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Sensor classification for the disturbance rejection by measurement feedback problem

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Abstract: In this paper, we consider dynamical models and we study the preservation of solvability for the Disturbance Rejection by Measurement Feedback (DRMF) problem under sensor failure in a structural framework. We consider a linear structured system and we wonder if the DRMF problem remains solvable in case of some sensor failure. More precisely we will characterize among the sensors some of those which are critical *i.e.* which failure leads to solvability loss, and some of those which are useless for solvability purpose.

Keywords: Linear structured systems, Disturbance rejection, Sensor failure, Sensor classification

1. INTRODUCTION

This paper is concerned with linear systems which are affected by unmeasurable disturbances and we look for a measurement feedback which rejects these disturbances. When this problem is solvable we wonder if the problem remains solvable in case of sensor failure. The problem of disturbance rejection by state feedback is a very well known problem. In the case where the state is not available for measurement the problem is more complex. The problem of disturbance rejection by measurement feedback has been solved in an elegant way in geometric terms, see Schumacher [1980], Willems and Commault [1981].
We consider here linear structured systems which represent a large class of parameter dependent linear systems. Generic properties for such systems can be obtained from a graph naturally associated with the system. This approach was pioneered by Lin [1974]. In this framework the Disturbance Rejection by Measurement Feedback (DRMF) problem has been solved via a graph approach in van der Woude [1993], Commault et al. [1997]. The graph approach generalizes the intuitive idea that the DRMF problem is solvable if and only if the measurements contain enough information to compute the effect of disturbances on the outputs, the control inputs are powerful enough to compensate these effects and in the context of discrete time systems, the time for the disturbances to affect controlled outputs should be longer than the time for measuring plus the time to annihilate the effect of the disturbance on the output. In this paper we focus our interest on the DRMF problem in case of potential sensor failures. It is clear that the solvability of this problem highly relies on the availability of the sensors. We will then tackle the problem of sensor classification with respect to their criticity for the DRMF problem under sensor failure. We wonder if the DRMF problem remains solvable in case of some sensor failure. More precisely we will characterize among the sensors some of those which are critical *i.e.* which failure leads to solvability loss, and some of those which are useless for solvability purpose. A similar sensor classification has already been studied for two other problems, the observability in Commault et al. [2006] and the Fault Detection and Isolation problem in Commault et al. [2007].
For the problem under consideration we determine using simple graph methods, sets of essential or useless sensors. The outline of this paper is as follows. First of all, we formulate the problem of sensor classification in section 2. The linear structured systems are presented in section 3. In section 4 we study the disturbance rejection by measurement feedback problem. The classification of sensor for disturbance rejection by measurement feedback problem is considered in section 5. In section 6 we work out an illustrative example. Some concluding remarks end the paper.

2. PROBLEM FORMULATION

In this paper we focus our interest on the Disturbance Rejection by Measurement Feedback (DRMF) problem in a structural framework. When the problem is solvable it is clear that the solution of this problem highly relies on the availability of the sensors. We will then tackle the problem of sensor classification regarding their criticity for the DRMF problem under sensor failure.
We define a *failing sensor* as a sensor which is down *i.e.* whose measure is no more available. We point out two main classes of sensors: the essential ones which are compulsory to preserve the property and the useless ones which do not play any role for solving the problem.

Definition 1. Let Σ be the linear system defined by:

$$\Sigma \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}, \tag{1}$$

where $u(t) \in \mathbb{R}^m$ is the input vector, $x(t) \in \mathbb{R}^n$ is the state vector and $y(t) \in \mathbb{R}^p$ the output vector provided by a set of sensors, and a property $\mathcal{P}$ which is satisfied for this system. We call solution for $\mathcal{P}$ a set of sensors $V \subset Y = \{y_1, y_2, \ldots, y_p\}$ such that $\mathcal{P}$ remains satisfied with the output set V. For $\mathcal{P}$, a sensor y^* can be classified as follows:

(1) y^* is called a *useless sensor* if for any solution V containing y^*, $\{V \backslash y^*\}$ is still a solution for $\mathcal{P}$. A sensor which is not useless is called a *useful sensor*.
(2) y^* is called an *essential sensor* if y^* belongs to any solution V. The set of essential sensors is a subset of the set of useful sensors.

Another classification of sensors has been introduced in Staroswiecki et al. [2004]. The authors introduce the notion of *minimal sensor set (MSS)*. A minimal sensor set is a set of sensors such that the property $\mathcal{P}$ is satisfied for this set of sensors but not for any proper subset. The authors define also critical sensor subsets which are sets of sensors whose simultaneous failure results in property loss. The notions presented in this paper are related with the above notions in the following way:

- A useless sensor is a sensor which does not belong to any minimal sensor set of the system.
- An essential sensor is a sensor which belongs to any minimal sensor set.
- An essential sensor can also be characterized as a critical sensor subset of cardinality one.

In the following, we will apply these notions to classify the sensors for the solvability of Disturbance Rejection by Measurement Feedback problem. More specifically, we determine some sets of useless sensors as well as some essential sensors.

3. LINEAR STRUCTURED SYSTEMS

In this part, we recall some definitions and results on linear structured systems. More details can be found in Dion et al. [2003]. We consider linear systems of type (1) with parameterized entries and denoted by Σ_Λ as follows:

$$\Sigma_\Lambda \begin{cases} \dot{x}(t) = A_\Lambda x(t) + B_\Lambda u(t) \\ y(t) = C_\Lambda x(t) \end{cases}, \qquad (2)$$

This system is called a linear structured system if the entries of the composite matrix $J_\Lambda = \begin{bmatrix} A_\Lambda & B_\Lambda \\ C_\Lambda & 0 \end{bmatrix}$ are either fixed zeros or independent parameters (not related by algebraic equations). $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ denotes the set of independent parameters of the composite matrix J_Λ.
For such systems, one can study generic properties *i.e.* properties which are true for almost all values of the parameters collected in Λ, see Murota [1987]. More precisely a property is said to be generic (or structural) if it is true for all values of the parameters (*i.e.* any $\Lambda \in \mathbb{R}^k$) outside a proper algebraic variety of the parameter space. A directed graph $G(\Sigma) = (V', W')$ can be associated with the structured system Σ_Λ of type (2):

- the vertex set is $V' = U \cup X \cup Y$ where U, X and Y are the input, state and output sets given by $\{u_1, u_2, \ldots, x_m\}$, $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_p\}$ respectively,
- the arc set is $W' = \{(u_i, x_j) | B_{\Lambda,ji} \neq 0\} \cup \{(x_i, x_j) | A_{\Lambda,ji} \neq 0\} \cup \{(x_i, y_j) | C_{\Lambda,ji} \neq 0\}$, where $A_{\Lambda,ji}$ (resp. $B_{\Lambda,ji}$, $C_{\Lambda,ji}$) denotes the entry (j,i) of the matrix A_Λ (resp. B_Λ, C_Λ).

Let V_1, V_2 be two nonempty subsets of the vertex set V' of the graph $G(\Sigma_\Lambda)$. We say that there exists a *path* from V_1 to V_2 if there are an integer q and vertices $i_0, i_1, \ldots, i_q$ such that $i_0 \in V_1$, $i_q \in V_2$, $i_t \in V'$ for $t = 0, 1, \ldots, q$ and $(i_{t-1}, i_t) \in W'$ for $t = 1, 2, \ldots, q$. We call the path *simple* if every vertex on the path occurs only once. The path is then denoted $(i_0, i_1), (i_1, i_2), \ldots, (i_{q-1}, i_q)$. If $i_0 \in X$ and, $i_q \in Y$, the path is called a state-output path. If $i_0 \in U$ and, $i_q \in Y$, the path is called a input-output path.
Two paths from V_1 to V_2 are said to be *disjoint* if they consist of disjoint sets of vertices. We call r paths from V_1 to V_2 disjoint if they are mutually disjoint, *i.e.* each two of them are disjoint. We call a set of r disjoint and simple paths from V_1 to V_2 a *linking from V_1 to V_2 of size r*. Since there are only a finite number of linkings, there obviously exist linkings consisting of a maximal number of disjoint paths. We call such linkings *maximal (size) linkings.*

4. DISTURBANCE REJECTION BY MEASUREMENT FEEDBACK

We consider a system of type (1) with an additional input $d(t) \in \mathbb{R}^q$ which is called disturbance and which we would like to have no effect on the output. We assume that the disturbance is completely unavailable for control purposes. In general the state is not completely available for feedback, but we only have access to a measured output $z = Hx$, which can be seen as a partial state. Hence, we consider the system Σ_{dz} given by:

$$\Sigma_{dz} \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y(t) = Cx(t) \\ z(t) = Hx(t) \end{cases}, \qquad (3)$$

where $z(t) \in \mathbb{R}^\nu$ and $d(t) \in \mathbb{R}^q$. For such a system, we have the transfer matrix:

$$\begin{bmatrix} y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} G(s) & K(s) \\ M(s) & N(s) \end{bmatrix} \begin{bmatrix} u(s) \\ d(s) \end{bmatrix} \qquad (4)$$

The problem of disturbance rejection then amounts to find a dynamical measured output feedback:

$$\Sigma_{zu} \begin{cases} \dot{w}(t) = Lw(t) + Mz(t) \\ u(t) = Nw(t) + Pz(t) \end{cases}, \qquad (5)$$

or in transfer matrix terms a dynamic compensator (see Fig 1) $u(s) = F(s)z(s)$ with $F(s)$ a proper rational matrix, such that the closed loop system transfer matrix from disturbance d to controlled output y is identically zero:

$$G(s)F(s)(I - M(s)F(s))^{-1}N(s) + K(s) = 0 \qquad (6)$$

This problem has a very elegant solution in geometric terms, see Schumacher [1980]. A necessary and sufficient solvability condition for disturbance rejection by measurement feedback is:

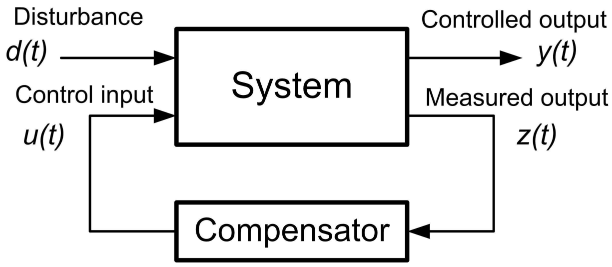

Fig. 1. Control with a dynamic feedback compensator

$$\mathbb{N}^* \subset \mathbb{V}^* \tag{7}$$

where $\mathbb{N}^*$ is the minimal (H, A)-invariant subspace containing $Im\ E$ and $\mathbb{V}^*$ is the maximal (A, B)-invariant subspace contained in $Ker\ C$.
In the context of linear structured system, we can transform the system of type (3) into a system with parameterized entries of type (2) as follows:

$$\Sigma_{\Lambda dz} \begin{cases} \dot{x}(t) = A_\Lambda x(t) + B_\Lambda u(t) + E_\Lambda d(t) \\ y(t) = C_\Lambda x(t) \\ z(t) = H_\Lambda x(t) \end{cases}, \tag{8}$$

With $\Sigma_{\Lambda dz}$, we can associate a graph $G(\Sigma_{\Lambda dz}) = (V, W)$ as previously with $V = V' \cup D \cup Z$ where D and Z are two news sets of vertices $D = \{d_1, d_2, \dots d_q\}$, $Z = \{z_1, z_2, \dots z_\nu\}$ and the corresponding arc set is $W = W' \cup \{(d_i, x_j) | E_{\Lambda,ji} \neq 0\} \cup \{(x_i, z_j) | H_{\Lambda,ji} \neq 0\}$ where $E_{\Lambda,ji}$ (resp. $H_{\Lambda,ji}$) denotes the entry (j, i) of the matrix E_Λ (resp. H_Λ).

Definition 2. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Denote I^* the set of vertices:
$I^* = \{x_i \in X \mid$ the maximal size of a linking in $G(\Sigma_{\Lambda dz})$ from $U \cup x_i$ to Y is the same as the maximal size of a linking in $G(\Sigma_{\Lambda dz})$ from U to Y, and the minimal number of vertices in $X \cup U$ is the same for both such maximal linkings $\}$

The set I^* corresponds to the states for which, a disturbance affecting directly these states, can be rejected by state feedback. In a dual way, we can define the set of vertices J^* which will be useful later:

Definition 3. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Denote J^* the set of vertices such that:
$J^* = \{x_j \in X \mid$ the maximal size of a linking in $G(\Sigma_{\Lambda dz})$ from D to $Z \cup x_j$ to is the same as the maximal size of a linking in $G(\Sigma_{\Lambda dz})$ from D to Z, and the minimal number of vertices in $X \cup Z$ is the same for both such maximal linkings$\}$

The set J^* corresponds to the states which can be estimated from the measured outputs through an observer independently from the disturbance. From the definitions of I^* and J^*, the geometric condition for disturbance rejection by measurement feedback can be translated for linear structured system as follows: van der Woude [1993], Commault et al. [1997]

Theorem 4. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. The problem of disturbance rejection by measurement feedback is generically solvable if and only if:

$$I^* \cup J^* = X \tag{9}$$

5. SENSOR CLASSIFICATION FOR THE DRMF PROBLEM

In this section we will classify the sensors with respect to the solvability of DRMF.

5.1 Useless sensors

In this subsection we will characterize some useless sensors for the DRMF problem. We will prove in particular that sensors measuring variables only out of I^* are useless. This proof will need some preliminary results.

Lemma 5. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Let L_{DZ} be a maximal linking in $G(\Sigma_{\Lambda dz})$ from D to Z having a minimal number of vertices in $X \cup Z$ denoted n_{XZ}. Let J^* be as in Definition 3. Then for any $x_i \in L_{DZ}$, $x_i \notin J^*$.

Proof: From the calculation of J^* it follows directly, by considering x_i as a new measurement, that the size μ of the maximal linking from D to Z cannot be reduced. So we have two cases:
1. If μ increases, $x_i \notin J^*$ by Definition 3.
2. If μ remains the same, the path from D to Z through x_i which is needed for the construction of the linking L_{DZ} is now ended at x_i, so n_{XZ} reduces at least by 1. Then $x_i \notin J^*$.■

Remark 6. The reverse is not true i.e. a vertex which is not in L_{DZ} may belong to $X \setminus J^*$.

We show now that discarding sensors measuring only variables of J^* does not change the set J^*.

Lemma 7. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$ and the set J^* of Definition 3. Assume that the disturbance rejection by measurement feedback has a solution. Let $z_j \in Z$ be such that for any x_i with $(x_i, z_j) \in W$, we have $x_i \in J^*$.
Consider the subsystem $\Sigma^{zj}_{\Lambda dz}$ obtained from $\Sigma_{\Lambda dz}$ by deleting in $G(\Sigma_{\Lambda dz})$ the vertex z_j and the adjacent edges. Denote $J^*(\Sigma^{zj}_{\Lambda dz})$ the set J^* of Definition 3 for $\Sigma^{zj}_{\Lambda dz}$. One has:

$$J^*(\Sigma^{zj}_{\Lambda dz}) = J^*$$

Proof: For $\Sigma_{\Lambda dz}$, we have a set J^* and a maximal linking L_{DZ} of size μ containing a minimum of n_{XZ} vertices belonging to $X \cup Z$. By Lemma 5 any x_i, x_k in J^* are such that $x_i, x_k \notin L_{DZ}$, so the suppression of z_j and adjacent edges does not modify L_{DZ} as all its previous vertices belong to J^*. In other words, J^* is not modified.■

We prove now that the sensors measuring only variables in J^* are not essential.

Proposition 8. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Assume that the Disturbance Rejection by Measurement Feedback problem has a solution. Let $z_j \in Z$ be such that for any $(x_i, z_j) \in W$, $x_i \in J^*$. Then z_j is not essential for the solvability of the problem.

Proof: As z_j measures only vertices in J^*, the suppression of z_j does not change J^* following Lemma 7. Furthermore, the set of vertices I^* is independent from Z, so the solvability of DRMF problem is held.■

The previous results allow us to prove that the sensors measuring only variables outside of I^* are indeed useless.

Theorem 9. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Assume that the Disturbance Rejection by Measurement Feedback problem has a solution. Let $z_j \in Z$ be such that for any $(x_i, z_j) \in W$, $x_i \in \bar{I}^* = X \setminus I^*$. Then z_j is useless for DRMF.

Proof: Let $z_j \in Z$ be such that for any $(x_i, z_j) \in W$, $x_i \in \bar{I}^*$. Let V, a subset of the sensor set containing z_j, be a solution of the problem in the sense of Definition 1. Denote J_V^* the corresponding set J^*, then

$$I^* \cup J_V^* = X \tag{10}$$

from Theorem 4 and because I^* does not depend on the sensor set. Therefore, z_j measures only vertices in $J_V^* \subset \bar{I}^*$. From Proposition 8, it follows that there exists another subset of V not containing z_j which is a solution. From Definition 1, this means that z_j is useless.■

5.2 Essential sensors

In this subsection we will characterize some essential sensors for the DRMF problem. In this part, we will assume that each sensor measures only one state.

Definition 10. Consider a structured system of type (8) with associated graph $G(\Sigma_{\Lambda dz})$. Denote F_{I^*}, the frontier of I^* as the set of vertices:
$F_{I^*} = \{x_i \in I^* \mid \exists (x_i, x_j) \in W,\, x_j \notin I^*\}$.

We prove now that a sensor measuring a state of F_{I^*} which is directly affected by a disturbance acting on this state, but not acting on another measured state, is essential.

Proposition 11. Consider $\Sigma_{\Lambda dz}$ a structured system of type (8) with associated graph $G(\Sigma_{dz})$. Assume that the disturbance rejection by measurement feedback has a solution. Let $x_i \in F_{I^*}$ and $z_j \in Z$ be such that there exists $(x_i, z_j) \in W$. Assume that there exists $d_k \in D$ such that there exists $(d_k, x_i) \in W$ and no $x_l \neq x_i$ such that $(d_k, x_l) \in W$ and $z_\xi \neq z_j$ such that $(x_l, z_\xi) \in W$. Then z_j is essential for DRMF problem.

Sketchy proof: By hypothesis, any path from d_k to any sensor different from z_j, must be longer than the path $d_k \to x_i \to z_j$. For example, on the Figure 2, one has the path $d_k \to x_r \to x_s \to \ldots \to x_t \to z_v$. Then, we can prove that the arc (x_i, z_j) belongs to any maximal linking L_{DZ} with minimal number of vertices in $G(\Sigma_{\Lambda dz})$.
By the definitions of I^* and F_{I^*}, each $x_i \in F_{I^*}$ has at least one successor denoted x_m such that $(x_i, x_m) \in W$, $x_m \notin I^*$. As the DRMF problem has a solution then $x_m \in J^*$.
From this point, for any solution $V \subset Y$ of the DRMF problem which contains the measure z_j, we can prove that discarding z_j rejects x_m out of J^* since in this case, x_m plays the role of a new sensor. Since I^* does not change, we still have $x_m \notin I^*$. Finally $x_m \notin I^* \cup J^*$ then $I^* \cup J^* \neq X$. In other words, if V is any solution of the DRMF problem which contains z_j, V minus z_j is no longer a solution, then z_j is essential.■

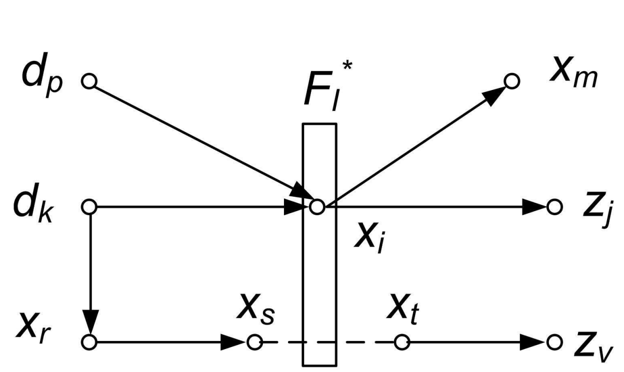

Fig. 2. Disturbance affecting directly a state of F_{I^*}

6. ILLUSTRATIVE EXAMPLE

6.1 The system and the solvability of the DRFM Problem

Consider the thermal process which is described in Figure 3. This process consists of five tanks, each tank is fed by a fixed water flow (F_1, F_2 and $F_1 + F_2$). The system control input is the heating power w. The regulated output is T_5, the temperature of the fifth tank. The disturbance is the feeding temperature T_0 and the measured output is $z = T_2$. This process can be linearized as a system of type (3) with

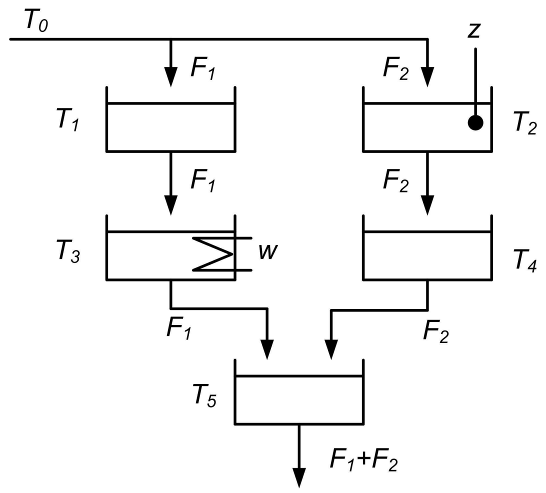

Fig. 3. The five tank system

$x = [\; T_1 \; T_2 \; T_3 \; T_4 \; T_5 \;]^T$, $d = T_0$, $u = w$ and $y = T_5$. We have the following state space matrices:

$$A = \begin{bmatrix} \frac{-F_1}{C_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{-F_2}{C_2} & 0 & 0 & 0 \\ \frac{F_1}{C_3} & 0 & \frac{-F_1}{C_3} & 0 & 0 \\ 0 & \frac{F_2}{C_4} & 0 & \frac{-F_2}{C_4} & 0 \\ 0 & 0 & \frac{F_1}{C_5} & \frac{F_2}{C_5} & \frac{-F_1 - F_2}{C_5} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/C_3 \\ 0 \\ 0 \end{bmatrix}, \quad \text{E}= \begin{bmatrix} F_1/C_1 \\ F_1/C_2 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [\,0\ 0\ 0\ 0\ 1\,], \text{H}= [\,0\ 1\ 0\ 0\ 0\,],$$

where C_i is the calorific capacity of the ith tank.
This model exhibits clearly the physical structure of the process. Notice that this model is not structured because some dependencies exist between the matrix entries. Nevertheless, to illustrate our approach, we will consider the following structured system of form (8):

$$A_\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ \lambda_3 & 0 & \lambda_4 & 0 & 0 \\ 0 & \lambda_5 & 0 & \lambda_6 & 0 \\ 0 & 0 & \lambda_7 & \lambda_8 & \lambda_9 \end{bmatrix}, B_\Lambda = \begin{bmatrix} 0 \\ 0 \\ \lambda_{10} \\ 0 \\ 0 \end{bmatrix},$$

$$E_\Lambda = \begin{bmatrix} \lambda_{11} \\ \lambda_{12} \\ 0 \\ 0 \\ 0 \end{bmatrix}, C_\Lambda = [\,0\ 0\ 0\ 0\ \lambda_{13}\,], \qquad (11)$$

$$H_\Lambda = [\,0\ \lambda_{14}\ 0\ 0\ 0\,]$$

This system has the same zero/nonzero structure as the physical system. The associated graph is depicted in Figure 4. From Definitions 2 and 3, we have $I^* = \{x_1, x_2\}$

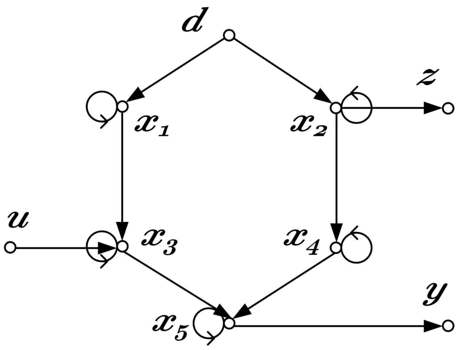

Fig. 4. Graph of the five tank system

and $J^* = \{x_3, x_4, x_5\}$. As $I^* \cup J^* = X$, the Disturbance Rejection by Measurement Feedback problem is solvable for this system. Indeed, on this model the measurement of x_2 allows to get an early information on the disturbance d and to compensate on time with u for the effect of this disturbance on the output y.

6.2 Sensor classification

In this case the classification is rather simple. Because the DRMF problem is solvable and we have only one sensor, it is clear that this unique sensor is essential.
As seen previously $I^* = \{x_1, x_2\}$ then the frontier F_{I^*} is also equal to $\{x_1, x_2\}$. From Proposition 11 the sensor $z = x_2$ is essential.
Assume now that all the variables $x_1, \ldots x_5$ are available for measurement. From Theorem 9, the measurements of x_3, x_4 and x_5 are useless for solving the DRMF problem. These measurements would have provided us with a too late information. From the previous theoretical results one cannot conclude directly concerning the classification of the measurements x_1 and x_2. But as previously shown, when only x_2 is available for measurement the DRMF problem is solvable and this measure is essential. The same analysis could be performed when only x_1 is available for measurement, the DRMF problem is solvable and this measure is also essential. Therefore when x_1 and x_2 are both available, x_1 and x_2 are both useful but not essential.

7. CONCLUDING REMARKS

In this paper, we revisited the Disturbance Rejection by Measurement Feedback (DRMF) problem for structured systems. Using a graph approach we focused our attention on solvability preservation of this problem under sensor failure and have determined sets of sensors which are essential and sets of sensors which are useless for the solvability of the DRMF problem. It is a first step towards a complete characterization and classification of the sensors respective to their criticity for solving the DRMF problem. The proposed approach is well suited for structural analysis prior to computation of the dynamic output feedback controller. More over, its numerical implementation is simple as the computation of useless and essential sensors can be performed in polynomial time. The interest of the proposed approach is illustrated on a simple example.

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