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Switching Control of a Modified Leader-Follower Team of Agents Under the Leader and Network Topological Changes

E. Semsar-Kazerooni * **K. Khorasani** *

* *Concordia University, Department of Electrical and Computer Engineering, 1455 de Maisonneuve Blvd. W., Montreal, Quebec, H3G 1M8, Canada (e-mails: e_semsar, kash@ece.concordia.ca)*

Abstract: In this paper the existence of a common Lyapunov function for stability is guaranteed in a switching network of agents. The objective of the team is to achieve consensus in a modified-leader follower team while the team structure is changing during the mission. The stability of the team dynamics is guaranteed for networks with both balanced or unbalanced describing graphs with directional communication links. Although the original design strategy is based on an optimal control approach, for determining a common Lyapunov function the optimal gains have to be reselected. However, by introducing a criterion for control gains selection, a desirable performance can still be achieved. In this paper, we concentrate on one of the many possible criteria, namely performance-control effort criterion in details. Simulation results are presented to illustrate the performance and capabilities of the team in presence of a switching structure and switching leader scenarios.

1. INTRODUCTION

There are many advantages for deploying autonomous network of unmanned systems. For instance, enhanced group robustness to individual failures, increased and improved instrument sensing and resolution, reduced cost of operation, and adaptive reconfigurability capabilities have been discussed in (Beard et al., 2000). Some applications that necessitate development of these systems are in satellite deployment for distributed earth or deep space observations; maneuvers of a group of unmanned aerial vehicles (UAVs) for intelligence, surveillance, and reconnaissance (ISR) missions; automated factories; unmanned underwater vehicles (UUVs) for search and rescue; and teams of mobile robots deployed in a hazardous environment where human involvement is dangerous. In order to fully take advantage of these large-scale networks and systems of systems, several prerequisites are required to be satisfied. Some of these are development of reliable communication, optimal power consumption management, and team cooperation as discussed in (Sinopoli et al., 2003). These issues are still open areas of research.

In many situations two agents in a team may not be able to obtain the status of each other, either through communication links or by means of onboard sensor measurements. This may arise due to large distances or appearance of obstacles among the team members. In this situation the communication network structure among the team members should no longer be fixed and should be characterized as a switching network architecture. This issue has been discussed in the literature from various perspectives. One of the underlying assumptions in many of the related work is that the graph describing the information exchange structure is a balanced graph. For example, one can refer to (Olfati-Saber and Murray, 2004) in which directed and undirected balanced networks with fixed and switching topologies are considered. The goal of that paper is consensus achievement for a connected graph subject to certain switching in the network structure. The main focus in (Jadbabaie et al., 2003) is on attitude alignment in undirected switching network of agents with discrete-time first order dynamical models. It is shown that the connectivity of the graph on average (jointly connected or connected through a finite interval) is sufficient for convergence of the heading angles of the agents. In (Tanner et al., 2007; Shi et al., 2006), switching control laws are designed using Fillippov and non-smooth system frameworks for stability analysis. The graph describing the network structure is assumed to be undirected and connected and the goal is consensus and formation achievement.

In (Xie and Wang, 2006; Xiao and Wang, 2007), the graph describing the network structure is assumed to be undirected (and therefore balanced) and connected and the goal is consensus and formation achievement. In (Kingston and Beard, 2006; Ren and Beard, 2005) consensus in a directed, jointly connected, and balanced network is discussed. The authors in (Sun et al., 2006) considered the balanced information graphs and proved the stability under switching time-delayed communication links. The analysis is performed by introducing a Lyapunov functional and then proving the feasibility of a set of linear matrix inequalities. In (Wang and Xiao, 2006) the concept of "pre-leader-follower" is introduced as a new approach to achieve consensus in a network of discrete-time systems. The basic properties of stochastic matrices are used to guarantee consensus achievement in a network with switching topology and time-delayed communication links. In (Ren, 2007), higher order consensus algorithms are discussed. The author's approach to handle the switching network structure with a spanning tree is to find a dwell time as the required time between any two switching instants. It is shown that the final consensus value depends on the information exchange structure as well as the controller weights.

In addition to the changes that may occur in the communication structure of a network, in some circumstances in the leader-follower structure, the assignment of the leader may also change during the mission. This can be either as a result of the fact that some agents are more accessible in different stages of the mission or for certain safety issues some agents are more reliable or safer to be assigned as the leader during some parts of the mission. In these conditions the leader assignment can be time-varying as well.

The main contribution of the present work is to introduce a strategy which can guarantee consensus achievement for a team of agents with a general underlying network topological graph subject to both types of changes in the network topology and leader assignment. Our proposed framework can handle strongly connected, directed, and unbalanced graphs under a switching network configuration. This work is an extension of another work by the authors presented in (Semsar-Kazerooni and Khorasani, 2007) to switching networks. In our previous work an optimal control strategy is suggested to achieve consensus in a team of agents. The team structure is a fixed modified leader-follower which is described in detail in that paper. In the current paper, our previous results are used to design a control strategy which guarantees stability and consensus achievement for a team with switching leader assignment and network structure.

Using the properties of balanced graphs and by assigning the eigenvectors of the closed-loop matrix which corresponds to the error dynamics of the team to a desirable vector, the existence of a common Lyapunov function and consequently the stability and consensus achievement are guaranteed. By utilizing this approach one requires that the gain matrices that are defined in the cost functions to take on specific values. This in turn results in a constraint on the optimal control law which has been designed initially for the fixed network topology. However, by introducing additional criteria, the desirable performance of the team can still be ensured and guaranteed. As a demonstration of such a criterion, the performance-control effort tradeoff is considered and is discussed in detail in this paper.

The organization of the paper is as follows: In Section 2, background preliminaries and problem formulations are presented. In Section 3, switching networks, switching control design and stability analysis as well as criteria for selection of the control gains are developed. Simulation results are presented in Section 4. Finally, conclusions are discussed in Section 5.

2. BACKGROUND PRELIMINARIES

2.1 Problem Formulation

Multi-agent teams: Assume a set of agents $A = \{a_i, i = 1, \dots, N\}$, where N is the number of agents in a team. Each member's dynamics are governed by the following model in which the interaction terms, i.e. $\breve{u}^i$, are incorporated (Semsar-Kazerooni and Khorasani, 2007), that is we have:

$$\begin{cases} \dot{r}^i = v^i \\ \dot{v}^i = u^i + \breve{u}^i \\ \breve{u}^i = \sum_{j \in N^i} F^{ij} v^j, \ Y^i = v^i, \ i = 1, \dots, N \end{cases} \tag{1}$$

where $r^i \in R^2$ and $v^i \in R^2$ are the position and velocity vectors of vehicles and F^{ij} is the interaction coefficient. In the literature, e.g. (Stipanović et al., 2004; Semsar-Kazerooni and Khorasani, 2007), it is shown that a double integrator model can be considered as the linearized dynamical representation of a ground vehicle, e.g. a mobil robot.

Information structure and neighboring sets: In order to ensure cooperation and coordination among team members, each member has to know the status of other members, and therefore members have to communicate with each other. For a given agent i, the set of agents connected to it via communication links is called a neighboring set N^i:

$$\forall \ i = 1, \dots, N, \quad N^i = \{j = 1, \dots, N | (a_i, a_j) \in E\} \tag{2}$$

where E is the edge set corresponds to the underlying graph of the network.

2.2 Graph Theory Preliminaries

The following are some definitions and properties from graph theory that we will use throughout the paper (Fax and Murray, 2004; Olfati-Saber and Murray, 2004).

- **Laplacian Matrix:** This matrix is used to describe the graph associated with information exchanges in a network of agents and is defined as $L = [l_{ij}]_{N \times N}$, where
$$l_{ij} = \begin{cases} d_{out}(i) & i = j \\ -1 & (a_i, a_j) \in E \ \text{ and } \ i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{3}$$
where $d_{out}(i)$ is equal to the cardinality of the set N^i (Olfati-Saber and Murray, 2003), $|N^i|$, or the number of links that depart the vertex i and is called the out-degree of vertex i (the number of nodes from which it receives information).
- **Balanced graphs:** If the Laplacian matrix of a graph, L, has the property that $\mathbf{1}^T L = 0$, then the graph is called a balanced graph.
- For balanced connected graphs we have the property that $L + L^T$ can be considered as the Laplacian matrix of an undirected and connected graph described in (Olfati-Saber and Murray, 2004).
- **Normalized adjacency matrix:** The normalized adjacency matrix of a graph, denoted by A is a square matrix of size N, defined by $a_{ij} = \frac{1}{d_{out}(i)}$ if $(a_i, a_j) \in E$ and $i \neq j$, and is zero otherwise.
- **Normalized Laplacian matrix:** The normalized Laplacian matrix $\hat{L}$ is defined similar to the Laplacian matrix of a graph, where $\hat{L} = [\hat{l}_{ij}]_{N \times N}$ and
$$\hat{l}_{ij} = \begin{cases} 1 & i = j \\ -1/d_{out}(i) & (a_i, a_j) \in E \ \text{ and } \ i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The following is the well-known Perron-Frobenius Theorem for nonnegative matrices.

Theorem 1. **Perron-Frobenius Theorem** (Horn and Johnson, 1990). Let $M \in M_n$ and suppose that M is irreducible and nonnegative. Then

1) $\rho(M) > 0$

2) $\rho(M)$ is an eigenvalue of M

3) There is a positive vector x such that $Mx = \rho(M)x$

4) $\rho(M)$ is an algebraically simple eigenvalue of M;

where M_n is the set of matrices of order n and $\rho(M)$ is the spectral radius of matrix M (Horn and Johnson, 1990).

Since certain concepts from the original optimal control approach developed in (Semsar-Kazerooni and Khorasani, 2007) is needed here, a brief description of our method is provided below.

2.3 Application of Semi-Decentralized Optimal Control to a Modified Leader-Follower Team of Vehicles

Our main goal is to make agents' output, e.g. velocity, converge to a desired value, i.e. $\forall i,\ Y^i \to Y^d$. It is assumed that this command is available to only the leader and the other vehicles should follow the leader through information exchanges among themselves and the leader.

Definition of cost functions: To achieve the above requirement, let us define the cost functions for the team as follows

$$\begin{aligned} d^i = \int_0^T \{ \sum_{j\in N^i} [(Y^i - Y^j)^T Q^{ij} (Y^i - Y^j)] + \\ (u^i)^T R^i u^i \} dt + Y^i(T)^T E^i Y^i(T) + (F^i)^T Y^i(T) + m^i \end{aligned} \tag{5}$$

$$\begin{aligned} d^1 = \int_0^T \{ \sum_{j\in N^1} [(Y^1 - Y^j)^T Q^{1j} (Y^1 - Y^j)] \\ + [(Y^1 - Y^d)^T \Gamma (Y^1 - Y^d)] + (u^1)^T R^1 u^1 \} dt \\ + Y^1(T)^T E^1 Y^1(T) + (F^1)^T Y^1(T) + m^1 \end{aligned} \tag{6}$$

Superscript 1 is used to denote the leader and $i = 2, \dots, N$ correspond to the followers, respectively. In the above definitions Q^{ij}, Γ, E^i and R^i are symmetric and positive definite matrices, F^i is a vector with proper dimension, and m^i is a scalar. By minimizing the above cost functions it can be shown that in steady state all agents in a neighboring set would reach to the same output vector.

The following two lemmas are presented and proven in (Semsar-Kazerooni and Khorasani, 2007).

Lemma 2. Consider a group of vehicles whose dynamics are governed by the double integrator equations given in (1) and that has a fixed topology. The leader is aware of the desired command while the followers operate through interactions among the vehicles based on the neighboring sets. The interaction coefficient terms and the control laws proposed below would minimize the cost functions (5) and (6), and moreover guarantee the alignment of the vehicles, where:

$$\breve{u}^i = \sum_{j\in N^i} F^{ij} v^j = \sum_{j\in N^i} 2(K^i)^{-1} Q v^j, i = 1, \dots, N \tag{7}$$

$$u^{i*} = -\frac{1}{2}(R^i)^{-1} K^i(t) v^i,\ i = 2, \dots, N \tag{8}$$

$$u^{1*} = -\frac{1}{2}(R^1)^{-1}(K^1(t)v^1 + g^1(t)) \tag{9}$$

We assume that all Q^{ij}'s are selected to be the same, i.e. $\forall i, j,\ Q^{ij} = Q$ and u^{i*} and u^{1*} denote the optimal values of u^i and u^1, respectively. Moreover, v^d is the command provided for the leader and the leader's parameter g^1 and the Riccati equations for finding K^i satisfy:

$$-\dot{K}^i = 2|N^i|Q - \frac{1}{2}K^i(R^i)^{-1}K^i, i = 2, \dots, N \tag{10}$$

$$-\dot{K}^1 = 2(|N^1|Q + \Gamma) - \frac{1}{2}K^1(R^1)^{-1}K^1 \tag{11}$$

$$K^i(T) = 2E^i$$

$$\dot{g}^1 = 2\Gamma v^d + \frac{1}{2}K^1(R^1)^{-1}g^1,\ g^1(T) = (F^1)^T \tag{12}$$

Lemma 3. **a. Modified consensus protocol:** For the group of vehicles described in Lemma 2 corresponding to an infinite horizon problem (i.e., $T \longrightarrow \infty$), the control law $\hat{u}^i$ reduces to the modified agreement protocol for the leader-follower structure. The protocol for a follower is given by

$$\hat{u}^i(v^i, v^j) = u^i(v^i) + \breve{u}^i(v^j) = \Gamma^i(v^i - \frac{\sum_{j\in N^i} v^j}{|N^i|}) \tag{13}$$

and for the leader is given by

$$\hat{u}^1(v^1, v^j) = \Gamma^1(v^1 - \frac{\sum_{j\in N^1} v^j}{|N^1|}) + \beta^1(v^1 - v^d) \tag{14}$$

$$\forall i,\ \Gamma^i = -2(K^i)^{-1}|N^i|Q,\ \beta^1 = -2(K^1)^{-1}\Gamma \tag{15}$$

b. Stability: The above protocol stabilizes the closed-loop system, i.e. the error dynamics of the entire team is asymptotically stable, implying that

$$e^i = v^i - v^d \to 0 \quad \text{as } t \to \infty, \quad i = 1, \dots, N \tag{16}$$

Using the results obtained in (Semsar-Kazerooni and Khorasani, 2007), the closed-loop error dynamics matrix can be found as follows. We assume that the desired command v^d is time-invariant. Therefore, the error dynamics for the entire team can be found as $\dot{e} = L_{cl}e$ using the agents' dynamical equations and the input commands for the leader and followers as given by (1), (13), and (14), respectively, where

$$L_{cl} = \begin{bmatrix} \Gamma^1 + \beta^1 & \frac{l_{12}}{|N^1|}\Gamma^1 & \dots & \frac{l_{1N}}{|N^1|}\Gamma^1 \\ \frac{l_{21}}{|N^2|}\Gamma^2 & \Gamma^2 & \dots & \frac{l_{2N}}{|N^2|}\Gamma^2 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{l_{i1}}{|N^i|}\Gamma^i & \dots & \frac{l_{ij}}{|N^i|}\Gamma^i & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{l_{N1}}{|N^N|}\Gamma^N & \dots & \frac{l_{N(N-1)}}{|N^N|}\Gamma^N & \Gamma^N \end{bmatrix}$$

$e = [(e^1)^T \ \dots \ (e^N)^T]^T$, l_{ij} are the elements of the Laplacian matrix, and Γ^i and β^1 are defined in (15). This can be further simplified as follows:

$$\begin{aligned} L_{cl} &= -2K^{-1}(L \otimes Q + \begin{bmatrix} \Gamma & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 0 \end{bmatrix}) \\ &= -2K^{-1}(L \otimes Q + G) \end{aligned} \tag{17}$$

where $K = Diag\{K^i, i = 1, \ldots, N\}$ and L is the Laplacian matrix of the underlying graph. We will use the expression for L_{cl} to investigate the stability properties of the switching network topologies in the next section.

3. SWITCHING NETWORK TOPOLOGIES

As discussed earlier, there are many situations in which two agents in a team cannot gather the status of each other due to several restrictions existing for the team members. In this situation team members have to find new neighbors in order to maintain the connectivity of the team information graph. This implies that the neighboring sets should more appropriately be defined as time-varying sets, namely $N^i(t)$. These neighboring sets would result in a set of information graphs with time-varying Laplacian matrix, for which the only assumed condition is their connectivity. Moreover, in some circumstances in a leader-follower structure the assignment of the leader may change during the mission. In these scenarios the team structure will not remain fixed any longer and, therefore, we have to analyze the team behavior with a switching topology.

3.1 Switching Control Input and Stability Analysis

Assume that we have a team of agents with a switching topology due to the time-varying neighboring sets $N^i(t)$ or time-varying leader assignment. The corresponding switching signal is denoted by $\sigma(t) : \mathbb{R}^+ \to \mathbb{N}$ which is a train of pulsed signals that has a constant integer value over each time interval τ. The communication links among the agents are assumed to be directional with a Laplacian matrix denoted by L. For switching networks, at each time interval matrix L is a function of the switching signal, i.e. $L_{\sigma(t)}$, where $L_{\sigma(t)}$ describes the Laplacian of a strongly connected graph. To represent that the leader assignment is time-varying we may assume that the matrix G in (17) is a function of the switching signal as well, i.e. G_σ.

Subsequently, we denote all the parameters corresponding to the switching case by a subscript $\sigma(t)$, i.e. $(.)_\sigma$. Hence, the closed-loop matrix defined in (17) can be rewritten as:

$$L_{cl,\sigma} = -2K_\sigma^{-1}(L_\sigma \otimes Q + G_\sigma)$$

where $L_{cl,\sigma}, K_\sigma, L_\sigma, G_\sigma$ are the matrices L_{cl}, K, L, G corresponding to the switching structure. Obviously, the controller coefficient matrix K depends on the switching state since it is a function of the neighboring sets $N^i(t)$. Now, we can split matrix $L_{cl,\sigma}$ into two parts $\bar{L}_\sigma = K_\sigma^{-1}(L_\sigma \otimes Q)$ and $K_\sigma^{-1}G_\sigma$. The first part, $\bar{L}_\sigma$ is itself the Laplacian of a directed weighted graph which is not necessarily balanced. However, if we could transform $\bar{L}_\sigma$ into the Laplacian of a balanced graph, then we will show below that a common Lyapunov function for the corresponding switching system can be found. One solution for achieving the above goal is to design a **switching control** such that $\bar{L}_\sigma$ becomes balanced for any switching network. This implies that one needs to design the matrix K_σ to satisfy this condition. One of the means by which K_σ can be designed to compensate for the switching structure is by selecting different Q^{ij}'s for different nodes in each structure (in contrast to the assumption in Lemma 2). If such a control design goal can be accomplished, not only undirected graphs but also directed and unbalanced graphs can be analyzed by utilizing our proposed method.

Hence, assuming that Q^{ij} is no longer equal to Q, it will have different values for different agents and this is denoted by Q_σ^i for each switching state. Moreover, $\bar{L}_\sigma$ can be written as

$$\bar{L}_\sigma = \begin{bmatrix} (K_\sigma^1)^{-1}Q_\sigma^1 l_{11} & (K_\sigma^1)^{-1}Q_\sigma^1 l_{12} & \ldots & (K_\sigma^1)^{-1}Q_\sigma^1 l_{1N} \\ (K_\sigma^2)^{-1}Q_\sigma^2 l_{21} & (K_\sigma^2)^{-1}Q_\sigma^2 l_{22} & \ldots & (K_\sigma^2)^{-1}Q_\sigma^2 l_{2N} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

where l_{ij} is the (ij)-th entry of the matrix L_σ which is time dependent (due to switching). In order to have a balanced $\bar{L}_\sigma$ matrix, we should have

$$\begin{aligned} &(\mathbf{1}^T \otimes I_n)\bar{L}_\sigma = \mathbf{0} \ \to \\ &\left[\, (K_\sigma^1)^{-1}Q_\sigma^1 l_{11} \ (K_\sigma^2)^{-1}Q_\sigma^2 l_{22} \ \ldots \ (K_\sigma^N)^{-1}Q_\sigma^N l_{NN} \,\right] \times \\ &(\begin{bmatrix} 1 & l_{12}/l_{11} & \ldots & l_{1N}/l_{11} \\ l_{21}/l_{22} & 1 & \ldots & l_{2N}/l_{22} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \otimes I_n) = \mu_\sigma^T(\hat{L}_\sigma \otimes I_n) = 0 \end{aligned} \tag{18}$$

where $\mathbf{1}^T = [\,1\ \ldots\ 1\,]$, n is the dimension of the agents' output and $\hat{L}_\sigma$ is the normalized Laplacian matrix of the graph. To ensure the above expression, Q^{ij} (Q_σ^i), R^i and Γ should be selected such that μ_σ in (18) is in the left null-space of $\hat{L}_\sigma \otimes I_n$. Assume that ω_σ is a normalized vector in this space (the eigenvector of $\hat{L}_\sigma$ corresponds to the zero eigenvalue), then we should have

$$\mu_\sigma = \kappa\omega_\sigma \otimes I_n \tag{19}$$

where κ is the scaling factor that should be selected by using a desired performance criterion (e.g. along the lines of what is provided in the next subsection).

We now state the following lemma which will be used in the subsequent discussions of our method.

Lemma 4. The Laplacian matrix of any strongly connected directed graph has a left eigenvector which corresponds to the zero eigenvalue and whose entries have the same sign, i.e. they are either all positive or all negative.

Proof: Since the adjacency matrix A (refer to section 2.2), is a nonnegative matrix (according to its definition), it would satisfy all the properties that are stated in Theorem 1. Also, from graph theory we know that $\rho(A) = 1$ for a normalized adjacency matrix. Hence, 1 is an eigenvalue of A and the corresponding eigenvector would have positive entries. This applies to both the right and the left eigenvectors of A since both A and A^T are nonnegative. Using the relationship between the normalized Laplacian matrix $\hat{L}$ and A (Godsil and Royle, 2001), that is $\hat{L} = I - A$, the zero eigenvalue of $\hat{L}$ corresponds to the 1 eigenvalue of A, and the corresponding right eigenvector is $\mathbf{1}$. The left eigenvector is not $\mathbf{1}$ for directed graphs in general, unless they are balanced. However, this vector would have the property that its entries have the same sign.

We are now in a position to summarize the above discussions in the following lemma:

Lemma 5. **Stability analysis.** For the team of vehicles described in Lemmas 2 and 3, and under the assumptions of switching network and switching leader, the control laws $\hat{u}_\sigma^i, i = 1, \ldots, N$ selected according to

$$\hat{u}^i_\sigma(v^i, v^j) = \Gamma^i_\sigma(v^i - \frac{\sum_{j\in N^i} v^j}{|N^i(t)|}), i = 2, \ldots, N \quad (20)$$

$$\hat{u}^1_\sigma(v^1, v^j) = \Gamma^1_\sigma(v^1 - \frac{\sum_{j\in N^1} v^j}{|N^1(t)|}) + \beta^1_\sigma(v^1 - v^d) \quad (21)$$

$$\begin{aligned}\Gamma^i_\sigma &= -2\kappa\rho_{i,\sigma} I_n,\ i = 1, \ldots, N,\\ \beta^1_\sigma &= -\gamma(\kappa\rho_{1,\sigma} r^1 + \sqrt{(\kappa\rho_{1,\sigma} r^1)^2 + \gamma r^1}) I_n\end{aligned} \quad (22)$$

will guarantee that for the family of the closed-loop error dynamics

$$\dot{e} = L_{cl,\sigma} e,\ L_{cl,\sigma} = -2K^{-1}_\sigma(Q_\sigma L_\sigma \otimes I_n + G_\sigma) \quad (23)$$

a common Lyapunov function exists. This function ensures that the closed-loop dynamics is asymptotically stable, where $Q_\sigma = diag\{Q^1_\sigma, \ldots, Q^N_\sigma\}$, and $\rho_{i,\sigma}$ is the i-th element of the vector ω_σ, i.e. an eigenvector of $\hat{L}_\sigma$ corresponding to its zero eigenvalue. Moreover, matrices R^i and Γ used in (5), (6) are chosen as $R^i = r^i I$, $\Gamma = \gamma I$, where r^i, γ are two constants and κ is a design parameter used for improving the team performance as shown below.

Proof: See the Appendix A for details.

Given that the performance of the optimal controller is now limited due to imposing some constraints on the cost function gains Q^i_σ (see Appendix A), one may compensate this performance degradation by introducing a new criterion for selecting the design parameter κ. This parameter can be considered as a scaling factor which can define the weight given to different design specifications. Various criteria can be considered in order to guarantee a specific closed-loop team behavior. One such criterion deals with a tradeoff between the performance and the control effort, i.e. the relationship between the matrices Q^i_σ and R^i as discussed in the following subsection.

3.2 Criterion for Selection of κ: Performance-Control Effort Tradeoff

An issue that we would like to consider here deals with defining the criterion for selecting the scaling factor κ. An example of this criterion is to achieve a tradeoff between the performance and the control effort. According to the definitions of the cost functions given in (5) and (6), Q^{ij} defines the weight assigned to the performance whereas R^i is the weight assigned to the control effort. Hence, depending on the specifics of a particular application the weights may be selected differently. For example, we may require a predefined ratio between the matrices Q^{ij} and R^i, i.e. to require that $\frac{\lambda_{max}(Q^{ij})}{\lambda_{max}(R^i)} > m_i$. The following lemma provides a sufficient condition to guarantee this requirement.

Lemma 6. To achieve a tradeoff between the performance-control effort as characterized according to $\frac{\lambda_{max}(Q^{ij})}{\lambda_{max}(R^i)} > m_i$, $i = 1, \ldots, N$, the design parameter κ defined in Lemma 5 should be selected according to

$$\kappa^2 > \frac{1}{4}\max\{\frac{m_1|N^1|}{\rho^2_{1,\sigma}}, \frac{\max_{i=2,\ldots,N}(m_i|N^i|)}{\min_{i=2,\ldots,N}(\rho^2_{i,\sigma})}\} \quad (24)$$

Proof: See the Appendix B for details.

4. SIMULATION RESULTS

Simulation results presented in this section are for a team of four agents. The team structure switches between 3 structures based on a specific switching signal pattern that is shown in Figure 1. It follows from this figure that the switching signal can take 3 different values at different time intervals, namely 1, 2, and 3. In other words, there are three different states for the team structure and the leader assignment during the mission. The leader assignment is changing at each switching instant and is defined to be according to agents 1, 4, and 2 corresponding to $\sigma(t) = 1, 2, 3$, respectively. Moreover, the leader command is a pulsed-like signal which has the same duration as the switching signal time interval, τ. The leader command values for $\sigma(t) = 1, 2, 3$ is $v^d = [15\ 14]^T$, $[7\ 20]^T$, $[20\ 6]^T$, respectively. The graphs describing the network structure are directional and the Laplacian matrices corresponding to the three switching states are as follows:

$$L_1 = \begin{pmatrix} 1 & -1 & 0 & 0\\ -1 & 2 & 0 & -1\\ -1 & 0 & 1 & 0\\ 0 & 0 & -1 & 1\end{pmatrix},\ L_2 = \begin{pmatrix} 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\\ -1 & 0 & 2 & -1\\ 0 & -1 & -1 & 2\end{pmatrix}$$
$$L_3 = \begin{pmatrix} 1 & 0 & 0 & -1\\ -1 & 1 & 0 & 0\\ -1 & -1 & 2 & 0\\ 0 & 0 & -1 & 1\end{pmatrix} \quad (25)$$

The simulation results are obtained by of applying the switching control laws given in Lemma 5 to the network of four agents with the dynamics governed by (1) and the switching topology as described above. In Figure 2, the $x-$component and in Figure 3, the $y-$component of the velocity profiles of the four-agent team are shown for the above configurations. Figure 4 shows the paths that are generated by the agents during the mission where the team members are switching to different structures and operating with different commands and leaders.

5. CONCLUSIONS

A novel control strategy for consensus seeking in a team with a switching structure characteristic is developed and investigated. In contrast to the common assumption in the literature where graphs are assumed to be balanced, in this paper it is assumed that the graph describing the communication topology is not necessarily a balanced graph. A criterion for selecting the controller parameters is suggested to guarantee a specific performance requirement. An extension of this work is to search for a solution that provides the required stability conditions while reducing the restrictions that are imposed on the optimal performance of the controller.

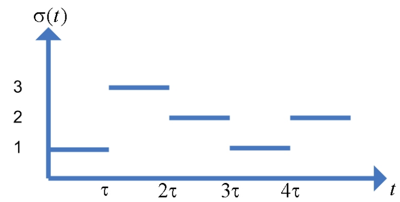

Fig. 1. Switching signal $\sigma(t)$

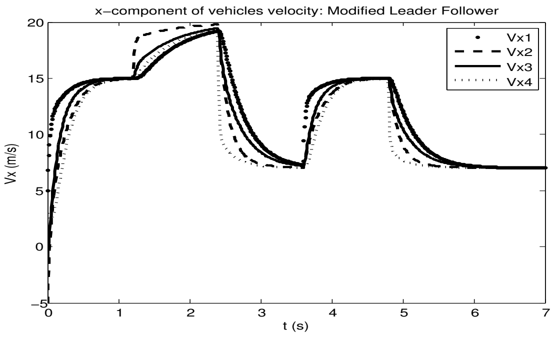

Fig. 2. The x-component of the velocity profile of a modified leader-follower (MLF) team of four agents with switching structure and switching leader resulting from the application of our proposed control strategy.

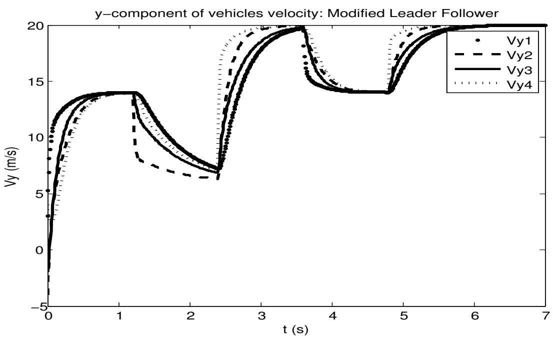

Fig. 3. The y-component of the velocity profile of a modified leader-follower (MLF) team of four agents with switching structure and switching leader resulting from the application of our proposed control strategy.

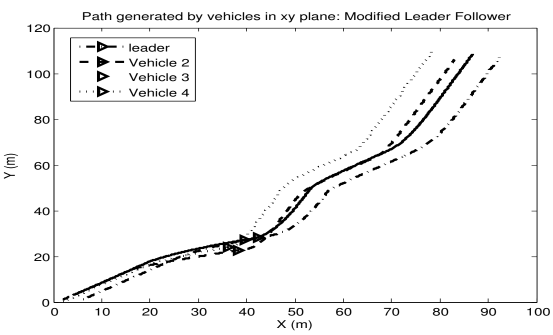

Fig. 4. The $x-y$ path trajectories of a modified leader-follower (MLF) team of four agents with switching structure and switching leader resulting from the application of our proposed control strategy.

APPENDIX A: PROOF OF LEMMA 5

First, we should show that the switching control laws given in (20) and (21) transform the matrix $\bar{L}_\sigma$ into a balanced matrix. For this to hold we have to design Q^{ij} (Q^i_σ) appropriately. Using the results achieved in Lemmas 2 and 3, we know that the following relationships hold between K^i (K^i_σ) and Q^{ij} (Q^i_σ) for an infinite horizon problem:

$$\begin{aligned} &2|N^i|Q^{ij} - \frac{1}{2}K^i(R^i)^{-1}K^i = 0,\ i = 2, \ldots, N \\ &2(|N^1|Q^{1j} + \Gamma) - \frac{1}{2}K^1(R^1)^{-1}K^1 = 0 \end{aligned} \tag{26}$$

Let us for sake of simplicity assume that all the design parameter matrices are homogenous and diagonal, i.e.:

$$Q^{ij} = q^i I,\ R^i = r^i I,\ \Gamma = \gamma I \tag{27}$$

The solutions to (26) is given by:

$$\begin{aligned} K^i &= 2\sqrt{|N^i|q^i r^i} I,\ i = 2, \ldots, N, \\ K^1 &= 2\sqrt{(|N^1|q^1 + \gamma)r^1} I \end{aligned} \tag{28}$$

Let us now denote the i-th element of the vector ω_σ as $\rho_{i,\sigma}$. Using the definition of μ_σ given in (18), we have:

$$\begin{aligned} &\mu_\sigma^T = \\ &\left(\frac{|N^1|q^1}{2\sqrt{(|N^1|q^1+\gamma)r^1}}\ \frac{|N^2|q^2}{2\sqrt{|N^2|q^2 r^2}} \cdots \frac{|N^N|q^N}{2\sqrt{|N^N|q^N r^N}} \right) \\ &\otimes I_n = \kappa\,[\,\rho_{1,\sigma}\ \rho_{2,\sigma}\ \ldots\ \rho_{N,\sigma}\,] \otimes I_n \end{aligned} \tag{29}$$

The following relationships would then follow

$$\begin{cases} \kappa\rho_{1,\sigma} = \dfrac{|N^1|q^1}{2\sqrt{(|N^1|q^1+\gamma)r^1}} \\ \kappa\rho_{i,\sigma} = \dfrac{|N^i|q^i}{2\sqrt{|N^i|q^i r^i}},\ i = 2, \ldots, N \end{cases} \tag{30}$$

In the first equation of (30), $\rho_{1,\sigma}$ and $|N^1|$ are given and κ, q^1, r^1, γ are parameters to be selected. Similarly, in the second equation κ, q^i, r^i are to be selected. It is assumed that r^i and γ are set to fixed values and q^i is then obtained that satisfies the above equations. Therefore, the following equations in terms of q^i should be satisfied:

$$\begin{cases} |N^1|^2(q^1)^2 - (4\kappa^2\rho_{1,\sigma}^2|N^1|r^1)q^1 - 4(\kappa\rho_{1,\sigma})^2\gamma r^1 = 0 \\ q^i = \dfrac{4(\kappa\rho_{i,\sigma})^2 r^i}{|N^i|},\ i = 2, \ldots, N \end{cases} \tag{31}$$

It is not difficult to show that the first equation in (31) always has a positive solution $q^1 = \frac{2\kappa\rho_{1,\sigma}}{|N^1|}(\kappa\rho_{1,\sigma}r^1 + \sqrt{(\kappa\rho_{1,\sigma}r^1)^2 + \gamma r^1})$. Also, from the second equation of (31), it is obvious that q^i is always positive. Hence, there is always a positive solution for $q^i, i = 1, \ldots, N$.

However, for the above results to be guaranteed, one should ensure a property in the left null space of $\hat{L}_\sigma$. Namely, due to the positive definiteness of $(K^i_\sigma)^{-1}Q^i_\sigma l_{ii}$, all the elements of the vector μ_σ are of the same sign, i.e. positive, which implies that the null space of $\hat{L}_\sigma$ should also enjoy this property. This can be shown by using the results provided in Lemma 4.

We are now in a position to use the above relationships for determining the switching control law. From Lemma 3, the control inputs can be calculated by using (13)-(15). By replacing q^i from (31), and K^i from (28) we obtain

$$\begin{aligned} \Gamma^i &= -2\frac{|N^i|q^i}{2\sqrt{|N^i|q^i r^i}} I_n = -2\kappa\rho_{i,\sigma} I_n,\ i = 2, \ldots, N, \\ \Gamma^1 &= -2\frac{|N^1|q^1}{2\sqrt{(|N^1|q^1+\gamma)r^1}} I_n = -2\kappa\rho_{1,\sigma} I_n, \\ \beta^1 &= -2\frac{\gamma}{2\sqrt{(|N^1|q^1+\gamma)r^1}} I_n \\ &= -\gamma(\kappa\rho_{1,\sigma}r^1 + \sqrt{(\kappa\rho_{1,\sigma}r^1)^2 + \gamma r^1}) I_n \end{aligned} \tag{32}$$

where $\rho_{i,\sigma}$ can be found if the Laplacian matrix of the team at each switching stage is given. The control laws provided in (32) are the same as the ones given in Lemma 5 and guarantee that matrix $\bar{L}_\sigma$ has $\mathbf{1}$ as its left eigenvector corresponding to the zero eigenvalue and is the Laplacian of a balanced graph.

For showing the stability of the closed-loop switching system we should suggest a common Lyapunov function which is valid for all the switching states. Let us select a Lyapunov function candidate to be $V = \frac{1}{2}e^T Pe$ and assume that $P = I$. Its derivative along the trajectories of (23) is given by $\dot{V} = \frac{1}{2}e^T(L_{cl,\sigma} + L_{cl,\sigma}^T)e = -e^T(\bar{L}_\sigma + \bar{L}_\sigma^T + K_\sigma^{-1}G_\sigma + G_\sigma^T K_\sigma^{-1})e$. Based on the above discussion $\bar{L}_\sigma = 2K_\sigma^{-1}Q_\sigma L_\sigma \otimes I_n$ can be considered as the Laplacian matrix of a weighted and a balanced graph. By using the results provided in (Olfati-Saber and Murray, 2004), $\bar{L}_\sigma + \bar{L}_\sigma^T$ is also a valid Laplacian matrix representing an undirected (due to its symmetry) and connected graph. Hence, it is a positive semi-definite (PSD) matrix. Moreover, the second term in the expression (23), i.e. $K_\sigma^{-1}G_\sigma$ is a diagonal matrix with one non-zero and positive element and so is PSD. Hence, $L_{cl,\sigma} + L_{cl,\sigma}^T$ is at least NSD. Also, the null space of the two matrices, $\bar{L}_\sigma + \bar{L}_\sigma^T$ and $K_\sigma^{-1}G_\sigma$ does not have any intersection and hence their summation is a PD matrix. Hence, $\dot{V} < 0$ and the proof is complete.

APPENDIX B: PROOF OF LEMMA 6

Similar to the proof of Lemma 5, and without loss of generality, assume that all the matrices involved are homogenous diagonal matrices. We would then have $\frac{\lambda_{max}(Q^{ij})}{\lambda_{max}(R^i)} = \frac{q^i}{r^i} = \frac{4(\kappa\rho_{i,\sigma})^2}{|N^i|} > m_i$, $i = 2, \ldots, N$ and given that $\forall i,\ \rho_{i,\sigma} \neq 0$ (Lemma 4), we have

$\kappa^2 > \frac{\max_{i=2,\ldots,N}(m_i|N^i|)}{4\min_{i=2,\ldots,N}(\rho_{i,\sigma}^2)}$

On the other hand for the leader, we have the following relationship:

$q^1 = \frac{2\kappa\rho_{1,\sigma}}{|N^1|}(\kappa\rho_{1,\sigma}r^1 + \sqrt{(\kappa\rho_{1,\sigma}r^1)^2 + \gamma r^1}) > \frac{4\kappa^2\rho_{1,\sigma}^2 r^1}{|N^1|}$

and therefore it is sufficient to select κ so that $\kappa^2 > \frac{m_1|N^1|}{4\rho_{1,\sigma}^2}$. Consequently, κ should be selected according to:

$$\kappa^2 > \frac{1}{4}\max\{\frac{m_1|N^1|}{\rho_{1,\sigma}^2}, \frac{\max_{i=2,\ldots,N}(m_i|N^i|)}{\min_{i=2,\ldots,N}(\rho_{i,\sigma}^2)}\} \tag{33}$$

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