

## A New Nonlinear Predictive Control Approach Using Hammerstein Models with Compensation Term

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**Abstract:** In this paper is presented a contribution for development and implementation of nonlinear predictive control based on Hammerstein models as well as to make properties evaluation. In this work, nonlinear predictive control development has been used the time-step linearity method and a compensation term is used with an objective to make better the controller performance. An example demonstrating the viability of the proposed methodology is presented.

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### 1. INTRODUCTION

The industrial processes have been affected for some changes in the last decade, mainly due to search for quality and efficiency in its productive processes. The industrial competition including (chemicals areas, food processing automotive, aerospace and metallurgy), and environmental factors make that looked for quality in the automation process (Al-Duwaish. & Naem 2000). It is looked for processes that minimize costs, losses and action (simplicity in control actions) and increase the production rate.

In this context, the predictive controllers appear in the end of the 70 decade (Model Based Predictive Controllers - MBPC). The MBPC is not only indicated for a specific control strategy, but, for a series of control methods that make explicit use of a processes model, with the purpose to get a signal control that minimizes an objective function (Camacho & Bordons, 1999).

The use of linear models in predictive control applications is very common, therefore, beyond the popularity of the MBPC, usually the use of a simplified model becomes necessary. Although the linear model is in the most part of the industrial processes that presents some nonlinearity degree. It has happened then, in few years, a great increasing in the industrial applications of nonlinear predictive control, being a promising strategy of control for some engineering areas.

When the process presents a very high nonlinearity degree, a possible solution is to work with nonlinear models. These should associate simplicity with a good capacity of process representation, besides a lot of studies related for nonlinear predictors. The Hammerstein models are given by a static nonlinearity and a linear dynamic system. The static nonlinearity gives when the time dependence is not quantified between the system variable. In this case, the model has static character, being represented for an algebraic equation.

In this work the properties of the Hammerstein model are presented, as well is considered a new algorithm for nonlinear Generalized Predictive Controller - GPC based on the approach to time-step linearity, single input single output (SISO), being considered the problem of nonlinear control of finite prediction horizon. A compensation term is added to the model with the objective to improve the performance of the considered algorithm.

An implementation result (simulation) of the considered algorithm is presented in this work.

### 2. HAMMERSTEIN MODEL

The Hammerstein Model appears as one of the representations of nonlinear models based on the use of interconnected blocks, which characterize the dynamics of the system through a static nonlinear preceding the block that contains the linear dynamics system (Boutayeb & Darouach 1995), as shows Fig.1.

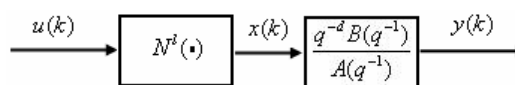


Fig. 1. Hammerstein Model

The Hammerstein model can be represented in a polynomial form, in other words, in a polynomial model NARX equivalent to the Hammerstein representation. Observe that the intermediate sign,  $x(k)$ , is obtained by the multiplication between the input sign,  $u(k)$ , and the function  $N^l$ , it is given that:

$$x(k) = N^l(u(k)) \quad (1)$$

Then, 
$$x(k-i) = N^l(u(k-i)), \quad \text{for } i = 1, \dots, n_u \quad (2)$$

The ARX model is obtained using the input and output,  $x(k)$  and  $y(k)$ , respectively, of the linear dynamic block, such that:

$$y(k) = \sum_{j=1}^{n_y} \theta_j y(k-j) + \sum_{i=1}^{n_u} \sigma_i x(k-i) \quad (3)$$

Where,

$n_y$  and  $n_u$  - maximum delay output and input of the ARX model respectively;

$\theta_j$  and  $\sigma_i$  - related parameters to each output and input regressor of the ARX model, respectively;

In practice, the intermediate signal  $x(k)$ , it is not available. Therefore, it is desirable to express the Hammerstein model in the polynomial form in relation to the input and output system information,  $u(k)$  and  $y(k)$ , respectively. Then, substituting the equation (2) in (3), it is

$$y(k) = \sum_{j=1}^{n_y} \theta_j y(k-j) + \sum_{i=1}^{n_u} \sigma_i N^l(u(k-i)) \quad (4)$$

The equation (4) shows that Hammerstein model is a particular case of the NARX polynomial model with nonlinearity degree  $l$ . Each term of the ARX model with  $m$  order, such what  $0 \leq m \leq 1$ , contains a order factor 1 in  $y(k-j)$  and an order factor  $m$  in  $u(k-i)$ . In brief, the NARX polynomial representation is equivalent to a Hammerstein model when:

- The nonlinearity to act only in the input regressors;
- There is not the presence of terms of the type  $n(k-i)^{m-q} u(k-i_1)^m$ , with  $i \neq i_1$ ,  $0 \leq q \leq m$  and  $(m-q) \leq 1$ , in other words, the nonlinearity doesn't act in terms with different delay.

The static nonlinearity representation for a polynomial happens when it doesn't have information regarding of nonlinearity nature. The representation is obtained by a finite polynomial approximating expansion of the type

$$x(k) = \gamma_1 u(k) + \gamma_2 u^2(k) + \dots + \gamma_l u^l(k) \quad (5)$$

In that,  $k$  it is the instant of time,  $x(k)$  is the output of the nonlinear block,  $u(k)$  is it the input variable and  $\gamma_i (i=1, \dots, l)$  represents the polynomial coefficients and  $l$  it is the nonlinearity degree of the Hammerstein model.

The Hammerstein model in it's parametric form can be written as:

$$A(q^{-1})y(k) = q^{-d} B(q^{-1})x(k-1) + C(q^{-1}) \frac{e(k)}{\Delta} \quad (6)$$

The popularity of Hammerstein model is due to it's simplicity in relation to representations as, for instance, of Volterra (Doyle et. al., 2002) among another, allied the representation

capacity of the nonlinearity of most of the practical processes, being able to represent processes with nonlinear actuators and variables gain.

### 3. MONOVARIABLE GPC BASED ON THE HAMMERSTEIN MODEL (APPROACH TO TIME-STEP QUASILINEAR)

In the last years there was a large growth in the industrial applications of Nonlinear Model Predictive Control - NMPC, which comes as a quite promising control strategy for several engineering areas (Qin and Badgwell, 2003). The main causes of this growth are the low performance of linear controllers in processes with high degrees of nonlinearity or in processes that works in a wide operation band.

So that can obtain a control law that minimizes a quadratic criterion for the nonlinear model and obtain an analytic solution for the problem, then it was adopted the linearization techniques to make possible the solution for the nonlinear predictive control.

Fig. 2 shows the blocks diagram of the process model represented by a Hammerstein model.

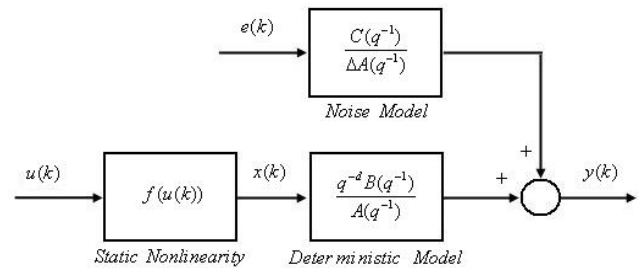


Fig. 2. Blocks diagram of the process model based on the Hammerstein Model

The static nonlinearity can be written as:

$$x(k-1) = \left( \sum_{j=1}^l \gamma_j u^{j-1}(k-1) \right) u(k-1) \quad (7)$$

substituting the equation (7) in the equation (6):

$$A(q^{-1})y(k) = q^{-d} B(q^{-1}) \left( \sum_{j=1}^l \gamma_j u^{j-1}(k-1) \right) u(k-1) + C(q^{-1}) \frac{e(k)}{\Delta} \quad (8)$$

The time-step quasi-linear approach consists of rewrite the model in the form:

$$A(q^{-1})y(k) = q^{-d} \left( \sum_{i=0}^{nb} \sum_{v=-1}^{l-1} \left( \sum_{j=1}^l b_i q^{-i} \gamma_j u^{j-i-v}(k-1) \right) \right) u(k-1) + C(q^{-1}) \frac{e(k)}{\Delta} \quad (9)$$

where:

$nb$  = polynomial degree  $B(q^{-1})$

Defining:

$$\bar{b}_i(q^{-1}, u) = \left( \sum_{v=1}^{l-1} \left( \sum_{j=1}^l b_j q^{-i} \gamma_j u^{j-i-v} (k-1) \right) \right) \quad (10)$$

and

$$\bar{B}(q^{-1}, u) = \sum_{i=0}^{nb} \bar{b}_i q^{-i} (u) \quad (11)$$

the model becomes:

$$\Delta A(q^{-1})y(k) = q^{-d} \bar{B}(q^{-1}, u) \Delta u(k-1) + C(q^{-1})e(k) \quad (12)$$

where,  $\Delta A(q^{-1}) = \tilde{A}(q^{-1}) = (1 - q^{-1})A$  (13)

the following model is obtained:

$$\boxed{\tilde{A}(q^{-1})y(k) = q^{-d} \bar{B}(q^{-1}, u) \Delta u(k-1) + C(q^{-1})e(k)} \quad (14)$$

This model is denominated timestep quasilinear NARIMAX. In this model, the polynomial coefficients  $\bar{B}(q^{-1}, u)$  depends of the last values of the  $u(k)$  that are known, considered constant until the following instant, after updating of its values.

#### 4. NEW NONLINEAR PREDICTIVE CONTROL APPROACH USING THE HAMMERSTEIN MODEL

This approach uses a nonlinear model (Hammerstein model) with a compensation term, whose objective is to correct the prediction error due to approach to time-step quasi-linear model, NARIMAX, used in the predictive controller presented for (Goodhart et. al. 1994).

The prediction error is obtained through the predictions  $i$ -step-ahead of the nonlinear model (Hammerstein model) and model quasilinear, being applied an aleatory sequence of inputs signs. With the prediction error, it is possible to obtain a term that compensates the generated error in this approach when it increases the prediction horizon. The compensation term is added to each prediction horizon, improving controller's performance.

It is important to observe that the presented approach has importance and interest degree, due to the fact that analytic solution doesn't exist for the problem. So, the effort to find a better solution, although sub-optimal, it is justified.

##### 4.1 Compensation Term and Properties

The compensation term consists if finding a linear, moving average model, whose order and parameters depend on the prediction error and of the prediction horizon (Fontes, 2002).

Consider the term  $L_i(q^{-1})$ , that corresponds to the linearized compensation term of the relationship of existent nonlinearity between  $x(\bullet)$  and  $e_i(\bullet)$ , where,  $x(\bullet)$  is the nonlinear input sequence and  $e_i(\bullet)$  is the prediction error vector for the horizon  $i$ . The Fig. 3 shows the linearized model diagram  $L_i(q^{-1})$ .

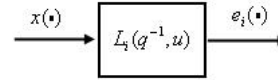


Fig. 3. Representation diagram of the compensation term

The term  $L_i(q^{-1})$  has two interesting properties (Fontes, 2002, to follow summarized). It is a polynomial in the way:

$$L_i(q^{-1}) = l_{0,i} + l_{1,i}q^{-1} + l_{2,i}q^{-2} + \dots + l_{nl,i}q^{-nl} \quad (15)$$

The order and the parameters of the compensation term depend on the prediction error and of the prediction horizon and the parameters are determined in way to minimize the prediction error variance. Therefore, it is used the following linear moving average model:

$$\varepsilon_i = L_i(q^{-1})x(k) \quad (16)$$

The parameters  $l_{j,i}$  with  $j = 1, \dots, l$  are determined using the last square algorithm.

The prediction error in the instant  $k$ , regarding the horizon  $i$  is given for:

$$\varepsilon_i(k) = y(k+i) - \hat{y}(k+i) \quad (17)$$

Where:

$y(k+i)$  it is the output of the nonlinear system;

$\hat{y}(k+i)$  it is the  $i$ -steps-ahead prediction obtained of the quasilinear model, with information until the instant  $k$ .

The polynomial  $L_i(q^{-1})$  is the dynamic compensation term, in that way, it is had that  $L_i(1) = 0$ , that won't modify the static gain of the compensated model. So, it is possible to conclude that:

$$\sum_{j=0}^{nl} l_{j,i} = 0, \quad \forall i \quad (18)$$

It must be chosen an order for the compensation term that satisfies the Akaike<sup>1</sup> criterion. Taken into the fact of the polynomial degree  $\tilde{A}(q^{-1})$  to be  $(na+1)$ , it is had, in agreement with Fontes (2002), the following structure of the compensation term:

$$L_i(q^{-1}) = l_{0,i} + l_{1,i}q^{-i} + l_{2,i}q^{-(i+1)} + \dots + l_{(na-1+i),i}q^{-(na-1+i)} \quad (19)$$

Considering the model presented on (14), it is had that the  $i$ -step-ahead dynamic representation, for  $i \geq 1$ , based on the compensated time-step quasilinear model is follow:

<sup>1</sup> The Akaike criterion is one of the best known technical for choice of the best order, in that the model is tested for a determined set of data in an identification process of a dynamic system.

$$\tilde{A}(q^{-1})y(k+1) = q^{-d} [\bar{B}(q^{-1}, u) + L_i(q^{-1})] \Delta u(k+i-1) + C(q^{-1})e(k+i) \quad (20)$$

The polynomial  $L_i(q^{-1})$  corresponds to a dynamic compensation term, which it compensates the prediction error, and the degree of this polynomial depends on the prediction horizon.

#### 4.2 GPC Hammerstein SISO Quasilinear with Compensation Term

Generalized Predictive Control Nonlinear Compensated, (GPCNC) such as the GPC algorithm, calculates a sequence of control actions to minimize an objective function, multi-step, defined on a prediction horizon, with consideration of the control action. This is obtained minimizing the objective function:

$$J = \sum_{i=N_1}^{NY} \delta(i) [\hat{y}(k+i) - r(k+i)]^2 + \sum_{i=1}^{NU} \lambda(i) [\Delta u(k+i-1)]^2 \quad (21)$$

Where:

$N_1$  and  $NY$  represents the minimum and maximum horizon of prediction;

$NU$  represents the control horizon;

$r(k+i)$  is the reference trajectory for the predicted output;

$\delta(i)$  and  $\lambda(i)$  are the weight factor on the error sign and control sign respectively.

It is important to observe that the output i-step-ahead prediction,  $\hat{y}(k+i)$  obtained by the process of quasilinear compensated prediction, it continues being a sub-optimal prediction, once this prediction is an approach of the exact prediction that would be obtained by the Hammerstein model. However, the quasilinear prediction with compensation term presents a smaller error in comparison with Hammerstein GPC. In the same way how shown previously, to minimize the objective function, it should be obtained the output sub-optimal prediction, i-step-ahead, in the interval  $N_1 \leq i \leq NY$ . Although the plant model is nonlinear, the used approach allows the same procedure to be used by GPC. With this, the concept of free response and forced response is also used for this case.

Starting from the exposed can be determined the output predicted i-step-ahead, defining:

$$\bar{B}_C(q^{-1}, u) = \bar{B}(q^{-1}, u) + L_i(q^{-1}) \quad (22)$$

It is given:

$$y(k+i) = \frac{\bar{B}_C(q^{-1}, u)}{\tilde{A}(q^{-1})} \Delta u(k+i-d-1) + \frac{C(q^{-1})}{\tilde{A}(q^{-1})} e(k+i) \quad (23)$$

In order to separate the dependence of  $y(k+i)$  of the last and future information, the Diophantine equation was introduced and the following predictor equation is obtained:

$$\hat{y}(k+i) = H_i(q^{-1}, u) \Delta u(k+i-d-1) + F_i'(q^{-1}) y(k) \quad (24)$$

The objective function shown in the equation (21) it will be minimized by a future control actions sequence and considering that the system has an equal dead time to  $d$  sampling periods, consequently, the system output will be influenced by the input  $u(k)$  after  $d+1$  periods. Therefore, the prediction minimum horizon will be:

$$N_1 = d+1, NY = d+N \text{ e } NU = N.$$

The set of predictions can be written in the matricial form as:

$$\mathbf{y} = \mathbf{H}(\mathbf{u}) \mathbf{u} + \mathbf{H}'(q^{-1}, \mathbf{u}) \Delta u(k-1) + \mathbf{F}''(q^{-1}) \mathbf{y}(k) \quad (25)$$

Where:

$$\mathbf{y} = \begin{bmatrix} \hat{y}(k+d+1) \\ \hat{y}(k+d+2) \\ \vdots \\ \hat{y}(k+d+N) \end{bmatrix} \quad \mathbf{H}(\mathbf{u}) = \begin{bmatrix} h_0(\mathbf{u}) & 0 & \dots & 0 \\ h_1(\mathbf{u}) & h_0(\mathbf{u}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1}(\mathbf{u}) & h_{N-2}(\mathbf{u}) & \dots & h_0(\mathbf{u}) \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N-1) \end{bmatrix} \quad \mathbf{F}''(q^{-1}) = \begin{bmatrix} F'_{d+1}(q^{-1}) \\ F'_{d+2}(q^{-1}) \\ \vdots \\ F'_{d+N}(q^{-1}) \end{bmatrix} \quad (26)$$

$$\mathbf{H}'(q^{-1}, \mathbf{u}) = \begin{bmatrix} [H_{d+1}(q^{-1}) - h_0] q \\ [H_{d+2}(q^{-1}) - h_0 - h_1 q^{-1}] q^{-2} \\ \vdots \\ [H_{d+N}(q^{-1}) - h_0 - h_1 q^{-1} - \dots - h_{N-1} q^{-(N-1)}] q^N \end{bmatrix}$$

The matrix elements  $\mathbf{H}(\mathbf{u})$  depend on  $u(k)$ .

In similar way to the previous case, the **Free Response Vector** ( $y_l$ ) is given by:

$$y_l = \mathbf{F}''(q^{-1}) \mathbf{y}(k) + \mathbf{H}'(q^{-1}, \mathbf{u}) \Delta u(k-1) \quad (27)$$

The predictor equation will be observed that the **Forced Response** ( $y_f$ ) is given by:

$$y_f = \begin{bmatrix} h_0(\mathbf{u}) & 0 & \dots & 0 \\ h_1(\mathbf{u}) & h_0(\mathbf{u}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1}(\mathbf{u}) & h_{N-2}(\mathbf{u}) & \dots & h_0(\mathbf{u}) \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N-1) \end{bmatrix} = \mathbf{H}(\mathbf{u}) \mathbf{u} \quad (28)$$

This way, we can say that the system complete response is given for:

$$y = \mathbf{H}(\mathbf{u}) \mathbf{u} + \mathbf{y}_l \quad (29)$$

The control law is obtained likely GPC. It must be observed that this is a sub-optimal solution, on way that the predictor is sub-optimal. Thus, the control law is given for:

$$u = (H(u)^T H(u) + \lambda I)^T H(u)^T (r - y_t) \quad (30)$$

The control signal that is really sent to the process is the first element of the vector  $u$ , due to control strategy of moving horizon, then:

$$\Delta u(k) = K (r - y_t) \quad (31)$$

being,  $K$  the first row of matrix  $(H(u)^T H(u) + \lambda I)^{-1} H(u)^T$

### 5. SIMULATION RESULTS

Consider the plant of 2<sup>nd</sup> order described by the following model:

$$y(k) = \frac{0.207 - 0.1464q^{-1}}{1 - 0.8q^{-1} + 0.2385q^{-2}} x(k-1)$$

with the static nonlinearity given by:

$$x(k-1) = 1.549u(k-1) + 1.732u^2(k-1)$$

the control sign was obtained, considering  $d = 0$ ,  $NY = NU = 3$  and  $\lambda = 5$ .

The predictive control based on the Hammerstein model, with compensation term, in this case, uses the following model:

$$y(k) = \frac{\bar{B}_c(q^{-1}, u)}{\bar{A}(q^{-1})} \Delta u(k-d-1) + \frac{C(q^{-1})}{\bar{A}(q^{-1})} e(k)$$

Using these parameters and the compensation term structure presented in (19) is given that the additional estimated terms of compensation, for  $i = 1, 2, 3$  obtained through the minimization of the prediction error variance by the method of the least square, are:

$$L = \begin{bmatrix} 0.1179 & -0.1179 & 0 & 0 \\ 0.1061 & 0 & -0.1061 & 0 \\ 0.0865 & 0 & 0 & -0.0865 \end{bmatrix}$$

Using these results, as well as the results of the same controller's simulation without the compensation term and with the same fittings parameters, it is verified through the Fig. 4 that Hammerstein GPC controller with the compensation term presents a better performance in comparison with GPC based on the Hammerstein model without the compensation term.

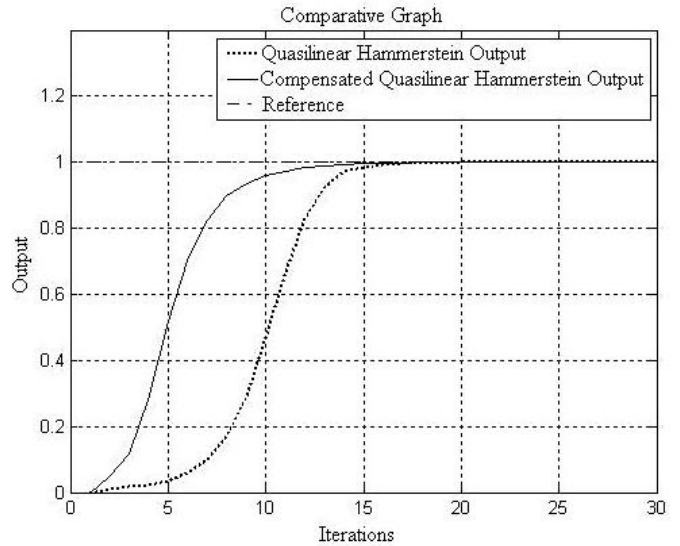


Fig. 4. Comparison between output Hammerstein's GPC and the output of Compensated Hammerstein GPC

Also, are shown in the Fig. 5 the control signals generated by the controller based on the quasilinear model and by the controller based on the compensated quasilinear model.

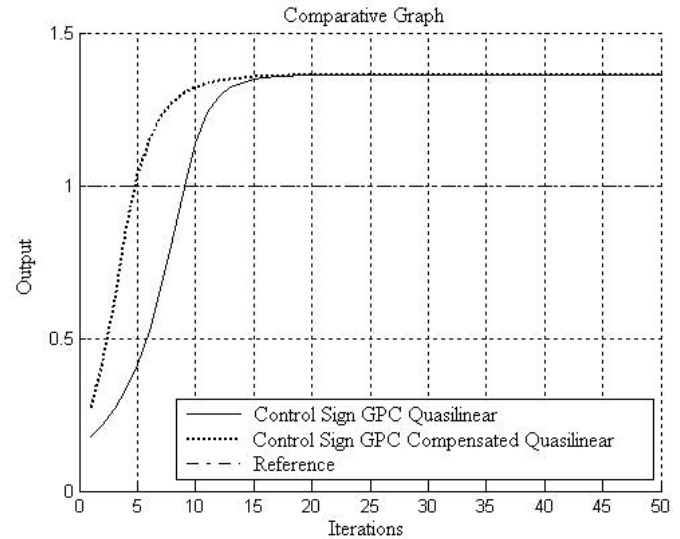


Fig. 5. Comparative graph between the control sign generated by the GPC quasilinear and GPC compensated quasilinear

#### 5.1 Prediction Error Analysis

A form of analyzing the predictive capacity of the predictor models, is using the relationship that compares the  $k$ -steps-ahead predictor performance  $\hat{y}()$ , with the performance  $k$ -steps-ahead quasilinear predictor  $\hat{y}(k)_{quasilinear}$ , that is to say, computing the prediction error with the measured data until the instant  $k$ .

$$e(k) = \hat{y}(k) - \hat{y}(k)_{quasilinear} \quad (32)$$

Considering the previous example, the output model results:

$$y(k) = 0.8y(k-1) - 0.2385y(k-2) + \bar{b}_0 u(k-1) + \bar{b}_1 u^2(k-1)$$

The prediction was implemented with an input  $u = 0.5$  varying 5%, as shows the Fig. 6.

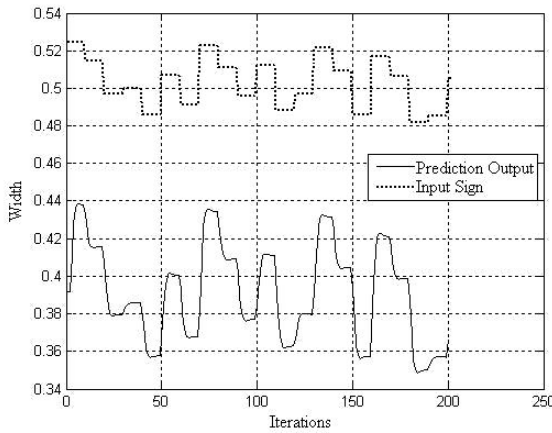


Fig. 6. Comparison between the input sign and prediction output for a horizon of 200 iterations.

The Fig. 7 shows a comparison between the original prediction, quasilinear and quasilinear with compensated term:

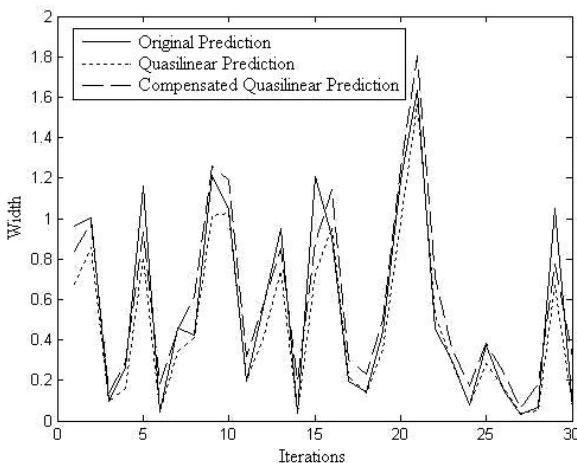


Fig. 7. – Comparative graph between original prediction, quasilinear and compensated quasilinear

It can be verified that there was an improvement with relationship to the quasilinear prediction with compensation term. It is observed that the error prediction between GPC compensated quasilinear and the original prediction is smaller than in comparison with GPC quasilinear and the original prediction.

Also we can do an analysis through the variance, that is given for:

$$\sigma^2 = E\{e^2\} - E^2\{e\} \quad (33)$$

where,  $E\{e^2\}$  is the expectation of the quadratic medium error, then:

$$E\{e^2\} = \frac{1}{N} \sum_{i=1}^N e_i^2 \quad (34)$$

and

$$E\{e\} = \frac{1}{N} \sum_{i=1}^N e_i \quad (35)$$

by the previous example, for a prediction with  $N = 30$  iterations, the variance between the predictor and the quasilinear predictor is given for:

$$\sigma^2_{quasilinear} = 0.0273$$

Now, in relation to the variance between the predictor and the compensated quasilinear predictor, is given:

$$\sigma^2_{Compensated} = 0.0230$$

It is noticed that the variance decreased, showing that the quasilinear prediction with compensation term is more exact than the prediction without the referring term, what proves a better performance of the controller.

## 6. CONCLUSIONS

In this work, it was introduced an analytical solution for the Nonlinear Predictive Control applied to the Hammerstein model. Due to model nonlinearity, it was necessary the use of linearization techniques for the obtaining of the explicit control law. The linearization method was approached by the approximation time-step quasilinear method, that was shown quite efficient. Based on the results, it can be ended that in spite of the approaches be sub-optimal, the results were satisfactory.

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