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A Reset State Estimator for Linear Systems to Suppress Sensor Quantization Effects

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Abstract: This paper presents a reset state estimator to improve the position estimation for motion control systems with sensor quantization. The reset scheme is guided by the idea that the actual output is known exactly to be at the mid-point of the two consecutive quantizer levels and is within the range of a quantizer level bounded by half of quantization step size. Hence, using this information to update the estimated state can give a better estimation under the influence of disturbance and quantization noise. We also show that the reset scheme will not destroy the stability of a baseline estimator system. The reset state estimator is applied to a linear motor control system with an optical encoder. Simulation and experiment demonstrate that the reset state estimator can achieve smaller position estimation error and more accurate tracking accuracy than those of a standard state estimator.

1. INTRODUCTION

Sensor quantization is a kind of typical measurement inaccuracy in motion control systems. For example, optical encoders are widely used to detect the position of the moving parts. The encoder is based on evenly spaced divisions or line counts on a glass or metal disk, which is simple in construction and easy to manufacture. Position control of the drive can be realized by direct feedback of the encoder signals to the controller. Additionally, velocity control is possible by estimating the velocity from the encoder position signals (Lorenz et al. [1991]). However, the interval of the divisions adversely leads to the so-called resolution limitation in position measurements. Thus, the encoder can not reflect the actual position continuously, but outputs the quantized signal of the actual position. When the encoder output is used as the feedback signal in a servo system, the encoder quantization noise will degrade the achievable position accuracy and even cause self-sustained oscillations (i.e., limit cycle) (Franklin [1998]). From the cost and effectiveness point of view, the designers generally need to select an encoder transducer of proper resolution consistent with the required performance. In digital control systems, quantization may be created not only by the sensor itself but also by the D/A converter (DAC). In most cases, quantization noise is ignored during control design process if it is substantially small compared to system noise and the desired position accuracy.

The quantization is inherently a nonlinear feature, whose model was studied in Widrow et al. [1996] and Kavanagh and Murphy [1998] from statistical perspective. In order to alleviate the sensor quantization effect, a straight and simple way is to replace the sensor with a higher-resolution one to provide more accurate measurement, although it greatly increases the implementation cost. Another cost-effective way is to remove the quantization noise by using some numerical algorithms, which can be simply implemented on a digital signal processor (DSP). For instance, the Kalman filter has been reported to suppress the sensor quantization effects under the assumption that quantization effects could be modeled as a random noise with a Gaussian form and a standard deviation (Luong-Van et al. [2004] and Chang and Perng [1996]). In motion control systems, signals tend to be more deterministic and exhibit stronger correlation over time. Thus, quantization behaves as highly colored noise, which makes the Kalman filter impractical to employ. In such circumstances, other approaches based on observer theory have been extensively studied in Hodel and Hung [2003], Sviestins and Wigren [2000], Sur and Paden [1998], and Delchamps [1989], in which extra useful information is extracted from the quantizer model and then used to better the estimation. This paper also presents a solution to this problem by resetting inaccurate estimated states to favorable ones, which are modulated in terms of the quantized output.

Our reset estimator design is based on a standard state estimator, where the estimator gain is determined to compromise the sensor noise and disturbance rejection. The reset scheme is guided by the idea that the actual output is known exactly to be at the mid-point of two consecutive quantizer levels and is within the range of a quantizer level bounded by half of quantization step size. Hence, we actually know the estimated output boundary from the quantizer output, which can be then used to avoid over-estimation of the system output in the case of large disturbance as well as plant uncertainty. In this paper, we have shown that the proposed reset scheme will not affect the stability of a stable state estimator. The reset state estimator is applied to an encoder-based linear motor control system. Simulation and experimental results demonstrate that it outperforms the standard state estimator in position estimations with low-resolution sensor. Moreover, we have shown improved tracking accuracy under the control scheme with the reset estimator.

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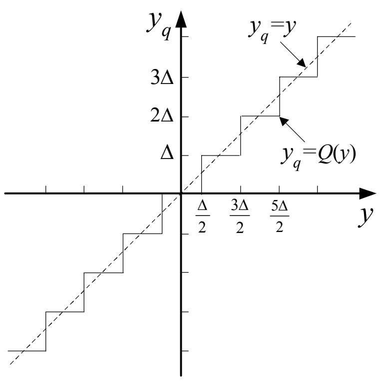

Fig. 1. Quantization characteristic; Δ: quantization step size (resolution).

2. SYSTEM MODEL

Consider a continuous linear time-invariant plant with process noise and quantized output, whose dynamics is given by the following state equation

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \\ y_q(t) = Q(y(t)) \end{cases} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B, G \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$ are constant system matrices, and (A, C) is an observable pair. $x(t)$, $y(t)$ and $y_q(t)$ are the state vector, controlled output and the measured quantized output, respectively. $w(t)$ indicates input disturbances and equivalent model uncertainties. The function $Q(\cdot)$ represents a uniform quantizer defined by

$$Q(y) = i \cdot \Delta, \quad \text{if } y \in [(i - 0.5)\Delta,\ (i + 0.5)\Delta] \tag{2}$$

where $i \in \mathbb{Z}$ and $\Delta > 0$ denotes the constant quantization step size. We assume that the quantization range is infinite. The input-output relationship of (2) is shown in Fig. 1. It can be seen that quantization is an operation on signals that is represented as a staircase function. In practical motion control systems, the quantizer (2) can be adopted to model the position sensor with quantization such as an incremental encoder, where Δ is also referred to as positioning resolution.

The quantizer introduces sensing error on the controlled output $y(t)$. We define this error as quantization error $\varepsilon(t)$, which is the difference between the output and the input of the quantizer. Hence,

$$\varepsilon(t) = y(t) - y_q(t). \tag{3}$$

Note that the characteristic of ε is related to y and it is bounded by

$$|\varepsilon(t)| \le \frac{\Delta}{2}. \tag{4}$$

The quantized output can accordingly be rewritten as

$$y_q(t) = Cx(t) - \varepsilon(t). \tag{5}$$

In order to achieve a smooth and actual estimate of the state as well as the controlled output using the quantized output only, the most common way is to use the state estimator as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\Big(y_q(t) - \hat{y}(t)\Big) \tag{6}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{7}$$

where $\hat{x}$ and $\hat{y}$ are respectively the estimate of the state and controlled output, and $L \in \mathbb{R}^{n\times 1}$ is the estimator gain. Conventionally, L can be artificially selected by using the pole placement method or optimally designed using the Kalman filter technique, where both the process noise and quantization error are assumed to be white Gaussian noise. However, we find that more accurate state estimate is possible if the quantization scheme is fully employed.

Though the quantized output y_q gives discretized measurements of the actual y, we can still extract some useful information given a measured quantized output. From Fig. 1, we can obtain the following observations:

Observation:

1) At the time when the quantized output transits from one quantization step to another, the actual position is measured exactly, which locates at the mid-points of the two consecutive quantization levels.
2) At the time when the quantizer holds its output equivalent to a certain quantization step, according to the fact of (4) the actual position relative to the quantized output is always bounded by $\Delta/2$. This implies that any estimate of y at these times must be bounded by $\Delta/2$ relative to the instantaneous quantized output.

We note that these observations can be used to improve the estimate of the state, which is presented in the next section.

3. RESET STATE ESTIMATOR

In this section, we present a state estimator with the use of reset technique to include the exact information from the measured quantized output and the stability of the estimator error system is analyzed.

3.1 Reset State Estimator

Firstly, we introduce a constant vector $H \in \mathbb{R}^{n\times 1}$, which is given by

$$H = P^{-1}C^T(CP^{-1}C^T)^{-1}, \tag{8}$$

where $P \in \mathbb{R}^{n\times n}$ is a positive definite symmetric matrix, which is the solution of the following Lyapunov function

$$(A - LC)^T P + P(A - LC) + I = 0, \tag{9}$$

where the estimator gain L is designed such that $A - LC$ is stable.

Next, we modify the standard state estimator (6) to incorporate the exacted information from the quantized output. Namely, we reset the estimated state in two cases:

1) At the reset time $t_{k,1}$, which is defined as

$$t_{k,1} : y_q(t_{k,1}) \ne y_q(t_{k,1}^-), \tag{10}$$

the estimated state is reset by

$$\hat{x}(t_{k,1}) = \hat{x}(t_{k,1}^-) - H\Big(\hat{y}(t_{k,1}^-) - \frac{1}{2}(y_q(t_{k,1}) + y_q(t_{k,1}^-)\Big). \quad (11)$$

It is easy to verify that the new estimated state leads to

$$\hat{y}(t_{k,1}) = \frac{1}{2}\Big(y_q(t_{k,1}) + y_q(t_{k,1}^-)\Big) = y(t_{k,1}). \quad (12)$$

2) At the pre-specified reset time $t_{k,2}$ defined by

$$t_{k,2} = kT, \quad k \in \mathbb{Z}^+ \quad (13)$$

where T indicates a predefined reset interval (e.g., sampling period). Ideally, a smaller T implies faster state update and thereby lessens the estimation error. The estimated state is then reset by

$$\hat{x}(t_{k,2}) = \hat{x}(t_{k,2}^-) - H\Big(\hat{y}(t_{k,2}^-) - y_q(t_{k,2}) - \text{Sat}_{\frac{\Delta}{2}}(\hat{y}(t_{k,2}^-) - y_q(t_{k,2}))\Big), \quad (14)$$

where $\text{Sat}(\cdot)$ is the saturation function with the saturation level of $\Delta/2$. We can see that the new estimated state can lead to

$$\hat{y}(t_{k,2}) = \begin{cases} y_q(t_{k,2}) + \text{Sat}_{\frac{\Delta}{2}}(\hat{y}(t_{k,2}^-) - y_q(t_{k,2})), & \text{if } |\hat{y}(t_{k,2}^-) - y_q(t_{k,2})| > \frac{\Delta}{2}; \\ \hat{y}(t_{k,2}^-), & \text{otherwise.} \end{cases} \quad (15)$$

Hence, it can be seen that over-estimation of the estimated output $\hat{y}$ is prevented while the estimated output is unchanged if over-estimation is not detected.

By incorporating the previous reset schemes into the standard state estimator (6), we can obtain a hybrid system named by reset state estimator, which has the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\Big(y_q(t) - \hat{y}(t)\Big), \quad t \notin \{t_{k,1}; t_{k,2}\}; \quad (16)$$

$$\hat{x}(t_{k,1}) = \hat{x}(t_{k,1}^-) - H\Big(\hat{y}(t_{k,1}^-) - \frac{1}{2}(y_q(t_{k,1}) + y_q(t_{k,1}^-)\Big), \quad t_{k,1} : y_q(t_{k,1}) \neq y_q(t_{k,1}^-); \quad (17)$$

$$\hat{x}(t_{k,2}) = \hat{x}(t_{k,2}^-) - H\Big(\hat{y}(t_{k,2}^-) - y_q(t_{k,2}) - \text{Sat}_{\frac{\Delta}{2}}(\hat{y}(t_{k,2}^-) - y_q(t_{k,2}))\Big), \quad t_{k,2} = kT; \quad (18)$$

$$\hat{y}(t) = C\hat{x}(t). \quad (19)$$

where $\hat{x}(t_{k,1})$ and $\hat{x}(t_{k,2})$ are the new estimated states at time $t_{k,1}$, $t_{k,2}$ respectively.

Remark 1: The selection of H (8) is motivated by Sur and Paden [1998], which is deduced using a projection approach. The benefits here are that the estimated controlled output is assured to be favorably updated at the reset times as shown in (12) and (15). It can be seen that the estimated output in (12) equals to the actual output and the estimated output in (15) avoids over-estimation. However, we can't claim other estimated states can be updated to actual states in this reset scheme except the estimated output.

3.2 Stability Analysis

In the following, we analyze the stability of the reset state estimator (16)-(19). Define the estimator error

$$e(t) \triangleq x(t) - \hat{x}(t), \quad (20)$$

and thus, subtracting the estimator (16)-(18) from the plant (1) derives the estimator error system with the following dynamic equations:

$$\begin{aligned} \dot{e}(t) &= A_e e(t) + Gw(t) + L\varepsilon(t),\ e(0) = e_0,\ t \notin \{t_{k,1}; t_{k,2}\} \\ e(t_{k,1}) &= De(t_{k,1}^-), \\ e(t_{k,2}) &= De(t_{k,2}^-) + H\psi(t_{k,2}), \end{aligned} \quad (21)$$

where

$$\begin{aligned} A_e &= A - LC, \\ D &= I - HC, \\ \psi(t_{k,2}) &= \varepsilon(t_{k,2}) - \text{Sat}_{\frac{\Delta}{2}}(\hat{y}(t_{k,2}^-) - y_q(t_{k,2})). \end{aligned}$$

We have the following result regarding to the stability of system (21):

Theorem 1. The estimator error system (21) is uniformly bounded-input bounded-state (UBIBS) stable. More specifically, for any $\alpha \geq 0$ and $\bar{w} \geq 0$, there exists $\mu > 0$ such that

$$||e(0)|| \leq \alpha, |w(t)| \leq \bar{w},\ \forall t \geq 0 \Rightarrow ||e(t)|| \leq \mu,\ \forall t \geq 0. \quad (22)$$

Proof. Firstly, we consider the case of $t \notin \{t_{k,1}; t_{k,2}\}$. Let $V(t) = e(t)^T Pe(t)$ be the Lyapunov function of the estimator error system (21). Thus, we have

$$\begin{aligned} \dot{V}(t) &= 2e^T P\dot{e} \\ &= 2e^T P(A_e e + Gw + L\varepsilon) \\ &= 2e^T PA_e e + 2e^T P^{\frac{1}{2}}(P^{\frac{1}{2}})^T(Gw + L\varepsilon) \\ &\leq -e^T e + \tau e^T Pe + \frac{1}{\tau}(Gw + L\varepsilon)^T P(Gw + L\varepsilon) \\ &\leq e^T(-I + \tau P)e + \frac{1}{\tau}(Gw + L\varepsilon)^T P(Gw + L\varepsilon), \end{aligned} \quad (23)$$

where $\tau > 0$ is a small scalar such that $-I + \tau P < 0$. Since $w(t)$ and $\varepsilon(t)$ are both bounded, we have $V(t)$ is bounded.

Next, we evaluate the Lyapunov function increment at $t \in \{t_{k,1}\}$.

$$\begin{aligned} \Delta V(t_{k,1}) &= V(t_{k,1}) - V(t_{k,1}^-) \\ &= e(t_{k,1})^T Pe(t_{k,1}) - e(t_{k,1}^-)^T Pe(t_{k,1}^-) \\ &= e(t_{k,1}^-)^T(D^T PD - P)e(t_{k,1}^-) \\ &= -e(t_{k,1}^-)^T\Big(C^T(CP^{-1}C^T)^{-1}C\Big)e(t_{k,1}^-) \\ &\leq 0. \end{aligned}$$

Lastly, we evaluate the Lyapunov function increment at $t \in \{t_{k,2}\}$.

$$\Delta V(t_{k,2}) = V(t_{k,2}) - V(t_{k,2}^-)$$

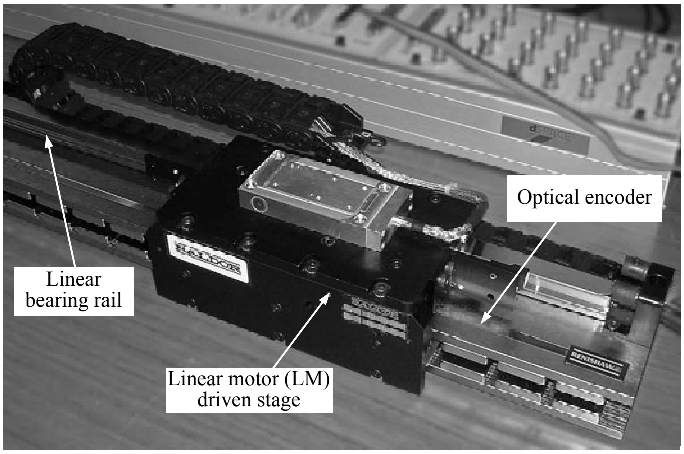

Fig. 2. Experimental setup.

$$
\begin{aligned}
&= e(t_{k,2})^T P e(t_{k,2}) - e(t_{k,2}^-)^T P e(t_{k,2}^-) \\
&= e(t_{k,2}^-)^T (D^T P D - P) e(t_{k,2}^-) \\
&\quad + \psi(t_{k,2})^T H^T P H \psi(t_{k,2}) \\
&= -e(t_{k,2}^-)^T \Big(C^T (C P^{-1} C^T)^{-1} C \Big) e(t_{k,2}^-) \\
&\quad + \psi(t_{k,2})^T (C P^{-1} C^T)^{-1} \psi(t_{k,2}) \\
&= -(C P^{-1} C^T)^{-1} \Big((y(t_{k,2}) - \hat{y}(t_{k,2}^-))^2 - \psi(t_{k,2})^2 \Big) \\
&= -(C P^{-1} C^T)^{-1} \Big((y(t_{k,2}) - \hat{y}(t_{k,2}^-) - \psi(t_{k,2}))^2 \\
&\quad + 2\Gamma \Big),
\end{aligned}
$$

where

$$\Gamma = \Big(\hat{y}(t_{k,2}^-) - \hat{y}(t_{k,2})\Big)\Big(\hat{y}(t_{k,2}) - y(t_{k,2})\Big). \tag{24}$$

From (15), it is implied that

$$
\begin{aligned}
&y(t_{k,2}) < \hat{y}(t_{k,2}) < \hat{y}(t_{k,2}^-), \text{ if } \hat{y}(t_{k,2}^-) > y_q(t_{k,2}) + \frac{\Delta}{2}, \\
&\hat{y}(t_{k,2}^-) < \hat{y}(t_{k,2}) < y(t_{k,2}), \text{ if } \hat{y}(t_{k,2}^-) < y_q(t_{k,2}) - \frac{\Delta}{2}.
\end{aligned}
$$

Accordingly, we have $\Gamma > 0$, and thus

$$\Delta V(t_{k,2}) \leq 0. \tag{25}$$

Therefore, following (23) we can show that there exist positive c_0, c_1 and c_2 such that

$$||e(t)||^2 \leq c_0 e^{-\eta t} ||e(0)||^2 + c_1 \bar{w}^2 + c_2 \Delta^2, \quad \forall t \geq 0. \tag{26}$$

where $\eta = \lambda_{min}(I - \tau P)/\lambda_{max}(P)$. We can thus take

$$\mu = \sqrt{c_0 \alpha^2 + c_1 \bar{w}^2 + c_2 \Delta^2} \tag{27}$$

for (22).

4. SIMULATION AND EXPERIMENT

This section presents the application of the reset estimator to a linear motor positioning system with optical encoder. Simulation and experimental results are shown to verify the effectiveness of the reset state estimator (RSE) over the standard state estimator (SSE) without reset actions.

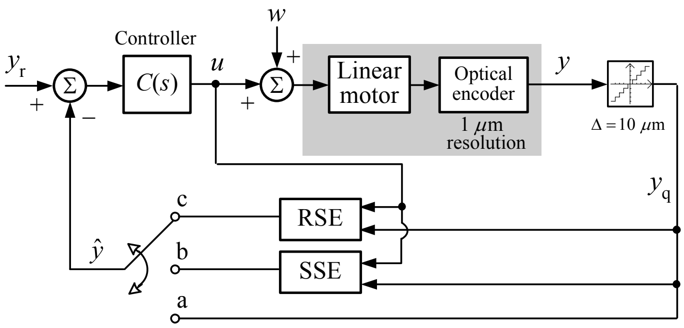

Fig. 3. Block diagram of linear motor control system with sensor quantization (SSE: Standard state estimator; RSE: Reset state estimator).

4.1 Linear Motor Position Control System

Fig. 2 shows the experimental setup. The linear motor driven stage has a 0.5 m travel range, a mounted optical encoder of 1 μm resolution , and a power amplifier. The linear motor is modeled from its physical parameters and the model in state space is given by

$$
\begin{aligned}
\begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u, \\
y_q &= Q(y),
\end{aligned} \tag{28}
$$

with $a = 7.5398$, $b = 1.5 \times 10^7$, and y is the position (in μm), v is the velocity of the stage (in μm/s), respectively.

The block diagram of the linear motor control system is shown in Fig. 3. The reference input y_r is set as sinusoidal signals with $y_r = 300 \times \mathrm{Sin}(2\pi \times 10t)$ μm. Since the amplitude of y_r is much larger than 1 μm, comparatively, the original output of the optical encoder with 1 μm resolution can be treated as having sufficiently high resolution and thus approximated as the actual position y. The measured positions y are passed through an artifical quantizer to simulate a lower resolution encoder by setting the quantizer step size as $\Delta = 10$ μm. The quantized outputs y_q together with the control input u are then used as the estimator inputs. To compare the performance of the estimators, the quantized output y_q, and the estimated positions $\hat{y}$ from the SSE and the RSE are respectively used as the feedback signals to the controller (see the switching points a, b, c in Fig. 3). When one estimator is under test, the position estimation error e_p and tracking error E_{tra} are respectively monitored by

$$e_p = y - \hat{y}, \tag{29}$$

$$E_{tra} = y_r - y. \tag{30}$$

In the setup, we assume that $w = 0$ and the initial conditions of the motor are

$$y(0) = 100\ \mu\text{m}; \quad v(0) = 60\ \text{mm/s}. \tag{31}$$

As we are only interested in the state estimation, the details of feedback control design are omitted. The controller is directly given by

$$C(s) = k_p + k_d \frac{s}{0.0001s + 1} + k_i \frac{1}{s}, \tag{32}$$

where $k_p = 0.1053$, $k_d = 1.2 \times 10^{-4}$, $k_i = 50$. The resulting closed-loop system can obtain a bandwidth of 200 Hz. The controller is then fixed throughout the tests.

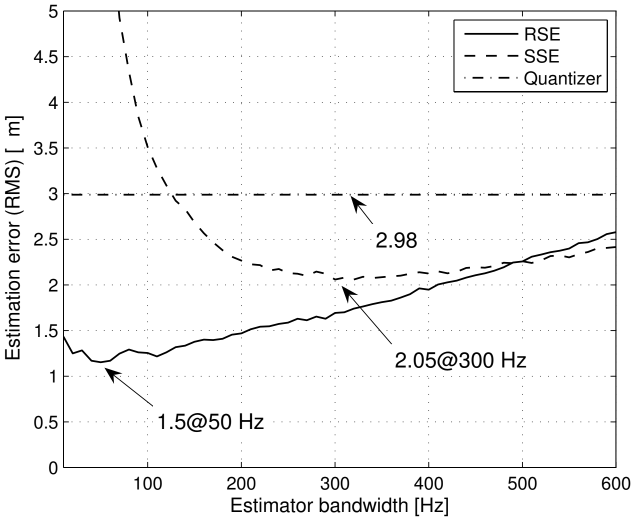

Fig. 4. Position estimation error versus estimator bandwidth.

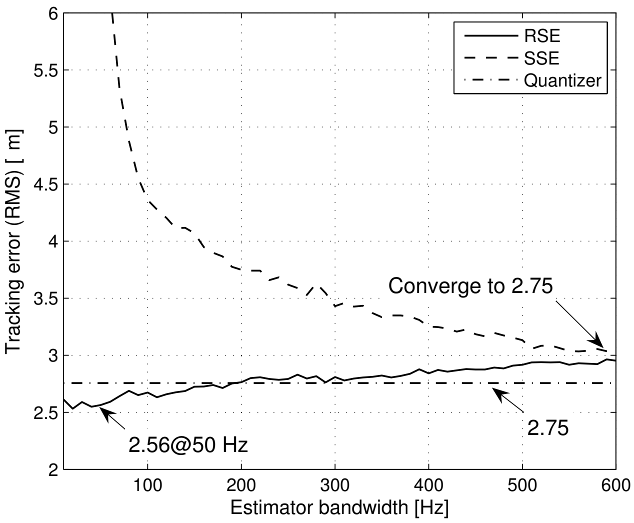

Fig. 5. Position tracking error versus estimator bandwidth.

Next, we select the estimator gain L such that the smallest RMS (root mean square) of position estimation error can be achieved. According to (21) the estimator gain should be selected to balance the effect of disturbance and quantization noise on the estimator error e. Here, we select the estimator gain in terms of the estimator bandwidth as follows:

$$L = [8.8w_n - 7.5 \quad 39.5w_n^2 - 66.9w_n + 56.8]^T, \qquad (33)$$

where w_n denotes the estimator bandwidth in Hz. Simulation results of the achievable estimation errors versus estimator bandwidth are shown in Fig. 4 and the corresponding tracking errors are shown in Fig. 5. It is obvious that RSE can achieve a smaller least estimation error than that of the SSE, while both estimators can achieve much smaller estimation error than the quantization error. However, the tracking error with SSE is not better than that using the quantized position as direct feedback no matter the bandwidth is tuned. Fortunately, the tracking error with RSE can be reduced by 7% compared to the case with quantized position as feedback.

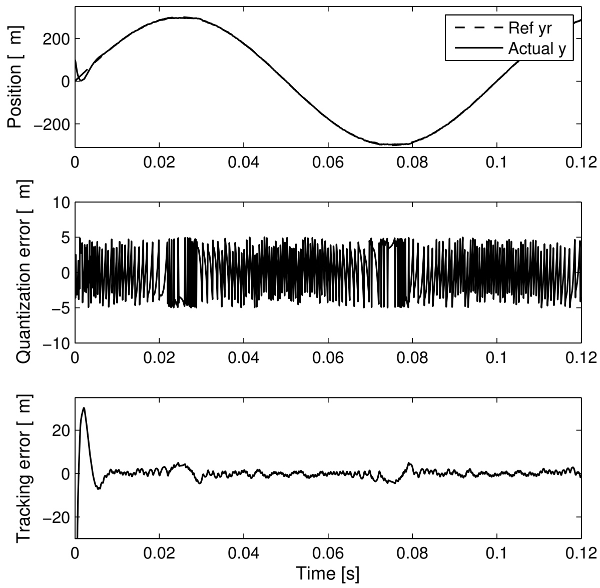

Fig. 6. Position tracking with quantized position as feedback signal.

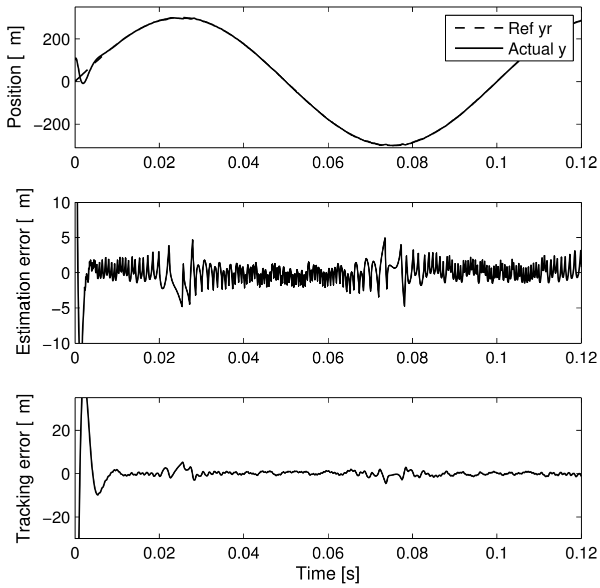

Fig. 7. Position tracking with estimated position from SSE as feedback signal.

4.2 Results and Discussions

We fix the estimator gains as those that can achieve the least estimation error. Figs. 6-8 show the time responses of the system with various position feedback. We can see that the profiles with RSE in Fig. 8 gives the best results in estimation error and tracking error. Moreover, Fig. 9 compares the estimated position profiles from SSE and RSE. It is shown that at the initial stages, the RSE converges to the actual position extremely faster than SSE due to its reset feature (see Figs. 9(a_1) and (b_1)). In Figs. 9(a_2), (b_2), the RSE is smoother and closer to the actual position. Particularly, we can easily observe that the estimated position is reset to the actual position at the quantizer transition (e.g. at 23 ms).

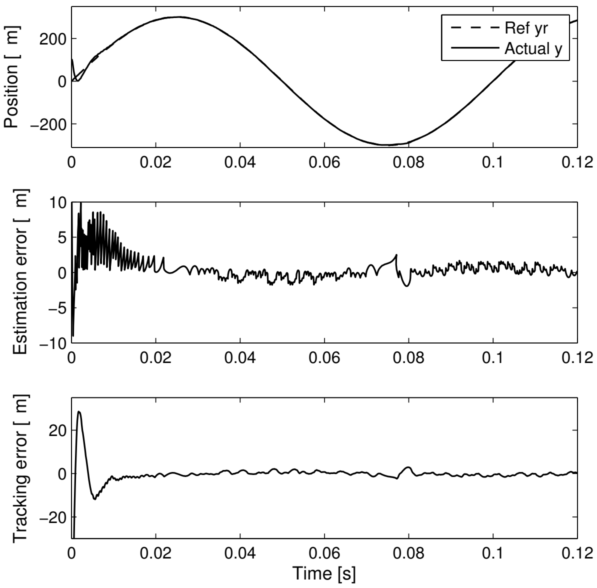

Fig. 8. Position tracking with estimated position from RSE as feedback signal.

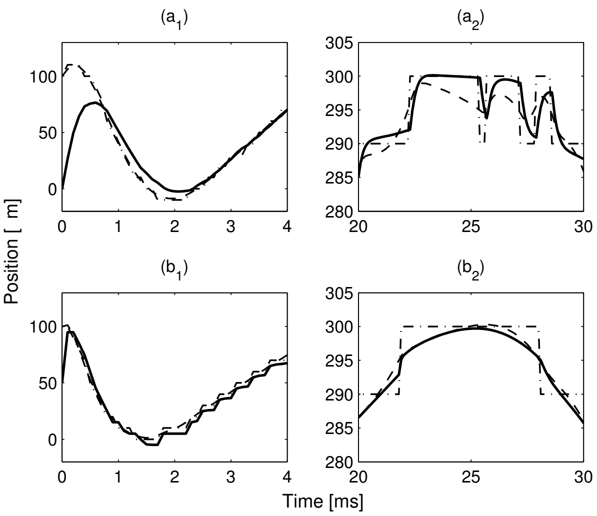

Fig. 9. Comparison of estimated position profiles. $(a_{1,2})$ from SSE. $(b_{1,2})$ from RSE. Solid lines: Estimated position, Dashed lines: actual position, Dashdot lines: quantized position.

Finally, we implemented the controller together with the estimators on a real-time DSP system (dSPACE-DS1103, dSPACE GmbH, Paderborn, Germany) with the sampling period 0.1 ms. The RSE was set with $T = 0.1$ ms and note that the time instants $t_{k,1}$ can only be captured as close as to some sampling instants. Fig. 10 shows the experimental results, which indicate that the RSE outperforms the SSE by 16% in the position estimation.

5. CONCLUSION

We have investigated a reset state estimator to suppress sensor quantization in motion control systems. The reset estimator can favorably update the estimated state by using the exacted information from the quantizer output, which contains partial knowledge of the actual position.

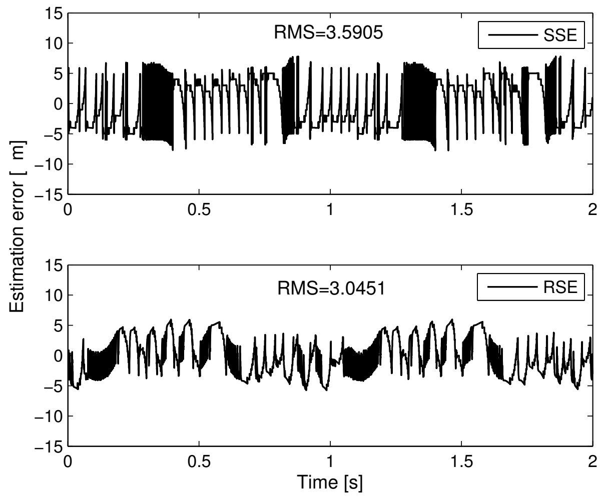

Fig. 10. Experimental results of position estimation errors.

We have shown that by embedding the reset scheme into a standard state estimator, the stability of the reset estimator system will not be destroyed. Moreover, the reset state estimator was applied to a linear motor control system with an optical encoder. Simulation and experimental results have shown that it can achieve smaller position estimation error than that of a standard state estimator.

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