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Joint PDF Tracking Control for a Class of Multivariate Time-varying Stochastic Descriptor Systems ⋆

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Abstract: This paper considers a new tracking control problem for a class of nonlinear stochastic descriptor systems, where the tracked target is a given joint probability density function (JPDF). The controlled plants can be represented by multivariate discrete-time descriptor systems with non-Gaussian disturbances and nonlinear output equations. The control objective is to find crisp algorithms such that the conditional output JPDFs can follow the given target JPDF. Rather than using statistic methods such as Bayesian estimation or Monte Carlo methods, we establish a direct relationship between the JPDFs of the transformed tracking error and the stochastic input. An optimization approach is applied to present recursive algorithms such that the distances between the output distributions and the desired one are minimized. Furthermore, a stabilization suboptimal control strategy is proposed by using of LMI-based Lyapunov theory. Simulations are provided to demonstrate the effectiveness of the stochastic tracking control algorithms.

Keywords: probability density function; descriptor system; tracking; stochastic control; optimal control; Lyapunov theory; simulation of stochastic system.

1. INTRODUCTION

The control problems for descriptor systems (or generalized systems, singular systems) have drawn considerable attention of many researchers due to their extensive applications in engineering. Many new feasible control approaches based on Lyapunov theory have extensively been presented for linear descriptor systems (see *e. g.* (Dai89, GM03, HL99, Tak98, WM94)). For stochastic descriptor systems, most of the existing results focused on generalized Kalman filtering theory, where linear plants with Gaussian noises were concerned (see e. g. (NCF99, YLM96, ZXS99)). It is shown that in the presence of descriptor (implicit) dynamics, the Kalman filtering problem for linear descriptor systems becomes much more complex than that for conventional systems.

On the other hand, for the conventional stochastic systems without descriptor dynamics, stochastic processes and stochastic control have been widely studied theoretically and applied in practice in the past decades. However, in most existing works only linear stochastic systems with Gaussian random variables were considered, and the control performance objectives were confined to be either expectation or variance of the stochastic output (see *e. g.* (Astrom70, Mao02, Pap91, Pet00, UP01, Wang00)). It has been shown that in many practical processes such as paper and board making systems, the related stochastic signals are non-Gaussian and/or the plants have strong non-linearity (see *e.g.* (Wang00)). For these stochastic systems with non-Gaussian inputs or outputs, the expectation and variance are not sufficient to characterize the statistics of the concerned random vectors. Generally speaking, for descriptor systems with non-Gaussian stochastic noises and nonlinear dynamics, few control and filtering applicable results have been obtained up to date.

For the past few years, new control strategies for the shape of the output probability density functions (PDF) have been developed for the conventional (non-descriptor) stochastic systems, where the stochastic variables are not confined to be only Gaussian and the control objective is the shape of output (see *e.g.* (Kar96, Wang00, Wang02)). Up to now, mainly there are two kinds of PDF control strategies having been proposed for some conventional stochastic

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systems. One concentrated on tuning the dynamic weights which correspond to the output PDFs after modeling the dynamics between the control input and the output PDFs using B-spline expansions (see *e. g.* (Wang00, Wang02)), another attempted to establish the relationships between the PDFs of the stochastic input and output, and made use of various optimization approaches (see, *e.g.* (Wang03)). However, most existing results on PDF control and minimum entropy control only considered single-input-single-output (SISO) conventional systems. It is noted that multivariate stochastic processes are much more complicated where in general some new methodologies involved in conditional joint probability density functions (JPDFs) should be discussed (see, *e.g.* (GW03)). Moveover, since stability can not be guaranteed with most of the proposed algorithms, stability analysis turns to be another obstacle in applications of these approaches.

In this paper, new PDF tracking control strategies are developed for a group of multivariate time-varying descriptor systems with non-Gaussian stochastic inputs. The control objective is to find crisp recursive design procedures such that the system outputs, which generally are non-Gaussian random vectors, can follow a target vector corresponding to a given joint distribution. After a new relationship is presented between the JPDFs of the tracking errors and inputs of the descriptor systems, the suboptimal control strategies are constructed based on gradient algorithms. Furthermore, a recursive stabilization suboptimal control law is proposed with which the stability of the closed loop system can be guaranteed. Finally simulations are provided to demonstrate the effectiveness of the proposed approaches.

2. PROBLEM FORMULATION

2.1 Process Model and Its transformation

We firstly consider the time-varying descriptor system described by

$$\Sigma_D : \begin{cases} Ex_{k+1} &= A_k x_k + B_{1k} u_k + B_{2k} w_k \\ y_k &= h(x_k, v_k, r) \end{cases} \tag{1}$$

where $x_k \in R^n$ is the state sequence, $y_k \in R^l$ is the controlled output sequence, $u_k \in R^p$ is the control sequence, $w_k \in R^q$ and $v_k \in R^l$ are the stochastic disturbance sequences, and r is the known reference input. E, A_k and B_{ik} $(i = 1, 2)$ are known time-varying matrices, where E satisfying $rank(E) = r < n$. It should be noted that the concerned plants can be generalized to more extensive nonlinear descriptor systems under some appropriate assumptions (see also *Remark 3*).

Definition 1. For a discrete-time linear descriptor system, a pair (E, A) is called regular if $\det(sE - A)$ is not identically zero and impulse-free if the degree of $\det(sE - A)$ is equal to $rank(E)$.

Similarly to most existing papers on descriptor systems, the following condition for admissibility is needed in this paper (see e. g. $(HL99, Tak98)$).

Assumption 1. (E, A) in Σ_D is regular and impulse-free at every sample time.

On the other hand, the involved stochastic disturbances w_k and v_k are supposed to be non-Gaussian but with known JPDFs. The following assumption is also quite general for the stochastic inputs, which have been shown to exist in many practical processes such as paper making systems (see e.g. $(Wang00)$).

Assumption 2. The random vectors w_k, v_k $(k = 0, 1, 2, \cdots)$ (and its elements) are bounded, continuous, independent mutually and with the known JPDFs $\gamma_w(\tau)$ and $\gamma_v(\tau)$ defined on $[a, b]^q$ and $[a, b]^l$, respectively.

The nonlinear function is required to satisfy the following condition.

Assumption 3. $h(\cdot, \cdot, \cdot)$ are known Borel measurable and smooth nonlinear functions of their augments, and $h(0, 0, 0) = 0$.

At each sample time k, random output variable y_k can be characterized by its JPDF $\gamma_{y_k}(\tau)$ defined on $[\alpha, \beta]^l$. Generally, y_k is non-Gaussian random vector even if w_k and v_k are Gaussian ones due to the existence of nonlinearity.

For any Σ_D satisfying *Assumption A.1*, there exist invertible matrices M and N such that

$$M^T EN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \; M^T B_{2k} = \begin{bmatrix} \widetilde{B}_{21k} \\ 0 \end{bmatrix}, \tag{2}$$

where x_k can be divided as $x_{ik} \in R^{n_i}$, $i = 1, 2$ with $n_1 + n_2 = n$, correspondingly. Thus, we can get the following result.

Lemma 1 If (E, A) is regular and impulse-free, then Σ_D can be transformed to be Σ_D^s, which is described by

$$\Sigma_D^s : \begin{cases} x_{1k+1} &= A_{1k} x_{1k} + B_{11k} u_k + B_{21k} w_k \\ x_{2k} &= A_{2k} x_{1k} + B_{12k} u_k \\ y_k &= H(x_{1k}, u_k, w_k, v_k, r) \end{cases} . \tag{3}$$

Proof. This proof is similar to Theorem 1 of (HL99) or (Tak98).

2.2 Problem formulation and calculations of JPDFs

In this paper, the tracking problem is considered for the multivariate stochastic descriptor systems described by (1) or (3). Different from a reference vector in classical tracking control theory, the tracked target is supposed to be a reference JPDF, which is denoted by $\gamma_d(\tau)$.

The control objective is to construct u_k such that the conditional JPDFs $\gamma_{y_k}(\tau|x_k, u_k)$, which is also a function of system output y_k and reference input r, satisfying

$$\gamma_{y_k}(\tau|x_k, u_k) \to \gamma_d(\tau), k \to +\infty.$$

Since the distance between $\gamma_{y_k}(\tau)$ and $\gamma_d(\tau)$ can be denoted by

$$\delta(\gamma_{y_k}, \gamma_d) := \int_\alpha^\beta \cdots \int_\alpha^\beta \left(\gamma_{y_k}(\tau) - \gamma_d(\tau)\right)^2 d\tau, \qquad (4)$$

the problem concerned in this paper can be formulated as follows.

The JPDF Tracking Control Problem is to find control strategies $u = g(x_k, r)$ such that

$$J_N := \sum_{k=0}^{N} \left[Q_k \delta(\gamma_{y_k}, \gamma_d) + \frac{1}{2} u_k^T R_k u_k\right] \qquad (5)$$

is minimized, and preferably, the closed loop system is stable, where $Q_k > 0$ and $R_k \geq 0$ are weighting matrices.

Remark 1 The control target using the distance of JPDFs is a new performance formulation existing in wide classes of engineering problems, where the desired output governs a known probabilistic distribution (see e. g. (Wang00)). It is noted that the measure for the distance in performance index J_N can be further generalized to L_p-distance, the Kullback-Leibler distance or entropy (Dev87, Pap91). Thus, the entropy optimization problem can also be dealt with similarly where $\delta(\gamma_{y_k}, \gamma_d)$ can be changed to entropy of the tracking errors. Also, the proposed problem can study the classical tracking problem for non-Gaussian systems.

To simplify the controller design procedures, we introduce the following assumption.

Assumption 4. It is supposed that the Jacobian $\Xi_k := \left|\det \frac{\partial H(x_k, u_k, w_k, v_k)}{\partial v_k}\right| \neq 0$.

Lemma 2 If *Assumptions 1˜4* hold, then we have

$$\gamma_{y_k}(\tau) = \int_a^b \gamma_v(H^{-1}(x_{1k}, u_k, \sigma, \tau))\gamma_w(\sigma) \cdot \left|\det \frac{\partial H(x_{1k}, u_k, \sigma, \tau)}{\partial \tau}\right|^{-1} d\sigma$$

where $H^{-1}(x_{1k}, u_k, \sigma, \tau)$ satisfies

$$\tau = H(x_{1k}, u_k, \sigma, H^{-1}(x_{1k}, u_k, \sigma, \tau)).$$

Proof. Based on Assumption 2 and Assumption 3, y_k $(k = 0, 1, 2, \cdots)$ are also continuous random vectors, then the proof can be given by using the total probability lemma in (pap91).

3. RECURSIVE JPDF TRACKING CONTROL STRATEGY

3.1 Design Based on Gradient Algorithms

In order to determine an optimal u_k, some optimization approaches can be applied to the JPDF control problem. For simplicity of notations, we suppose $Q_k = 1$ in (5) and only consider the explicit linear control strategies.

To provide recursive procedures, we denote

$$u_k = u_{k-1} + \Delta u_k, k = 1, 2, \cdots, N, \cdots, +\infty. \qquad (6)$$

Based on (4) and (2.2), as a conditional JPDF with respect to the previous states x_{ij}, $i = 1, 2$; $j = 0, 1, 2, \cdots, k-1$, disturbance inputs w_k, v_k and control inputs u_k, the function $\delta(\gamma_{y_k}, \gamma_d)$ can be approximated via

$$\delta(\gamma_{y_k}, \gamma_d) = \delta_0 + \delta_1 \Delta u_k + \Delta u_k^T \delta_2 \Delta u_k + o(\Delta u_k^2) \qquad (7)$$

where $\delta_0 := \delta(\gamma_{y_k}, \gamma_d)\big|_{u_k = u_{k-1}}$, $\delta_1 := \left.\frac{\partial \delta(\gamma_{y_k}, \gamma_d)}{\partial u_k}\right|_{u_k = u_{k-1}}$, $\delta_2 := \frac{1}{2} \left.\frac{\partial^2 \delta(\gamma_{y_k}, \gamma_d)}{\partial u_k^2}\right|_{u_k = u_{k-1}}$.

Theorem 1. Under *Assumptions 1∼4*, the recursive suboptimal JPDF control strategy to minimize the cost function J_N subject to descriptor system (3) is given by

$$\Delta u_k^* = -\frac{\delta_1 + R_k u_{k-1}}{2\delta_2 + 2R_k} \qquad (8)$$

where R_k satisfies $\delta_2 + R_k > 0$.

Proof. The optimal control strategy can be obtained via

$$\frac{\partial \left[\delta(\gamma_{y_k}, \gamma_d) + \frac{1}{2} u_k^T R_k u_k\right]}{\partial \Delta u_k} = 0. \qquad (9)$$

Based on Bellman's principle of optimality, the resulting control law leads to the optimal one for the whole process. From (6), it can be seen that

$$R_k u_k^2 = R_k u_{k-1}^2 + 2R_k u_{k-1} \Delta u_k + R_k \Delta u_k^2. \qquad (10)$$

Substituting (7) and (10) into (9), we can obtain recursive suboptimal control law (8) for $k = 1, 2, \cdots, N, \cdots, +\infty$.

It is noted that the above algorithm only results from a necessary condition for optimization. To guarantee sufficiency, the following second-order derivative should be satisfied

$$\frac{\partial^2 \left[\delta(\gamma_{y_k}, \gamma_d) + \frac{1}{2} u_k^T R_k u_k\right]}{\partial \Delta u_k^2} = 2\left(\delta_2 + R_k\right) > 0 \qquad (11)$$

which can be guaranteed if R_k is selected sufficient large.

3.2 Stabilization Control Strategy

It is noted that the proposed JPDF tracking control strategy resulting from the gradient approach leads to a nonlinear closed loop system. Generally stability analysis and synthesis are relatively difficult for stochastic nonlinear systems (see *e. g.* (Mao02, UP01, Wang02)), especially for multi-variable cases. In this section, an improved suboptimal optimization strategy will be proposed with which the closed loop system can be guaranteed to be stable.

For this purpose, under *Assumption 1*, we can also transform the state equation of system (1) to be (see also *e.g.*(LC99)

$$Ex_{k+1} = \overline{A}_k x_k + \overline{B}_{1k} u_k + \overline{B}_{2k} w_k\,, \qquad (12)$$

where $x_k := \left[\, x_{1k}^T \;\; x_{2k}^T \,\right]^T$ and

$$E := \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},\ \overline{A}_k := \begin{bmatrix} A_{1k} & 0 \\ 0 & -I \end{bmatrix},$$
$$\overline{B}_{1k} := \begin{bmatrix} B_{11k} \\ B_{12k} \end{bmatrix},\ \overline{B}_{2k} := \begin{bmatrix} B_{21k} \\ B_{22k} \end{bmatrix},$$

which leads to

$$E\Delta x_{k+1} = \overline{A}_k \Delta x_k + \overline{B}_{1k} \Delta u_k + \overline{B}_{2k} \Delta w_k\,, \qquad (13)$$

where

$$\Delta x_k = x_k - x_{k-1}, \Delta u_k = u_k - u_{k-1}, \Delta w_k = w_k - w_{k-1}.$$

Equation (12) is also called Weierstrass form of Σ_D. To consider the stabilization problem, we introduce the following concept.

Definition 2 If there exists a gain matrix C_k together with feedback law $u_k = C_k x_{1k}$ such that $Ex_{k+1} = \overline{A}_k x_k + \overline{B}_{1k} u_k$ is stable, regular and impulse-free, the triple $(E, \overline{A}_k, \overline{B}_{1k})$ denoted in (12) is called *generalized stabilizable* and $u_k = C_k x_{1k}$ is called *generalized stabilization control law.*

The following assumption is necessary for stabilization problems, which corresponds to the stabilizable condition.

Assumption 5. The triple $(E, \overline{A}_k, \overline{B}_{1k})$ in (12) is supposed to be *generalized stabilizable.*

Lemma 3. Under *Assumption 5*, $u_k = C_k x_{1k}$ is the *generalized stabilization control law* of (12) if and only if there exists a symmetric matrix P and a gain matrix C_k satisfying

$$\widetilde{A}_k^T P \widetilde{A}_k < E^T P E \qquad (14)$$

and $E^T P E \geq 0$, where

$$\widetilde{A}_k := \overline{A}_k + \overline{B}_{1k}\overline{C}_k, \overline{C}_k := [\, C_k \;\; 0\,]$$

which is compatible to x_k.

Proof. It can be given by connecting Definition 1 with Theorem 2 of (HL99).

Lemma 4. For the transformed system (12) satisfying *Assumption 5*, at each sample time k, both of the following statements hold: (i) linear matrix inequality (LMI)

$$\begin{bmatrix} -Q & QA_{1k}^T + S_1^T B_{11k}^T \\ A_{1k}Q + B_{11k}S_1 & -Q \end{bmatrix} < 0 \qquad (15)$$

is solvable for definite positive matrix $\dot{Q}$ and invertible matrix S_1. (ii) For given matrices Γ_{k1} and Γ_{k2},

$$\begin{bmatrix} -Q & P_k^T \\ P_k & -Q \end{bmatrix} < 0 \qquad (16)$$

where $P_k = A_{1k}Q + B_{11k}S_1 + B_{11k}S_2\Gamma_{k2}$ is solvable for definite positive matrices $\dot{Q}$ and S_2, and invertible matrix S_1. In both cases, $\Delta u_k = S_1 Q^{-1} \Delta x_{1k}$ is the generalized stabilization control law.

Proof. See (HL99).

3.3 Stabilization JPDF Tracking Control Strategy

At this stage, the tracking control strategy can be constructed by

$$\Delta u_k = C_{1k}\Delta x_{1k} + \Delta u_k^* \qquad (17)$$

where C_{1k} is the generalized stabilization control law given by *Lemma 4* , and Δu_k^* is the part for JPDF tracking control to be further determined.

Noting that $\delta(\gamma_{y_k}, \gamma_d)$ is also a function of Δx_{1k}, u_{k-1} and Δu_k, we consider the expansion

$$\delta(\gamma_{y_k}, \gamma_d) = \alpha_{k0} + \alpha_{k1}\Delta u_k + \alpha_{k2}\Delta x_{1k} + \frac{1}{2}\Delta u_k^T \Gamma_{k1} \Delta u_k$$
$$+ \Delta x_{1k}^T \Gamma_{k2} \Delta u_k + \frac{1}{2}\Delta x_{1k}^T \Gamma_{k3} \Delta x_{1k} + o(\Delta u_k^2, \Delta x_{1k}) \qquad (18)$$

where

$$\alpha_{k0} = \delta(\gamma_{y_k}, \gamma_d)|_{k-1}, \ \Gamma_{k1} = \left.\frac{\partial^2 \delta(\gamma_{y_k}, \gamma_d)}{\partial u_k^2}\right|_{k-1},$$
$$\alpha_{k1} = \left.\frac{\partial \delta(\gamma_{y_k}, \gamma_d)}{\partial u_k}\right|_{k-1}, \ \Gamma_{k2} = \left.\frac{\partial^2 \delta(\gamma_{y_k}, \gamma_d)}{\partial x_{1k} \partial u_k}\right|_{k-1}, \quad (19)$$
$$\alpha_{k2} = \left.\frac{\partial \delta(\gamma_{y_k}, \gamma_d)}{\partial x_{1k}}\right|_{k-1}, \ \Gamma_{k3} = \left.\frac{\partial^2 \delta(\gamma_{y_k}, \gamma_d)}{\partial x_{1k}^2}\right|_{k-1}$$

are functions with respect to sample value τ.

For flexibility of design, in the following, R_k will be determined at every sample step to guarantee stability of the closed loop systems.

Theorem 2. For the transformed system (12) satisfying *Assumptions 1˜5*, at each sample time k, if we select

$$R_k = -S_2^{-1} I - \Gamma_{k1} \geq 0 \quad (20)$$

where S_2 is calculated via (16), then the closed loop descriptor system is stable, and

$$\Delta u_k^* = -(\Gamma_{k1} + R_k)^{-1} \left(\Gamma_{k2}^T \Delta x_{1k} + \alpha_{k1}^T + R_k u_{k-1}\right) \quad (21)$$

together with (17) forms the stabilization JPDF tracking control strategy, where C_{1k} can be calculated by *Lemma 4*.

Proof. Equation(21) can be obtained by substituting (18) into (9) and removing the second-order terms. Substituting (21) into (13) yields the closed loop system described by

$$\begin{cases} \Delta x_{1k+1} = D_{1k} \Delta x_{1k} + B_{21k} \Delta w_k - p_{1k} \\ \Delta x_{2k} = D_{2k} \Delta x_{1k} + B_{22k} \Delta w_k - p_{2k} \end{cases} \quad (22)$$

where

$$D_{1k} = A_{1k} + B_{11k} C_k - B_{11k} (\Gamma_{k1} + R_k)^{-1} \Gamma_{k2}^T,$$
$$D_{2k} = A_{2k} + B_{21k} C_k - B_{21k} (\Gamma_{k1} + R_k)^{-1} \Gamma_{k2}^T$$

and $p_{ik} = B_{i1k} (\Gamma_{k1} + R_k)^{-1} (\alpha_{k1}^T + R_k u_{k-1}), i = 1, 2$, can be regarded as an additive bounded inputs. Simalarly to proof of *Lemma 4*, it can be claimed that stability of descriptor system (22) depends on

$$A_{1k} + B_{11k} C_k - B_{11k} (\Gamma_{k1} + R_k)^{-1} \Gamma_{k2}^T$$
$$= A_{1k} + B_{11k} C_k + B_{11k} S_2 \Gamma_{k2}^T$$

whoes stability has been guaranteed based on *Lemma 4*.

Remark 2. The recursive suboptimal stabilization JPDF tracking control algorithms can be summarized as follows:

- Initialize x_0 and u_0;
- At the sample time k, compute C_k and S_{2k} by using (16);
- Formulate $\gamma_{y_k}(\tau)$ using Lemma 2 and $\delta(\gamma_{y_k}, \gamma_d)$ using (4);
- Calculate Δu_k^* and u_k using equation (21) and (6);
- Increase k by 1 to the next step.

Remark 3. In practice, generally nonlinearity also exists in many descriptor systems. In this paper, besides the above linear plants (1) or (3), the following nonlinear descriptor systems

$$\begin{cases} x_{1k+1} = f_{1k}(x_{1k}) + g_{11}(x_{1k}) w_k + g_{21}(x_{1k}) u_k \\ x_{2k} = f_{2k}(x_{1k}) + g_{21}(x_{1k}) w_k + g_{22}(x_{1k}) u_k \end{cases} \quad (23)$$

with a linear controlled output

$$y_k = H_k(x_{1k}, w_k, u_k, v_k, r) \quad (24)$$

can also be studied similarly, where x_k, x_{ik} $(i = 1, 2)$, w_k, u_k, v_k, y_k and r are defined as above.

4. A NUMERAL EXAMPLE

In this section we give an example to demonstrate how to compute the JPDF tracking control laws. The considered descriptor system is given by

$$\begin{cases} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11k+1} \\ x_{12k+1} \\ x_{2k+1} \end{bmatrix} = \begin{bmatrix} 0.95 & 0 & 0 \\ 1 & -0.90 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_{11k} \\ x_{12k} \\ x_{2k} \end{bmatrix} \\ \qquad + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ -0.50 \\ 0 \end{bmatrix} w_k, \\ y_k = \dfrac{1}{1 + x_{11k}^3} u_k + x_{12k} - 2x_{2k} + \sqrt{v_k}. \end{cases}$$

This system has been transferred to be a Weierstrass form. Random variables w_k and v_k $(k = 0, 1, 2, \cdots)$ are assumed to be mutually independent with known PDFs. The asymmetric PDF of v_k is defined by

$$\gamma_v(x) = \begin{cases} \dfrac{-6(x - 0.25)}{\sqrt{x}} & x \in [0, 0.25], \\ 0 & else. \end{cases} \quad (25)$$

while the PDF of w_k is given by

$$\gamma_w(x) = \begin{cases} -48(x^2 - x + \dfrac{3}{16}) & x \in [0.25, 0.75], \\ 0 & else. \end{cases} \quad (26)$$

In simulations, the initial conditions are set to be $u_0 = 0, y_0 = -0.4$. For every sample time k, the sequence of the suboptimal control input is demonstrated in *Figure 1*. To illustrate the effectiveness of the proposed method, the practical output JPDFs under control at several different sample times as well as the tracked target JPDF are given in *Figure 2* for comparisons. *Figure 3* displays the outputs and the tracking errors where it can be seen that the tracking errors decrease to a small value in a short time. These simulation results show that satisfactory dynamical tracking performance can be obtained using the proposed approach.

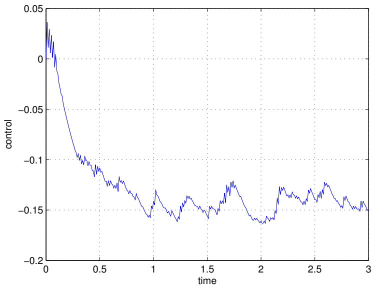

Fig. 1. JPDF control

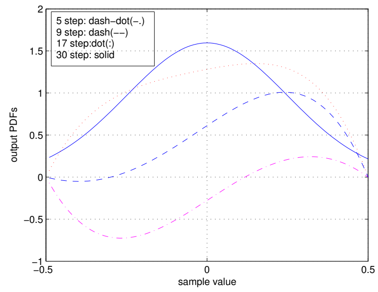

Fig. 2. the JPDF of the output

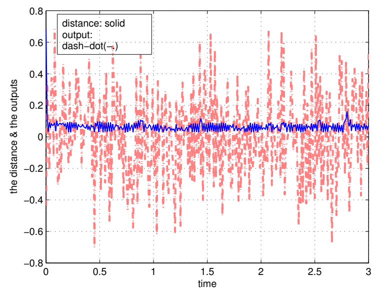

Fig. 3. the outputs and the tracking errors

5. CONCLUSION

In this paper, a new JPDF tracking control problem is considered for multivariate descriptor systems with non-Gaussian random inputs. Optimization approach is applied to present recursive algorithms such that the distances between the output distributions and the desired one are minimized. Furthermore, a stabilization suboptimal control strategy can be given by using of LMI-based Lyapunov theory. Simulation shows that the proposed approach can achieve satisfactory performance for non-Gaussian descriptor systems.

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