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Spatial-based Output Feedback Adaptive Feedback Linearization Repetitive Control of Uncertain Rotational Motion Systems Subject to Spatially Periodic Disturbances

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Abstract: In this paper, we propose a new design of spatial-based repetitive control for rotational motion systems required to operate at varying speeds and subject to spatially periodic disturbances. The system has known model structure with uncertain parameters. To synthesize a repetitive controller in spatial domain, a linear time-invariant system is reformulated with respect to a spatial coordinate (e.g., angular displacement), which results in a nonlinear system. A nonlinear state observer is then established for the system. Adaptive feedback linearization is applied to the system with the state observer so as to minimize the tacking error. Moreover, a spatial-based repetitive controller is added and operates in parallel with the adaptively feedback linearized system, which not only further reduces the tracking error but also improves parameter adaptation. The overall output feedback adaptive feedback linearization repetitive control system is robust to structured parameter uncertainty, capable of rejecting spatially periodic disturbances under varying process speeds, and can be proved to be stable and produce bounded tracking error. Finally, feasibility and effectiveness of the proposed scheme is verified by simulation.

1. INTRODUCTION

Rotational motion systems have found their application in many industry products. For most application, the systems are required to operate at varying speeds while following repetitive trajectories and/or rejecting disturbances with sinusoidal/periodic components. For example, the brushless dc motor in a typical laser printer may need to operate at different speed when driving the photosensitive drum for printing tasks of different media or resolution. Also, laser printing systems often suffer from a type of print artifacts (known as banding), which is mostly due to periodic disturbances affecting the angular velocity of the photosensitive drum (see e.g., Chen *et al.*, 2003). Repetitive control systems have been shown to work well for tracking periodic reference commands or for rejecting periodic disturbances in regulation applications. Typical repetitive controllers are time-based controllers since they are synthesized and operate in temporal or time domain. For example, to synthesize the repetitive controller proposed by Hara *et al.* (1988), one of the key steps is to determine the temporal period or frequency of the tracking or disturbance signal. To ensure effectiveness of the design, an underlying assumption is that the tracking or disturbance signal is stationary, i.e., the frequency/period of the tracking or disturbance signal does not vary with time. This assumption can easily be satisfied for many cases where the objective of the design is to track pre-specified repetitive trajectory. However, it might be violated for disturbance rejection problems where periods or frequencies of the disturbances are mostly time-varying.

Recent researches started studying control problems of rejecting/tracking spatially periodic disturbances/references for rotational motion systems using spatial-based repetitive controllers. A spatial-based repetitive controller has its repetitive kernel (i.e. e^{-Ls} with positive feedback) synthesized and operate with respect to a spatial coordinate, e.g., angular position or displacement. Hence its capability for rejecting or tracking spatially periodic disturbances or references will not degrade when the controlled system operates at varying speed. Note that a typical repetitive controller consists of repetitive (i.e., a repetitive kernel) and non-repetitive (e.g., a stabilizing controller) portions. With the repetitive kernel synthesized in spatial domain and given a time-domain open-loop system, design of the non-repetitive portion that properly interfaces the repetitive kernel and the open-loop system actually poses a challenge. For rejection of spatially periodic disturbances, Nakano *et al.* (1996) reformulated a given open-loop linear time-invariant (LTI) system with respect to angular position, and linearized the resulting nonlinear system with respect to a constant operating speed. A stabilizing controller with built-in repetitive kernel was then synthesized for the obtained linear model using coprime factorization. A more recent and advanced design based on linearization using robust control was proposed in (Chen et al., 2006). Although design methods for the linearized system are simple and straightforward, it is unclear whether the overall control system (which is nonlinear) will operate at varying speed or could sustain large velocity fluctuation without stability concern. For tracking of spatially periodic references, Mahawan and Luo (2000) proposed and proved the feasibility of operating the repetitive kernel in spatial domain and the stabilizing controller in time domain. Thus, no reformulation of the open-loop system is required. For practical implementation, however, the proposed method requires solving an optimization problem in real-time to synchronize the hardware and software interrupts corresponding to time and angular position, respectively. Also, the function between time and angular position needs to be known a priori, which further limits the applicability of the proposed method. Both Nakano (1996) and Mahawan (2000) assumed the simplest scenario when making problem formulation. Namely, the open-loop system was assumed to be free of modeling uncertainty and nonlinearity. Chen and Chiu (2007) showed that the nonlinear plant model can be formulated into a quasi-linear parameter varying (quasi-LPV) system. Then, an LPV gain-scheduling controller was obtained which addresses unstructured/bounded modeling uncertainties, actuator saturation and spatially periodic disturbances. The proposed approach, however, could lead to conservative design if the number of varying parameters increases, the varying parameter space is nonconvex, or the size of the modeling uncertainties becomes significant. To relieve the constraint and conservatism of modeling uncertainties imposed on controller design and control performance, Chen and Yang (2007) formulated a spatial-based repetitive control system which combines adaptive feedback linearization (Sastry and Bodson, 1989) and repetitive control. However, this method requires full-state feedback and is thus not applicable to systems of which measurements of states are not available in real-time.

In this paper, we propose a new design of spatial-based repetitive control system which evolves from our previous work (Chen and Yang, 2007). The proposed design resolves the major shortcoming in our former design, i.e., which requires full-state feedback, by incorporation of a nonlinear state observer known as the K-filters (Kreisselmeier, 1977; Kanellakopoulos, 1991; Krstic *et al.*, 1994; Krstic *et al.*, 1995; Yang et al., 2004; Yao and Xu, 2006). The proposed output feedback adaptive feedback linearization repetitive control (AFLRC) system is robust to structured uncertainty of system parameters and capable of rejecting spatially periodic disturbances under variable process speed. Also, the overall system can be proved to be stable under bounded disturbance and parameter uncertainty. Furthermore, addition of the repetitive controller not only improves the tracking error but also reduces the dead zone in the parameter update law. A brushless dc motor of second-order is used for demonstration and derivation of the control algorithm. Simulation is performed to verify the feasibility and effectiveness of the proposed scheme.

This paper is organized as follows: Reformulation of an LTI rotational motion system with respect to angular displacement will be presented in Section 2. Design of the state estimator is described in Section 3. Section 4 will cover derivation and stability analysis of the proposed output feedback AFLRC scheme. Simulation verification for the proposed scheme will be presented in Section 5. Conclusion and future work are given in Section 6.

2. PROBLEM STATEMENT

Suppose that a 2nd order LTI model for a rotational motion system is expressed as

$$Y(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} U(s) + d(s) \tag{1}$$

where a_1, a_0, b_1 and b_0 are coefficients whose values depend on system parameters and are unknown (but might have known upper/lower bounds). $U(s)$ and $Y(s)$ correspond to control input and measured output angular velocity of the system, respectively. $d(s)$ represents a class of bounded output disturbances which are spatially periodic. The only available information of the disturbances is the number of distinctive spatial frequencies which need to be rejected. $Y(s)$ is the motor rotational velocity, and $U(s)$ is the motor input voltage. If no pole/zero cancellation occurs, a possible state space realization of (1) is

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \ \ y(t) = \Psi x(t) + d(t), \tag{2}$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T, \begin{bmatrix} A \mid B \end{bmatrix} = \left[\begin{array}{cc|c} -a_1 & 1 & b_1 \\ -a_0 & 0 & b_0 \end{array}\right], \ \Psi = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Following the same procedure as described in our previous work (Chen and Yang, 2007), we may rewrite (2) as

$$\begin{aligned} \hat{\omega}(\theta)\frac{d\hat{x}(\theta)}{d\theta} &= A\hat{x}(\theta) + B\hat{u}(\theta), \\ \hat{y}(\theta) &= \Psi\hat{x}(\theta) + \hat{d}(\theta). \end{aligned} \tag{3}$$

Equation (3) can be viewed as a nonlinear position-invariant (as opposed to the definition of time-invariant) system with the angular displacement θ as the independent variable. Note that the concept of transfer function is still valid for linear position-invariant systems if we define the Laplace transform of a signal $\hat{g}(\theta)$ in the angular displacement domain as

$$\hat{G}(\tilde{s}) = \int_0^\infty \hat{g}(\theta) e^{-\tilde{s}\theta} d\theta .$$

3. NONLINEAR STATE OBSERVER

In this section, we will establish a state estimator for (3). To allow us to present the proposed design in a simpler context, we will focus on the case in which (2) has relative degree equal to two, i.e., $b_1 = 0$.

As first step, drop the θ notation and rewrite (3) in the form

$$\dot{\hat{x}}_1 = -a_1 + \hat{x}_2/\hat{x}_1, \ \dot{\hat{x}}_2 = -a_0 + b_0\hat{u}/\hat{x}_1, \ \hat{y} = \hat{\omega} + \hat{d} = \hat{x}_1 + \hat{d} \tag{4}$$

where the state variables have been specified such that the angular velocity $\hat{\omega}$ is equal to $\hat{x}_1$, i.e., the undisturbed output. Suppose that both states in (4) cannot be measured in real time. To design a state estimator or the K-filters (Krstic *et al.*, 1995), we proceed as follows. First, rewrite the state equations in (4) as

$$\dot{\hat{x}} = A_0\hat{x} + \bar{k}\hat{x}_1 + \eta(\hat{x}_1)a + \varphi(\hat{x}_1) + \begin{bmatrix} 0 & b_0 \end{bmatrix}^T \sigma(\hat{x}_1)\hat{u}, \tag{5}$$

where

$$\hat{x}=\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix},\ A_0=\begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix},\ \bar{k}=\begin{bmatrix}k_1\\ k_2\end{bmatrix},\ \eta(\hat{x}_1)=\begin{bmatrix}-\hat{x}_1 & 0\\ 0 & -1\end{bmatrix},$$
$$a=\begin{bmatrix}a_1\\ a_0\end{bmatrix},\ \varphi(\hat{x}_1)=\begin{bmatrix}\dot{\hat{x}}_1(-\hat{x}_1+1)\\ 0\end{bmatrix},\ \sigma(\hat{x}_1)=\frac{1}{\hat{x}_1}.$$

By properly choosing k_1 and k_2, the matrix A_0, which determines the properties of the K-filters, can be made Hurwitz. Next, we decide on the following observer structure:

$$\dot{\bar{x}}=A_0\bar{x}+\bar{k}\hat{y}+\eta(\hat{y})a+\varphi(\hat{y})+\begin{bmatrix}0 & b_0\end{bmatrix}^T\sigma(\hat{y})\hat{u}, \tag{6}$$

where $\bar{x}=\begin{bmatrix}\bar{x}_1 & \bar{x}_2\end{bmatrix}^T$ is the state estimates of $\hat{x}$,

$$\eta(\hat{y})=\begin{bmatrix}-\hat{y} & 0\\ 0 & -1\end{bmatrix},\ \varphi(\hat{y})=\begin{bmatrix}\dot{\hat{y}}(-\hat{y}+1)\\ 0\end{bmatrix},\ \sigma(\hat{y})=\frac{1}{\hat{y}}.$$

Equation (6) can be further expressed as

$$\dot{\bar{x}}=A_0\bar{x}+\bar{k}\hat{y}+\varphi(\hat{y})+F(\hat{y},\hat{u})^T\Theta, \tag{7}$$

where $\Theta=\begin{bmatrix}b_0 & a^T\end{bmatrix}^T\in\mathbb{R}^3$ is a parameter vector and

$$F(\hat{y},\hat{u})^T=\begin{bmatrix}0 & \\ \sigma(\hat{y})\hat{u} & \eta(\hat{y})\end{bmatrix}\in\mathbb{R}^{2\times 3}.$$

Define the state estimated error as $\varepsilon\triangleq\begin{bmatrix}\varepsilon_{\hat{x}_1} & \varepsilon_{\hat{x}_2}\end{bmatrix}^T\triangleq\hat{x}-\bar{x}$. Then the state space description of the estimated error can be obtained by subtracting (7) from (5), i.e.,

$$\dot{\varepsilon}=A_0\varepsilon+\Delta, \tag{8}$$

where

$$\Delta=-\bar{k}\hat{d}+\begin{bmatrix}\hat{d} & 0\\ 0 & 0\end{bmatrix}a+\begin{bmatrix}-\hat{d}-\hat{d}\dot{\hat{d}}+\dfrac{d}{d\theta}\left(\hat{d}\hat{y}\right)\\ 0\end{bmatrix}+\begin{bmatrix}0\\ b_0\end{bmatrix}\frac{\hat{d}}{\hat{y}\left(\hat{y}-\hat{d}\right)}\hat{u}$$

Define the state estimate $\bar{x}\triangleq\xi+\Omega^T\Theta$ such that $\xi=\begin{bmatrix}\xi_{11} & \xi_{12}\end{bmatrix}^T\in\mathbb{R}^2$ and $\Omega\in\mathbb{R}^{2\times 3}$. Substituting this definition into (7) gives

$$\dot{\xi}+\dot{\Omega}^T\Theta=A_0\xi+\bar{k}\hat{y}+\varphi(\hat{y})+\left(A_0\Omega^T+F(\hat{y},\hat{u})^T\right)\Theta$$

Thus the following two filters may be employed

$$\dot{\xi}=A_0\xi+\bar{k}\hat{y}+\varphi(\hat{y}),\ \dot{\Omega}^T=A_0\Omega^T+F(\hat{y},\hat{u})^T \tag{9}$$

Define $\Omega^T=\begin{bmatrix}v_0 & \Xi\end{bmatrix}$, i.e., the first column $v_0\triangleq\begin{bmatrix}v_{01} & v_{02}\end{bmatrix}^T\in\mathbb{R}^2$ and the rest as $\Xi\in\mathbb{R}^{2\times 2}$. The second equation in (9) can be split into two filters:

$$\dot{v}_0=A_0v_0+e_2\sigma(\hat{y})\hat{u},\ \dot{\Xi}=A_0\Xi+\eta(\hat{y}) \tag{10}$$

where $e_2=\begin{bmatrix}0 & 1\end{bmatrix}^T$ denotes one of the basis vector for $\mathbb{R}^2$. Expressing $\Xi=\begin{bmatrix}\Xi_1 & \Xi_2\end{bmatrix}^T$, where $\Xi_1\in\mathbb{R}^2$ and $\Xi_2\in\mathbb{R}^2$, and with the definition of the state estimates, we obtain

$$\bar{x}_1=\xi_{11}+v_{01}b_0+\Xi_1^Ta,\ \bar{x}_2=\xi_{12}+v_{02}b_0+\Xi_2^Ta \tag{11}$$

Equation (10) and (11) will be used in the subsequent design.

4. OUTPUT FEEDBACK ADAPTIVE FEEDBACK LINEARIZATION REPETITIVE CONTROL SYSTEM

With the definition of the state estimated error ε, the output equation in (4) can be expressed as

$$\hat{y}=\hat{x}_1+\hat{d}=\bar{x}_1+\varepsilon_{\hat{x}_1}+\hat{d} \tag{12}$$

Substituting the first equation of (11) into (12), we have

$$\hat{y}=\xi_{11}+v_{01}b_0+\Xi_1^Ta+\varepsilon_{\hat{x}_1}+\hat{d}.$$

To apply adaptive feedback linearization, differentiate the output $\hat{y}$ until the term involving the input $\hat{u}$ appears,

$$\ddot{\hat{y}}=\ddot{\xi}_{11}-\left(k_1\dot{v}_{01}+k_2v_{01}\right)b_0+b_0\sigma(\hat{y})\hat{u}+\ddot{\Xi}_1^Ta+\ddot{\varepsilon}_{\hat{x}_1}+\ddot{\hat{d}} \tag{13}$$

Define $\tilde{a}=\begin{bmatrix}\tilde{a}_1 & \tilde{a}_0\end{bmatrix}^T$, where $\tilde{a}_1$ and $\tilde{a}_0$ are the estimates of a_1 and a_0, and $\tilde{b}_0$ is the estimate of b_0. The control law using the estimated parameters and states can be specified as

$$\hat{u}=\frac{1}{\tilde{b}_0\sigma(\hat{y})}\left(-\ddot{\xi}_{11}+\left(k_1\dot{v}_{01}+k_2v_{01}\right)\tilde{b}_0-\ddot{\Xi}_1^T\tilde{a}+\tilde{\hat{v}}_d+\hat{u}_{\hat{R}}\right), \tag{14}$$

where $\tilde{\hat{v}}_d$ is the estimate of a designable input $\hat{v}_d$, $\hat{u}_{\hat{R}}$ is another designable input which will be used to target rejection of the periodic disturbances. Choose $\tilde{\hat{v}}_d$ as

$$\tilde{\hat{v}}_d=\ddot{\hat{y}}_m+\alpha_1(\dot{\hat{y}}_m-\tilde{\dot{\hat{y}}})+\alpha_2(\hat{y}_m-\hat{y}), \tag{15}$$

where α_1 and α_2 are adjustable parameters, $\hat{y}_m$ is the reference command, and $\tilde{\dot{\hat{y}}}$ is the estimates of $\dot{\hat{y}}$. Note that $\dot{\hat{y}}$ cannot be measured directly because of parameter uncertainties. Let $\tilde{\Theta}=\begin{bmatrix}\tilde{b}_0 & \tilde{a}^T\end{bmatrix}^T$ and define the parameter error vector as $\Phi\triangleq\Theta-\tilde{\Theta}=\begin{bmatrix}\Phi_{b_0} & \Phi_a\end{bmatrix}^T$, where $\Phi_{b_0}=b_0-\tilde{b}_0$ and $\Phi_a=\begin{bmatrix}a_1-\tilde{a}_1 & a_0-\tilde{a}_0\end{bmatrix}^T$. Substituting (14) into (13), we obtain

$$\ddot{\hat{y}}=\Phi^TW_1+\tilde{\hat{v}}_d+\hat{u}_{\hat{R}}+\ddot{\varepsilon}_{\hat{x}_1}+\ddot{\hat{d}} \tag{16}$$

where $W_1=\begin{bmatrix}-\left(k_1\dot{v}_{01}+k_2v_{01}\right)+P(.)/\tilde{b}_0 & \ddot{\Xi}_1^T\end{bmatrix}^T\in\mathbb{R}^3$ with

$$P(.)=-\ddot{\xi}_{11}+\left(k_1\dot{v}_{01}+k_2v_{01}\right)\tilde{b}_0-\ddot{\Xi}_1^T\tilde{a}+\tilde{\hat{v}}_d+\hat{u}_{\hat{R}}.$$

Defining the tracking error $\hat{e}\triangleq\hat{y}-\hat{y}_m$ and calculating the mismatch between $\hat{v}_d$ and $\tilde{\hat{v}}_d$, we may arrive at the error equation, i.e.,

$$\ddot{\hat{e}}+\alpha_1\dot{\hat{e}}+\alpha_2\hat{e}=\Phi^TW+\hat{u}_{\hat{R}}+\alpha_1\left(\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}\right)+\ddot{\varepsilon}_{\hat{x}_1}+\ddot{\hat{d}}, \tag{17}$$

where

$$W = \left[-\left(k_1 \dot{v}_{01} + k_2 v_{01}\right) + \alpha_1 \left(-k_1 v_{01} + v_{02}\right) + \frac{P(.)}{\tilde{b}_0} \quad \ddot{\Xi}_1^T + \alpha_1 \dot{\Xi}_1^T \right]^T .$$

If we denote $M(\tilde{s}) = 1/(\tilde{s}^2 + \alpha_1 \tilde{s} + \alpha_2)$, (17) implies that

$$\hat{E}(\tilde{s})/M(\tilde{s}) = \Phi^T W + \hat{U}_{\hat{R}}(\tilde{s}) + \left(\tilde{s}^2 + \alpha_1 \tilde{s}\right)\left(\varepsilon_{\hat{x}_1} + \hat{d}\right). \quad (18)$$

The designable input $\hat{u}_{\hat{R}}$ provides additional degree of freedom of control for the system to deal with spatially periodic disturbances, i.e., $\hat{d}$ in (18). A control loop is connected between $\hat{E}(\tilde{s})$ and $\hat{U}_{\hat{R}}(\tilde{s})$. As shown in Figure 1, the tracking error $\hat{E}(\tilde{s})$ and the control input $\hat{U}_{\hat{R}}(\tilde{s})$ is related by

$$\hat{U}_{\hat{R}}(\tilde{s}) = -\hat{R}(\tilde{s})\hat{C}(\tilde{s})\hat{E}(\tilde{s}), \quad (19)$$

where we have chosen $\hat{R}(\tilde{s})$ as a low-order and attenuated-type repetitive controller (Lee and Smith, 1998), i.e.,

$$\hat{R}(\tilde{s}) = \prod_{i=1}^{k} \frac{\tilde{s}^2 + 2\zeta_i \omega_{ni} \tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i \omega_{ni} \tilde{s} + \omega_{ni}^2},$$

where k is the number of periodic frequencies to be rejected, ω_{ni} is determined based on the i^{th} disturbance frequency in rad/rev, and ξ_i and ζ_i are two damping ratios that satisfy $0 < \xi_i < \zeta_i < 1$. Substituting (19) into (18) gives

$$\left[1/M(\tilde{s}) + \hat{R}(\tilde{s})\hat{C}(\tilde{s})\right]\hat{E}(\tilde{s}) = \Phi^T W + \left(\tilde{s}^2 + \alpha_1 \tilde{s}\right)\left(\varepsilon_{\hat{x}_1} + \hat{d}\right). \quad (20)$$

Define

$$\bar{M}(\tilde{s}) \triangleq \left[1/M(\tilde{s}) + \hat{R}(\tilde{s})\hat{C}(\tilde{s})\right]^{-1}. \quad (21)$$

Equation (20) becomes

$$\hat{e} = \bar{M}(\tilde{s})\Phi^T W + \hat{d}_{\bar{M}}, \quad (22)$$

where $\hat{d}_{\bar{M}} \triangleq \bar{M}(\tilde{s})\left(\tilde{s}^2 + \alpha_1 \tilde{s}\right)\left(\varepsilon_{\hat{x}_1} + \hat{d}\right)$. Since $\dot{\hat{e}}$ can not be measured directly, the so-called augmented error scheme will be used. The augmented error is defined as

$$\hat{e}_1 = \hat{e} + \left(\Phi^T \bar{M}(\tilde{s})W - \bar{M}(\tilde{s})\Phi^T W\right). \quad (23)$$

Substituting (22) into (23) gives $\hat{e}_1 = \Phi^T \bar{\varsigma} + \hat{d}_{\bar{M}}$, where $\bar{\varsigma} = \bar{M}(\tilde{s})W$. The parameter update law to be used is a modified version (French and Rogers, 2000) based on the normalized gradient method proposed in (Sastry and Isidori, 1989). In our case, the update law can be described by

$$\dot{\tilde{\Theta}} = -\dot{\Phi} = \begin{cases} P_R\left(\dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1 + \bar{\varsigma}^T \bar{\varsigma}}\right) & \text{if } |\hat{e}_1| > \hat{d}_{\bar{M}_0}, \\ 0 & \text{if } |\hat{e}_1| \le \hat{d}_{\bar{M}_0}. \end{cases} \quad (24)$$

where

$$P_R\left(\frac{\rho \hat{e}_1 \bar{\varsigma}}{1 + \bar{\varsigma}^T \bar{\varsigma}}\right) = \begin{cases} 0 & \text{if } \tilde{b}_0 = \tilde{b}_{0\_\min} \text{ and } \dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1 + \bar{\varsigma}^T \bar{\varsigma}} < 0, \\ 0 & \text{if } \tilde{b}_0 = \tilde{b}_{0\_\max} \text{ and } \dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1 + \bar{\varsigma}^T \bar{\varsigma}} > 0, \\ \dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1 + \bar{\varsigma}^T \bar{\varsigma}} & \text{otherwise.} \end{cases}$$

where $\hat{d}_{\bar{M}_0}$ is an upper bound for $\hat{d}_{\bar{M}}$, and ρ is the adaptation rate which affects the convergence property. The following theorem summarizes the main result of this paper:

Theorem Consider the control law of (14), (15) and (19) applied to a nonlinear system with a state observer as given by (4) and (6), respectively. Assume that $\hat{y}_m, \dot{\hat{y}}_m$ are bounded, $\hat{d}_{\bar{M}}$ is bounded with an upper bound $\hat{d}_{\bar{M}_0}$, $\tilde{b}_0$ is bounded away from zero, and W has bounded derivative with respect to $\hat{x}$ and $\tilde{\Theta}$. Furthermore, suppose that a stable and proper controller $\hat{C}(\tilde{s})$ is designed such that $\bar{M}(\tilde{s})$ is stable. Then the modified parameter adaptation law as given by (24) yields the bounded tracking error, namely $|\hat{y}(\theta) - \hat{y}_m(\theta)| < \hat{d}_{\bar{M}_0}$ as $\theta \to \infty$.

Proof: Omit for brevity. Note that the theorem can be extended so that it is applicable for nth order system.

5. SIMULATION RESULTS

The proposed output feedback AFLRC scheme is applied to brushless dc motor system. The actual system is a 2^{nd} order system as described in (1) with $a_0 = 5155$, $a_1 = 1138$, $b_0 = 140368$, and $b_1 = 0$. The parameters are specified in accordance with the system identification results for an actual motor system from Shinano Kenshi Corp. The parameters of the K-filters are set to $k_1 = 2000$ and $k_2 = 2500$ for fast convergence. For verification purpose, the output disturbance is assumed to be a rectangular periodic signal (with amplitude switching between -0.05 and 0.05) plus some white noise, i.e.,

$$\hat{d} = 0.08\left(\frac{1}{\dfrac{\tilde{s}}{20} + 1}\right)^2 \left[\sum_{l=-\infty}^{\infty} (-1)^l \Pi(\theta - 1 - l)\right] + \left(\frac{1}{\dfrac{\tilde{s}}{200} + 1}\right)^2 N_0,$$

where

$$\Pi(\theta) = \begin{cases} 1 & |\theta| < 1, \\ 0.5 & |\theta| = 1, \\ 0 & \text{otherwise,} \end{cases}$$

and N_0 is white noise with zero mean and variance equal to 10^{-6}. Note that both disturbances have been low-pass filtered so that they are continuously differentiable. Parameters of repetitive controller are specified to target the fundamental frequency and the first three harmonic frequencies of the periodic disturbance, i.e.,

$$\hat{R}(\tilde{s}) = \prod_{i=1}^{4} \frac{\tilde{s}^2 + 2\zeta_i \omega_{ni}\tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i \omega_{ni}\tilde{s} + \omega_{ni}^2},$$

where

$$\varsigma_i = 0.2,\ \xi_i = 0.0002,$$
$$\omega_{n1} = \pi, \omega_{n2} = 3\times\pi, \omega_{n3} = 5\times\pi, \omega_{n4} = 7\times\pi.$$

A simple lead compensator

$$\hat{C}(\tilde{s}) = 150000(\tilde{s}/50+1)/(\tilde{s}/1000+1)$$

is sufficient to stabilize the overall output feedback AFLRC system. Suppose a motion control task demands the system to initially run at 60 rev/s and then speed up to 65 rev/s and finally speed down to 55 rev/s. To avoid getting infinite value when taking derivative, the reference command is specified to have smooth (instead of instant) change. Figure 2 compares the frequency responses of $M(\tilde{s})(\tilde{s}^2 + \alpha_1\tilde{s})$ or $\bar{M}(\tilde{s})(\tilde{s}^2 + \alpha_1\tilde{s})$ corresponding to three different designs. The dashed line labeled 'w/o RC#1' is the design with $\hat{u}_{\hat{R}} = 0$ and $M(\tilde{s}) = 1/(\tilde{s}^2 + 1200\tilde{s} + 90000)$ (i.e., without repetitive control); the solid line labeled 'with RC' is the proposed output feedback AFLRC design; the dash-dot line labeled 'w/o RC#2' is the design with $\hat{u}_{\hat{R}} = 0$ but having a different $M(\tilde{s}) = 1/(\tilde{s}^2 + 108000\tilde{s} + 8.1\times10^9)$ to produce approximately the same magnitude reduction at the four frequencies as specified in the case with repetitive control (the solid line). Figure 3 compares the tracking performance and control input for the three designs. The tracking error and the dead zone for the parameter update law, i.e., $\hat{d}_{\bar{M}_0}$ in (24), are significantly reduced for the output feedback AFLRC design when compared to those for the design labeled 'w/o RC#1'. The design labeled 'w/o RC#2' has even better tracking performance, but the corresponding control input becomes large and goes negative most of the time, which will cause problems if the actuator has saturation limits.

6. CONCLUSION AND FUTURE WORK

This paper presents the design of a new spatial-based repetitive control system, which can be applied to rotational motion systems with uncertain parameters operating at varying speeds and subject to spatially periodic disturbances. The proposed design combines two control paradigms, i.e., adaptive feedback linearization and repetitive control. The overall output feedback AFLRC system can be proved to be stable and have bounded tracking error. Feasibility and effectiveness of the proposed design are further justified by simulation. Although this paper only presents the design method for a 2nd order system, the proposed design may be extended to higher order systems, which is currently under our investigation.

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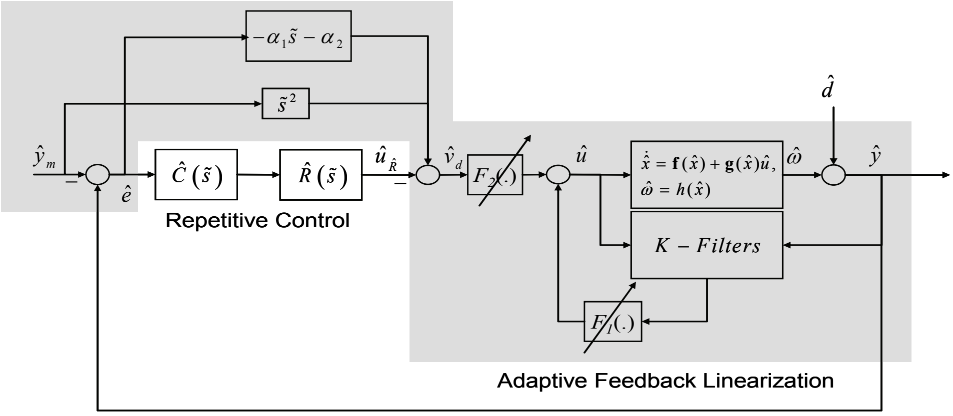

Figure 1: The control structure for the proposed adaptive robust repetitive control system.

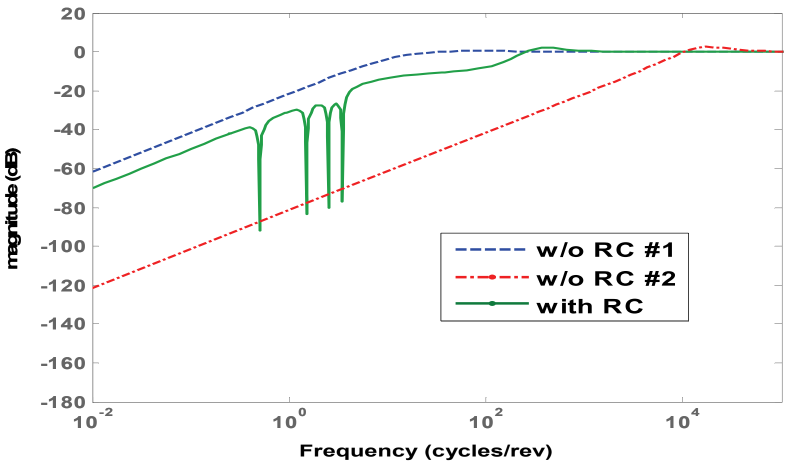

Figure 2: Frequency response from disturbance to tracking error.

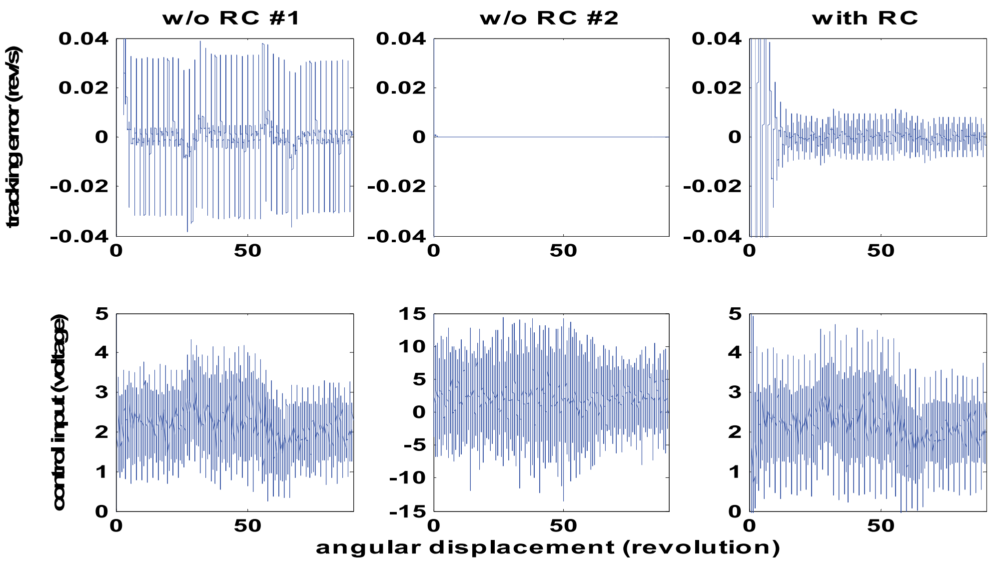

Figure 3: Comparison of tracking performance and control input for three different designs.