The effect of the reset of the system is reflected on the calculation of the line reactance and the power delivered to the system. Consequently, a PSS is designed independently for each machine. This procedure is a considerable approximation that is made possible because fuzzy modelling allows imprecision.

5-1 The single-machine infinite bus model

The study in this section will be carried on a single machine infinite-bus system whose data are given in Soliman et al. (2000). P, Q (at the generator terminals) and X_e are assumed to vary independently over the following ranges provided that all points included have a steady state load flow solution: $P \in [0.4 \ 1.0]$, $Q \in [-0.2 \ 0.5]$ and $X_e \in [0.2 \ 0.4]$. These ranges encompass the practical operating conditions and very weak to very strong transmission

networks. Fig. 2 shows the system open loop poles for 1000 plants as P, Q and X_e vary over the specified ranges. It is noted that, most of the plants in this poly-tope do not have adequate damping and some plants are unstable. It is required to design a stabilizer that shifts these poles to the open left half plane and guarantee a certain response speed. The design is carried out for an LMI region bounded by $\alpha_L = -0.5, \alpha_R = -25$ and $\theta = 168^\circ$.

The optimization problem defined by (15)-(17) is solved to calculate the observer gains and feedback gains. Fig. 3 shows the efficacy of the observer-based design in clustering the system roots in the pre-defined LMI region. The time response of the rotor angle following a 10% step change in the reference voltage is shown in Fig. 4. The results are based on a fourth-order nonlinear model simulation of the power system.

5-2 A four-machine two-area system

Fig. 5 shows the multi-machine system which is used in the simulation study. The test system consists of two fully symmetrical areas linked together by two 230 KV lines of 220 Km length. It is specifically designed in Kundur (1994) to study low frequency electromechanical oscillations in large interconnected power systems. Each area is equipped with two identical round rotor generators rated 20 KV/900 MVA. The synchronous machines have identical parameters except for the inertias which are $H = 6.5s$ in area 1 and $H = 6.175s$ in area 2. Thermal plants having identical speed regulators are further assumed at all locations, in addition to fast static exciter with a gain of 200. The load is represented as constant impedance and split between the areas. Each generator is equipped with a PSS.

A three phase to ground fault is applied for 100 msec. at the terminals of G4 in Fig. 5. A PSS designed as proposed here is compared to the conventional stabilizer (Kundur, 1994) at two possible tie-line power values of 400 MW and 600 MW. The results are depicted in Figs. 6-7. It is clear that the conventional stabilizer fails to maintain stability at 600 MW tie-line power.

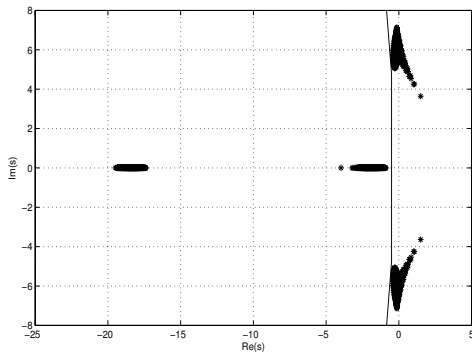


Fig. 2. The pole distribution of the open loop for different operating points

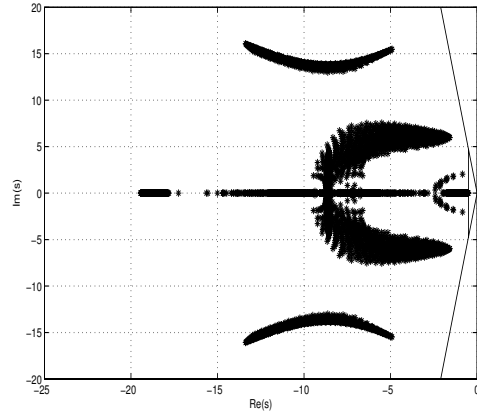


Fig. 3. Poles of the closed loop systems for different operating points.

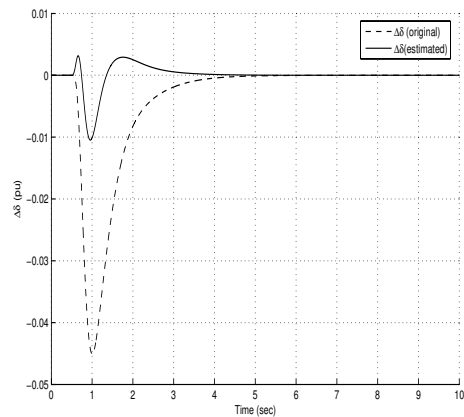


Fig. 4. Rotor angle response due to 10% step change in V_{ref} at $P=1.25, Q=-0.5$ and $X_e=0.2$ with full recovery after 100 ms

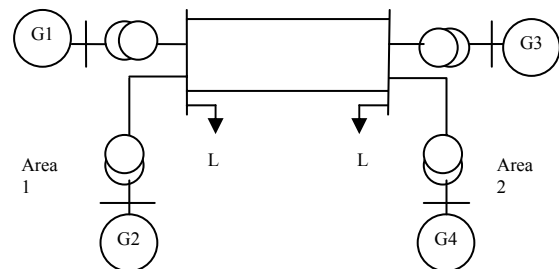


Fig. 5. A schematic diagram of the 4-machine 2-area system

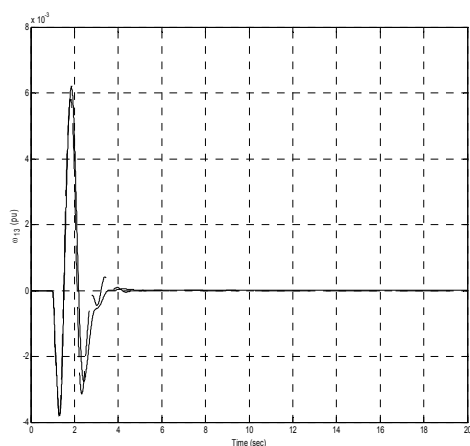


Fig. 6. Relative speed between machines 1 and 3 when fault occurs at 400 MW tie-line power (The solid line represents the proposed controller and the dashed line represents the conventional one)

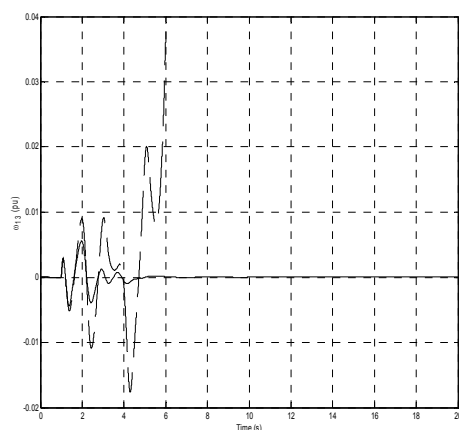


Fig. 7. Relative speed between machines 1 and 3 when fault occurs at 600 MW tie-line power (The solid line represents the proposed controller and the dashed line represents the conventional one)

6. Conclusions

A design of a power system stabilizer that can cope with a wide range of loading conditions and external disturbances has been the objective of the power industry. This paper has provided a step towards this goal. One of the contributions here has been to show that a nonlinear model of a power system can be systematically represented in the form of a T-S fuzzy system. This has allowed us to use an approximate design model of the power system to develop a stabilizer that cope with different operating conditions and disturbances. A fuzzy observer has been designed to estimate the system states assuming speed measurements only. LMI conditions have been derived to facilitate the design of the observer and the controller gains such that the closed loop poles lie in a pre-defined design region. The combined design of the fuzzy stabilizer and the observer in the PSS framework has been another contribution in the present work. Simulation results of different power

systems have confirmed the capability of the proposed algorithm.

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