

# An Optimal Adaptive Control Approach for Power Systems<sup>1</sup>

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**Abstract:** An optimal adaptive control method for power systems is concerned in this paper. The control approach adopts a weighted controller and a type of extended recursive identification algorithm, of which the unbiased estimation deduction is also presented. Furthermore, the proposed control approach is applied to power system stabilizer (PSS) and some simulation experiments are carried out to test its control effect. The results prove conclusively that the optimal adaptive control approach can effectively impact the stability of the power system operation under disturbance.

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## 1. INTRODUCTION

Adaptive control methods based on system identification have many outstanding qualities, such as real-time operation and good adaptability. There is no doubt that a good identification method is an important precondition to improve control performance in many cases. In recent years, system identification research has received substantial attention and many methods have been proposed, such as Chen and Guo (1991) and Vander (1994). On the other hand, system modelling techniques including system identification and adaptive control have been playing a more and more important role in industrial processes, such as spacecraft control and power systems. Compared with traditional means, these new methods based on adaptive techniques exhibit more remarkable performance.

The recursive least-squares (RLS) algorithm is a widely used identification method, but it can only provide the optimal estimation of the model parameters on the assumption that the disturbance affecting the system has the normal (or Gaussian) distribution  $N(0, \sigma^2)$ . If the real working conditions do not conform to this assumption, the estimation will be seriously deteriorated. To resolve this problem, many modified RLS algorithms have been proposed (see, for example, Lai and Wei (1986), Lo and Kwon (2003), and Lo and Kimura (2003)). This paper describes a new algorithm that improves the anti-noise performance of the traditional RLS algorithm by extending it to consider two-step estimation errors while adjusting the last estimation of the model parameters. Thus, it is called the extended recursive algorithm hereafter in this paper. As shown in the methods described in Lai and Wei (1986), Lo and Kwon (2003) and

Soos and Malik (2002), most of the system identification algorithms modified from the traditional RLS algorithm only use a one-step estimation error, which makes the on-line parameter estimation relatively sensitive to large momentary disturbances. Lo and Kimura (2003) considers all the estimation errors for the past steps, however, this will lead to the rapidly increasing calculation and very slow convergence. Therefore, in this paper, we propose an extended recursive algorithm that considers the previous two steps' estimation errors. The extended recursive algorithm can be used to obtain an unbiased estimation of the system parameters based on some assumption. After this, a weighted optimal control method is adopted to output the controlling value.

A simulation experiment to test the optimal adaptive control algorithm was carried out on a typical power system, in which the optimal adaptive control method was used as a Power System Stabilizer (PSS). The PSS is an important component of the modern large generator, and can adjust the generator's excitation to effectively restrain oscillation when the plant suffers from disturbance. These traditional standard PSSs (see Huang (2002)) all adopt a PID control structure, and they tend to lack adaptability. In this simulation, an optimal adaptive power system stabilizer (OAPSS) was designed and run against every kind of typical disturbance. Finally, the simulation results were compared with a conventional PSS (CPSS). The results illustrated that the OAPSS can stabilize power systems more effectively than the CPSS under many typical disturbances, and also proved that the optimal adaptive control approach based on the extended recursive identification method has important theoretical and practical value.

## 2. DESCRIPTION OF ALGORITHMS

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<sup>1</sup> This work is supported by the Funds NSFC 60672110, NSPC 60474026, and the JSPS Foundation.

### 2.1 System Parameter Estimation

Linear discrete systems can be described according to the following ARMAX model:

$$A(z^{-1})y_n = B(z^{-1})u_n + d_n, \quad (1)$$

where  $\{u_n\}$  and  $\{y_n\}$  are system input and system output, and  $\{d_n\}$  is the error in the model, the so-called prediction error at sampling n.  $A(z^{-1}) = 1 + a_1z^{-1} + \dots + a_mz^{-m}$ , and  $B(z^{-1}) = b_1z^{-1} + b_2z^{-2} + \dots + b_mz^{-m}$ .

Generally, we can rewrite (1) in the following form:

$$y_n = \varphi_n^T \theta + d_n, \quad (2)$$

Here  $\varphi_n = (y_{n-1}, \dots, y_{n-m}, u_{n-1}, \dots, u_{n-m})^T$ , which is the called regression vector, and  $\theta = (-a_1, -a_2, \dots, -a_m, b_1, b_2, \dots, b_m)^T$ , which is the system parameter vector. The  $\theta$  is considered to be unknown and is determined using a certain identification algorithm such as the traditional recursive least-squares (RLS) algorithm described in Ljung (1999):

$$\theta_n = \theta_{n-1} + L_n (y_n - \varphi_n^T \theta_{n-1}) \quad (3)$$

$$L_n = \frac{P_{n-1} \varphi_n}{1 + \varphi_n^T P_{n-1} \varphi_n} \quad (4)$$

$$P_n = P_{n-1} - L_n \varphi_n^T P_{n-1}, \quad (5)$$

where  $\theta_n$  is the estimation value of  $\theta$  at n-step and (3) suggests that the estimation value of  $\theta$  at n-step is obtained from the (n-1)-step estimation value adjusted by the prediction error at the n sampling time.

The traditional RLS algorithm has been widely applied in the identification of linear systems because of its low computational complexity and clear physical signification. But it is obvious that only the current estimation error is considered (shown in (3)), which results in great sensitivity to sudden, large disturbances or data error. Inspired by Lo and Kimura (2003), we consider not only the current step but also the last step estimation error; thereby we can obtain the following extended recursive identification method:

$$\theta_n = \theta_{n-1} + b_n^{-1} P_{n-1}^{-1} (\alpha_n \varphi_{n-1} + a_n^{-1} \sigma_n \varphi_n) \times (y_n - \varphi_n^T \theta_{n-1}) \quad (6)$$

$$+ \alpha_n b_n^{-1} P_{n-1}^{-1} \times (\varphi_n - \alpha_n a_n^{-1} \varphi_n^T P_{n-1}^{-1} \varphi_n \varphi_{n-1}) \times (y_{n-1} - \varphi_{n-1}^T \theta_{n-1})$$

$$P_n^{-1} = P_{n-1}^{-1} - \alpha_n b_n^{-1} \times P_{n-1}^{-1} (\varphi_n \varphi_{n-1}^T + \varphi_{n-1} \varphi_n^T) P_{n-1}^{-1} \quad (7)$$

$$+ (\alpha_n b_n)^{-1} P_{n-1}^{-1} (\alpha_n^2 \varphi_n^T P_{n-1}^{-1} \varphi_n \times \varphi_{n-1} \varphi_{n-1}^T - \sigma_n \varphi_n \varphi_n^T) P_{n-1}^{-1}$$

$$a_n = 1 + \alpha_n \varphi_n^T P_{n-1}^{-1} \varphi_{n-1} \quad (8)$$

$$\sigma_n = q_n - \alpha_n^2 \varphi_{n-1}^T P_{n-1}^{-1} \varphi_{n-1} \quad (9)$$

$$b_n = a_n + \alpha_n^{-1} \sigma_n \varphi_n^T P_{n-1}^{-1} \varphi_n, \quad (10)$$

where  $\{q_n\}$  is a weight sequence;  $\alpha_n$ , whose value is  $0 \leq \alpha_n < 1$ , denotes the impact factor of the last step prediction error, which can be determined according to the actual situation. This algorithm is degraded to the normal RLS when  $\alpha_n = 0$ .

Because of the diversionary effect of the last step prediction error on the adjustment of parameter estimation, it is predictable that the parameter estimation of the extended recursive identification approach would be affected less when the system is disturbed badly. This could be a rather good attenuation effect for big disturbances such as random load changes and unpredictable noises in actual industrial environments.

### 2.2 Unbiased Estimation Property

In this section, we consider the unbiased estimation property of this extended recursive identification method. The discussion reveals the precondition that can assure its unbiased estimation property.

Lemma 1 (Matrix Inversion Lemma) (see Ljung (1999)): Suppose that matrices  $A \in R^{n \times n}$  and  $C, D \in R^{n \times 1}$ ; then the inverse matrix  $B = A + CD^T$  is:

$$(A + CD^T)^{-1} = A^{-1} - a^{-1} A^{-1} CD^T A^{-1}, \quad (11)$$

where  $a = 1 + D^T A^{-1} C$ .

Then we have the following result:

Theorem 1: If matrix  $\alpha_n \varphi_n \varphi_{n-1}^T + \alpha_n \varphi_{n-1} \varphi_n^T + q_n \varphi_n \varphi_n^T$  is positive definite and  $P_0$  is symmetrical and positive definite, the extended recursive identification method shown in (6)-(10) can achieve an unbiased estimation under a white noise sequence  $\{w_n\}$  whose expectation is zero.

Proof: From Lo *et al.* (2007) it can be obtained that:

$$P_n \tilde{\theta}_n = P_{n-1} \tilde{\theta}_{n-1} - (\alpha_n \varphi_n w_{n-1} + (q_n \varphi_n + \alpha_n \varphi_{n-1}) w_n), \quad (12)$$

where  $\tilde{\theta}_n$  is the estimation error of  $\theta$  at n-step.

Since  $\{w_n\}$  is white-noise and its expectation is zero,

$$\begin{aligned} E\{P_n \tilde{\theta}_n\} &= E\{P_{n-1} \tilde{\theta}_{n-1} - (\alpha_n \varphi_n w_{n-1} + (q_n \varphi_n + \alpha_n \varphi_{n-1}) w_n)\} \\ &= E\{P_{n-1} \tilde{\theta}_{n-1}\} - E\{\alpha_n \varphi_n w_{n-1}\} - E\{(q_n \varphi_n + \alpha_n \varphi_{n-1}) w_n\} \\ &= E\{P_{n-1} \tilde{\theta}_{n-1}\}. \end{aligned}$$

It can be rewritten as:

$$E\{\tilde{\theta}_n\} = P_n^{-1} P_{n-1} E\{\tilde{\theta}_{n-1}\} \quad (13)$$

To achieve  $\lim_{n \rightarrow \infty} E(\tilde{\theta}_n) = 0$ , we can make:

$$\|P_n^{-1} P_{n-1}\| < 1 - \varepsilon \quad (14)$$

where  $\varepsilon$  is an arbitrary small positive number.

From Lo *et al.* (2007) it can be obtained that:

$$P_n = P_{n-1} + \alpha_n \varphi_n \varphi_n^T + \alpha_n \varphi_{n-1} \varphi_n^T + q_n \varphi_n \varphi_n^T, \quad (15)$$

It's obvious that  $\alpha_n \varphi_n \varphi_n^T + \alpha_n \varphi_{n-1} \varphi_n^T + q_n \varphi_n \varphi_n^T$  is a symmetrical matrix, so it can be denoted as  $S$ , and  $S = DD^T$ , where  $D \in R^{n \times 1}$ . Because  $P_0$  is also symmetrical,  $P_{n-1}$  is a symmetrical matrix and can be denoted as  $P_{n-1}^{-1} = XX^T$ , and  $X \in R^{n \times 1}$ . Then formula (11) can be used to obtain:

$$P_n^{-1} = P_{n-1}^{-1} - a^{-1} P_{n-1}^{-1} DD^T P_{n-1}^{-1}, \quad (16)$$

where  $a = 1 + D^T P_{n-1}^{-1} D$ .

So the Eigen function  $f(\lambda)$  of matrix  $P_n^{-1} P_{n-1}$  is:

$$\begin{aligned} f(\lambda) &= \det(\lambda I - P_n^{-1} P_{n-1}) = \det(\lambda I - (I - a^{-1} P_{n-1}^{-1} S)) \\ &= \det[(\lambda - 1)I + a^{-1} XX^T S] \\ &= (\lambda - 1)^{n-2} \det[(\lambda - 1)I + a^{-1} (X^T S)X]. \end{aligned}$$

It follows that:

$$\lambda_1 = 1 - a^{-1} (X^T S X)$$

$$\lambda_2 = \lambda_3 = \dots = 1.$$

Let  $\lambda_1 = 1 - a^{-1} (X^T S X) < 1$ ; then  $a^{-1} X^T S X > 0$ , and we can see that the precondition of an unbiased estimation is:  $S$  is positive definite, which means that the  $\{\alpha_n\}$  and  $\{q_n\}$  sequences can cause  $\alpha_n \varphi_n \varphi_n^T + \alpha_n \varphi_{n-1} \varphi_n^T + q_n \varphi_n \varphi_n^T$  to be positive definite.

### 2.3 Optimal Controls

Here we introduce an optimal adaptive control design as in Jiang and Lo (2006). Consider:

$$J(u_n) = E\{(y_{n+1} - y_{n+1}^*)^2 | F_n\} + \lambda u_n^2 \quad (17)$$

as the performance index, where  $\{y_n^*\}$  is a given reference signal sequence and  $F_{n-1}$ -measurable is a  $\sigma$ -algebra generated by the information of  $y_k$  and  $u_k$  ( $k < n$ ).  $\lambda$  is a weight factor related to the control energy, and  $\lambda \geq 0$ .

By minimizing  $J(u_n)$  in (17), we can get the controller:

$$u_n = b_{1n} (b_{1n}^2 + \lambda)^{-1} (y_{n+1}^* - \varphi_n^T \theta_n), \quad (18)$$

where  $b_{1n}$  is the n-step estimated value  $b_1$  in  $\theta$ .

Thus, the optimal adaptive control approach based on extended recursive identification is obtained as (6)-(10) and (18). The structure of the optimal adaptive control approach is shown in Fig. 1.

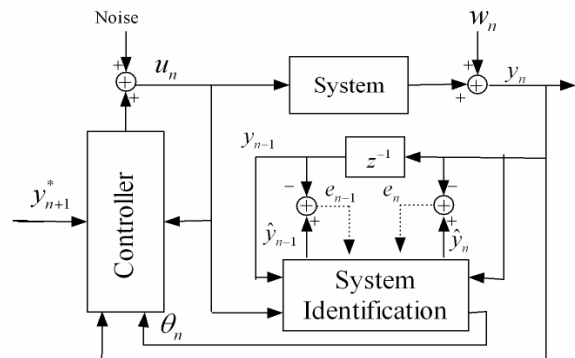


Fig.1. Structure of Optimal Adaptive Control

### 3. APPLICATION IN POWER SYSTEM

#### 3.1 Experiment System Configuration

As previously stated, the optimal adaptive control method should have a prominent anti-disturbance performance in theory. To verify this point, the control algorithm was applied in a single machine infinite-bus power system as stabilizer, and a simulation experiment was carried out based on that. The single machine infinite-bus power system is a simple but effective model widely used to test anti-disturbance performance. It is made up of a generator connected to an infinite bus, and the generator is often configured with an automatic voltage regulator (AVR) and exciter as shown in Fig. 2.

Fig. 2 shows the experimental system in which the optimal adaptive control approach was used as a power system stabilizer, named OAPSS for short. Another experiment in which the OAPSS was replaced with a kind of CPSS (conventional power system stabilizer) was carried out under identical conditions. In Fig. 2,  $y_n$  is the input of this stabilizer, which can be measured from the generator's power or speed deviation. The output control signal  $u_n$  is used to feed the exciter and adjust the generator's excitation to stabilize its power or speed.

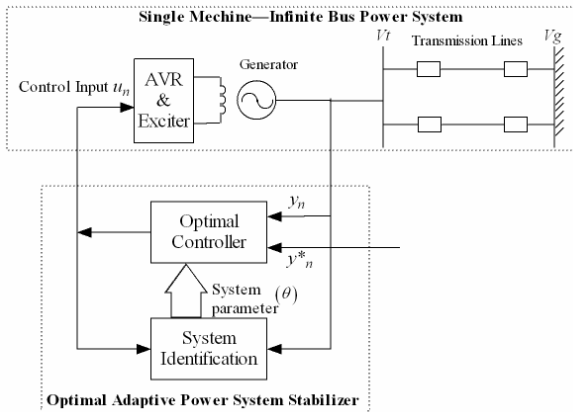


Fig. 2. System configuration

In the design of the optimal adaptive power system stabilizer, the generator and exciter are both treated as black-box models. Once the system model has been established, the identification algorithm of OAPSS can recursively estimate the system parameter  $\theta$  using the measured data  $u_n$  and  $y_n$ , and then the optimal controller can determine the next control signal  $u_{n+1}$  with  $\theta_n$ ,  $y_n$  and the reference output  $y_n^*$  of the plant.

#### 3.2 Simulation

Normally, the single machine infinite-bus power system can be treated as a single input and single output system, of which the operating characteristics are very near to the linear

discrete model displayed with (2). Applying the ability to linearize a continuous system operating in a given small range state, the studied power system in this paper can be approximately expressed according to (2), and then the effect of the optimal adaptive control approach can be applied as a power system stabilizer as shown in Fig. 2. Here is the configuration of the simulation experiment:

Synchronous generator: Nominal Power  $P_n = 2.5$  MVA , frequency  $f_n = 50$  Hz , and terminal voltage  $V_n = 6.3$  KV .  
 Stator:  $R_s = 0.00285$  ,  $L_l = 0.114$  ,  $L_{md} = 1.19$  , and  $L_{mq} = 0.36$  . Field:  $R_f = 0.000579$  and,  $L_{fd} = 0.114$  .  
 Dampers:  $R_{kd} = 0.0117$  ,  $L_{ikd} = 0.182$  ,  $R_{kq1} = 0.0197$  ,  $L_{ikq1} = 0.384$  , and  $H = 4$  .

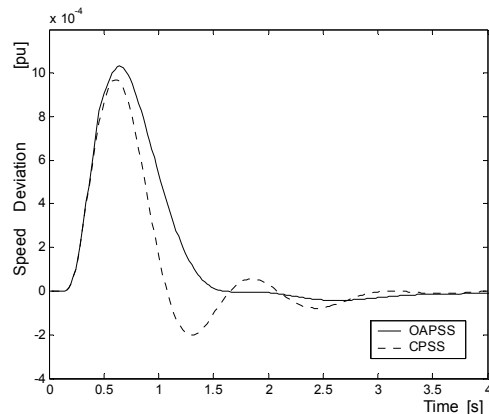
Excitation system:  $K_a = 50$  ,  $T_a = 0.1$  s ,  $K_c = 1$  ,  $T_c = 0$  s ,  $T_r = 0.02$  s ,  $K_f = 0.001$  , and  $T_f = 0.1$  s .

The generator speed deviation was selected as the input  $y_n$  of the PSS, and obviously the reference signal  $y_n^*$  of  $y_n$  should be a zero signal. According to the operating conditions of real power systems, several simulation experiments were carried out in different disturbances and states. Furthermore, the simulation results for the OAPSS were compared with the IEEE standard PSS1A type conventional PSS (denoted as OAPSS and CPSS respectively in the simulation figures).

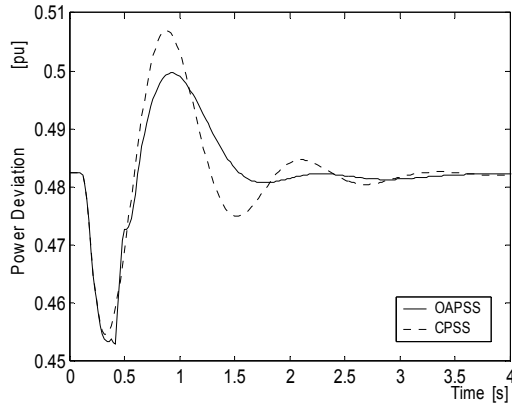
First, these parameters in the two algorithms were tuned to minimize the time of system recovery from disorder in the Experiment No.1. In the CPSS: the gain=40 and the wash-out time constant=10; in the OAPSS:  $q_n = 1.5$  ,  $\alpha_n = 0.6$  , and  $\lambda = 3$  .

#### A. Voltage Reference Step Decrease

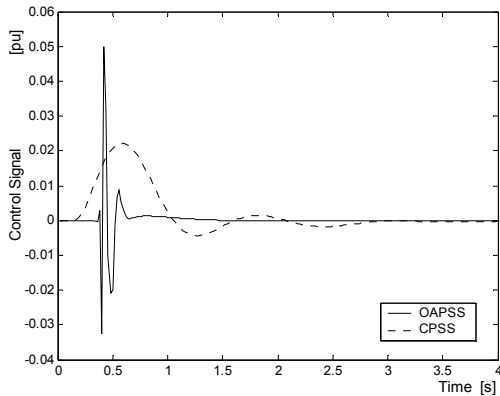
With the generator operating at active power  $P=0.48$  pu and reactive power  $Q=0.8$  pu, a very large (10%) step decrease in the reference voltage of the generator was applied at 0.1 s. The speed deviation, power deviation and the control actions with the OAPSS and CPSS under this condition are shown respectively in Fig. 3(a) to Fig. 3(c).



(a)



(b)



(c)

Fig. 3. (a) Speed deviation, (b) power deviation and (c) control for reference voltage 10% decrease,  $P=0.48$  pu,  $Q=0.8$  pu

As illustrated in Fig. 3(a), both the speed and power deviations under the control of OAPSS can achieve the comeback more quickly than those of CPSS. It's due to the strong control action of the OAPSS after disturbance occurs, which is shown in Fig. 3(c). It should be pointed out that the control signal of OAPSS in Fig. 3(c) doesn't come back to zero immediately at about 0.7 s, but keeps fine-tuning the system output until it restores stability completely.

### B. Voltage Reference Step Increase

With the generator operating at active power  $P=0.64$  pu and reactive power  $Q=0.4$  pu, a very large (10%) step increase in the reference voltage of the generator was applied at 0.1 s. The system response with these PSSs under this condition is shown in Fig. 4.

### C. Ground Fault Test

To verify the behavior of the proposed adaptive algorithm under extreme conditions, an A-phase to ground short was applied at 0.1 s and cleared 100 ms later by the disconnection of the faulted line. The response is shown in Fig. 5. For this test, the operating conditions of the generator were set to active power  $P=0.8$  pu and reactive power  $Q=0.1$  pu.

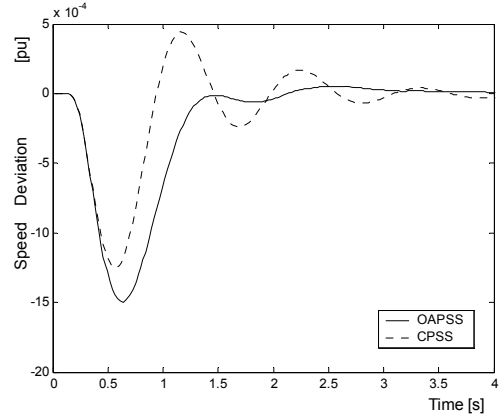


Fig. 4. Reference voltage 10% increase,  $P=0.64$  pu,  $Q=0.4$  pu

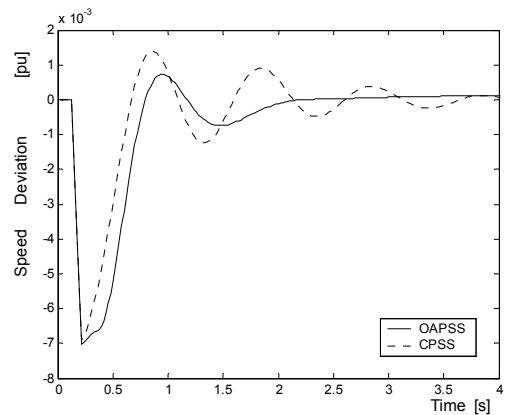


Fig. 5. A-phase to ground fault between 0.1 s-0.2 s,  $P=0.8$  pu,  $Q=0.1$  pu

The three simulation experiments were carried out in different states, with different kinds of disturbances; however, the power system with the OAPSS was always able to damp the concussion effectively and recover to a secure operating state more quickly than the CPSS after disordering. This means the OAPSS not only can improve the dynamic performance of the power system but also can enhance its transient performance.

### 3.3 Simulation Experiment Review

The control algorithm is usually designed in advance aiming at the system disorder in a certain operation state range. The anti-jamming performance under these working states beyond the range design will be deteriorated. Because of the OAPSS using an adaptive technique in which the optimal control effect is based on the former recursive identification, it has better adjustability and good robust for identification of a disturbed system.

Moreover, unlike other adaptive PSS algorithms (Soos and Malik 2002), the controller of OAPSS does not require the *Riccati Equation*. It results in less computation and reduced computer performance requirements. On the other hand, the simplified computation accelerates the operation, the

sampling frequency can be set to a higher value, which can increase the precision.

Compared with the PID control algorithm of CPSS and other adaptive PSS algorithms, OAPSS method can be displayed some advantages. It's less sensitivity to large momentary disturbances and simplified computation. From part 2.1, in fact, the diversionary effect of the last step prediction error makes the on-line parameter estimation process impacted less when the plant suffers transient disturbances. This ensures the OAPSS less sensitive to the large momentary disturbances compared with those adaptive power system stabilizer based on the traditional RLS.

#### 4. CONCLUSION

In this paper, the recursive revision of the tradition RLS algorithm is analyzed. To improve its robustness and stability, an extended recursive estimation algorithm is proposed. It was proven that the new identification method can obtain an unbiased estimation under some premises. As a result, an optimal adaptive control approach was obtained combined the extended recursive estimation method and a weighted controller. To test its stability and robustness, we applied the optimal adaptive control method as a power system stabilizer, and carried out a simulation experiment. Compared with a conventional power system stabilizer, the application result also shows that the optimal adaptive control method has outstanding stability and robustness. This means the optimal adaptive control method has important theoretical and practical values.

Furthermore, the RLS algorithm used widely in linear systems has been successfully applied in a power system through some modification in this study. This will have some significance to applying the relatively mature linear system theories to power system.

#### REFERENCES

- Chen, H.F. and Guo, L. (1991). The Astrom-Wittenmark self-tuning regulator revisited and ELS-based adaptive trackers. *IEEE Transactions on Automatic Control*, **36**, 802-812.
- Huang, S.L. (2002). Three kinds of power system stabilizers (PSSs) in international standards and foreign standards. *Electric Power Standardization & Measurement*, **40**, 8-11.
- Jiang, R. and Lo, K.M. (2006). Optimal adaptive controller for stochastic systems based on weighted least-squares algorithm. *ACTA Automatica Sinica*, **42**, 140-147.
- Lo, K.M. and Kwon, W.H. (2003). New identification approaches for disturbed models. *Automatica*, **39**, 1627-1634.
- Lo, K.M. and Kimura, H. (2003). Recursive estimation methods for discrete systems. *IEEE Transactions on Automatic Control*, **48**, 2019-2024.
- Lo, K.M., *et al.* (2008). Recursive identification algorithms based on minimizing estimation error, *Proceedings of IFAC'08*, Korea.
- Ljung, L. (1999). *System Identification Theory for the User*. Prentice-Hall, NJ.
- Lai, T.L. and Wei, C.Z. (1986). Extended least squares and their applications to adaptive control and prediction in linear systems. *IEEE Transactions on Automatic Control*, **AC-31**, 898-906.
- Soos, A. and Malik, L.P. (2002). An H2 Optimal Adaptive Power System Stabilizer. *IEEE Transactions on Energy Conversion*, **17**, 147-149.
- Vander, S.P. (1994). Minimization methods for training feed forward neural networks. *Neural Networks*, **7**, 1-11.