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Stabilizability of Uncertain Switched Systems via Static/Dynamic Output Feedback Sliding Mode Control ⋆

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Abstract: The output feedback stabilizability problem of a class of uncertain switched systems is investigated using sliding mode control and a synthesis design solution derived. Firstly, a common sliding surface is constructed such that the system restricted to the sliding surface is asymptotically stable and completely invariant to matched and mismatched uncertainties under arbitrary switching. Secondly, static and dynamic output feedback variable structure controllers are designed that can drive the state of the switched system to reach the common sliding surface in finite time and remain them thereafter. A illustrative example is given to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Switched systems represent one of the most active topics research recently. For, these are an important class of hybrid systems that are consist of a family of subsystems and a switching law specifying at each instant of time which of the subsystems is activated along the system trajectory. Switched systems are an appealing class of systems for both theoretical investigation as well as development of practical applications. To switching between different internal or external system structures appears an essential feature of many systems, for instance, power systems and power electronics (Williams, Hoft [1991]).

A great deal of the research carried out recently has been devoted to stability analysis and synthesis design of switched systems (Mancilla-Aguilar, Garcia [1999], Zhao, Dimirovski [2004]) with the goal to achieve either asymptotic or quadratic stability. Asymptotic stability of a switched system under arbitrary switching law is a most desirable property, which may be guaranteed by a common Lyapunov function shared by subsystems (Liberson, Morse [1999]). There exist a number of ways to have a common Lyapunov function. For example, when two stable matrices are commutative, it was proved in Narendra, Balakrishnan [1994] that they share a common Lyapunov function. Existence conditions for a common Lyapunov function were also studied in Cheng, Gao, Huang [2003], Liberson, Hespanhan, Morse [1999], Vu, Liberzon [2005].

On the other hand, handling uncertainties is one of the key issues too in the study of switched systems. Robust control and stabilization of uncertain switched linear systems are addressed on the grounds of the multiple Lyapunov function approach in Ji, Wang, Xie [2006]. Sun [2004] addressed robustness issues for a class of switched linear systems with perturbations, and proposed a state feedback switching law with a non-zero level set.

Over the years, much attention has been pain to investigate sliding mode control of uncertain systems without switching in the essential meaning work (Choi [2003], Shyu, Tsai, Lai [2001], Choi [2002], Goncalves [2001], Goncalves [2003]). However, due to of the complexity of the systems themselves and the excess burden of design, there are no results for output feedback sliding mode control of switched systems in the current literature.

In this paper, we consider the output feedback sliding mode control problem for a class of uncertain switched systems. A co-ordinate transformation matrix is defined to change the system into the regular form. Using the output information, we construct a common sliding surface in terms of constrained LMIs, such that the equivalent sliding mode dynamics restricted to the common sliding surface is asymptotically stable and completely invariant to matched and mismatched uncertainties under arbitrary switching. Static and dynamic output feedback variable structure controllers are given that can drive the state of the switched system to reach the common sliding surface in finite time and maintain it thereafter. A numerical example shows the effectiveness of proposed design method.

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Throughout this paper, $\|\cdot\|$ denotes the Euclidean norm for a vector or the matrix induced norm for a matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following uncertain switched system

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + \Delta A_\sigma x(t) + B[u_\sigma(t) \\ &\quad + Z_\sigma(t)u_\sigma(t) + f_\sigma(x,t)], \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the system state, $\sigma : [0,\infty) \to \Xi = \{1,2,\ldots,l\}$ is the piecewise constant switching signal that might depend on time t or state x, $u_i \in R^m$ is the control input of the $i-th$ subsystem, $y(t) \in R^p$ is the measurement output, A_i, B, C are constant matrices with appropriate dimensions, $\triangle A_i, Z_i(t), f_i(x)$ represent the system parameter uncertainty, input matrix uncertainty and nonlinearity of the system, respectively. The following assumptions are introduced.

Assumption 1. $rank(B) = m$ and $rank(C) = p$ with $m \leq p < n$.

Assumption 2. The parameter uncertainties can be composed as follows

$$\triangle A_i = D\Sigma_i(t)E, i \in \Xi,$$

where D and E are known constant matrices with appropriate dimensions, Σ_i is unknown matrices with Lebesgue measurable elements and satisfy $\Sigma_i^T\Sigma_i \leq I$.

Assumption 3. There exist known nonnegative constants $\varphi_i, i \in \Xi$ such that $\|Z_i(t)\| \leq \varphi_i < 1$ for all t.

Assumption 4. There exist known nonnegative constants $b_i, i \in \Xi$ and known nonnegative scalar-valued functions $\rho_i(y,t), i \in \Xi$ such that $\|f_i(x,t)\| \leq \|b_i\| + \rho_i(y,t)$.

Remark 1. Assumptions 1∼4 are standard assumptions in the study of variable structure control.

The common sliding surface is defined as follows

$$\zeta(t) = Fy(t) = FCx(t) = Sx(t) = 0. \tag{2}$$

The object of design is to determine matrices F, S and variable structure controllers $u_i, i \in \Xi$ for arbitrary switching signal such that:

1). the equivalent sliding mode dynamics restricted to the common sliding surface (2) is asymptotically stable and completely invariant to any uncertainties satisfying Assumptions 2-4;

2). the common sliding surface (2) can be reached in finite time.

In the derivations that follow, the following presented lemmas will be needed.

Lemma 1. Let real-valued matrices $\bar{A}, \bar{H}, \bar{F}(t)$, and $\bar{E}$ of appropriate dimensions be given and suppose $\bar{F}^T(t)\bar{F}(t) \leq I$. Then

(i) (Petersen [1987]). For any positive scalar ϑ, it holds

$$\bar{H}\bar{F}(t)\bar{E} + \bar{E}^T\bar{F}^T(t)\bar{H}^T \leq \vartheta\bar{H}\bar{H}^T + \frac{1}{\vartheta}\bar{E}^T\bar{E}.$$

(ii) (Wang, Xie, Souza [1992]). For any matrix $P_0 > 0$ and scalar $\gamma_0 > 0$ such that $P_0 - \gamma_0\bar{H}\bar{H}^T > 0$, it holds

$$\begin{aligned} &(\bar{A} + \bar{H}\bar{F}(t)\bar{E})^T P_0(\bar{A} + \bar{H}\bar{F}(t)\bar{E}) \leq \\ &\quad \bar{A}^T(P_0 - \gamma_0\bar{H}\bar{H}^T)^{-1}\bar{A} + \gamma_0^{-1}\bar{E}^T\bar{E}. \end{aligned}$$

Lemma 2 (Iwasaki, Skelton [1994]). Let matrices $\bar{B} \in R^{n\times m}$ and $\bar{Q} \in R^{n\times n}$ be given. Suppose $rank(\bar{B}) < n$ and $\bar{Q} = \bar{Q}^T$. Let $(\bar{B}_R, \bar{B}_L)$ be any full rank factor of $\bar{B}$, i.e. $\bar{B} = \bar{B}_L\bar{B}_R$, and define $\bar{D} := (\bar{B}_R\bar{B}_R^T)^{-1/2}B_L^+$. Then

$$\phi\bar{B}\bar{B}^T - \bar{Q} > 0$$

holds for some $\phi \in R$ if and only if

$$\bar{P} := \bar{B}^\perp\bar{Q}\bar{B}^{\perp T} < 0$$

holds, in which case, all such ϕ are given by

$$\phi > \phi_{\min} := \lambda_{\max}\{\bar{D}(\bar{Q} - \bar{Q}\bar{B}^{\perp T}\bar{P}^{-1}\bar{B}^\perp\bar{Q})\bar{D}^T\}.$$

Lemma 3. Assume $C' \geq 0, r(t), h(t)$ and $g(t)$ nonnegative-valued continuous functions of time. If

$$r(t) \leq C' + \int_{t_0}^{t} h(\tau)r(\tau)d\tau + \int_{t_0}^{t} g(\tau)d\tau,$$

then

$$r(t) \leq C'\exp(f(t)) + \int_{t_0}^{t} g(\tau)\exp\{f(t) - f(\tau)\}d\tau,$$

where $f(t) = \int_{t_0}^t h(\tau)d\tau$.

Proof of Lemma 3 is similar to that of Lemma in Shyu, Tsai, Lai [2001], and therefore omitted.

3. MAIN RESULTS

In this section, we give the design method. The design procedure is divided into two phases. First, the sliding surface is designed, so that the controlled system will yield the desired dynamic performance in the sliding surface. The second phase is to design the variable structure controllers such that the trajectory of the system arrive and remain on the sliding surface for all subsequent time.

3.1 Design of a common sliding surface

Refer to paper Choi [2003], we define an symmetric matrix Γ satisfying

$$\Gamma = \begin{cases} I, & if\tilde{B}^TD = 0, \\ I - E^gE, & if\tilde{B}^TD \neq 0, \end{cases} \tag{3}$$

where $\tilde{B}$ is an orthogonal complement of the matrix B , E^g is the Moore-Penrose inverse of the matrix E.

We design the common sliding surface for the system (1) as

$$\begin{aligned} \zeta(t) &= Sx(t) \\ &= B^T(\Gamma X\Gamma + BYB^T)^{-1}x(t) = 0, \end{aligned} \tag{4}$$

where X and Y are symmetric matrices which will be determined later.

Theorem 1. If there exist matrix F and symmetric matrices X and Y satisfying the following constrained LMIs

$$\begin{aligned} &\Gamma X\Gamma + BYB^T > 0, \\ &\tilde{B}^T(A_i\Gamma X\Gamma + \Gamma X\Gamma A_i^T)\tilde{B} < 0, i \in \Xi, \\ &B^T(\Gamma X\Gamma + BYB^T)^{-1} = FC. \end{aligned} \tag{5}$$

Then the system (1) is asymptotically stable and completely invariant to matched and mismatched uncertainties for arbitrary switching signal on the common sliding surface (4).

Proof. To get a regular form of the system (1), we define a nonsingular matrix G and an associated vector ξ as follows:

$$G = \begin{bmatrix} \tilde{B}^T \\ S \end{bmatrix} = \begin{bmatrix} \tilde{B}^T \\ B^T P^{-1} \end{bmatrix} \tag{6}$$

and

$$\xi(t) = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = Gx(t) = \begin{bmatrix} \tilde{B}^T \\ B^T P^{-1} \end{bmatrix} x(t), \tag{7}$$

where $\xi_1 \in R^{n-m}, \xi_2 \in R^m$ and $P = \Gamma X \Gamma + BYB^T$. It is easy to see that the matrix G is invertible with

$$G^{-1} = \left[P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1} \ B(SB)^{-1} \right]. \tag{8}$$

Via the state transformation (7), the system (1) is transformed into the following regular form

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} \bar{A}_{\sigma 11} & \bar{A}_{\sigma 12} \\ \bar{A}_{\sigma 21} & \bar{A}_{\sigma 22} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \zeta \end{bmatrix} + \begin{bmatrix} 0 \\ SB \end{bmatrix} (u_\sigma(t) + Z_\sigma u_\sigma(t) + f_\sigma(x,t)), \tag{9}$$

where
$\bar{A}_{\sigma 11} = \tilde{B}^T[A_\sigma + D\Sigma_\sigma(t)E]P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}$,
$\bar{A}_{\sigma 12} = \tilde{B}^T[A_\sigma + D\Sigma_\sigma(t)E]B(SB)^{-1}$,
$\bar{A}_{\sigma 21} = S[A_\sigma + D\Sigma_\sigma(t)E]P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}$,
$\bar{A}_{\sigma 22} = S[A_\sigma + D\Sigma_\sigma(t)E]B(SB)^{-1}$.
The system (9) implies that if $\zeta = \dot{\zeta} = 0$ then the dynamics restricted to the sliding surface (4) can be described by the following $(n-m)$ dimensional switched system

$$\begin{aligned} \dot{\xi}_1 &= \tilde{B}^T A_\sigma P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}\xi_1 \\ &\quad + \tilde{B}^T D\Sigma_\sigma(t)EP\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}\xi_1. \end{aligned} \tag{10}$$

For the case when $\tilde{B}^T D = 0$, i.e., the uncertain $D\Sigma_i E$ satisfy the matched condition, we have

$$\Gamma = I, P = X + BYB^T > 0.$$

There $\tilde{B}^T P\tilde{B} = \tilde{B}^T X\tilde{B} > 0$. Then the system (10) can be reduced to the following form

$$\begin{aligned} \dot{\xi}_1 &= \tilde{B}^T A_\sigma P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}\xi_1 \\ &= \tilde{B}^T A_\sigma X\tilde{B}(\tilde{B}^T X\tilde{B})^{-1}\xi_1. \end{aligned} \tag{11}$$

Hence the LMIs (5) imply that there exists a common Lyapunov function $V_1 = \xi_1^T(\tilde{B}^T X\tilde{B})^{-1}\xi_1$ for all subsystems of system (11). Therefore the system (1) is asymptotically stable and completely invariant to matched uncertainties for arbitrary switching signal on the sliding surface (4).

On the other hand, if $\tilde{B}^T D \neq 0$, i.e., the uncertain $D\Sigma_i E$ does not satisfy the matching condition, we have

$$\begin{aligned} &\Gamma = I - E^g E, \\ &P = (I - E^g E)X(I - E^g E) + BYB^T > 0. \end{aligned} \tag{12}$$

There $EP\tilde{B} = E[(I - E^g E)X(I - E^g E) + BYB^T]\tilde{B} = 0$. Then the system (10) can be represented by means of equation

$$\begin{aligned} \dot{\xi}_1 &= \tilde{B}^T A_\sigma P\tilde{B}(\tilde{B}^T P\tilde{B})^{-1}\xi_1 \\ &= \tilde{B}^T A_\sigma (I - E^g E)X(I - E^g E)\tilde{B} \\ &\quad \times [\tilde{B}^T(I - E^g E)X(I - E^g E)\tilde{B}]^{-1}\xi_1. \end{aligned} \tag{13}$$

Hence the LMIs (5) imply that there exists a common Lyapunov function $V_2 = \xi_1^T(\tilde{B}^T(I - E^g E)X(I - E^g E)\tilde{B})^{-1}\xi_1$ for all subsystems of system (13). Therefore the system (1) is asymptotically stable and completely invariant to matched and mismatched uncertainties for arbitrary switching signal on the sliding surface (4). This completes the proof.

Remark 2. The solvability of (5) can be checked by the method in Galimidi, Barmish [1986].

Remark 3. We can see that by using the sliding mode control method, uncertainties $\triangle A_i$ and f_i disappear in the sliding motion (11), (13) and the order of the considered system (1) is reduced. Therefore we only need to study stability of the $(n-m)$ switched system (11) and (13) without uncertainties.

3.2 Design of static output feedback variable structure controllers

We now turn to the design of variable structure controllers of subsystems by using reachability condition of sliding surface. We assume that only the measurement output y rather than the state x is available for our design.

Theorem 2. Suppose (5) have solutions X , Y , F and the common sliding surface is given by (4). Then the state of the system (1) can enter the sliding surface in finite time, and subsequently remain on it by employing the following static output feedback controllers

$$u_i = -\gamma_i \zeta - \frac{1}{1-\varphi_i}(\rho_i(y,t) + \eta)sign(\zeta), i \in \Xi, \tag{14}$$

where η is a positive scalar, $\gamma_i = \frac{1}{2(1-\varphi_i)}(\frac{b_i^2}{\varepsilon_i} + \beta_i)$, ε_i are positive constants such that

$$\varepsilon_i \tilde{B}^T P^2 \tilde{B} < -\tilde{B}^T(A_i P + PA_i^T)\tilde{B}, \tag{15}$$

β_i are positive constants such that

$$\beta_i = \begin{cases} \lambda_{\max}[(B^T B)^{-1}B^T(W_i + DD^T + PE^T EP \\ -W_i\tilde{B}(\tilde{B}^T W_i\tilde{B} + \tau_i\tilde{B}^T PE^T EP\tilde{B})^{-1}\tilde{B}^T W_i \\ +\dfrac{1}{\tau_i}DD^T)B(B^T B)^{-1}], if \tilde{B}^T D = 0 \\ \lambda_{\max}[(B^T B)^{-1}B^T(W_i + DD^T + PE^T EP \\ -W_i\tilde{B}(\tilde{B}^T W_i\tilde{B} + \tau_i\tilde{B}^T DD^T\tilde{B})^{-1}\tilde{B}^T W_i, \\ +\dfrac{1}{\tau_i}PE^T EP)B(B^T B)^{-1}], if \tilde{B}^T D \neq 0 \end{cases} \tag{16}$$

τ_i are small positive constants such that $\tilde{B}^T W_i\tilde{B} + \tau_i\tilde{B}^T PE^T EP\tilde{B} < 0, if \tilde{B}^T D = 0$, and $\tilde{B}^T W_i\tilde{B} + \tau_i\tilde{B}^T DD^T\tilde{B} < 0, if \tilde{B}^T D \neq 0$, where $W_i = A_i P + PA_i^T + \varepsilon_i P^2$.

Proof. If (5) is feasible, then there always exist positive constants ε_i satisfying (15). Consider the following Lyapunov function

$$V = x^T P^{-1} x. \tag{17}$$

Then the derivative of the Lyapunov function (17) along the trajectory of the system (1) is

$$\begin{aligned} \dot{V} &= 2x^T P^{-1}(A_i x(t) + \Delta A_i x(t) + Bu_i(t) \\ &\quad + BZ_i u_i(t) + Bf_i(x,t)). \end{aligned} \tag{18}$$

Substituting (14) into (18), one can get

$$\dot{V} = x^T P^{-1}(A_i P + P A_i^T) P^{-1} x - 2\gamma_i x^T P^{-1} B B^T P^{-1} \times x - \frac{2}{1-\varphi_i}(\rho_i(y,t)+\eta)\|\zeta\| + 2x^T P^{-1} D\Sigma_i E x + 2x^T P^{-1} B Z_i u_i + 2x^T P^{-1} B f_i. \quad (19)$$

Using (14) and $\zeta = B^T P^{-1}$, we obtain

$$2x^T P^{-1} B Z_i u_i \leq 2\varphi_i(\gamma_i \|\zeta\| + \frac{\rho_i+\eta}{1-\varphi_i})\|\zeta\| \quad (20)$$

and

$$\begin{aligned} &2x^T P^{-1} B f_i \\ &\leq \frac{b_i^2}{\varepsilon_i} x^T P^{-1} B B^T P^{-1} x + \varepsilon_i x^T x + 2\|\zeta\|\rho_i(y,t) \\ &\leq \frac{b_i^2}{\varepsilon_i}\|\zeta\|^2 + \varepsilon_i x^T x + 2\|\zeta\|\rho_i(y,t). \end{aligned} \quad (21)$$

Let it be denoted

$$\hat{W}_i = A_i P + P A_i^T + D\Sigma_i E P + P E^T \Sigma_i^T D^T + \varepsilon_i P^2.$$

Then we have

$$\dot{V} \leq x^T P^{-1}(\hat{W}_i - \beta_i B B^T) P^{-1} x - 2\eta\|\zeta\|. \quad (22)$$

It can be shown fairly easy that

$$\begin{aligned} \tilde{B}^T \hat{W}_i \tilde{B} &= \tilde{B}^T (A_i P + P A_i^T + D\Sigma_i E P \\ &\quad + P E^T \Sigma_i^T D^T + \varepsilon_i P^2)\tilde{B} \\ &= \tilde{B}^T (A_i P + P A_i^T + \varepsilon_i P^2)\tilde{B} \\ &= \tilde{B}^T W_i \tilde{B}. \end{aligned} \quad (23)$$

From Lemma 1, condition (i) we find

$$\begin{aligned} \hat{W}_i &= A_i P + P A_i^T + D\Sigma_i E P + P E^T \Sigma_i^T D^T + \varepsilon_i P^2 \\ &\leq A_i P + P A_i^T + D D^T + P E^T E P + \varepsilon_i P^2. \end{aligned} \quad (24)$$

Using Lemma 1, condition (ii), if $\tilde{B}^T D = 0$, then one can finds

$$\begin{aligned} &-\hat{W}_i \tilde{B}(\tilde{B}^T W_i \tilde{B})^{-1} \tilde{B}^T \hat{W}_i \\ &= -(W_i \tilde{B} + D\Sigma_i E P \tilde{B})(\tilde{B}^T W_i \tilde{B})^{-1}(\tilde{B}^T W_i \\ &\quad + \tilde{B}^T P E^T \Sigma_i^T D^T) \\ &\leq -W_i \tilde{B}(\tilde{B}^T W_i \tilde{B} + \tau_i \tilde{B}^T P E^T E P \tilde{B})^{-1} \tilde{B}^T W_i \\ &\quad + \frac{1}{\tau_i} D D^T, \end{aligned} \quad (25)$$

and if $\tilde{B}^T D \neq 0$, then one finds

$$\begin{aligned} &-\hat{W}_i \tilde{B}(\tilde{B}^T W_i \tilde{B})^{-1} \tilde{B}^T \hat{W}_i \\ &= -(W_i \tilde{B} + P E^T \Sigma_i^T D^T \tilde{B})(\tilde{B}^T W_i \tilde{B})^{-1} \\ &\quad \times (\tilde{B}^T W_i + \tilde{B}^T D\Sigma_i E P) \\ &\leq -W_i \tilde{B}(\tilde{B}^T W_i \tilde{B} + \tau_i \tilde{B}^T D D^T \tilde{B})^{-1} \tilde{B}^T W_i \\ &\quad + \frac{1}{\tau_i} P E^T E P. \end{aligned} \quad (26)$$

It follows from Lemma 2 that $\hat{W}_i - \beta_i B B^{\mathrm{T}} < 0$ holds, which in turn yields $\dot{V} < 0$.
Now we introduce another Lyapunov function as follows

$$V_s = \zeta^T (B^{\mathrm{T}} P^{-1} B)^{-1} \zeta. \quad (27)$$

Then its time derivative along the trajectory of the system (1) is

$$\begin{aligned} \dot{V}_s \leq \|\zeta\|\{2(&\|(B^T P^{-1} B)^{-1} B^T P^{-1} A_i\| \\ &+ \|(B^T P^{-1} B)^{-1} B^T P^{-1} D\|\|E\| + b_i)\|x\| - \eta\}. \end{aligned} \quad (28)$$

Next we define the set

$$\begin{aligned} \Theta_s = \min\{x \in R^n : 2(&\|(B^T P^{-1} B)^{-1} B^T P^{-1} A_i\| \\ &+ \|(B^T P^{-1} B)^{-1} B^T P^{-1} D\|\|E\| + b_i)\|x\| \\ &- \eta < -\bar{\eta}, i \in \Xi\} \end{aligned} \quad (29)$$

with $0 < \bar{\eta} < \eta$. Notice that $\dot{V} < 0$ implies the system (1) with the controllers (14) is asymptotically stable under arbitrary switching, thus, the state of the system (1) in finite time will come into the domain in which the following inequality holds

$$\dot{V}_s \leq -\bar{\eta}\|\zeta\|. \quad (30)$$

Therefore, the state of the system (1) will enter the common sliding surface (4), and remain on it subsequently. This completes the proof.

3.3 Design of dynamic output feedback variable structure controllers

The static output feedback variable structure controllers (14) are simple in structure yet imply high control efforts thus may not be acceptable or be costly. For this reason, we also introduce dynamic output feedback which are complex in structure but imply lower control efforts Choi [2002], Shyu, Tsai, Lai [2001].
The following lemma is important to develop results of dynamic output feedback variable structure control.
Lemma 4. Consider the first equation of system (9)

$$\begin{aligned} \dot{\xi}_1 &= \bar{A}_{\sigma 11}\xi_1 + \bar{A}_{\sigma 12}\zeta \\ &= \tilde{B}^T [A_\sigma + D\Sigma_\sigma(t)E] P \tilde{B}(\tilde{B}^{\mathrm{T}} P \tilde{B})^{-1}\xi_1 \\ &\quad + \tilde{B}^T [A_\sigma + D\Sigma_\sigma(t)E] B (SB)^{-1}\zeta. \end{aligned} \quad (31)$$

Then, for all time $\|\xi_1\|$ is bounded by $w(t)$ which is the solution of

$$\dot{w}(t) = \lambda w(t) + (G_M + \|\tilde{B}^T D\|\|EB(SB)^{-1}\|)\|\zeta\| \quad (32)$$

with $\lambda = \bar{\lambda} + \|\tilde{B}^T D\|\|P\tilde{B}(\tilde{B}^T P \tilde{B})^{-1}\| < 0$, $G_M = \max\{\|\tilde{B}^T A_i B\ (SB)^{-1}\|, i \in \Xi\}$, where $\bar{\lambda} = \max\{\lambda_{imax}, i \in \Xi\}$, λ_{imax} is the maximum eigenvalue of $\tilde{B}^T A_i P \tilde{B}(\tilde{B}^T P \times \tilde{B})^{-1}$.
Proof. Denote $\hat{G}_1 = P\tilde{B}(\tilde{B}^T P \tilde{B})^{-1}$, $\hat{G}_2 = B(SB)^{-1}$. We have $\exp(\tilde{B}^T A_i \hat{G}_1 t) < \exp(\bar{\lambda} t)$. Suppose $i-th$ subsystem is active for the $j-th$ time in the interval $[t_j^i, t_{j'}^i)$. Solving (31) yields

$$\begin{aligned} \|\xi_1\| \leq &\|e^{\tilde{B}^T A_i \hat{G}_1 (t-t_j^i)}\|\|\xi_1(t_j^i)\| + \int_{t_j^i}^{t} e^{\tilde{B}^T A_i \hat{G}_1 (t-\tau)} \\ &\times \|\tilde{B}^T D\Sigma_i(t) E \hat{G}_1 \xi_1 + \tilde{B}^T A_i \hat{G}_2 \zeta \\ &+ \tilde{B}^T D\Sigma_i(t) E \hat{G}_2 \zeta\| d\tau \\ \leq &\exp(\bar{\lambda}(t-t_j^i))\|\xi_1(t_j^i)\| + \int_{t_j^i}^{t} \exp(\bar{\lambda}(t-\tau)) \\ &\times (\|\tilde{B}^T D\|\|E\hat{G}_1\|\|\xi_1\| + (\|\tilde{B}^T A_i \hat{G}_2\| \\ &+ \|\tilde{B}^T D\|\|E\hat{G}_2\|)\|\zeta\|) d\tau. \end{aligned} \quad (33)$$

Multiply the term $\exp(-\bar{\lambda}(t-t_j^i))$ to both sides of (33) gives rise to

$$\begin{aligned}&\|\xi_1\|\exp(-\bar{\lambda}(t-t_j^i))\\ &\le \|\xi_1(t_j^i)\| + \int_{t_j^i}^{t}\exp(-\bar{\lambda}(\tau-t_j^i))\left\|\tilde{B}^TD\right\|\\ &\times\left\|E\hat{G}_1\right\|\|\xi_1\|d\tau + \int_{t_j^i}^{t}\exp(-\bar{\lambda}(\tau-t_j^i))\\ &\times(\left\|\tilde{B}^TA_i\hat{G}_2\right\| + \left\|\tilde{B}^TD\right\|\left\|E\hat{G}_2\right\|)\|\zeta\|d\tau\end{aligned} \quad (34)$$

Let it be denoted:$r(t) = \|\xi_1\|\exp(-\bar{\lambda}(t-t_j^i))$, $C' = \|\xi_1(t_j^i)\|$, $h(t) = \left\|\tilde{B}^TD\right\|\left\|E\hat{G}_1\right\|$, $g(t) = \exp(-\bar{\lambda}(t-t_j^i))(\left\|\tilde{B}^TA_i\hat{G}_2\right\| + \left\|\tilde{B}^TD\right\|\left\|E\hat{G}_2\right\|)\|\zeta\|$, $f(t) = \int_{t_j^i}^t\left\|\tilde{B}^TD\right\| \times\left\|E\hat{G}_1\right\|d\tau = \left\|\tilde{B}^TD\right\|\left\|E\hat{G}_1\right\|(t-t_j^i)$.

By virtue of Lemma 4, we have

$$\begin{aligned}&\|\xi_1\|\exp(-\bar{\lambda}(t-t_j^i))\\ &\le \|\xi_1(t_j^i)\|\exp(\left\|\tilde{B}^TD\right\|\left\|E\hat{G}_1\right\|(t-t_j^i))\\ &+\int_{t_j^i}^{t}\exp(-\bar{\lambda}(\tau-t_j^i))(\left\|\tilde{B}^TA_i\hat{G}_2\right\|+\left\|\tilde{B}^TD\right\|\\ &\times\left\|E\hat{G}_2\right\|)\|\zeta\|\exp(\left\|\tilde{B}^TD\right\|\left\|E\hat{G}_1\right\|(t-\tau))d\tau.\end{aligned} \quad (35)$$

Thus, if $w(t_j^i) \ge \left\|\xi_1(t_j^i)\right\|$, then we have

$$\begin{aligned}\|\xi_1(t)\| &\le w(t_j^i)\exp\{\lambda(t-t_j^i)\} + \int_{t_j^i}^{t}\exp\{\lambda(t-\tau)\}\\ &\times(G_M + \left\|\tilde{B}^TD\right\|\left\|E\hat{G}_2\right\|)\|\zeta\|d\tau.\end{aligned} \quad (36)$$

It is that $w(t) \ge \|\xi_1(t)\|$ holds in the interval $[t_j^i, t_{j'}^i)$. Hence $\|\xi_1(t)\|$ is bounded by $w(t)$ for all the time if and only if $w(0) \ge \|\xi_1(0)\|$. This completes the proof.

Now let us focus on the design of the dynamic output feedback variable structure controllers for the system (1) by reaching condition of sliding surface.

Theorem 3. Suppose (5) have solutions X , Y , F and the common sliding surface is given by (4). If the following conditions

$$\begin{aligned}k_{1i} &\ge \left\|SA_i\hat{G}_2\right\| + \|SD\|\left\|E\hat{G}_2\right\| + b_i\|SB\|\left\|\hat{G}_2\right\|,\\ k_{2i} &\ge \left\|SA_i\hat{G}_1\right\| + \|SD\|\left\|E\hat{G}_1\right\| + b_i\|SB\|\left\|\hat{G}_1\right\|\end{aligned} \quad (37)$$

are satisfied, then the state of the closed-loop system (1) reach the common sliding surface and subsequently remain on it by employing the following dynamic output feedback controllers

$$\begin{aligned}u_i = -\frac{(SB)^{-1}}{1-\varphi_i}k_{1i}\zeta - \frac{(SB)^{-1}}{1-\varphi_i}(k_{2i}w(t)\\ + \|SB\|\rho_i(y,t) + \eta)sign(\zeta), i \in \Xi,\end{aligned} \quad (38)$$

where $w(t)$ is the solution of (32), η is a positive scalar to adjust the convergent rate.

Proof. Since $x = \hat{G}_1\xi_1 + \hat{G}_2\zeta$, due to Lemma 4 we have $\|x\| \le \left\|\hat{G}_1\right\|w(t) + \left\|\hat{G}_2\right\|\|\zeta\|$. We introduce a Lyapunov function as follows

$$V_d = \zeta^{\mathrm{T}}\zeta.$$

Its time derivative along the trajectory of the system (1) is

$$\begin{aligned}\zeta^{\mathrm{T}}\dot{\zeta} &\le \left\|SA_i\hat{G}_2\right\|\|\zeta\|^2 + \|SD\|\left\|E\hat{G}_2\right\|\|\zeta\|^2\\ &+\left\|SA_i\hat{G}_1\right\|\|\zeta\|\|\xi_1\| + \|SD\|\left\|E\hat{G}_1\right\|\|\zeta\|\|\xi_1\|\\ &+\zeta^{\mathrm{T}}SB(u_i + Z_iu_i) + b_i\|SB\|\left\|\hat{G}_1\right\|\|\zeta\|\|\xi_1\|\\ &+ b_i\|SB\|\left\|\hat{G}_2\right\|\|\zeta\|^2 + \rho_i(y,t)\|SB\|\|\zeta\|.\end{aligned} \quad (39)$$

Applying the dynamic output feedback controllers (38) to the inequality (39) results in $\zeta^{\mathrm{T}}\dot{\zeta} \le -\eta\|\zeta\|$. Hence the state of the system (1) will reach the common sliding surface (4) in finite time and subsequently remain on it. This completes the proof.

4. EXAMPLES

In this section, we present a numerical example to illustrate the usage of the presented new result and to demonstrate the effectiveness of the proposed design method. Consider the following uncertain switched system

$$\begin{aligned}\dot{x}(t) &= A_\sigma x(t) + \Delta A_\sigma x(t) + B[u_\sigma(t)\\ &\quad + Z_\sigma u_\sigma(t) + f_\sigma(x,t)],\\ y(t) &= Cx(t),\end{aligned} \quad (40)$$

where $\sigma \in \Xi = \{1,2\}$, $A_1 = \begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & -1\\ 0 & -1 & -1\end{bmatrix}$, $A_2 = \begin{bmatrix}-1 & -1 & -1\\ 0 & 2 & 1\\ 0 & 2 & -1\end{bmatrix}$, $B = \begin{bmatrix}1\\1\\0\end{bmatrix}$, $C = \begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$, uncertainties $\Delta A_i = D\Sigma_i(t)E$, where $D = \begin{bmatrix}1\\0\\-1\end{bmatrix}$, $E = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\Sigma_1 = \nu_1 \in [-1,1]$, $\Sigma_2 = \nu_2 \in [-1,1]$ and $Z_1 = Z_2 = 0$, $f_1 = f_2 = 0$.

We select the following constants $\tau_1 = \tau_2 = 0.1$ and $\varepsilon_1 = \varepsilon_2 = 1$. The initial state adopted is $x_0 = [1,2,-1]^T$. By solving LMIs (5), we obtain the following solutions:

$$X = \begin{bmatrix}0.6226 & 0 & 0.152\\ 0 & 0 & 0\\ 0.152 & 0 & 0.7538\end{bmatrix}, Y = 0.5373, F = [1.861, -1.861].$$

By virtue of (4), the common sliding surface is

$$\begin{aligned}\zeta(t) = Fy &= Sx(t)\\ &= [0, 1.861, 0]x(t).\end{aligned}$$

According to (14), the obtained control laws are

$$u_1 = -3.9979\zeta - 1.5sign(\zeta),$$

$$u_2 = -9.9946\zeta - 1.5sign(\zeta).$$

It is easy to verify that the conditions of Theorem 1, 2 are satisfied. The simulation results are depicted in Figure 1 and Figure 2 by using Theorem 1, 2. It is clearly seen from these simulated time histories that by applying the proposed static output feedback controllers (14) the closed-loop system of the switched system (40) is asymptotically stable under arbitrary switching.

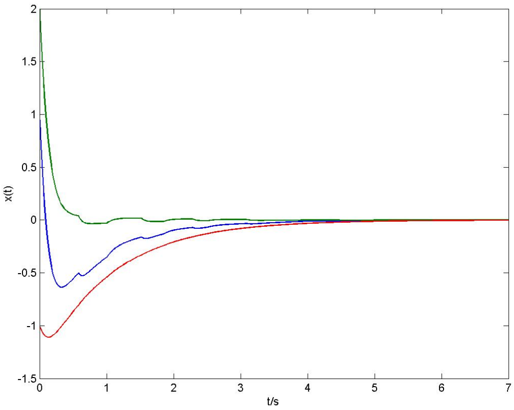

Fig. 1. The state responses of the switched controlled system (40)

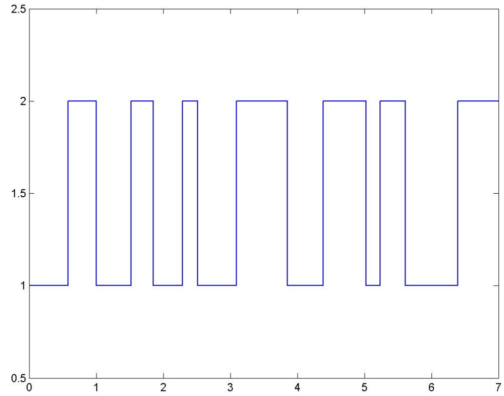

Fig. 2. The switching signal of the system (40)

5. CONCLUSION

In this paper, the problem of robust output feedback sliding mode variable structure control has been considered for a class of uncertain switched systems. The sufficient conditions for the existence of the common sliding surface are derived in terms of constrained LMIs. These guarantee that the switched system is asymptotically stable and completely invariant to matched and mismatched uncertainties for arbitrary switching signal on the common sliding surface. Furthermore, static and dynamic output feedback variable structure controllers are designed to guarantee the state of the switched system to reach the sliding surface in finite time and remain on it. Thus the system is guaranteed to reach the equilibrium state.

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