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Neural Network Based Control for a Class of Uncertain Robot Manipulator with External Disturbance ⋆

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Abstract: In this paper, a neural network based continuous control mechanism that can compensate for system uncertainties is developed for a class of robot manipulators under both repeating and non-repeating disturbances. With limited assumptions about the systems' dynamics, Lyapunov techniques are utilized to show that a semi-global asymptotic tracking result is achieved while all the closed-loop states remain bounded. Numerical simulation results are provided to demonstrated that the proposed control design achieves a good tracking performance.

1. INTRODUCTION

The control objective in many robot manipulator applications is to command the manipulator motion to track some desired trajectories with the control inputs being applied to the manipulator joints. Under most working environment, the robot systems will face different kinds of additive disturbance, and hence, the system performance and stability can not be directly predicted a priori. To make the matters even more difficult, for many robot system, the mathematical models are uncertain. In the case the model uncertainty is caused by the unknown constant system parameters and can be linearly parameterized, adaptive control design (Kristic [1995], Sastry [1989]) is often considered to be the method of choice. However, an adaptive control strategy designed for a disturbance-free system might not be able to compensate for the disturbance, and may even become unstable under certain condition. On the other hand, some manipulator systems do not meet the parameter linearizable condition, which prevent the application of standard non-linear adaptive design strategy. Neural network has been employed to compensate for the systems's uncertainty via its learning ability(Lewis [1999]). Lots of existing neural-network based control system can only provide a bounded tracking result in the case with external disturbance Kwan [1998]. There are different methods to deal with additive disturbance. If the disturbance is periodic and the period is known, learning control scheme can be designed (Antaskalis [1993]). If the disturbance can be upper bounded by a norm-based inequality, slide-mode control (SMC) (Slotine [1991]), Utkin [1992] (i.e., discontinuous control design) can be employed.

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Inspired by the control strategy present in (Xian [2004], Tatlicioglu [2007]), and (Braganza [2006]), we consider a n-link robot manipulator system with uncertainties under repeating and non-repeating external disturbances. A neural network control design combing with learning feed-forward term and robust term are used to compensate for system uncertainty, repeating disturbance with known period and non-repeating disturbance respectively. Compared with other robust control design methods, the proposed control strategy need less control effort because the system uncertainty is compensated by the neural network based feed-forward and the periodic disturbance is compensated by the learning component. Lyapunov-based techniques are utilized to ensure that the proposed control law achieve semi-global tracking result while all the closed-loop system states remain bounded.

This paper is organized as follows. Section 2 presents the dynamic model for the robot manipulator system. The problem statement and error system development are provided in Section 3. Section 4 and Section 5 present the control design and stability analysis, respectively. Some numerical simulation results for a two-link robot manipulator are provided in Section 6 to demonstrate the performance of proposed control design. Finally, conclusion remarks are provided in Section 7.

2. ROBOT MANIPULATOR'S DYNAMIC MODEL

The dynamic model for a n-link, revolute, direct-drive robot manipulator under external disturbance is assumed to be of the following form (Lewis [1993], Xian [2004])

$$M(q)\ddot{q} = -C(q,\dot{q})\dot{q} - G(q) - F(\dot{q}) - D_1(t) - D_2(t) + \tau(t) \qquad (1)$$

where $q(t) \in \mathbb{R}^n$ is the link position, $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ represents the gravity effects on robot manipulator, $F(\dot{q}) \in \mathbb{R}^n$ denotes the viscous friction force, $D_1(t) \in \mathbb{R}^n$ is the unknown external repeating disturbance, $D_2(t) \in \mathbb{R}^n$ is the unknown general external disturbance, and $\tau(t) \in \mathbb{R}^n$ is the torque input vector. The subsequent control design is based on the assumption that link position $q(t)$ and link velocity $\dot{q}(t)$ are measurable. $M(q)$, $C(q,\dot{q})$, $G(q)$, $F(\dot{q})$ are unknown nonlinear function matrixes or vectors. Moreover, the following properties and assumptions (Lewis [1993], Nicosia [1990], Xian [2004]) will be utilized in the subsequent analysis:

Property 1 The inertia matrix $M(\theta)$ is symmetric and positive-definite, and satisfies the following inequalities

$$m_1 \|x\|^2 \le x^T M(x,\theta) x \le m_2(\|x\|)\|x\|^2 \qquad (2)$$

for all $x \in \mathbb{R}^n$, where m_1 is a positive constant, $m_2(\cdot)$ is a positive scalar function, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2 The external repeating disturbance $D_1(t)$ satisfies the following properties

$$\begin{aligned} D_1(t+T) &= D_1(t) \\ \dot{D}_1(t+T) &= \dot{D}_1(t) \end{aligned} \qquad (3)$$

where $T \in \mathbb{R}^+$ is the period for $D_1(t)$, and T is known.

Assumption 1 The nonlinear function $M(q)$, $C(q,\dot{q})$, $G(q)$ and $F(\dot{q})$ are continuous differentiable up to their second derivatives (i.e., $M(q)$, $C(q,\dot{q})$, $G(q)$ and $F(\dot{q}) \in \mathcal{C}^2$).

Assumption 2 The additive disturbance signal $D_1(t)$ and $D_2(t)$ are assumed to be continuous differentiable and bounded up to their second derivatives (i.e. $D_i(t) \in \mathcal{C}^2$ and $D_i(t)$, $\dot{D}_i(t)$, $\ddot{D}_i(t) \in \mathcal{L}_\infty$, for $i = 1, 2$).

3. ERROR SYSTEM DEVELOPMENT

The control objective is to ensure that the robot manipulator tracks a time-varying, reference trajectory in presence of system uncertainty and additive disturbance.

Let $q_d(t) \in \mathbb{R}^n$ be a $\mathcal{C}^3$ reference trajectory such that

$$q_d^{(i)}(t) \in \mathcal{L}_\infty \qquad (4)$$

for $i = 0, 1, 2, 3$, and let the output tracking error $e_1(t) \in \mathbb{R}^n$ being defined as follows

$$e_1(t) := q_d(t) - q(t). \qquad (5)$$

Then the control objective is to make $\lim e_1(t) = 0$ as $t \to \infty$ via a continuous control torque input $\tau(t)$ with state feedback $q(t)$ and $\dot{q}(t)$, and keep all the signals in the closed-loop system to be bounded. To facilitate the control design and stability analysis, filtered error signals $e_1(t) \in \mathbb{R}^n$ and $r(t) \in \mathbb{R}^n$ are defined as follows

$$\begin{aligned} e_2(t) &= \dot{e}_1(t) + e_1(t) \\ r(t) &= \dot{e}_2(t) + \Lambda e_2(t) \end{aligned} \qquad (6)$$

where $\Lambda \in \mathbb{R}^{n\times n}$ is a positive-definite, diagonal, constant gain matrix. The filtered error signal $e_2(t)$ is measurable but $r(t)$ is not measurable due to the unmeasurable signal $\dot{e}_2(t)$. After differentiating $r(t)$ with t, multiplying both sides of the resulting equation by $M(q)$, and substituting from the second derivative of (1), it can be obtained that

$$\begin{aligned} M(q)\dot{r} = {} & M(q)\dddot{q}_d + \dot{M}(q)\ddot{q} + \dot{C}(q,\dot{q})\dot{q} + C(q,\dot{q})\ddot{q} + \\ & \dot{G}(q) + \dot{F}(q) + \dot{D}_1(t) + \dot{D}_2(t) + M(q)\ddot{e}_1 + \\ & M\Lambda\dot{e}_2 - \dot{u} \\ = {} & -\frac{1}{2}\dot{M}(q)r - e_2 + N + \Phi(t) + \\ & \dot{D}_2(t) - \dot{u} \end{aligned} \qquad (7)$$

where the auxiliary nonlinear function $N(\cdot) \in \mathbb{R}^6$ is defined as follows

$$\begin{aligned} N(q,\dot{q},\ddot{q},t) := {} & M(q)\left(\dddot{q}_d + \Lambda e_2 + \ddot{e}_1\right) + \\ & \dot{M}(q)(\ddot{q} + \frac{1}{2}r) + \dot{C}(q,\dot{q})\dot{q} + \\ & C(q,\dot{q})\ddot{q} + \dot{G}(q) + \dot{F}(\dot{q}) - e_2 \end{aligned} \qquad (8)$$

and $\Phi(t) := \dot{D}_1(t)$. Let $N_d(t) := N(q_d, \dot{q}_d, \ddot{q}_d, t)$; hence, from (8), it can be obtained that

$$\begin{aligned} N_d(t) = {} & M(q_d)\dddot{q}_d + \dot{M}(q_d)\ddot{q}_d + \dot{C}(q_d,\dot{q}_d)\dot{q}_d + \\ & C(q_d,\dot{q}_d)\ddot{q}_d + \dot{G}(q_d) + \dot{F}(\dot{q}_d). \end{aligned} \qquad (9)$$

Note that $N_d(t)$ and $\dot{N}_d(t) \in \mathcal{L}_\infty$ due to (4) and the $\mathcal{C}^2$ condition in assumption 2. Now, after adding and subtracting $N_d(t)$ to the right-side of (7), (7) can be written as follows

$$\begin{aligned} M(q)\dot{r} = {} & -\frac{1}{2}\dot{M}(q)r - e_2 + \tilde{N} + N_d + \Phi(t) + \\ & \dot{D}_2(t) - \dot{u} \end{aligned} \qquad (10)$$

where

$$\tilde{N}(q,\dot{q},\ddot{q},t) := N - N_d. \qquad (11)$$

Remark 1 Since $N(\cdot)$ defined in (8) is continuously differentiable, $\tilde{N}(\cdot)$ can be upper bounded as follows (Xian [2004])

$$\left\|\tilde{N}\right\| \leqslant \rho(\|z\|)\|z\| \qquad (12)$$

where $\|\cdot\|$ is the Euclidean norm and $z(t) \in \mathbb{R}^{18}$ is defined as

$$z := \left[\begin{array}{ccc} e_1^T & e_2^T & r^T \end{array}\right]^T, \qquad (13)$$

and $\rho(\cdot): \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is some globally invertible, nondecreasing function.

4. DESIGN OF CONTROL LAW

Based on the open-loop dynamics of $r(t)$ in (10), the control torque input $\tau(t)$ is designed as follows

$$u(t) = \hat{\Psi}(t) + \bar{u}(t) + \int_{t_0}^{t} \hat{n}(\tau)d\tau \qquad (14)$$

where $\hat{\Psi}(t) \in \mathbb{R}^n$ represents the learning component utilized to compensate for the periodic signal $\Phi(t)$ in (10), $\hat{n}(t) \in \mathbb{R}^n$ denotes a neural network based forward component to compensate for the system uncertainty, and $\bar{u}(t)$ is the auxiliary torque input.

$\hat{\Psi}(t)$ is defined as follows

$$\hat{\Psi}(t) = \hat{\Psi}(t-T) + k_L \int_{t_0}^{t} \Lambda e_2(\tau) d\tau + k_L e_2(t) - k_L e_2(t_0) \quad (15)$$

where k_L is a positive control gain. Note that the initial value of $\hat{\Psi}(t)$ is $\hat{\Psi}(t_0) = 0_{n\times 1}$ from (15). The auxiliary function $\hat{\Phi}(t) \in \mathbb{R}^n$ is defined as

$$\hat{\Phi}(t) = \dot{\hat{\Psi}}(t). \quad (16)$$

Taking the time derivative of $\hat{\Psi}(t)$ along (15), and substituting the result into (16), it can be obtained that

$$\hat{\Phi}(t) = \hat{\Phi}(t-T) + k_L r(t). \quad (17)$$

According to the approximation properties of neural work (Lewis [2002], Spooner [2002]), the continuous, bounded, uncertain nonlinear function vector N_d in (10) can be approximated by two-layer (i.e., a hidden-layer with p neurons and an output layer with 1 neurons) neural network such that

$$N_d(\cdot) = W^T \sigma(V^T \chi) + o(\chi) \quad (18)$$

where $\chi = \left[\, q_d \; \dot{q}_d \; \ddot{q}_d \; \dddot{q}_d \,\right]^T \in \mathbb{R}^{4n}$ represents the bounded input to the neural network, $W \in \mathbb{R}^{p\times n}$ and $V \in \mathbb{R}^{4n\times p}$ denote the ideal input layer and output layer weights, $\sigma(\cdot)$ is the corresponding neural activation function vector, $o(\cdot) \in \mathbb{R}^n$ denotes the bounded reconstruction error (i.e., $|o(\chi)| \leq \bar{o}$ where $\bar{o}$ is some positive constant). The output of neural network is designed as follows

$$\hat{n}(t) = \hat{W}^T \sigma(\bar{V}^T \chi) \quad (19)$$

where $\hat{W}(t) \in \mathbb{R}^{4n\times p}$ is the weight estimation of W. Note, here for simplicity reason, only the estimation for W will be designed, and $\bar{V}$ is set to some constant matrix. There are many choices for the activation function $\sigma(\cdot)$, such as radial basis function $\sigma(z_0) = \exp(-\frac{z_0^2}{\gamma})$ where γ is some positive constant, hyperbolic tangent function $\sigma(z_0) = -\frac{1-\exp(-z_0)}{1+\exp(-z_0)}$, sigmoid function $\sigma(z_0) = \frac{1}{1+\exp(-z_0)}$ and so on (Spooner [2002]). Here sigmoid function will be utilized as the neurons' activation function. Inspired by the augmented back-propagation algorithm in (Lewis [1999]) and the update law design in (Braganza [2006]), the $\hat{W}(t)$ is generated via the following tuning law

$$\begin{aligned} \hat{W} &= -\kappa_1 \hat{W} + \Gamma \sigma(\bar{V}^T \chi) \mathbf{sat}(e_2 + \omega_1) \\ \omega_1 &= \frac{1}{\kappa_2}(-\omega_2 + e_2) \\ \dot{\omega}_2 &= \frac{1}{\kappa_2}(-\omega_2 + e_2) \end{aligned} \quad (20)$$

where $\kappa_1, \kappa_2 \in \mathbb{R}$ are some small positive constant, $\Gamma \in \mathbb{R}^{p\times p}$ is a diagonal, positive definite, update gain matrix, $\omega_1(t), \omega_2(t) \in \mathbb{R}$ are auxiliary filter signals, $\mathbf{sat}(\cdot) \in \mathbb{R}^n$ is defined as $\mathbf{sat}(x) = \left[\, sat(x_1),\, sat(x_2),\, \cdots, sat(x_n) \,\right]^T$ $\forall x \in \mathbb{R}^n$ where $sat(x_i)$ is the standard saturation function. Based on the structure of (20), it is easy to check that $\hat{W}(t)$, $\dot{\hat{W}}(t) \in \mathcal{L}_\infty$, thus it can be shown that $\hat{n}(t)$ and $\dot{\hat{n}}(t) \in \mathcal{L}_\infty$.

Based on the dynamics of $r(t)$ in (10), the auxiliary control input $\bar{u}(t)$ is designed as follows

$$\bar{u}(t) = (K + I_n)e_2(t) - (K + I_n)e_2(t_0) + \int_{t_0}^{t} \left[(K + I_n)\Lambda e_2(\tau) + \beta \mathbf{sgn}(e_2(\tau))\right] d\tau \quad (21)$$

where K, $\beta \in \mathbb{R}^{n\times n}$ are positive definite, diagonal, constant gain matrix, $I_n \in \mathbb{R}^{n\times n}$ represents a identity matrix, and the function vector $\mathbf{sgn}(e_2(t)) := \left[\, sgn(e_{21}(t)) \; sgn(e_{22}(t)) \; ... \; sgn(e_{2n}(t)) \,\right]^T \in \mathbb{R}^n$ with the function $sgn(\cdot)$ being the standard signum function, $e_{2i}(t)$ represents the i-th entry of the vector $e_2(t)$ for $i = 1, 2, \cdots, n$. It is easy to check that $\bar{u}(t_0) = 0_{n\times 1}$. Thus it can be obtained that $u(t_0) = 0_{n\times 1}$ from (14). After taking the time derivative of (21), it can be obtained that

$$\dot{\bar{u}} = (K + I_n)r(t) + \beta \mathbf{sgn}(e_2(t)). \quad (22)$$

Thus the time derivative of control torque input $u(t)$ is

$$\dot{u} = (K + I_n)r(t) + \beta \mathbf{sgn}(e_2(\tau)) + \hat{\Phi}(t) + \hat{n}(t) \quad (23)$$

where (14), (16), (17), and (22) have been utilized.

By substituting (23) into (10), the closed-loop dynamics of $r(t)$ can be obtained as follows

$$\begin{aligned} M(q)\dot{r}(t) = &-\frac{1}{2}\dot{M}(q)r(t) - e_2(t) + \tilde{N}(t) + \tilde{\Phi}(t) + \\ &\tilde{N}_d(t) + \dot{D}_2(t) - (K + I_n)r(t) - \\ &\beta \mathbf{sgn}(e_2(\tau)) \end{aligned} \quad (24)$$

where $\tilde{\Phi}(t) := \Phi(t) - \hat{\Phi}(t)$, and $\tilde{N}_d(t) = N_d(t) - \hat{n}(t)$. It is easy to know that $\tilde{N}_d(t)$, $\dot{\tilde{N}}_d(t) \in \mathcal{L}_\infty$. By utilizing (17) and the condition that $\Phi(t) = \Phi(t-T)$, it can be obtained that

$$\tilde{\Phi}(t) = \tilde{\Phi}(t-T) - k_L r(t). \quad (25)$$

5. STABILITY ANALYSIS

Before presenting the main result of this section, a lemma which will be invoked later is stated first.

Lemma 1 Let the auxiliary function $L(t) \in \mathbb{R}$ be defined as follows

$$L(t) := r^T(t)(\dot{D}_2(t) + \tilde{N}_d(t) - \beta \mathbf{sgn}(e_2(\tau)). \quad (26)$$

If the control gain matrix β is selected to satisfy the following condition

$$\beta_i > \left\| \dot{D}_{2i} + \tilde{N}_{di} \right\|_{\mathcal{L}\infty} + \frac{1}{\Lambda_{ii}} \left\| \ddot{D}_{2i} + \dot{\tilde{N}}_{di} \right\|_{\mathcal{L}\infty} \quad (27)$$

where $\|\cdot\|_{\mathcal{L}\infty}$ is the $\mathcal{L}\infty$ norm (Khalil [2002]), $\dot{D}_{2i}(t)$, $\ddot{D}_{2i}(t)$, $\tilde{N}_d(t)_i$, $\dot{\tilde{N}}_d(t)_i$, and Λ_{ii} represent the i-th entry of the corresponding vector respectively, then

$$\int_{t_0}^{t} L(\tau) d\tau \leqslant \zeta_b \quad (28)$$

with the positive constant ζ_b being defined as follows

$$\zeta_b = \beta \left\| e_2(t_0) \right\| - e_2^T(t_0)(\dot{D}_2(t_0) + \tilde{N}_d(t)). \quad (29)$$

Proof: Please refer to (Xian [2004]).

The stability result for the proposed controller is stated by the following theorem.

Theorem 1 The controller given in (14), (15), (19), and (21) ensures that all system signals are bounded under closed-loop operation and $\lim e_1(t) = 0$ as $t \to \infty$ provided that the control gain matrix β is adjusted according to (27), the minimum eigenvalue value of matrix Λ satisfies that $\lambda_{\min}(\Lambda) > \frac{1}{2}$, and the control gain matrix K is selected sufficiently large relative to the system initial condition (details of the selection for K is provided in the following analysis).

Proof: Let the auxiliary function $P(t) \in \mathbb{R}$ be defined as follows

$$P(t) = \zeta_b - \int_{t_0}^{t} L(\tau)d\tau \tag{30}$$

where ζ_b and $L(t)$ are defined in Lemma 1. It is easy to see that the use of Lemma 1 ensures $P(t) \geqslant 0$. Let a non-negative auxiliary function $V_g(t) \in \mathbb{R}_{\geqslant 0}$ be defined as follows

$$V_g(t) = \frac{1}{2k_L} \int_{t-T}^{t} \tilde{\Phi}^T(\tau)\tilde{\Phi}(\tau)d\tau. \tag{31}$$

Taking the time derivative of (31) along (25) results in

$$\begin{aligned} \dot{V}_g(t) &= \frac{1}{2k_L}(\tilde{\Phi}^T(t)\tilde{\Phi}(t) - \tilde{\Phi}^T(t-T)\tilde{\Phi}(t-T)) \\ &= -r^T(t)\tilde{\Phi}(t) - \frac{1}{2}k_L r^T(t)r(t). \end{aligned} \tag{32}$$

Now let the non-negative function $V(y(t), t) \in \mathbb{R}$ be defined as follows

$$\begin{aligned} V(y,t) :&= e_1^T(t)e_1(t) + e_2^T(t)e_2(t) + \\ &\frac{1}{2}r^T(t)M(q)r(t) + P(t) + V_g(t) \end{aligned} \tag{33}$$

where $y(t) \in \mathbb{R}^{(3n+2)}$ is defined as

$$y(t) := \begin{bmatrix} z^T & \sqrt{P(t)} & \sqrt{V_g(t)} \end{bmatrix}^T. \tag{34}$$

According to Property 1, (33) can be bounded by

$$W_1(y) \leqslant V(t) \leqslant W_2(y) \tag{35}$$

where the non-negative scalar auxiliary function $W_1(y)$, $W_2(y)$ are defined as follows respectively

$$\begin{aligned} W_1(y) &:= \lambda_1 \|y\|^2 \\ W_2(y) &:= \lambda_2(\|y\|) \|y\|^2 \end{aligned} \tag{36}$$

with $\lambda_1 := (\frac{1}{2})\min\{1, m_1, \lambda_{mi}(\Gamma^{-1})\}$ and $\lambda_2(\|y\|) := \max\{\frac{1}{2}m_2((\|y\|), \frac{1}{2}\lambda_{\max}(\Gamma^{-1}), 1\}$. Here $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of the corresponding matrix respectively. Note it can be shown that $\|q\| \leqslant g(\|y\|)$ where $g(\cdot)$ is some positive function by using (4), (5), (6), and (34), thus $m_2(\|q\|) \leqslant m_2(\|y\|)$. By differentiating (33) with time, $\dot{V}(y,t)$ can be expressed as

$$\begin{aligned} \dot{V}(y,t) &= e_1^T \dot{e}_1 + e_2^T \dot{e}_2 + \frac{1}{2}r^T \dot{M}(q)r + r^T M(q)\dot{r} + \\ &\dot{V}_g + \dot{P}. \end{aligned} \tag{37}$$

After substituting (6), (24), (26), (30), and (32) into (37), $\dot{V}(y,t)$ can be simplified as follows

$$\begin{aligned} \dot{V}(y,t) &= -e_1^T e_1 - e_2^T \Lambda e_2 + e_1^T e_2 - r^T(K + I_6)r + \\ &r^T \tilde{N} - \frac{1}{2}k_L r^T r. \end{aligned} \tag{38}$$

By using the fact that $e_1^T e_2 \leqslant \frac{1}{2}(\|e_1\|^2 + \|e_2\|^2)$, an upper bound of (38) can be obtained as

$$\begin{aligned} \dot{V}(y,t) &\leqslant -\lambda_3 \|z\|^2 + \|r\| \rho(\|z\|) \|z\| - k_s \|r\|^2 \\ &\leqslant -(\lambda_3 - \frac{1}{4k_s}\rho^2(\|z\|)) \|z\|^2 \end{aligned} \tag{39}$$

where (12) was used, $\lambda_{\min}(\Lambda) > \frac{1}{2}$, $\lambda_3 := \min\{\frac{1}{2}, \lambda_{\min}(\Lambda) - \frac{1}{2}\}$, and $k_s := \lambda_{\min}(K) + \frac{1}{2}k_L$. Based on (39), it can be stated that

$$\begin{aligned} &\dot{V}(y,t) \leqslant -\gamma \|z\|^2 \; for k_s > \frac{1}{4\lambda_3}\rho^2(\|z\|) \\ &or \|z\| < \rho^{-1}(2\sqrt{k_s \lambda_3}) \end{aligned} \tag{40}$$

where γ is some positive constant. Let the non positive scalar auxiliary functions $W(y)$, $\bar{(W)}$ be defined as $W(y) := -\gamma \|z\|^2$ and $\bar{W}(y) = -\gamma \|e_1\|^2$. It is easy to check that $W(y) \leqslant \bar{W}(y)$. By utilizing the inequality in (40), a region $\mathcal{D}$ can be defined as follows

$$\mathcal{D} := \{y(t) \in \mathbb{R}^{3n+2} \Big| \|y(t)\| < \rho^{-1}(2\sqrt{k_s \lambda_3})\}. \tag{41}$$

The inequalities in (35) and (40) can be employed to show that $V(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t)$, $e_2(t)$, $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given the results that $e_1(t)$, $e_2(t)$ and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, it can be proven that $\dot{e}_1(t)$, $\dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (6). Thus the condition that $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are bounded can be used with (5) and (6) to conclude that $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Assumption 1 can be employed to show that $M(q)$, $C(q, \dot{q})$, $G(q)$, and $F(\dot{q}) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Now it can be proven that $u(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (1) and the condition that $D_1(t)$, $D_2(t)$ are bound. All the closed-loop system states are bounded in $\mathcal{D}$.

Using the aforementioned boundedness statements, it is clear that $\dot{\bar{W}}(y) \in \mathcal{L}_\infty$ in $\mathcal{D}$, which is a sufficient condition for $\bar{W}(y)$ being uniformly continuous. Let the region $\mathcal{S}$ be defined as follows

$$\mathcal{S} := \{y(t) \in \mathcal{D} \Big| W_2(y) < \lambda_1(\rho^{-1}(2\sqrt{k_s \lambda_3}))^2\} \tag{42}$$

By applying the Theorem 8.4 in Khalil [2002] and follow the similar steps in Xian [2004], it can be proven that $e_1(t) \to 0$ as $t \to \infty$ $\forall y(t_0) \in \mathcal{S}$. Note that the attraction region in (42) can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e., a semi-global stability result). Specifically, (36) and (42) can be used to calculate the region of attraction as follows

$$\|y(t_0)\| < \sqrt{\frac{\lambda_1}{\lambda_2(\|y(t_0)\|)}}\rho^{-1}(2\sqrt{\lambda_3 k_s}) \tag{43}$$

which can be arranged as

$$k_s > \frac{1}{4\lambda_3}\rho^2(\sqrt{\frac{\lambda_2(\|y(t_0)\|)}{\lambda_1}}\|y(t_0)\|). \quad (44)$$

Based on (34), (30), and (31), an explicit expression of $\|y(t_0)\|$ can be written as follows

$$\|y(t_0)\| = \sqrt{\|e_1(t_0)\|^2 + \|e_2(t_0)\|^2 + \|r(t_0)\|^2 + \zeta_b^2}. \quad (45)$$

6. NUMERICAL SIMULATION

The proposed control law was simulated for a two link planar robot of the dynamic structure in Lewis [1993] with the inertia matrix being set as

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (46)$$

where $m_{11} = (m_1+m_2)a_1^2+m_2a_2^2+2m_2a_1a_2\cos(q_2)$, $m_{12} = m_2a_2^2 + m_2a_1a_2\cos(q_2)$, $m_{21} = m_{12}$, and $m_{22} = m_2a_2^2$, the Centripetal-Coriolis matrix being set as

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \quad (47)$$

where $c_{11} = -m_2a_1a_2\sin(q_2)\dot{q}_2$, $c_{12} = -m_2a_1a_2\sin(q_2)(\dot{q}_1+\dot{q}_2)$, $c_{21} = -c_{12}$ and $c_{22} = 0$, the gravity force vector being set as

$$G(q) = \begin{bmatrix} (m_1+m_2)ga_1\cos(q_1) + m_2ga_2\cos(q_1+q_2) \\ m_2ga_2\cos(q_1+q_2) \end{bmatrix}, (48)$$

the friction force vector being set as

$$F(\dot{q}) = \begin{bmatrix} F_{d1}\dot{q}_1 \\ F_{d2}\dot{q}_2 \end{bmatrix}. \quad (49)$$

External disturbance are set as $D_1(t) = \begin{bmatrix} sin(t) \\ cos(t) \end{bmatrix}$ $(N.m)$ and $D_2(t) = \begin{bmatrix} \cos(2t) + e^{-0.5t} + 0.5 \\ \sin(5t) + 1.2e^{-0.4t} + 0.3 \end{bmatrix}$ $(N.m)$ respectively. The manipulator's parameters are chosen as $m_1 = 1kg$, $m_2 = 1kg$, $a_1 = 1m$, $a_2 = 1m$, $g = 9,8N/\sec^2$, $F_{d1} = 0.2$, and $F_{d2} = 0.3$. The reference trajectory $q_d(t)$ is selected as

$$q_d = \begin{bmatrix} 0.5\sin(t)(1-e^{-0.2t^3}) \\ 0.4\cos(t)(1-e^{-0.25t^3}) \end{bmatrix} rad. \quad (50)$$

The system's initial condition is set as $q(t_0) = [\,0.1\ 0.1\,]^T$ (rad). The number of neuron is selected as $p = 10$ with the input for the neural network in (19) being set as $\chi_1 = [\,1\ q_d\ \dot{q}_d\ \ddot{q}_d\ \dddot{q}_d\,]^T$ where the first term "1" is used to provide a basis. $\bar{V}$ in (19) is chosen as

$$\bar{V} = [\,1\ 0.1\ 0.2\ 0.3\ 0.11\ 0.25\ 0.6\ 0.27\ 1\,]^T \quad (51)$$

to provide a basis Lewis [2002]. The control gains in (15) and (21) are adjusted to the following value $k_L = 1$, $K = diag\{18, 20\}$, $\beta = diag\{12, 12\}$, $\Lambda = \{4, 4\}$. The upper and lower value for the saturation function in (20) are set as 100 and -100 respectively. The tuning gains for neural network in (20) are tuned by trial-and-error until a good tracking performance was achieved. This results in the following gain values $\kappa_1 = 0.001$, $\kappa_2 = 0.001$, $\Gamma = diag\{200, 420, 220, 300, 300, 500, 500, 500, 320, 320\}$. Figure(1) and Figure (2) show the simulation results for the tracking error $e_1(t)$ and control torque input $\tau(t)$.

7. CONCLUSION

This paper considered the tracking control problem of a n-link robot manipulator system under system uncertainty and external disturbance. A continuous control strategy was proposed to ensure the semi-global asymptotic tracking under very limited restrictions on the uncertainties and disturbance. The neural network based component was used to compensated the system uncertainty. With the help of robust term and learning term, the controller compensated for both non-repeating and repeating disturbance with unknown period. Lyapunov-based methods were employed to ensure the system stability and the tracking error being driven to zero. Numerical simulation results show that the proposed control design achieves good tracking performance. Future research work will examine the extensions of proposed control mechanism to the output feedback problem and redundant robots.

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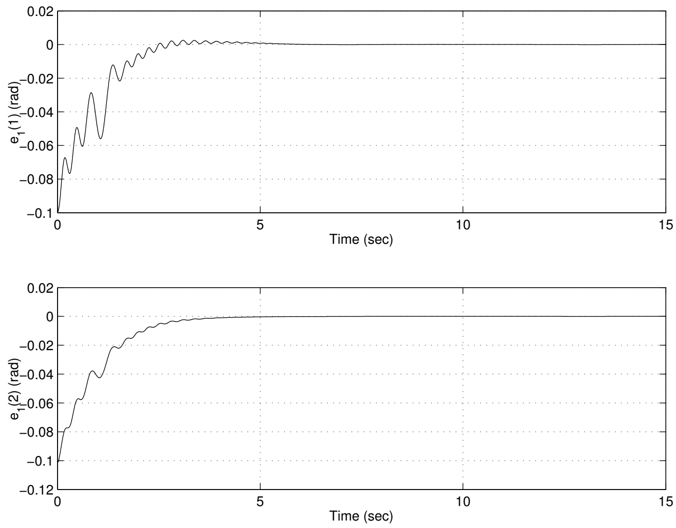

Fig. 1. Tracking error $e_1(t)$

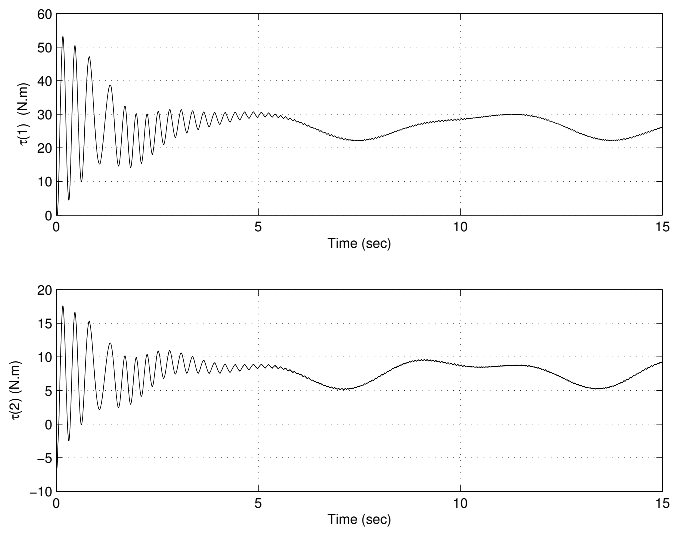

Fig. 2. Control torque inputs $\tau(t)$