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Fault Detection and Isolation of Retarded Time-Delay Systems Using A Geometric Approach

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Abstract: This paper investigates development of Fault Detection and Isolation (FDI) filters for retarded time-delay systems. A bank of residual generators is designed based on the linear geometric approach so that each residual is affected by one fault and is decoupled from the others while the H_∞ norm of the transfer function between the disturbance and the residual signals are less that a prespecified value. Simulation results presented in the paper demonstrate the effectiveness of our proposed FDI algorithm.

1. INTRODUCTION

Modern control systems are becoming increasingly more complex and issues of availability, efficiency, reliability, operating safety, and environmental protection concerns are receiving more attention. This requires a fault diagnosis system that is capable of reliably detecting plant, actuator and sensor faults when they occur, and of identifying and isolating the faulty component in the system. In the past three decades, a number of fundamental results on fault detection and isolation (FDI) have been developed see e.g. Massoumnia (1986); White and Speyer (1987); Frank and Wunnenberg (1989); Seliger and Frank (1991); Massoumnia et al. (1989); Douglas and Speyer (1995); Chung and Speyer (1998a,b); Chen and Patton (1999); Meskin and Khorasani (2006, 2007). However, limited results exist on designing FDI strategies for time-delay systems. Time-delay is an inherent characteristic of many physical systems, such as rolling mills, chemical processes, water resources, biological, economical and traffic control systems, to name a few. In this paper, we investigate development and design of a fault detection and isolation scheme for retarded time-delay systems.

In recent years, only a few results on FDI of retarded time-delay systems have been developed. In Yang and Saif (1996, 1998), an unknown input observer (UIO) is designed for FDI and Yang and Saif (1997) proposed a robust UIO approach for uncertain time-delay systems with bounded uncertainty. In this work, some assumptions on the system structures are considered. Both approaches are based on determining a suitable state transformation and designing a reduced order observer for the transformed system. Parity space approach is also developed in Kratz et al. (1998) for fault detection of retarded time-delay systems. In Liu and Frank (1999); Ding et al. (2001); Zhong et al. (2003); Jiang et al. (2003); Zhong et al. (2004); Leishi et al. (2006, 2007), a robust fault detection problem for linear time-delay systems is investigated by solving an H_∞ optimization problem. In this approach one attempts to keep the sensitivity of the residual signal to unknown inputs (disturbances) less than a specific bound while increase the sensitivity of the residual signal to the fault over the frequency range of the fault. In Jiang et al. (2002); Fuqiang et al. (2004); C. Jiang (2005), an adaptive observer approach is developed for estimating the fault signal in time-delay systems. In Nguang et al. (2006) a robust fault estimation for uncertain time delay Takagi-Sugeno (TS) fuzzy models is developed ensuring a prescribed H_∞ performance level for fault estimation error. In Zhu and Cheng (2004), a robust fault detection and isolation observer for uncertain singular time delay systems is developed. However, the problem of fault isolation for a general retarded time-delay systems has not been completely addressed in the above methods.

In this paper, a set of residuals that are based on the dedicated residual scheme (Chen and Patton (1999)) is generated by extending the geometric FDI results in Massoumnia et al. (1989) to retarded time-delay systems. Using the unobservability subspace properties of linear systems, a set of residuals is generated such that each residual is affected by one fault and is decoupled from others. At the same time the effects of disturbances on the residuals are attenuated by using an H_∞ optimization technique and the LMI approach is used for solving this optimization problem. The main contribution of this work is in extending geometric FDI methods to retarded time-delay linear systems.

The remainder of this paper is organized as follows. In section II, a brief background on geometric properties of linear systems and an H_∞ control for retarded time-delay systems are reviewed. The problem formulation and framework of our proposed fault detection and isolation strategy are presented in section III. In section IV, a robust fault detection and isolation strategy for time-delay systems is presented. In section V, the effectiveness and capabilities of our proposed algorithm are shown

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through simulation results. Conclusions and future work are presented in section VI.

The following notation is used throughout this paper. Script letters $\mathcal{X}, \mathcal{U}, \mathcal{Y}, ...$, denote real vector spaces. Matrices and linear maps are denoted by capital italic letters $A, B, C, ...$; the same symbol is used both for a matrix and its map; the zero space, zero vector ,..., are denoted by 0. $\mathcal{B} = \text{Im } B$ denotes the image of B; Ker C denotes the kernel of C. If a map C is epic, then C^{-r} denotes a right inverse of C (i.e., $CC^{-r} = I$)). A subspace $\mathcal{S} \subseteq \mathcal{X}$ is termed A-invariant if $A\mathcal{S} \subseteq \mathcal{S}$. For A-invariant subspace $\mathcal{S} \subseteq \mathcal{X}$, $A : \mathcal{S}$ denotes the restriction of A to $\mathcal{S}$, and $A : \mathcal{X}/\mathcal{S}$ denotes the map induced by A on the factor space $\mathcal{X}/\mathcal{S}$. For a linear system (C, A, B), $<\text{Ker } C|A>$ denotes the unobservable subspace of (C, A).

2. BACKGROUND

Consider the linear system

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

where $x \in \mathcal{X}$ is the state of the system with dimension n, $u \in \mathcal{U}, y \in \mathcal{Y}$ are input and output signals with dimensions m and q, respectively.

Definition 1. (Massoumnia (1986)). A subspace $\mathcal{S}$ is a (C, A) *unobservability subspace* (u.o.s.) if $\mathcal{S} =<\text{Ker } HC|A+DC>$ for some output injection map $D : \mathcal{Y} \to \mathcal{X}$ and measurement mixing map $H : \mathcal{Y} \to \mathcal{Y}$.

Given an u.o.s. $\mathcal{S}$, a measurement mixing map H can be computed from $\mathcal{S}$ by solving the equation $\text{Ker}HC = \text{Ker}C + \mathcal{S}$. Let $\underline{D}(\mathcal{S})$ denote the class of all maps $D : \mathcal{Y} \to \mathcal{X}$ such that $(A + DC)\mathcal{S} \subseteq \mathcal{S}$. The notation $\underline{\mathcal{S}}(\mathcal{L})$ refers to the class of (C, A) u.o.s. containing $\mathcal{L} \subseteq \mathcal{X}$. The class of u.o.s. is closed under intersection; therefore, it contains an infimal element $\mathcal{S}^* = inf\underline{\mathcal{S}}(\mathcal{L})$. In Massoumnia et al. (1989) an algorithm for computing $\mathcal{S}^*$ is proposed.

Given the matrices $A_i, i = 0, ..., N$ and C, a subspace $\mathcal{S}^{0,...,N}$ is called a common u.o.s. for the pairs $(C, A_i), i = 0, ..., N$ if

$$\mathcal{S}^{0,...,N} =<\text{Ker } HC|A_i + D_iC>, \quad i = 0, ..., N \tag{1}$$

for some output injection maps $D_i : \mathcal{Y} \to \mathcal{X}$ and measurement mixing map $H : \mathcal{Y} \to \mathcal{Y}$.

The notation $\mathcal{S}^{0,...,N}(\mathcal{L})$ refers to a common u.o.s. containing $\mathcal{L} \subseteq \mathcal{X}$. The following algorithm can be used for finding the smallest common u.o.s. $\mathcal{S}^{(0,...,N)^*}(\mathcal{L})$ for the pairs $(C, A_i), i = 0, ..., N$ containing $\mathcal{L}$

$$CUOS : \begin{cases} \mathcal{S}_0 = \mathcal{X} \\ \mathcal{S}_k = \mathcal{W}^* + (\cap_{i=0}^N A_i^{-1} S_{k-1}) \cap \text{Ker } C \end{cases}$$

where $\mathcal{S}^{(0,...,N)^*}(\mathcal{L}) = lim\ \mathcal{S}_k$ and $\mathcal{W}^* = lim\ \mathcal{W}_k$ where $\mathcal{W}_k$ can be obtained from following algorithm

(1) $\mathcal{W}_0 = \mathcal{L}$
(2) $\mathcal{W}_k = \mathcal{W}_{k-1} + \sum_{i=0}^N A_i(\mathcal{W}_{k-1} \cap \text{Ker } C)$

For details see Massoumnia (1986) and Balas et al. (2002, 2003). As it will be shown in section IV, a central role is played by common unobservability subspaces $\mathcal{S}^{0,...,N}$ in the *geometrical approach* to fault detection and isolation of retarded time-delay systems.

Let $\mathcal{S} \subset \mathcal{X}$ be an u.o.s. , i.e., $\mathcal{S} =<\text{Ker } HC|A + D_0C>$, then the factor system of Σ which is denoted by $\Sigma : \mathcal{X}/\mathcal{S}$ is defined as

$$\Sigma : \mathcal{X}/\mathcal{S} \begin{cases} \dot{x}(t) = A_{\mathcal{S}}x(t) + B_{\mathcal{S}}u(t) \\ y(t) = C_{\mathcal{S}}x(t) \end{cases}$$

where $A_{\mathcal{S}} = A + D_0C : \mathcal{X}/\mathcal{S}$, $B_{\mathcal{S}} = PB$, $C_{\mathcal{S}}$ is the unique solution of $C_{\mathcal{S}}P = HC$, $D_0 \in \underline{D}(\mathcal{S})$ and $P : \mathcal{X} \to \mathcal{X}/\mathcal{S}$ is the canonical projection.

In the following, certain results on H_∞ disturbance attenuation of retarded time-delay systems are reviewed. Consider a linear time-delay system

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \sum_{i=1}^N A_ix(t - \tau_{xi}(t)) + Dd(t) \\ y(t) &= Cx(t), x(t) = 0 \quad (t \leq 0) \end{aligned} \tag{2}$$

where $\tau_{xi}(t)$ are assumed to satisfy $\tau_{xi}(t) \leq \tau_i < \infty$, $\dot{\tau}_{xi}(t) \leq \bar{\tau}_i \leq 1$ and $d(t)$ represents the unknown input vector including modeling errors and uncertain disturbances. Without loss of generality, it is assumed that d is L_2-norm bounded. The next lemma provides a sufficient condition for asymptotic stability of system (2) while the H_∞ norm of the transfer function between the disturbance d and the output signal y is less that a given positive value γ.

Lemma 2. (Kim and Park (1999)). Given $\gamma > 0$ and the time-delay system (2), if there exist positive-definite matrices P and Q such that the following Riccati inequality is satisfied

$$\begin{aligned} &A_0^TP + PA_0 + C^TC + Q + \gamma^{-2}(PD)(PD)^T \\ &+ N\sum_{i=1}^N c_iPA_iQ^{-1}A_i^TP < 0 \end{aligned}$$

where $c_i = \frac{1}{1-\bar{\tau}_i}$, then system (2) is asymptotically stable and its L_2 gain is not greater than γ, i.e.

$$\int_0^\infty y^T(t)y(t)dt \leq \gamma^2 \int_0^\infty d^T(t)d(t)dt \tag{3}$$

3. PROBLEM FORMULATION

Consider the following linear retarded time-delay system

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \sum_{i=1}^N A_ix(t - \tau_{xi}(t)) + B_0u(t) \\ &+ \sum_{j=1}^L B_ju(t - \tau_{uj}(t)) + \sum_{l=1}^k L_lm_l(t) \\ &+ Dd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

with the continuous initial condition $x(\theta) = \phi(\theta), \theta \in [-\tau, 0]$ where $x \in \mathcal{X}$ is the state of the system with dimension n, $u \in \mathcal{U}, y \in \mathcal{Y}$ are input and output signals with dimensions m and q, respectively, $m_i \in \mathcal{M}_i$ are the fault modes with dimension k_i and L_i's are fault signatures and $\tau = max_i\tau_{xi}(0)$. The fault modes together with the fault signatures may be used to model the effects of actuator faults, sensor faults and system faults on the dynamics of the system. For modeling a fault in the i-th actuator, $L_i = [b_{0i}, b_{1i}, ..., b_{Li}]$ and the fault mode m_i is chosen to model the type of a fault where $b_{ji}, j = 0, ..., L$ denote the i-th column of matrices $B_j, j = 0, ..., L$. For example a complete failure of an actuator can be represented and modeled by $m_i(t) = [-u_i(t), -u_i(t-\tau_{u1}(t)), ..., -u_i(t-\tau_{uL}(t))]^T$. A system fault can be represented by a potential variation in the parameters of the A_i's matrices as shown below:

$$\begin{aligned}\dot{x}(t) =&(A_0 + \Delta A_0)x(t) + \sum_{i=1}^{N}(A_i + \Delta A_i)x(t-\tau_{xi}(t)) \\ &+ B_0u(t) + \sum_{j=1}^{L} B_j u(t-\tau_{uj}(t)) \\ y(t) =&Cx(t)\end{aligned}$$

As an example, a change in the i-th row and j-th column of matrix A_1 can be modeled as $\Delta A_1 x(t-\tau_{x1}(t)) = I_i \Delta a_{1_{ij}} x_j(t-\tau_{x1}(t))$ where x_j is the j-th element of the vector x and I_i is an n-dimensional vector with all zero elements except one in the i-th element. Define the signal $m_i(t) \triangleq \Delta a_{1_{ij}} x_j(t-\tau_{x1}(t))$ as an external input and fault signature $L_i = I_i$, then this fault can be modeled as in equation (4).

It should be noted that sensor faults can initially be modeled as additive inputs in the measurement equation $y = Cx + \sum_{j=1}^{q} E_j n_j$ where E_j is an $q \times 1$ unit vector with a one at the j-th position and $n_j \in \mathbb{R}$ is a sensor fault mode, which corresponds to a fault in the j-th sensor. For example, a complete failure of the j-th sensor can be represented and modeled by $n_j = -c_j x$ where c_j is the j-th row of the matrix C. The sensor fault signature can also be modeled as an input to the system (Chung and Speyer (1998a); Hashtrudizad and Massoumnia (1999)). Following Chung and Speyer (1998a), let f_j be the solution to $E_j = Cf_j$. The new states can be defined according to $\bar{x}(t) = x(t) + \sum_{j=1}^{q} f_j n_j(t)$, where the state space representation for the new states can be written as

$$\begin{aligned}\dot{\bar{x}}(t) &= A_0\bar{x}(t) + \sum_{i=1}^{N} A_i\bar{x}(t-\tau_{xi}(t)) + B_0u(t) \\ &+ \sum_{j=1}^{L} B_j u(t-\tau_{uj}(t)) + \sum_{j=1}^{q} L_j m_j(t) \\ y(t) &= C\bar{x}(t)\end{aligned} \tag{5}$$

where $L_j = [\, f_j \ A_0 f_j \ A_1 f_j \ \cdots \ A_N f_j \,]$ and $m_j(t) = [\dot{n}_j(t), -n_j(t), -n_j(t-\tau_{x1}(t)), \cdots, -n_j(t-\tau_{xN}(t))]^T$.

We are now in a position to formally introduce the robust fault detection and isolation problem considered in this paper.

4. ROBUST FAULT DETECTION AND ISOLATION OF RETARDED TIME-DELAY SYSTEMS

The Robust Extended Fundamental Problem in Residual Generation (REFPRG) for the retarded time-delay system (4) is to design a set of filters that generate k residuals $r_i(t)$ such that a fault in the i-th component L_i can only affect the residual $r_i(t)$ and no other residual $r_j(t)(i \neq j)$ and

$$\int_0^{\infty} r_i^T(t) r_i(t)dt \leq \gamma^2 \int_0^{\infty} d^T(t)d(t)dt, \ i = 1, ..., k \tag{6}$$

Specifically, the residual signals $r_i(t)$ are generated according to the following filters:

$$\begin{aligned}\dot{w}_i(t) =&F_i w_i(t) + \sum_{l=1}^{N} F_{il} w_i(t-\tau_{xl}(t)) - E_i y(t) + K_i u(t) \\ &- \sum_{l=1}^{N} E_{il} y(t-\tau_{xl}(t)) + \sum_{j=1}^{L} K_{ij} u(t-\tau_{uj}(t)) \\ r_i(t) =& M_i w_i(t) - H_i y(t)\end{aligned} \tag{7}$$

The following theorem summarizes our proposed strategy.

Theorem 3. The REFPRG problem defined by expressions (6) and (7) has a solution for the linear retarded time-delay system (4) if there exist the following common unobservability subspaces

$$\mathcal{S}_i = \mathcal{S}^{(0,...,N)^*}(\sum_{j\neq i} \mathcal{L}_j), \quad i = 1, ..., k \tag{8}$$

such that $\mathcal{L}_i \cap \mathcal{S}_i = 0, i = 1, ..., k$ as well as the gain matrices $G_{ij}, j = 0, ..., N, i = 1, ..., k$ and positive-definite matrices R_i and Q_i, $i = 1, ..., k$ such that

$$\begin{aligned}&(A_{0_{\mathcal{S}_i}} + G_{i0}M_{\mathcal{S}_i})^T R_i + R_i(A_{0_{\mathcal{S}_i}} + G_{i0}M_{\mathcal{S}_i}) \\ &+ M_{\mathcal{S}_i}^T M_{\mathcal{S}_i} + Q_i + \gamma^{-2}(R_i D_{u_\mathcal{S}})(R_i D_{u_\mathcal{S}})^T \\ &+ N\sum_{l=1}^{N} c_i R_i (A_{l_{\mathcal{S}_i}} + G_{il}M_{\mathcal{S}_i}) Q_i^{-1} (A_{l_{\mathcal{S}_i}} + G_{il}M_{\mathcal{S}_i})^T R_i < 0\end{aligned} \tag{9}$$

where P_i is the canonical projection of $\mathcal{X}$ on $\mathcal{X}/\mathcal{S}_i$, $D_{u_\mathcal{S}} = -P_i D$, and the pairs $(M_{\mathcal{S}_i}, A_{l_{\mathcal{S}_i}}), l = 0, ..., N$ are the factor system of the pairs $(C, A_l), l = 0, ..., N$ on $\mathcal{X}/\mathcal{S}_i$, respectively.

Proof: Given the unobservability subspaces $\mathcal{S}_i$, there exist output map injections $D_{i0}, D_{i1}, ..., D_{iN}$ and measurement mixing map H_i such that

$$\begin{aligned}\mathcal{S}_i &=< \text{Ker } H_iC | A_0 + D_{i0}C > \\ \mathcal{S}_i &=< \text{Ker } H_iC | A_1 + D_{i1}C > \\ &\vdots \\ \mathcal{S}_i &=< \text{Ker } H_iC | A_N + D_{iN}C >\end{aligned}$$

where H_i is the solution to Ker $H_iC = \mathcal{S}_i +$ Ker C and is common for all A_i's. Let $M_{\mathcal{S}_i}$ be a unique solution to $M_{\mathcal{S}_i}P_i = H_iC$ and

$$\begin{aligned}A_{0_{\mathcal{S}_i}} &= (A_0 + D_{i0}C : \mathcal{X}/\mathcal{S}_i) \\ A_{1_{\mathcal{S}_i}} &= (A_1 + D_{i1}C : \mathcal{X}/\mathcal{S}_i) \\ &\vdots \\ A_{N_{\mathcal{S}_i}} &= (A_N + D_{iN}C : \mathcal{X}/\mathcal{S}_i)\end{aligned}$$

where

$$\begin{aligned}P_i(A_0 + D_{i0}C) &= A_{0_{\mathcal{S}_i}} P_i \\ P_i(A_1 + D_{i1}C) &= A_{1_{\mathcal{S}_i}} P_i \\ &\vdots \\ P_i(A_N + D_{iN}C) &= A_{N_{\mathcal{S}_i}} P_i\end{aligned} \tag{10}$$

Let $G_{ij}, j = 0, ..., N$ denote the solution to the inequality (9) and define

$$\begin{aligned}F_i &= A_{0_{\mathcal{S}_i}} + G_{i0}M_{\mathcal{S}_i} \\ F_{i1} &= A_{1_{\mathcal{S}_i}} + G_{i1}M_{\mathcal{S}_i} \\ &\vdots \\ F_{iN} &= A_{N_{\mathcal{S}_i}} + G_{iN}M_{\mathcal{S}_i}\end{aligned}$$

$$\begin{bmatrix} A_{0_{\mathcal{S}_i}}^T R_i + R_i A_{0_{\mathcal{S}_i}} + M_{\mathcal{S}_i}^T T_0^T + T_0 M_{\mathcal{S}_i} + M_{\mathcal{S}_i}^T M_{\mathcal{S}_i} + Q_i & R_i D_{us} & R_i A_{1_{\mathcal{S}_i}} + T_1 M_{\mathcal{S}_i} & \cdots & R_i A_{N_{\mathcal{S}_i}} + T_N M_{\mathcal{S}_i} \\ * & -\gamma^2 I & 0 & \cdots & 0 \\ * & * & -\frac{1}{N} c_1^{-1} Q_i & 0 & 0 \\ \vdots & * & \vdots & \ddots & \vdots \\ * & * & * & * & -\frac{1}{N} c_N^{-1} Q_i \end{bmatrix} < 0 \quad (12)$$

and

$$\begin{aligned} E_i &= P_i(D_{i0} + P_i^{-r} G_{i0} H_i) \\ E_{i1} &= P_i(D_{i1} + P_i^{-r} G_{i1} H_i) \\ &\vdots \\ E_{iN} &= P_i(D_{iN} + P_i^{-r} G_{iN} H_i) \end{aligned}$$

Let $M_i = M_{\mathcal{S}_i}$, $K_i = P_i B_0$, and $K_{ij} = P_i B_j, j = 1, ..., L$. Define $e_i(t) = w_i(t) - P_i x(t)$, then using (7) we have

$$\begin{aligned} \dot{e}_i(t) =& F_i w_i(t) + \sum_{l=1}^{N} F_{il} w_i(t - \tau_{xl}(t)) - E_i y(t) + K_i u(t) \\ & - \sum_{l=1}^{N} E_{il} y(t - \tau_{xl}(t)) + \sum_{j=1}^{L} K_{ij} u(t - \tau_{uj}(t)) \\ & - P_i(A_0 x(t) + \sum_{l=1}^{N} A_l x(t - \tau_{xl}(t)) + B_0 u(t) \\ & + \sum_{j=1}^{L} B_j u(t - \tau_{uj}(t)) + \sum_{l=1}^{k} L_l m_l(t) + D(t) d(t)) \\ =& F_i w_i(t) + \sum_{l=1}^{N} F_{il} w_i(t - \tau_{xl}(t)) \\ & - P_i(A_0 + D_{i0} C) x(t) - G_{i0} M_i P_i x(t) \\ & - \sum_{l=1}^{N} P_i(A_l + D_{il} C) x(t - \tau_{xl}(t)) - P_i D d(t) \\ & - \sum_{l=1}^{N} G_{il} M_i P_i x(t - \tau_{xl}(t)) - P_i L_i m_i(t) \\ =& F_{0i} e_i(t) + \sum_{l=1}^{N} F_{il} e_i(t - \tau_{xl}(t)) \\ & - P_i L_i m_i(t) - P_i D d(t) \end{aligned}$$

Note that $P_i L_j = 0, j \neq i$, since $\mathcal{L}_j \in \mathcal{S}_i, j \neq i$. Also

$$\begin{aligned} r_i(t) &= M_i w_i(t) - H_i y(t) = M_i w_i(t) - H_i C x(t) \\ &= M_i e_i(t) \end{aligned}$$

Consequently, the error dynamics can be written as

$$\begin{aligned} \dot{e}_i(t) =& F_{0i} e_i(t) + \sum_{l=1}^{N} F_{il} e_i(t - \tau_{xl}(t)) \\ & - P_i L_i m_i(t) + D_{u\mathcal{S}} d(t) \\ r_i(t) =& M_i e_i(t) \end{aligned} \quad (11)$$

Using Lemma 2 and the inequality (9), it follows that the inequality (6) holds. Moreover, from the error dynamics (11), it follows that $r_i(t)$ is only affected by L_i and is decoupled from other fault signatures. ■

Using the Schur complement and change of variables $T_j = R_i G_{ij}, j = 0, ..., N$, the inequality (9) can be written as given by the inequality (12) which is in an LMI form in terms of R_i, Q_i and T_j's, and that can be solved by using standard LMI tools. The observer gains can be calculated from the $G_{ij} = R_i^{-1} T_j$.

The generic conditions for existence of the unobservability subspaces of Theorem 3 can be stated as follows.

Proposition 4. Let $A_i, i = 0, ..., N$, C and L_i be arbitrary matrices of dimensions $n \times n$, $q \times n$ and $n \times k_i$, respectively, let $v = \sum_{i=1}^{k} k_i$. The unobservability subspaces of Theorem 3 generically exist if and only if

$$v \leq n \quad (13)$$

and

$$v - \min\{k_i, i = 1, ..., k\} < q \quad (14)$$

Proof: The proof is the same as in the EFPRG problem for linear systems Massoumnia et al. (1989) and is omitted due to space limitations. ■

After constructing the residual signals $r_i(t), i = 1, ..., k$, the last step is to determine the threshold J_{th_i} and the evaluation function $J_{r_i}(t)$. In this paper, the following thresholds and evaluation functions are selected

$$J_{r_i}(t) = \int_{t-T_0}^{t} r_i^T(t) r_i(t) dt, \quad i = 1, ..., k \quad (15)$$

$$J_{th_i} = \sup_{d \in L_2, m_j = 0, j = 1, ..., k} (J_{r_i}), \quad i = 1, ..., k \quad (16)$$

where T_0 is the length of the evaluation window. Based on the above thresholds and evaluation functions, the occurrence of a fault can be detected and isolated by using the following decision logics

$$J_{r_i}(t) > J_{th_i} \Longrightarrow m_i \neq 0, \quad i = 1, ..., k \quad (17)$$

5. NUMERICAL EXAMPLE

To illustrate the effectiveness and capabilities of our proposed FDI algorithm, a numerical example is provided in this section. Consider the time-delay system (4) that is specified with parameters

$$A_0 = \begin{bmatrix} 2 & -1.5 & 1 & 1 \\ 1 & -1 & 0.5 & 2 \\ 1 & 2 & -3 & 0 \\ 2 & 0 & -1 & 1 \end{bmatrix}, B_0 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -1 & 2 & 0.2 & 0 \\ -0.1 & 1.3 & 0.5 & 1 \\ 0.1 & -1 & 2 & 0.1 \\ 1 & 0.1 & 1 & 2 \end{bmatrix}, B_1 = 0$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0.2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.3 \\ 0.2 \\ 0 \\ 0.6 \end{bmatrix}$$

and $\tau_{x1}(t) = 0.5 + 0.2sin(t)$ and $N = 1$, $k = 2$ and $L = 0$. The fault signatures L_1 and L_2 are selected as the first and second columns of the matrix B, and hence they represent actuator faults for the time-delay system.

The subspaces in Theorem 3 for the above time-delay system can be determined using the *CUOS* algorithm and are given by $\mathcal{S}_1 = \mathcal{L}_1, \mathcal{S}_2 = \mathcal{L}_2$. After determining the subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$, the maps D_{i0}, $D_{i1}, H_i, M_i, i = 1,2$ and matrices $A_{0_{\mathcal{S}_1}}, A_{1_{\mathcal{S}_1}}, A_{0_{\mathcal{S}_2}}, A_{1_{\mathcal{S}_2}}$ can be found according to Theorem 3. Using the LMI tools, the gain matrices G_{10}, G_{11}, G_{20} and G_{21} are computed by solving the LMI inequality (15) for $\gamma = 0.07$. An H_∞ robust state feedback control $u(t) = Kx(t)$ is also designed for the closed-loop system to ensure its stability.

A disturbance input $d(t)$ is assumed to be a band-limited white-noise with power of 0.5. The thresholds are calculated as $J_{th_1} = 0.01$ and $J_{th_2} = 0.015$ for $T_0 = 5$ seconds. Figure 1 shows the residuals and their evaluation functions corresponding to the healthy operation of the system. As shown in this figure, no false alarm is generated during normal operation of the system. Figure 2 shows the residuals and the evaluation functions corresponding to a fault in the second actuator (u_2) of the system where the gain of the actuator is decreased by 60% at $t = 10$ seconds. This fault can be modeled as $m_2(t) = -0.6u_2(t)$, where $m_2(t)$ is the fault mode of the second actuator. As shown in this figure, the fault is detected and isolated at $t = 19.7$ seconds and the evaluation function of r_1 (i.e. $J(r_1)$) remains below the threshold. Figure 3 shows the residuals and evaluation functions corresponding to a fault in the first actuator where the gain of the actuator is decreased by 70% at $t = 10$ seconds. This fault can be modeled as $m_1(t) = -0.7u_1(t)$, where $m_1(t)$ is the fault mode of the first actuator. As shown in this figure, this fault is detected and isolated at $t = 12.1$ seconds and the evaluation function of r_2 (i.e. $J(r_2)$) remains below the threshold. Figure 4 shows the residuals and the evaluation functions corresponding to simultaneous faults in both actuators where 60% loss of effectiveness (gain) is occurred in the first actuator at $t = 5$ seconds and 50% loss of gain is occurred in the second actuator at $t = 10$ seconds. According to this figure, the fault in the first actuator is detected at $t = 7.2$ seconds and the fault in the second actuator is detected at $t = 13$ seconds. It should be noted that in all above scenarios the time-delay system remains stable and well-behaved, which makes the FDI problem more challenging.

Remark: It should be emphasized that the presently available FDI algorithms in the literature cannot generate the residual signals with the above decoupling properties. In those algorithms, faults that one needs to be decoupled are considered as unknown inputs and the algorithms seeks to attenuate the effects of faults on the residual. Therefore, those type of algorithms cannot decouple fault effects from the residuals. However, in our proposed approach, the residual signals that can decouple the faults from each other and are robust with respect to disturbances are constructed where one can easily use these residuals for both fault detection and isolation.

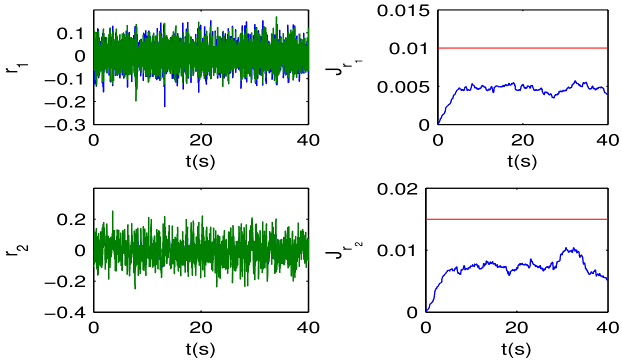

Fig. 1. Residual signals and their evaluation functions corresponding to the normal mode (healthy operation)

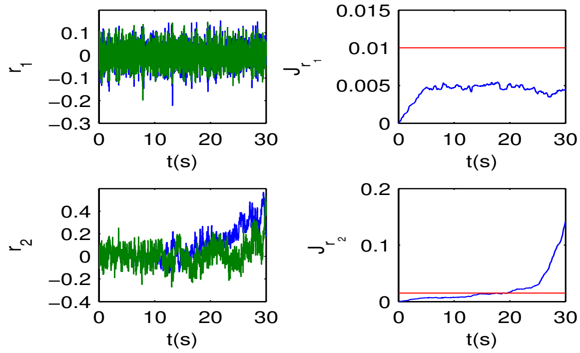

Fig. 2. Residual signals and their evaluation functions corresponding to a fault in the second actuator

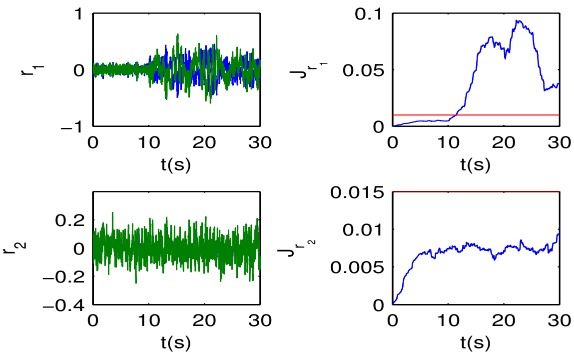

Fig. 3. Residual signals and their evaluation functions corresponding to a fault in the first actuator

6. CONCLUSIONS

A geometric approach to fault detection and isolation for linear retarded time-delay systems is developed in this paper. A set of residual signals are generated so that each residual is only affected by one fault and is decoupled from the others while the H_∞ norm of the transfer function between the unknown input (disturbances, uncertainties and modeling errors) and residual signals is less than a given positive value. Simulation results demonstrate and illustrate the effectiveness and capabilities of our proposed method.

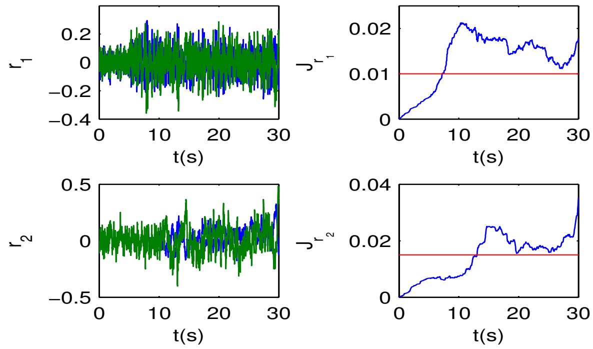

Fig. 4. Residual signals and their evaluation functions corresponding to simultaneous faults in both actuators

REFERENCES

G. Balas, J. Bokor, and Z. Szabo. Failure detection for LPV systems- a geometric approach. *American Control Conference*, pages 4421–4426, 2002.

G. Balas, J. Bokor, and Z. Szabo. Invariant subspaces for LPV systems and their applications. *IEEE Trans. Autom. Control*, 48(11):2065–2068, 2003.

D.H. Zhou C. Jiang. Fault detection and identification for uncertain linear time-delay systems. *Computer and Chemical Engineering*, 30:228–242, 2005.

J. Chen and R. J. Patton. *Robust model-based fault diagnosis for dynamic systems*. Kluwer Academic Publishers, Boston/Dordrecht/London, 1999.

W. H Chung and J. L. Speyer. A game theoretic fault detection filter. *IEEE Trans. Autom. Control*, 43(2): 143–161, 1998a.

W. H Chung and J. L. Speyer. A decectralized fault detection filter. *American Control Conference*, pages 2017–2021, 1998b.

S. X. Ding, M. Zhong, T. Bingyong, and P. Zhang. An LMI approach to the design of fault detection filter for time-delay LTI systems with unknown inputs. *American Control Conference*, pages 2137–2142, 2001.

R. K. Douglas and J. L. Speyer. Robust fault detection filter design. *American Control Conference*, pages 91–96, 1995.

P. M. Frank and J. Wunnenberg. *Robust Fault Diagnosis using Unknown Input Observer Scheme*. Prentice Hall, 1989.

Y. Fuqiang, T. Zuohua, and S. Songjiao. Actuator fault diagnosis for a class of time-delay systems. *Proceeding of* 5^{th} *World Congress on Intelligent Control and Automation*, pages 1798–1802, 2004.

S. Hashtrudizad and M. A. Massoumnia. Generic solvability of the failure detection and identification. *Automatica*, 35(5):887–893, 1999.

B. Jiang, M. Staroswiecki, and V. Cocquempot. Fault identification for a class of time-delay systems. *Proceeding of the American Control Conference*, pages 2239–2244, 2002.

B. Jiang, M. Staroswiecki, and V. Cocquempot. H_∞ fault detection filter design for linear discrete-time systems with multiple time delays. *International Journal of System Science*, 34(5):365–373, 2003.

J. H. Kim and H. B. Park. H_∞ state feedback control for generalized continuous/discrete time delay system. *Automatica*, 35(8):1443–1451, 1999.

F. Kratz, W. Nuninger, and S. Ploix. Fault detection for time-delay systems: a parity space approach. *American Control Conference*, pages 2009–2011, 1998.

B. Leishi, T. Zuohua, and S. Songjiao. Design of H_∞ robust fault detection filter for linear uncertain time delay systems. *The Instrumentation, Systems and Automation Transaction*, 45(4):491–502, 2006.

B. Leishi, T. Zuohua, and S. Songjiao. RFDF design for linear time-delay systems with unknown inputs and parameter uncertainties. *International Journal of System Science*, 38(2):139–149, 2007.

J. H. Liu and P. M. Frank. H_∞ detection filter for state delayed linear systems. *Proc.* 14^{th} *IFAC World Congress Conference*, pages 229–233, 1999.

M. A. Massoumnia. A geometric approach to the synthesis of failure detection filters. *IEEE Trans. Autom. Control*, 31(9):839–846, 1986.

M. A. Massoumnia, G. C. Verghese, and A. S. Willsky. Failure detection and identification. *IEEE Trans. Autom. Control*, 34(3):316–321, 1989.

N. Meskin and K. Khorasani. Fault detection and isolation of actuator faults in spacecraft formation flight. *Proceeding of the* 45^{th} *IEEE Conference on Decision and Control*, pages 1159–1164, 2006.

N. Meskin and K. Khorasani. Fault detection and isolation of actuator faults in overactuated systems. *American Control Conference*, pages 2527 – 2532, 2007.

S. K. Nguang, P. Shi, and S. Ding. Delay dependent fault estimation for uncertain time delay nonlinear systems: An LMI approach. *American Control Conference*, pages 5135–5140, 2006.

R. Seliger and P. M. Frank. Fault diagnosis by disturbance decoupled nonlinear observer. *Proceedings of IEEE Conference on Decision and Control*, pages 2248–2253, 1991.

J. E. White and J. L. Speyer. Detection filter design: spectral theory and algorithms. *IEEE Trans. Autom. Control*, 32(7):593–603, 1987.

H. Yang and M. Saif. Observer design and fault diagnosis for retarded dynamical systems. *Proceeding of the* 35^{th} *Conference on Decision and Control*, pages 1149–1154, 1996.

H. Yang and M. Saif. Robust observation and fault diagnosis in a class of time-delay control systems. *American Control Conference*, pages 478–482, 1997.

H. Yang and M. Saif. Observer design and fault diagnosis for state-retarded dynamical systems. *Automatica*, 34 (2):217–227, 1998.

M. Zhong, S. X. Ding, J. Lam, and C. Zhang. Fault detection filter design for LTI systems with time delays. *Proceeding of the* 42^{th} *Conference on Decision and Control*, pages 1467–1472, 2003.

M. Zhong, H. Ye, and G. Wang. Multi-freedom design of fault detection filter for linear time-delay systems. 8^{th} *International conference on control, Automation, Robotics and Vision*, pages 1630–1634, 2004.

S. Zhu and Z. Cheng. Design of robust fault detection and isolation observers for singular time delay systems. *American Control Conference*, pages 5064–5065, 2004.