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Design of adaptive sliding mode controller (ASM) for a distillation column

Pinak Pani Biswas*. Subhabrata Ray**
Amar Nath Samanta***

*Indian Institute of Technology Kharagpur
India (e-mail: biswaspinakin@che.iitkgp.ernet.in).
** Indian Institute of Technology Kharagpur
India (Tel: +91 - 3222 - 283948; e-mail: sray@che.iitkgp.ernet.in).
*** Indian Institute of Technology Kharagpur
India (Tel: +91 - 3222 - 283944; e-mail: amar@che.iitkgp.ernet.in).

Abstract: In this work adaptive sliding mode controller is designed and implemented on a simulated high purity binary distillation column. The sliding mode controller design procedure is composed of approximate linearization and recursive backstepping approach. This makes the controller capable of eliminating the destabilizing effect of unknown structured plant parameter and uncertainty due to process model mismatch. Each of the first n-1 virtual control law is designed using zero order sliding mode controller to eliminate unmatched uncertainty. In the final step general sliding mode controller is used for eliminating the matched uncertainty of the process. The proposed control law also guarantees the exact output tracking in the presence of unknown unstructured process parameter.

1. Introduction

Variable structure control (VSC) with sliding mode, commonly known as sliding mode control (SMC), was first proposed in the early 1950's in Russia by (Emel'yanov., S.V., 1959) During the last decade, researchers showed significant interest in the design and implementation of sliding mode control strategies for a wide spectrum of system types including Non-linear, Multi–Input/Multi–Output systems (MIMO) and Stochastic Systems (DeCarlo and Matthews., 1988;Edwards and Spurgeon., 1998; Utkin.,1992). The basic philosophy of Sliding mode control is to move the system states from any initial state on to a user-defined surface in the state space (switching surface) using high-speed switching control law and to maintain the states on that surface. This results in a system whose dynamics is governed only by the parameters that describe the sliding surface and is insensitive to parametric uncertainties and external disturbances.

2. Process description

Dual composition control of distillation column has been studied extensively by many researchers. The rigorous distillation column model is described in the work of (Choe and luyben, 1987, Skogetad, 1997) and is not included here. This distillation column, separating methanol water mixture, is simulated here and used as process for implementing the proposed controller.

3. Representation of uncertainty

For uncertain system, we can write the system as:

$$\dot{x} = f(x) + g(x)u + \psi\theta + \Delta(x); \qquad y = h(x) \tag{1.1}$$

Here $\theta \in R^s$ are the unknown parameter vectors. $\psi : R^n \to R^{n\times s}$ are the known functions. $\Delta : R^n \to R^n$ are the unknown vectors whose magnitude can be approximated by looking at the process uncertainty. Parameter independent diffeomorphism results in:

$$\begin{aligned}
\dot{z}_{11} &= z_{12} + \theta^T\psi_{11} + \Delta_{11} \\
&\vdots \\
\dot{z}_{1(\gamma_1-1)} &= z_{1\gamma_1} + \theta^T\psi_{1(\gamma_1-1)} + \Delta_{1(\gamma_1-1)} \\
\dot{z}_{1\gamma_1} &= f_1(z,\eta) + g_1(z,\eta)u + \theta^T\psi_{1\gamma_1} + \Delta_{1\gamma_1} \\
\dot{z}_{21} &= z_{22} + \theta^T\psi_{21} + \Delta_{21} \\
&\vdots \\
\dot{z}_{2(\gamma_2-1)} &= z_{2\gamma_2} + \theta^T\psi_{2(\gamma_2-1)} + \Delta_{2(\gamma_2-1)} \\
\dot{z}_{2\gamma_2} &= f_2(z,\eta) + g_2(z,\eta)u + \theta^T\psi_{2\gamma_2} + \Delta_{2\gamma_2} \\
\dot{z}_{31} &= z_{32} + \theta^T\psi_{31} + \Delta_{31} \\
&\vdots \\
\dot{z}_{p\gamma_p} &= f_p(z,\eta) + g_p(z,\eta)u + \theta^T\psi_{p\gamma_p} + \Delta_{p\gamma_p} \\
\dot{\eta}_1 &= \mathrm{T}_1(z,\eta) + \Upsilon_1(z,\eta)u + \theta^T\phi_1 \\
\dot{\eta}_2 &= \mathrm{T}_2(z,\eta) + \Upsilon_2(z,\eta)u + \theta^T\phi_2 \\
&\vdots \\
\dot{\eta}_{(n-m)} &= \mathrm{T}_{(n-m)}(z,\eta) + \Upsilon_{(n-m)}(z,\eta)u + \theta^T\phi_{(n-m)} \\
y_1 &= z_{11} \\
y_2 &= z_{21} \\
&\vdots \\
y_p &= z_{p\gamma_p}
\end{aligned} \tag{1.2}$$

4. Sliding mode controller for uncertain system

The design procedure is recursive (Zinober and Liu, 1996). At its ij-th step the respective subsystem is stabilized with respect to a Lyapunov function V_{ij}. The stabilizing function

α_{ij} and a tuning function τ_{ij} are obtained by the stabilization of Lyapunov function. The overall Lyapunov function can be viewed as $V = \sum_{i=1}^{p}\sum_{j=1}^{\gamma_i} V_{ij} \le -\sum_{i=1}^{p}\sum_{j=1}^{\gamma_i-1} c_{ij}\xi_{ij}^2$.

Assume co-ordinate transformation ξ as

$$\begin{aligned}
\xi_{11} &= z_{11} - y_{1r} \\
\xi_{12} &= z_{12} - \alpha_{11} \\
\xi_{13} &= z_{13} - \alpha_{12} \\
&\vdots \\
\xi_{1(\gamma_1-1)} &= z_{1(\gamma_1-1)} - \alpha_{1(\gamma_1-2)} \\
\xi_{1\gamma_1} &= z_{1\gamma_1} \\
\xi_{21} &= z_{21} - y_{2r} \\
\xi_{22} &= z_{22} - \alpha_{21} \\
&\vdots \\
\xi_{p\gamma_p} &= z_{p\gamma_p} - \alpha_{p(\gamma_p-1)}
\end{aligned} \tag{1.3}$$

Where $\{y_{1r} \quad \cdots \quad y_{pr}\}$ are the reference trajectory.

Step 1: according to Eqn. (1.3)

$\xi_{11} = z_{11} - y_{1r}$ and $\xi_{12} = z_{12} - \alpha_{11}$

$$\dot{z}_{11} = z_{12} + \theta^T \psi_{11} + \Delta_{11} \tag{1.4}$$

Substituting the value of z_{11} we get

$$\dot{\xi}_{11} = \xi_{12} + \alpha_{11} + \theta^T\psi_{11} + \Delta_{11} - \dot{y}_{1r} \tag{1.5}$$

consider the first Lyapunov function as

$$V_{11} = \frac{1}{2}\xi_{11}^{\ 2} + \frac{1}{2}\tilde{\theta}^T \Gamma \tilde{\theta} \tag{1.6}$$

differentiating above we get

$$\begin{aligned}
\dot{V}_{11} &= \xi_{11}\left(\xi_{12} + \hat{\theta}^T\psi_{11} + \Delta_{11} + \alpha_{11} - \dot{y}_{1r}\right) \\
&+ \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} + \Gamma\psi_{11}\xi_{11}\right)
\end{aligned} \tag{1.7}$$

Here $\hat{\theta}$ is the estimated value of adaptive parameter, the error between actual and estimated parameter can be expressed as $\tilde{\theta} = \hat{\theta} - \theta$ and the derivative of that becomes $\dot{\tilde{\theta}} = \dot{\hat{\theta}} - 0$.

Let $\alpha_{11} = -\hat{\theta}^T\psi_{11} - c_{11}\xi_{11} + \dot{y}_{1r} - \rho_{11} sign(\xi_{11})$ (1.8)

and $\tau_{11} = \Gamma\psi_{11}\xi_{11}$ (1.9)

$$where, \quad sign(\xi_{11}) = \begin{cases} 1 & for\, \xi_{11} > 0 \\ 0 & for\, \xi_{11} = 0 \\ -1 & for\, \xi_{11} < 0 \end{cases}$$

Parameter update law becomes

$$\dot{\hat{\theta}} = \tau_{11} \tag{1.10}$$

using (1.8) and (1.9) we get

$$\begin{aligned}
&\dot{V}_{11} = -c_{11}\xi_{11}^{\ 2} + \xi_{11}\xi_{12} + \tilde{\theta}^T\Gamma^{-1}\left(\tilde{\theta} + \tau_{11}\right) + \left(\rho_{11}\xi_{11} - \rho_{11}\xi_{11} sign(\xi_{11})\right) \\
&we\ know\ \xi_{11} sign(\xi_{11}) = |\xi_{11}| \\
&\dot{V}_{11} = -c_{11}\xi_{11}^{\ 2} + \xi_{11}\xi_{12} + \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} + \tau_{11}\right) + \left(\rho_{11}\xi_{11} - \rho_{11}|\xi_{11}|\right) \\
&\dot{V}_{11} \le -c_{11}\xi_{11}^{\ 2} + \xi_{11}\xi_{12} + \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} + \tau_{11}\right)
\end{aligned} \tag{1.11}$$

Step 2: $V_{12} = V_{11} + \frac{1}{2}\xi_{12}^{\ 2}$ (1.12)

$$\begin{aligned}
\dot{V}_{12} &\le -c_{11}\xi_{11}^{\ 2} + \xi_{11}\xi_{12} + \tilde{\theta}^T\Gamma^{-1}\left(\tilde{\theta} + \tau_{11}\right) + \xi_{12}\left(\dot{z}_{12} - \dot{\alpha}_{11}\right) \\
&\le \dot{V}_{11} + \xi_{12}\left(\xi_{13} + \alpha_{12} + \Delta_{12} + \theta^T\psi_{12} - \dot{\alpha}_{11}\right) \\
&\le \dot{V}_{11} + \xi_{12}\left(\xi_{13} + \alpha_{12} + \rho_{12} + \theta^T\psi_{12} - \dot{\alpha}_{11}\right)
\end{aligned} \tag{1.13}$$

Where

$$\begin{aligned}
&\dot{\alpha}_{11} = \sum_{i=1}^{p}\sum_{j=1}^{\gamma_i-1}\frac{\partial\alpha_{11}}{\partial z_{ij}}\left(z_{i,(j+1)} + \hat{\theta}^T\psi_{ij} + \Delta_{ij}\right) + \sum_{i=1}^{p}\sum_{j=\gamma_i}^{\gamma_i}\frac{\partial\alpha_{11}}{\partial z_{ij}}\left(f_i + g_i u + \Delta_{ij} + \hat{\theta}^T\psi_{ij}\right) + \\
&\sum_{l=1}^{n-m}\frac{\partial\alpha_{11}}{\partial\eta_l}\left(\mathrm{T}_l(\cdot) + \Upsilon_l(\cdot)u + \theta^T\phi_l(x,u)\right) + \frac{\partial\alpha_{11}}{\partial\hat{\theta}}\tau_{12} + \frac{\partial\alpha_{11}}{\partial y_{1r}}\dot{y}_{1r} \\
&\tau_{12} = \tau_{11} + \left(\Gamma\psi_{12} + \left(\sum_{i=1}^{p}\sum_{j=1}^{\gamma_i}\frac{\partial\alpha_{11}}{\partial z_{ij}}\psi_{ij} + \sum_{l=1}^{n-m}\frac{\partial\alpha_{11}}{\partial\eta_l}\phi_l\right)\right)\xi_{12} \\
&\dot{\hat{\theta}} = \tau_{12}
\end{aligned}$$

(1.14)

Now the choice of

α_{12} is $\alpha_{12} = -\xi_{11} - \hat{\theta}^T\psi_{12} - c_{12}\xi_{12} - \rho_{12} sign(\xi_{12}) + \dot{\alpha}_{11}$ (1.15)

If we substitute value of (1.15) in (1.13) the final form is

$$\begin{aligned}
\dot{V}_{12} &\le -c_{11}\xi_{11}^{\ 2} - c_{12}\xi_{12}^{\ 2} + \xi_{12}\xi_{13} + \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} + \tau_{12}\right) \\
&\quad + \left(\rho_{12}\xi_{12} - \rho_{12} sign(\xi_{12})\right)
\end{aligned} \tag{1.16}$$

$$=> \dot{V}_{12} \le -c_{11}\xi_{11}^{\ 2} - c_{12}\xi_{12}^{\ 2} + \xi_{12}\xi_{13} + \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} + \tau_{12}\right)$$

Step 3: $i = 1, j = 1\ to\ \gamma_1 - 1$

$$\begin{aligned}
&\alpha_{1(j-1)} = -\xi_{1(j-2)} - \hat{\theta}^T\psi_{1(j-1)} - c_{1(j_1-1)}\xi_{1(\gamma_1-1)} - \rho_{1(\gamma_1-1)} sign(\xi_{1(j-1)}) + \dot{\alpha}_{1(j_1-2)} \\
&where\ \dot{\alpha}_{1(j-1)} = \sum_{m=1}^{p}\sum_{n=1}^{\gamma_m-1}\frac{\partial\alpha_{1(j-1)}}{\partial z_{mn}}\left(z_{m,(n+1)} + \hat{\theta}^T\psi_{mn} + \Delta_{mn}\right) + \\
&\sum_{m=1}^{p}\sum_{n=\gamma_m}^{\gamma_m}\frac{\partial\alpha_{1(j-1)}}{\partial z_{mn}}\left(f_m + g_m u + \Delta_{mn} + \hat{\theta}^T\psi_{mn}\right) + \\
&\sum_{l=1}^{n-m}\frac{\partial\alpha_{1(j-1)}}{\partial\eta_l}\left(\mathrm{T}_l(\cdot) + \Upsilon_l(\cdot)u + \theta^T\phi_l(x,u)\right) + \frac{\partial\alpha_{1(j_1-1)}}{\partial\hat{\theta}}\tau_{1j} \\
&\tau_{1j} = \tau_{1(j-1)} + \left(\Gamma\psi_{1j} + \left(\sum_{m=1}^{p}\sum_{n=1}^{\gamma_m}\frac{\partial\alpha_{1(j-1)}}{\partial z_{mn}}\psi_{mn} + \sum_{l=1}^{n-m}\frac{\partial\alpha_{1(j-1)}}{\partial\eta_l}\phi_l\right)\right)\xi_{1j} \\
&\dot{\hat{\theta}} = \tau_{1j}
\end{aligned} \tag{1.17}$$

The Lyapunov function becomes

$$V_{1j} = -\sum_{r=1}^{j-1} c_{1r}\xi_{1r}^{\ 2} + \tau_{1j} + \xi_{1j}\xi_{1(j-1)} \tag{1.18}$$

Step 4: $i = 1$, $j = \gamma_1$:

Now we start designing sliding surface which appears in the final equation recursively. The control algorithm can be deduced easily by maintaining stable sliding surface.

$$\begin{aligned}
V_{1j} &= V_{1(j-1)} + \frac{1}{2}s_1^{\ 2} \\
\dot{V}_{1j} &= \dot{V}_{1(j-1)} + s_1\dot{s}_1
\end{aligned} \tag{1.19}$$

Where s_1 is the sliding surface with respect to output y_1.

The sliding surface is chosen such that

$$s_1 = \xi_{1\gamma_1} + \sum_{i=1}^{\gamma_1 - 1} k_{1i}\xi_i \tag{1.20}$$

Putting it in equation (1.19) we get

$$\dot{V}_{1j} = \dot{V}_{1(j-1)} + s_1\dot{s}_1$$

$$s_1\dot{s}_1 = s_1\begin{pmatrix} f_1 + g_1 u + \hat{\theta}^T\psi_{1\gamma_1} + \Delta_{1\gamma_1} \\ +\sum_{i=1}^{\gamma_1-1}\left(z_{1(i+1)} + \hat{\theta}^T\psi_{1i} + \Delta_{1i} - \dot{\alpha}_{1(i-1)}\right) - k_{11}\dot{y}_{1r}\end{pmatrix} \tag{1.21}$$

$$\alpha_{1(\gamma_1-1)} = \begin{pmatrix} -\xi_{1(\gamma_1-2)} - \hat{\theta}^T\psi_{1(\gamma_1-1)} - c_{1(\gamma_1-1)}\xi_{1(\gamma_1-1)} - \\ \rho_{1(\gamma_1-1)} sign(\xi_{1(\gamma_1-1)}) + \dot{\alpha}_{1(j_1-2)}\end{pmatrix}$$

where

$$\dot{\alpha}_{1(\gamma_1-1)} = \begin{pmatrix} \left(\sum_{m=1}^{p}\sum_{n=1}^{\gamma_m-1}\frac{\partial\alpha_{1(\gamma_1-1)}}{\partial z_{mn}}\left(z_{m,(n+1)} + \hat{\theta}^T\psi_{mn} + \Delta_{mn}\right) + \right. \\ \left.\sum_{m=1}^{p}\sum_{n=\gamma_m}^{\gamma_m}\frac{\partial\alpha_{1(\gamma_1-1)}}{\partial z_{mn}}\left(f_m + g_m u + \Delta_{mn} + \hat{\theta}^T\psi_{mn}\right)\right) \\ +\sum_{l=1}^{n-m}\frac{\partial\alpha_{1(\gamma_1-1)}}{\partial\eta_l}\left(\mathrm{T}_l(\cdot) + \Upsilon_l(\cdot)u + \theta^T\phi_l(x,u)\right) + \frac{\partial\alpha_{1(\gamma_1-1)}}{\partial\hat{\theta}}\tau_{1\gamma_1}\end{pmatrix} \tag{1.22}$$

$$\tau_{1\gamma_1} = \tau_{1(\gamma_1-1)} + \left(\Gamma\psi_{1\gamma_1} + \left(\sum_{i=1}^{\gamma_1-1}k_{1i}\psi_{1i}\right)\right)s_1$$

$$\dot{\hat{\theta}} = \tau_{1\gamma_1}$$

Equation (1.21) becomes

$$\dot{V}_{1\gamma_1} = -\sum_{r=1}^{\gamma_1-1}c_{1r}\xi_{1r}^{\ 2} + \tau_{1\gamma_1} + s_1\dot{s}_1$$

Step 5: $i = 1\ to\ p,\ j = 1\ to\ \gamma_1 - 1$

Similarly from above argument we get.

$$\alpha_{i(j-1)} = -\xi_{i(\gamma_1-2)} - \hat{\theta}^T\psi_{i(j-1)} - c_{i(j-1)}\xi_{i(j-1)} - \rho_{i(j-1)}sign(\xi_{i(j-1)}) + \dot{\alpha}_{i(j-2)}$$

$$\begin{aligned}\dot{\alpha}_{i(j-1)} &= \sum_{m=1}^{p}\sum_{n=1}^{\gamma_m-1}\frac{\partial\alpha_{i(j-1)}}{\partial z_{mn}}\left(z_{m,(n+1)} + \hat{\theta}^T\psi_{mn} + \Delta_{mn}\right) + \\ &\sum_{m=1}^{p}\sum_{n=\gamma_m}^{\gamma_m}\frac{\partial\alpha_{i(j-1)}}{\partial z_{mn}}\left(f_m + g_m u + \Delta_{mn} + \hat{\theta}^T\psi_{mn}\right) \\ &+\sum_{l=1}^{n-m}\frac{\partial\alpha_{i(j-1)}}{\partial\eta_l}\left(\mathrm{T}_l(\cdot) + \Upsilon_l(\cdot)u + \theta^T\phi_l(x,u)\right) + \frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}}\tau_{1j}\end{aligned} \tag{1.23}$$

$$\tau_{ij} = \tau_{i(j-1)} + \left(\Gamma\psi_{i\gamma j} + \left(\sum_{m=1}^{p}\sum_{n=1}^{\gamma_m}\frac{\partial\alpha_{i(j-1)}}{\partial z_{mn}}\psi_{mn} + \sum_{l=1}^{n-m}\frac{\partial\alpha_{i(j-1)}}{\partial\eta_l}\phi_l\right)\right)\xi_{ij}$$

So the Lyapunov function becomes

$$\dot{V}_{i(\gamma_1-1)} = -\sum_{i=1}^{p}\sum_{r=1}^{\gamma_1-1}c_{ir}\xi_{1r}^{\ 2} + \tau_{i(\gamma_1-1)} + \xi_{i(\gamma_1-1)}\xi_{i(\gamma_1-2)} \tag{1.24}$$

Step 6: $i = p,\ j = \gamma_p$

$$\begin{aligned}V_{p\gamma_p} &= V_{p(\gamma_p-1)} + \frac{1}{2}\sum_{i=1}^{p}s_i^2 \\ \dot{V}_{p\gamma_p} &= \dot{V}_{p(\gamma_p-1)} + \sum_{i=1}^{p}\dot{s}_i s_i \\ \dot{V}_{p\gamma_p} &= \begin{pmatrix} -\sum_{i=1}^{p}\sum_{j=1}^{\gamma_i-1}c_{ij}\xi_{ij}^2 + \\ \begin{bmatrix}\sum_{i=1}^{p}\left(f_{i\gamma_i} + \left(\sum_{l=1}^{p}g_{l\gamma_r}u_l\right) + \hat{\theta}^T\psi_{i\gamma_i} + \Delta_{i\gamma_i} - \dot{\alpha}_{i(\gamma_i-1)}\right) + \sum_{i=1}^{p}\sum_{j=1}^{\gamma_i-1}k_{ij}\left(\xi_{ij} + \alpha_{i(j-1)} + \Delta_{i(j-1)} + \hat{\theta}^T\psi_{ij} - \dot{\alpha}_{i(j-2)}\right) \\ +k_{i(\gamma_{(i-1)}+1)}\dot{y}_{ir}\end{bmatrix}s_i\end{pmatrix}\end{aligned} \tag{1.25}$$

Now

$$\alpha_{p(\gamma_p-1)} = \begin{pmatrix} -\xi_{p(\gamma_p-2)} - \hat{\theta}^T\psi_{p(\gamma_p-1)} - c_{p(\gamma_p-1)}\xi_{p(\gamma_1-1)} \\ -\rho_{p(\gamma_p-1)}sign(\xi_{p(\gamma_p-1)}) + \dot{\alpha}_{p(\gamma_p-2)}\end{pmatrix}$$

$$\dot{\alpha}_{p(\gamma_p-1)} = \begin{pmatrix} \sum_{m=1}^{p}\sum_{n=1}^{\gamma_m-1}\frac{\partial\alpha_{p(\gamma_p-1)}}{\partial z_{mn}}\left(z_{m,(n+1)} + \hat{\theta}^T\psi_{mn} + \Delta_{mn}\right) + \\ \sum_{m=1}^{p}\sum_{n=\gamma_m}^{\gamma_m}\frac{\partial\alpha_{p(\gamma_p-1)}}{\partial z_{mn}}\left(f_m + g_m u + \Delta_{mn} + \hat{\theta}^T\psi_{mn}\right) \\ +\sum_{l=1}^{n-m}\frac{\partial\alpha_{p(\gamma_p-1)}}{\partial\eta_l}\left(\mathrm{T}_l(\cdot) + \Upsilon_l(\cdot)u + \theta^T\phi_l(x,u)\right) \\ +\frac{\partial\alpha_{p(\gamma_p-1)}}{\partial\hat{\theta}}\tau_{1j}\end{pmatrix} \tag{1.26}$$

$$\tau_{p\gamma_p} = \tau_{p(\gamma_p-1)} + \sum_{i=1}^{p}\left(\Gamma\psi_{p\gamma_p} + \sum_{l=1}^{\gamma_l-1}k_{il}\psi_{il}\right)s_i$$

We know for stable sliding surface $\sum_{i=1}^{p}\dot{s}_i s_i$ should be less than zero. So keeping it mind we can deduce control law such that $\sum_{i=1}^{p}\dot{s}_i s_i < 0$ and the equation (1.25) becomes

$$\dot{V}_{p\gamma_p} \le \dot{V}_{p(\gamma_p-1)} \tag{1.27}$$

$$=> \dot{V}_{p\gamma_p} \le 0$$

The control law obtained as

$$u_{l(l=1,..,p)} = \begin{bmatrix}u_1 \\ \vdots \\ u_p\end{bmatrix} = \left(\begin{bmatrix}g_{11} & \cdots & g_{1\gamma_1} \\ \vdots & \ddots & \vdots \\ g_{p1} & \cdots & g_{p\gamma_p}\end{bmatrix}^{-1}\left\{\begin{matrix}-\begin{bmatrix}k_{11}\xi_{12} & \cdots & k_{1(\gamma_1-1)}\xi_{1\gamma_1} \\ \vdots & \ddots & \vdots \\ k_{p1}\xi_{p2} & \cdots & k_{p(\gamma_p-1)}\xi_{p\gamma_p}\end{bmatrix} - \begin{bmatrix}\dot{\alpha}_{1(\gamma_1-1)} \\ \vdots \\ \dot{\alpha}_{p(\gamma_p-1)}\end{bmatrix}\hat{\theta}^T\psi - K\alpha + \dot{\bar{\alpha}} - KY + K\dot{\alpha} - K_\rho \\ -\rho sign(S) - \begin{bmatrix}f_{11} \\ \vdots \\ f_{p\gamma_p}\end{bmatrix}\end{matrix}\right\}\right)$$

$$where\ K = \begin{bmatrix}k_{11} & \cdots & k_{1(\gamma_1-1)} \\ \vdots & \ddots & \vdots \\ k_{p1} & \cdots & k_{p(\gamma_p-1)}\end{bmatrix}, \alpha = \begin{bmatrix}0 & \alpha_{11} & \cdots & \alpha_{1(\gamma_1-1)} \\ \vdots & & \ddots & \vdots \\ 0 & \alpha_{p1} & \cdots & \alpha_{p(\gamma_p-1)}\end{bmatrix}, \dot{\alpha} = \begin{bmatrix}0 & 0 & \dot{\alpha}_{12} & \cdots & \dot{\alpha}_{1(\gamma_1-1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dot{\alpha}_{p2} & \cdots & \dot{\alpha}_{p(\gamma_p-1)}\end{bmatrix}, \dot{\bar{\alpha}} = \begin{bmatrix}\alpha_{1(\gamma_1-1)} \\ \alpha_{2(\gamma_2-1)} \\ \vdots \\ \alpha_{p(\gamma_p-1)}\end{bmatrix}$$

$$K_\rho = \begin{bmatrix} k_{11}sign(\xi_{11}) & \cdots & k_{1(\gamma_1-1)}sign(\xi_{1(\gamma_1-1)}) \\ \vdots & \ddots & \vdots \\ k_{p1}sign(\xi_{p1}) & \cdots & k_{p(\gamma_p-1)}sign(\xi_{p(\gamma_p-1)}) \end{bmatrix}, \quad (1.28)$$

$$Y_r = \begin{bmatrix} \dot{y}_{1r} & 0 & \cdots & 0 \\ 0 & \dot{y}_{2r} & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \dot{y}_{pr} \end{bmatrix} \hat{\theta}^T\psi = \begin{bmatrix} \hat{\theta}^T\psi_{11} \\ \hat{\theta}^T\psi_{22} \\ \vdots \\ \hat{\theta}^T\psi_{p\gamma_p} \end{bmatrix},$$

$$\rho sign(S) = \begin{bmatrix} \rho_{1\gamma_1}sign(s_1) \\ \rho_{2\gamma_2}sign(s_2) \\ \vdots \\ \rho_{p\gamma_p}sign(s_p) \end{bmatrix}$$

And the parameter update law is

$$\dot{\hat{\theta}}^T = \tau_{p\gamma_p}, \tau_{p\gamma_p} = \tau_{p(\gamma_p-1)} + \sum_{i=1}^{p}\left(\Gamma\psi_{p\gamma_p} + \sum_{l=1}^{\gamma_i-1}k_{il}\psi_{il}\right)s_i \quad (1.29)$$

The above control law turns the Final Lyapunov function to our goal

$$V \le -\sum_{i=1}^{p}\sum_{j=1}^{\gamma_i-1}c_{ij}\xi_{ij}^2 \quad (1.30)$$

5. Sliding mode controller for Distillation Column

The sliding control law for the distillation column is derived based on the theory proposed in section 5. Assumptions of constant hold-ups and ideal VLE simplify the system model considerably. This will evidently create considerable mismatch in process and the system model used in the controller.

The system model can be written as

Bottom and reboiler

$$\frac{d(M_b x_b)}{dt} = L_2x_2 - Bx_b - Vy_b \quad (1.31)$$

Condenser and Accumulator

$$\frac{d(M_{nt}x_{nt})}{dt} = (V + (1-q_F)F)y_{nt-1} - Rx_{nt} - Dx_{nt} \quad (1.32)$$

$$\frac{d(M_{nt})}{dt} = (V + (1-q_F)F) - R - D = 0$$

Top Tray

$$\frac{d(M_{nt-1}x_{nt-1})}{dt} = V(y_{nt-2} - y_{nt-1}) + Rx_{nt} - L_{nt-1}x_{nt-1} \quad (1.33)$$

$$\frac{d(M_{nt-1})}{dt} = R - L_{nt-1}$$

Feed Tray

$$\frac{d(M_{nf}x_{nf})}{dt} = V(y_{nf-1} - y_{nf}) + L_{nf+1}x_{nf+1} - L_{nf}x_{nf} + Fz_f \quad (1.34)$$

$$\frac{d(M_{nf})}{dt} = L_{nf+1} - L_{nf} + Fz_f$$

Other Tray

$$\frac{d(M_i x_i)}{dt} = V(y_{i-1} - y_i) + L_{i+1}x_{i+1} - L_i x_i \quad (1.35)$$

$$\frac{d(M_i)}{dt} = L_{i+1} - L_i$$

$$y_i = \alpha x_i \quad (1.36)$$

$$L_i = \begin{cases} R & i > nf \\ R + q_F F & i \le nf \end{cases} \quad (1.37)$$

$$V = \begin{cases} V & i < nf \\ V + (1-q_F)F & i \ge nf \end{cases}$$

The possible sources of uncertainty of the process are

a. For considering $V_{ac} = (\theta_1 + V_o)$ where V_o is nominal vapor flow rate.

b. Relative Volatility $(\alpha = \alpha_o + \theta_2)$ as we consider linear vapor-liquid equilibrium $y = \alpha_0 x$ here.

c. Feed flow rate $(F = F_o + \theta_3)$.

d. Liquid fraction in feed $(q_F = q_{F_o} + \theta_4)$.

e. Feed composition (z_F).

f. Feed Temperature (T_{nf})

The constant parameters $F_o, \alpha_o, q_{F_o}, V_o$ are the nominal values of feed flow rate, liquid vapor equilibrium relation, vapor flow rate, liquid fraction in feed and feed composition respectively.

The parameter independent diffeomorphism

$$Z = \Phi(x) = [z_{11} \quad z_{12} \quad z_{21}] = \left[x_{nt} \quad \frac{V_o(\alpha_o x_{nt-1} - x_{nt})}{M_{nt}} \quad x_b\right]$$

is used because the overall relative degree of the system is $m = (\gamma_1 + \gamma_2) = (2+1) = 3$ for output $[y_{1r} \quad y_{2r}] = [z_{11} \quad z_{21}] = [x_{nt} \quad x_b]$.

By applying recursive law derived in section 4, the final control law becomes

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \left[-\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} - \begin{pmatrix} \rho_{12}sat(\rho_{12}/\varepsilon) \\ \rho_{21}sat(\rho_{21}/\varepsilon) \end{pmatrix} - \begin{pmatrix} \hat{\theta}^T\psi_{12} \\ \hat{\theta}^T\psi_{21} \end{pmatrix} + \begin{pmatrix} \dot{\alpha}_{11} \\ 0 \end{pmatrix} - \begin{pmatrix} k_{11}\left(\xi_{12} + \alpha_{11} + \hat{\theta}^T\psi_{11} + \rho_{11}sat(\rho_{11}/\varepsilon) - \dot{y}_{1r}\right) \\ k_{11}\xi_{11} - \dot{y}_{2r} \end{pmatrix} \right] \quad (1.38)$$

here ε is the design parameter.

6. Results & Discussion

The proposed adaptive sliding mode (ASM) controller is implemented on a simulated high purity distillation column and the performance of the controller is observed. The controller is further evaluated by forcing structured and unstructured uncertainty in the system. The sliding gains considered here depend on the choice of ρ. The nominal values of the adaptive parameters which are used in this study are $F_o = 4320,\quad \alpha_0 = 6,\quad z_{F_0} = .73,\quad q_{F_o} = 1,\quad T_{nf} = 395$.

Fig. 1 shows that the set point tracking performance for the set point changes in distillate compositions by adaptive design. The composition set points are changed at every 20 hour interval. Adaptive Sliding mode (ASM) tracked the set points efficiently.

6.1. Effect of uncertain feed flow rate

Disturbances in feed flow rate is considered as a deviation of a parameter value from its nominal value in the model used by the controller and therefore is an uncertainty to the system. Although disturbance in feed rate is expected to be having a small / limited magnitude of deviation with randomness, a fairly large (+22%) step change in feed, not uncommon as a load change, is considered here for severity. The effect of measured disturbance on controller performance is studied by implementing step increases in feed flow rate at time 50 hour. The sliding mode controller immediately estimated the uncertainty in terms of increased feed flow rate by around 930 units and accordingly took corrective action. It is evident from figure 2 that the product compositions stayed at their respective setpoints indicating a very good disturbance rejection capability by the controller.

6.2. Effect of uncertain feed composition

The effect of unmeasured disturbance on controller performance is being studied by implementing periodic change in feed composition. The amount of variation has given by setting the periodic signal at amplitude of +25% from its nominal value and a frequency of .1 hour. Feed compositions are generally considered as unstructured uncertainty as it never appears in the controller equation. Since the feed condition is liquid; the bottom composition was affected immediately by this periodic change. As evident from figure 3, the ASM controller shows good disturbance rejection capability in spite of no feedback information on feed composition.

6.3. Effect of uncertain parameters

Robustness in the parameter (α) can be achieved by changing the operating pressure in the column. The pressure set point is changed at 30 hours in the pressure controller (PI) which manipulates coolant flow rate. The main aspect of this can be viewed by estimation of α by the adaptive sliding mode controller. Figure 4 shows that the ASM controller performs a rapid adaptation of by increasing its value by nearly 50% for maintaining close loop stability.

6.4. Effect of uncertain feed liquid fraction

In Figure 5, the adaptive sliding mode controller shows the performance against the uncertainty in feed liquid fraction. This effect is studied by implementing periodic change in feed liquid fraction (q_F) with an amplitude of +18% from its nominal value and a frequency of .1 hour. Since the feed condition is liquid (q_F =1), the bottom composition was affected immediately by this variation. Figure 5 shows the estimated parameter value for tackling the above uncertainty. The estimated value is not exactly the same as periodic variation in q_F and therefore, the algorithm can not be used as an exact estimator of process parameter. However, its main objective of maintaining overall closed loop stability with desired performance is always achieved by this adaptation.

6.5. Effect of uncertain feed temperature

The effect of periodic disturbance in feed temperature on controller performance is depicted in figure 6. Feed temperature variation is also a case of unstructured uncertainty as this information is not known to the ASM controller. The variation in feed temperature is implemented by introducing a periodic wave signal having amplitude of 30% from its nominal value and a frequency of .1 hour. Robustness of the ASM controller did not allow the compositions to move away from their setpoints in spite of this uncertainty. However, the manipulated variables become wavy in nature because of the rapid adjustments made by the controller to tackle the feed temperature variation.

7. Conclusions

A systematic design of adaptive sliding mode controller for MIMO system has been presented. The controller algorithm can be applied for a wide class of nonlinear MIMO system. Finally the controller algorithm is applied to a high purity Distillation Column. This column is simulated using a rigorous model. ASM controller designed based on the simplified model showed excellent performances for both servo and regulatory situations in presence of structured and unstructured uncertainty.

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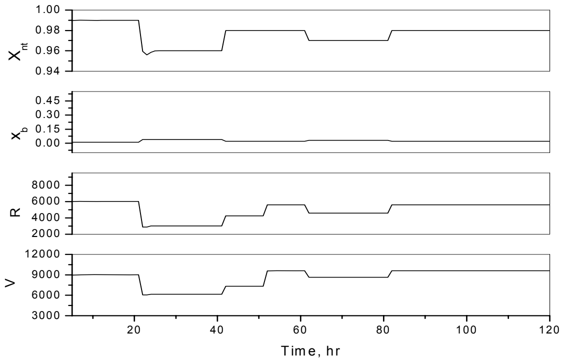

Fig. 1. servo performance of ASM algorithm.

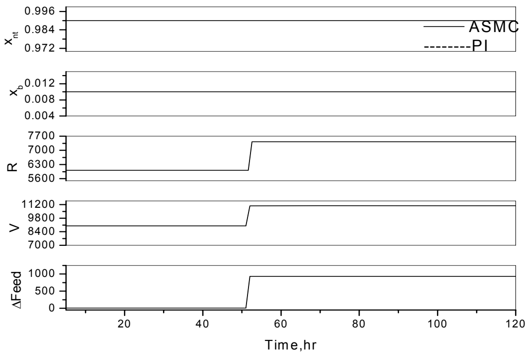

Fig. 2. Comparisons between ASM controller under (22%) feed flow rate change.

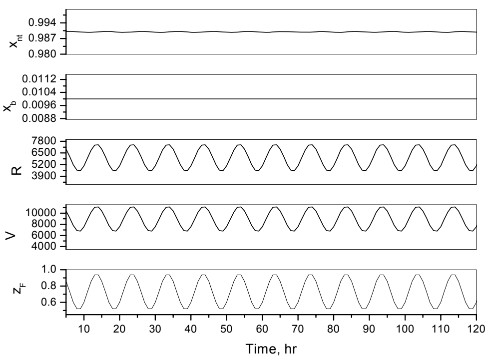

Fig. 3. comparisons under the cyclic change in feed compositions.

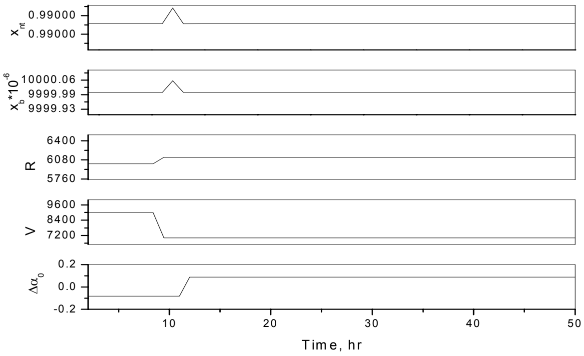

Fig. 4. Comparisons under the change of relative volatility.

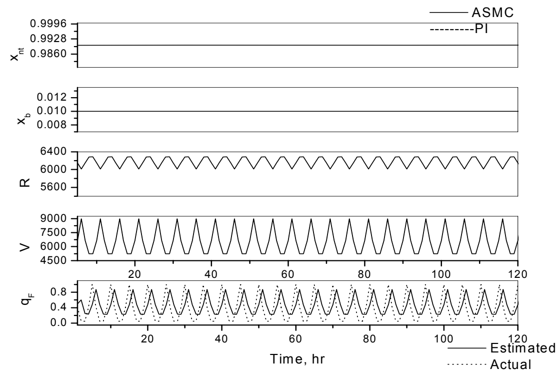

Fig.5. comparisons under periodic disturbance in feed liquid fraction.

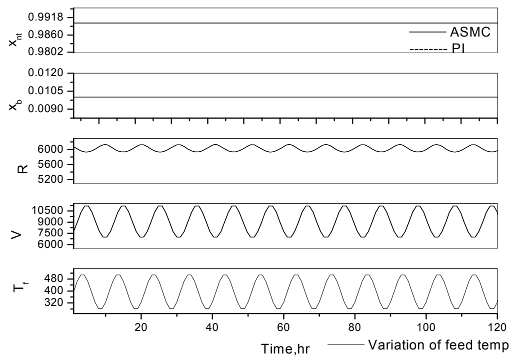

Fig. 6. comparisons under periodic change in feed Temperature.