Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Identification for a kind of Disturbed multi-dimensional Wiener System ⋆

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Abstract: Multi-Input Single-Output (MISO) Wiener system is comprised of a multi-dimensional linear subsystem and a memoryless nonlinear block. In this paper a disturbed MISO Wiener system is concerned, of which the nonlinearity is discontinuous piece-wise linear characteristic. A recursive algorithm is proposed for identifying all of unknown system parameters. It is shown that the algorithm is convergent. Finally, some simulation results illustrate the identification theoretic results.

Keywords: convergency, interference noise, multi-dimensional, recursive estimation, system identification,Wiener system

1. INTRODUCTION

Hammerstein and Wiener models(Ljung [1999], Hsia [1977]) as kinds of Grey Box models are widely used in engineering practice, such as modeling the pH neutralization process(Kalafatis et al. [2005]), distillation columns(Bloemen et al. [2001]), chromatographic separation process(Visala et al. [2001]), adaptive precompensation of nonlinear distortions(Kang et al. [1998]), and so on. Although it is possible for these models to simulate the structure and behavior of whole systems on the basis of the input-output measurements, the signals between their two subsystems cannot be measured, which makes the identification of such systems difficult. Due to the different connection sequence, identification for Wiener models is more complicated than that of Hammerstein models. As a result, in the past few decades, the research of Hammerstein models is more advanced, whereas the studies of Wiener models should be given more attention. Although there already have many approaches on the identification of Wiener models (neural networks(Visala et al. [2001]), correlation analysis(Billings et al. [1982]), parametric regression (Bai [2003], Gomez et al. [2004]), and nonparametric regression(Greblicki [2001]), very few of them are about multi-dimensional systems.

In the previous studies, it is found that using piece-wise linear functions to characterize the nonlinear subsystem of the Wiener model can be used in some practical applications. To deal with these special models, we can use some particular methods because the identification of the nonlinear block could be reduced to the estimation of the unknown parameters in these piece-wise linear functions (Vörös [2001, 2007], Chen [2006]). A key term separation principle is introduced, which is used with an iterative algorithm for the identification of ARMA Wiener systems (Vörös [2001]). The same method is used in (Vörös [2007]) for the identification of another similar kind of Winer model. For better results in estimating the nonlinearity, a recursive estimation algorithm is proposed in(Chen [2006]), and most importantly, its convergence has been proven without no restrictive conditions except the structural assumptions. However, both of the two methods are not for multi-dimensional systems.

Thus, the model treated in this paper is a multi-dimensional Wiener model with such a nonlinear system. Specified real systems will inevitably be affected by noise. So noise has been taken into consideration during the identification also. The identification problem is explicitly defined and the assumptions for the identification convergency are given in section 2 and 3, respectively. Then the detailed identification algorithms are proposed in section 4. Section 5 illustrates the identification accuracy and the convergence rate by means of simulation studies. At last, a conclusion is given in section 6.

2. PROBLEM DESCRIPTION

Consider system in Fig. 1.

Here $U(k) = [u_1(k), u_2(k), \ldots, u_n(k)]^T$ is the input vector at time k, $v(k)$ is intermediate signal, $e(k)$ is the noise and $y(k)$ is the output of the whole system.

Assume the difference equation of the linear subsystem is:

⋆ This work is supported by the Funds of NSFC60672110, NSFC60474026, and the JSPS Foundation.

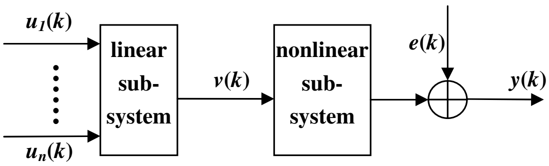

Fig. 1. Multi-Input Single-Output Wiener system

$$v(k) = B(z)U(k) \tag{1}$$

where

$$B(z) = A + B_1 z^{-1} + \ldots + B_q z^{-q},$$

In which, A is an n-dimensional vector of ones, z is the unit delay operator and q is the upper boundary of the real order, and

$$B_j = [b_{j1}, b_{j2}, \ldots, b_{jn}]^T \quad (j = 1, 2, \ldots, q).$$

In fact, (1) can be rewritten as:

$$v(k) = \sum_{i=1}^{n} u_i(k) + \theta^T \phi(k). \tag{2}$$

where $\phi(k) = [U^T(k-1), U^T(k-2), \ldots, U^T(k-q)]^T$ is the regressor vector, and $\theta = [B_1^T, B_2^T, \ldots, B_q^T]^T$ denotes the unknown coefficient matrix of the linear subsystem.

Assume the nonlinear subsystem is characterized by the function $f(v(k))$, then the output of the Wiener model can be written as:

$$y(k) = f(v(k)) + e(k). \tag{3}$$

3. ASSUMPTIONS

Noting that the wiener systems with a memory-less nonlinear block which is made of discontinuous asymmetric piece-wise linear function are often used in practical application, here the nonlinear function is assumed to be

$$f(v(k)) = \begin{cases} c^+(v(k) - d^+) + b^+, v(k) > d^+, c^+ \geq 0 \\ 0, \quad -d^- \leq v(k) \leq d^+ \\ c^-(v(k) + d^-) - b^-, v(k) < -d^-, c^- \geq 0 \end{cases} \tag{4}$$

where b^+, b^-, c^+, c^-, and d^+, d^- are the corresponding preloads, slopes, and dead zones, which need to be estimated. This function is shown in Fig. 2

From fig2, we notice that it is nonlinear in the whole domain but partially asymmetric linear. So we can use traditional estimation method for linear systems on these parts. In addition to the above structure assumptions, we should also assume the following conditions for the proofs of convergence:

(A.1) The input vector $U(k)$ is an n-dimensional iid random vector, and $U(k) \sim N(\mathbf{0}, I)$, which means $U(k)$ obeys a normal distribution;

(A.2) $m_b > 0$ is the lower boundary of all the preloads, that is, $b^+ > m_b$, and $b^- > m_b$;

(A.3) Noise vector $e(k)$ is an n-dimensional iid random vector, and the mathematical expectation and variance of each dimension are 0 and σ_e respectively, and $| e(k) | < m_e$, where m_e is known and $0 < m_e \leq m_b/2$.

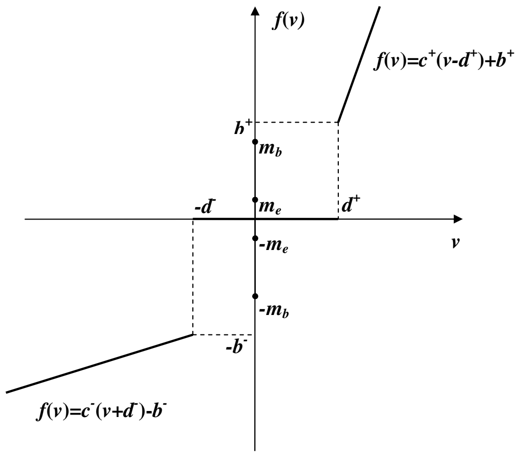

Fig. 2. Nonlinear block

4. ESTIMATION OF THIS WIENER SYSTEM

According to the equations above, in order to identify this kind of Wiener model, the key problem is to choose a suitable input vector and q to estimate the unknown parameters in both the linear and nonlinear subsystems, that is, (θ) and $b^+, b^-, c^+, c^-, d^+, d^-$ on the basis of the measurement of the input-output vectors.

By the characteristics of the normal random vector(Rotar [1997]) we know that, under (A.1),

$$v(k) = u(k) + \theta^T \phi(k)$$

must be Gaussian stationary and ergodic, and its mathematical expectation, variance and marginal density function should be:

$$\mu_v = 0, \ \sigma_v^2 = n + \parallel \theta \parallel^2, \ p(v) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{v^2}{2\sigma_v^2}}. \tag{5}$$

For the sake of convenience, we predefine several interim variables that will be estimated before the unknown parameters of the model:

$$\alpha^+ = \frac{d^+}{\sigma_v}, \ \alpha^- = \frac{d^-}{\sigma_v}, \ \beta^+ = c^+\sigma_v, \ \beta^- = c^-\sigma_v, \tag{6}$$

$$h^+ = c^+d^+ - b^+, \ h^- = c^-d^- - b^-. \tag{7}$$

So in this paper, we use a five-step estimation method, which can be expressed as follows:

Step 1: Estimate the interim variables;

Step 2: Knowing the interim variables, do the estimation of b^+ and b^-;

Step 3: Also use the results of step 1 to estimate σ_{v_i};

Step 4: Calculate to get c^+, c^-, d^+, and d^-;

Step 5: With all parameters got above, estimate θ using the Least Squares Algorithm.

Step 1: Estimating the interim variables.

Lemma 1. For the system described by (1)–(4), $y(k) > m_e$ is equivalent to $v(k) > d^+$, and similarly, $y(k) < -m_e$ is equivalent to $v(k) < -d^-$, where $i = 1, 2, \ldots, n$.

Proof: On the one hand, supposing $v(k) > d^+$, by the definition of $f(\cdot)$ and Fig. 1 together with (A.2) we can see that it must be correct that $f(v(k)) > b^+ > m_b$.

Considering (A.3), we know that$|\ e\ | < m_e$, and then it is clear that

$$y(k) = f(v(k)) + e(k) > m_b + (-m_e) = m_b - m_e > m_e.$$

On the other hand, let us assume that $y(k) > m_e$. Since $c^+ \geq 0$ and $c^- \geq 0$, which is to say, in the intervals $(-\infty, -d^-)$ and (d^+, ∞), respectively, $f(\cdot)$ is an increasing function, so it is inevitably right that $v(k) > d^+$. Because if not, we should have

$$y(k) = f(v(k)) + e(k) < 0 + m_e = m_e,$$

which contradicts the assumption $y(k) > m_e$.

Through these two aspects, we can conclude that $y(k) > m_e$ is equivalent to $v(k) > d^+$. In the same way it can be obtained that $y(k) < -m_e$ is equivalent to $v(k) < -d^-$.

Define

$$g^+(k) = (1-k)g^+(k-1) + \frac{1}{k}I_{[y(k)>m_e]}, \tag{8}$$

$$g^-(k) = (1-k)g^-(k-1) + \frac{1}{k}I_{[y(k)<-m_e]}. \tag{9}$$

These will be used to estimate α^+ and α^-.

It is clear that with the arbitrary initial values $g^+(0)$ and $g^-(0)$, $g^+(k)$ and $g^-(k)$ can be obtained. Then according to the characteristics of the normal random vector, we can figure out $\alpha^+(k)$ and $\alpha^-(k)$ by using the following two equations, and later we will prove that they converge to their true values respectively.

$$g^+(k) = 1 - \Phi(\alpha^+(k)),\ g^-(k) = \Phi(-\alpha^-(k)). \tag{10}$$

Lemma 2. For the system described by (1)–(4), if (A.1)–(A.3) are satisfied, when $k \to \infty$, we have

$$\alpha^+(k) \to \alpha^+, \quad \alpha^-(k) \to \alpha^- \quad a.s. \tag{11}$$

where $\alpha^+(k)$ and $\alpha^-(k)$ are calculated by (10).

Proof: As mentioned earlier, $v(k)$ is a Gaussian stationary and ergodic normal random vector, and the noise vector $e(k)$ is also ergodic. Therefore the sum of the two which is the output of the whole system $y(k)$, is ergodic too. Then we have

$$g^+(k) = (1-k)g^+(k-1) + \frac{1}{k}I_{[y(k)>m_e]} = \frac{1}{k}\sum_{l=1}^{k} I_{[y(l)>m_e]} \stackrel{k\to\infty}{\longrightarrow} EI_{[y(1)>m_e]}, \tag{12}$$

$$g^-(k) = (1-k)g^-(k-1) + \frac{1}{k}I_{[y(k)<-m_e]} = \frac{1}{k}\sum_{l=1}^{k} I_{[y(l)<-m_e]} \stackrel{k\to\infty}{\longrightarrow} EI_{[y(1)<-m_e]}. \tag{13}$$

Furthermore, according to lemma 1 and the characteristics of the normal random vector,we can have:

$$EI_{[y(1)>m_e]} = P(y(1) > m_e) = P(v(1) > d^+) = \int_{d^+}^{+\infty} p(v)dv = 1 - \Phi(\alpha^+), \tag{14}$$

$$EI_{[y(1)<-m_e]} = P(y(1)<-m_e) = P(v(1)<-d^-) = \int_{-\infty}^{d^-} p(v)dv = \Phi(-\alpha^-), \tag{15}$$

where $p(v)$ is shown in (6).

Since $\Phi(x)$ is continuous and increasing, then from (12)–(15) we know that

$$\alpha^+(k) \stackrel{k\to\infty}{\longrightarrow} \alpha^+,\ \alpha^-(k) \stackrel{k\to\infty}{\longrightarrow} \alpha^- \text{ a.s.}$$

where $\alpha^+(k)$ and $\alpha^-(k)$ are calculated by (10).

Let us continue to estimate β^+, β^-, h^+, and h^-. Similarly, we need to define:

$$\bar{y}^+(k) = (1-k)\bar{y}^+(k-1) + \frac{1}{k}y(k)I_{[y(k)>m_e]}, \tag{16}$$

$$\bar{y}^-(k) = (1-k)\bar{y}^-(k-1) + \frac{1}{k}y(k)I_{[y(k)<-m_e]}, \tag{17}$$

$$\underline{y}^+(k) = (1-k)\underline{y}^+(k-1) + \frac{1}{k}y^2(k)I_{[y(k)>m_e]}, \tag{18}$$

$$\underline{y}^-(k) = (1-k)\underline{y}^-(k-1) + \frac{1}{k}y^2(k)I_{[y(k)<-m_e]}. \tag{19}$$

Then $\beta^+(k)$, $h^+(k)$, $\beta^-(k)$, and $h^-(k)$ can be obtained by solving the following (20)–(23).

$$\bar{y}^+(k) = \frac{\beta^+(k)}{\sqrt{2\pi}}e^{-\frac{(\alpha^+(k))^2}{2}} - h^+(k)g^+(k), \tag{20}$$

$$\underline{y}^+(k) = (\beta^+(k))^2(\frac{\alpha^+(k)}{\sqrt{2\pi}}e^{-\frac{(\alpha^+(k))^2}{2}} + g^+(k)) - \frac{2}{\sqrt{2\pi}} \times\beta^+(k)h^+(k)e^{-\frac{(\alpha^+(k))^2}{2}} + ((h^+(k))^2 + \sigma_e^2)g^+(k), \tag{21}$$

$$\bar{y}^-(k) = \frac{\beta^-(k)}{\sqrt{2\pi}}e^{-\frac{(\alpha^-(k))^2}{2}} - h^-(k)g^-(k), \tag{22}$$

$$\underline{y}^-(k) = (\beta^-(k))^2(\frac{\alpha^-(k)}{\sqrt{2\pi}}e^{-\frac{(\alpha^-(k))^2}{2}} + g^-(k)) - \frac{2}{\sqrt{2\pi}} \times\beta^-(k)h^-(k)e^{-\frac{(\alpha^-(k))^2}{2}} + ((h^-(k))^2 + \sigma_e^2)g^-(k). \tag{23}$$

The calculation results are as follows.

$$h^+(k) = \frac{1}{g^+(k)}(\beta^+(k)\gamma^+(k) - \bar{y}^+(k)), \tag{24}$$

$$\beta^+(k) = \sqrt{\frac{\underline{y}^+(k) - (\bar{y}^+(k))^2/g^+(k) - \sigma_e^2 g^+(k)}{\alpha^+(k)\gamma^+(k) + g^+(k) - (\gamma^+(k))^2/g^+(k)}}, \tag{25}$$

$$h^-(k) = \frac{1}{g^-(k)}(\beta^-(k)\gamma^-(k) - \bar{y}^-(k)), \tag{26}$$

$$\beta^-(k) = \sqrt{\frac{\underline{y}^-(k) - (\bar{y}^-(k))^2/g^-(k) - \sigma_e^2 g^-(k)}{\alpha^-(k)\gamma^-(k) + g^-(k) - (\gamma^-(k))^2/g^-(k)}}. \tag{27}$$

where

$$\gamma^+(k) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(\alpha^+(k))^2}{2}},\ \gamma^-(k) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(\alpha^-(k))^2}{2}}.$$

Lemma 3. If the conditions of Lemma 2 holds, then we have the following results about the above $\beta^+(k)$, $h^+(k)$, $\beta^-(k)$, and $h^-(k)$.

$$h^+(k) \stackrel{k\to\infty}{\longrightarrow} h^+,\ \beta^+(k) \stackrel{k\to\infty}{\longrightarrow} \beta^+,$$
$$h^-(k) \stackrel{k\to\infty}{\longrightarrow} h^-,\ \beta^-(k) \stackrel{k\to\infty}{\longrightarrow} \beta^- \text{ a.s.}$$

Proof: Since we have determined that $y(k)$ is ergodic, from the definition in (16)–(19) we have:

$$\bar{y}^+(k) = \frac{1}{k}\sum_{l=1}^{k} y(l)I_{[y(l)>m_e]} \stackrel{k\to\infty}{\longrightarrow} Ey(1)I_{[y(1)>m_e]} \text{ a.s.,} \quad (28)$$

$$\underline{y}^+(k) = \frac{1}{k}\sum_{l=1}^{k} y^2(l)I_{[y(l)>m_e]} \stackrel{k\to\infty}{\longrightarrow} Ey^2(1)I_{[y(1)>m_e]} \text{ a.s.} \quad (29)$$

Note that $e(k)$ and $v(k)$ are independent of each other, so we have:

$$\begin{aligned} &Ey(1)I_{[y(1)>m_e]} \\ &= E(c^+v(1) - h^+ + e(1))I_{[v(1)>d^+]} \\ &= \frac{\beta^+}{\sqrt{2\pi}}e^{-\frac{(\alpha^+)^2}{2}} - h^+(1-\Phi(\alpha^+)), \end{aligned} \quad (30)$$

and

$$\begin{aligned} &Ey^2(1)I_{[y(1)>m_e]} \\ &= E(c^+v(1) - h^+ + e(1))^2 I_{[v(1)>d^+]} \\ &= (\beta^+)^2[\frac{\alpha^+ e^{-\frac{(\alpha^+)^2}{2}}}{\sqrt{2\pi}} + (1-\Phi(\alpha^+))] - \frac{2}{\sqrt{2\pi}}\beta^+ \\ &\times h^+ e^{-\frac{(\alpha^+)^2}{2}} + ((h^+)^2 + \sigma_e^2)(1-\Phi(\alpha^+)). \end{aligned} \quad (31)$$

Then from the above four equations and Lemma 2 it is clear that

$$\beta^+(k) \stackrel{k\to\infty}{\longrightarrow} \beta^+,\ h^+(k) \stackrel{k\to\infty}{\longrightarrow} h^+ \text{ a.s.}$$

In the same way we can obtain

$$\beta^-(k) \stackrel{k\to\infty}{\longrightarrow} \beta^-,\ h^-(k) \stackrel{k\to\infty}{\longrightarrow} h^- \text{ a.s.}$$

Step 2: Estimating b^+ and b^-.

Now we have completed the estimation for all the interim variables. Then from (6) and (7) it is clear that

$$b^+(k) = \alpha^+(k)\beta^+(k) - h^+(k), \quad (32)$$
$$b^-(k) = \alpha^-(k)\beta^-(k) - h^-(k). \quad (33)$$

Theorem 4. According to Lemmas 2 and 3, it is clear that $b^+(k)$ and $b^-(k)$ calculated by (32) and (33) are the consistent estimates of b^+ and b^-, respectively, that is

$$b^+(k) \stackrel{k\to\infty}{\longrightarrow} b^+,\ b^-(k) \stackrel{k\to\infty}{\longrightarrow} b^- \text{ a.s.}$$

(The proof is obvious and omitted.)

Step 3: Estimating σ_v.

Now that we have obtained β^+, β^-, α^+, and α^-, according to (6), for estimating the rest unknown parameters of the nonlinearity(c^+, c^-, d^+, and d^-), we should calculate σ_v next. For this we need the help of the kernel function described in(Chen [2006]).

First define the kernel function as follows.

$$\omega(k) = k^{2\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2},\ \varepsilon \in (0, \frac{1}{4}) \quad (34)$$

And define

$$\begin{aligned} G(k) &= \frac{1}{k}\sum_{l=1}^{k}\omega(l)y(l)I_{[y(l)>m_e]} \\ &= (1-k)G(k-1) + \frac{1}{k}\omega(k)y(k)I_{[y(k)>m_e]}. \end{aligned} \quad (35)$$

Lemma 5. Under the conditions of Lemma 2, we have

$$\begin{aligned} \lim_{k\to\infty} G(k) &= \frac{-h^+}{\sqrt{2}}(1-\Phi(\frac{d^+}{\|\theta\|})) \\ &+\frac{c^+\|\theta\|}{2\sqrt{\pi}}e^{-\frac{1}{2}(\frac{d^+}{\|\theta\|})^2} \text{ a.s.} \end{aligned} \quad (36)$$

Proof: By Lemma 1, (2)– (4) and (34) we have

$$\begin{aligned} \omega(k)y(k)I_{[y(k)>m_e]} &= k^{2\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}[c^+\sum_{i=1}^{n}u_i(k) \\ &+c^+\theta^T\phi(k) - h^+ + e(k)]I_{[v(k)>d^+]}. \end{aligned} \quad (37)$$

Introduce G to denote the right side of (36). Since

$$\begin{aligned} &\frac{1}{k}\sum_{l=1}^{k} l^{2\varepsilon}e^{-l^{4\varepsilon}(\sum_{i=1}^{n}u_i(l))^2}[c^+\sum_{i=1}^{n}u_i(l) + c^+\theta^T\phi(k) \\ &-h^+]I_{[v(l)>d^+]} \stackrel{k\to\infty}{\longrightarrow} G \quad \text{a.s.} \end{aligned} \quad (38)$$

which has been proven in the Appendix of (Chen [2006]), then we only need to prove that

$$\frac{1}{k}\sum_{l=1}^{k} l^{2\varepsilon}e^{-l^{4\varepsilon}(\sum_{i=1}^{n}u_i(l))^2}e(l)I_{[v(l)>d^+]} \stackrel{k\to\infty}{\longrightarrow} 0 \quad \text{a.s.} \quad (39)$$

Suppose $\phi(k)$ is generated by the σ-algebra $\{U(1), U(2), \ldots, U(k), e(1), e(2), \ldots, e(k)\}$. Since

$$\begin{aligned} &\sup_k E[(k^{\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k))^2 \\ &\quad \cdot I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]} \mid F_{k-1}] \\ &= \frac{k^{2\varepsilon}}{2\pi\sigma_e\sqrt{n}}\int_{d^+-\theta^T\phi(k)}^{+\infty}\int_{-\infty}^{+\infty} e^{-k^{4\varepsilon}x^2}e^{-\frac{x^2}{2n}}e^{-\frac{y^2}{2\sigma_e^2}}ydxdy \\ &\le \frac{1}{2\pi\sigma_e\sqrt{2n}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{s^2}{2}}e^{-\frac{k^{-4\varepsilon}s^2}{4n}}e^{-\frac{y^2}{2\sigma_e^2}}ydsdy < \infty \end{aligned} \quad (40)$$

and $\sum_{k=1}^{\infty}(1/k^{1-\varepsilon})^2 < \infty$, then by the Convergence Theorem for Martingale Difference Sequences(Chen et al. [1991]), we have

$$\begin{aligned} &\sum_{k=1}^{\infty}\frac{1}{k^{1-\varepsilon}}\{k^{\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k)I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]} \\ &-E[k^{\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k)I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]} \end{aligned}$$

$$| F_{k-1}]\} < \infty. \quad (41)$$

By the Kronecker Lemma(Chen et al. [1991]) we know

$$\frac{1}{n}\sum_{k=1}^{n} k^{2\varepsilon}\{e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k) \cdot I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]} - E[e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k) \cdot I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]} \mid F_{k-1}]\} \xrightarrow{k\to\infty} 0 \quad \text{a.s.} \quad (42)$$

Noting that $e(k)$ is independent of $\theta^T\phi(k)$ and $u_i(k)$, then

$$E[k^{2\varepsilon}e^{-k^{4\varepsilon}(\sum_{i=1}^{n}u_i(k))^2}e(k)I_{[\sum_{i=1}^{n}u_i(k)>d^+-\theta^T\phi(k)]}|F_{k-1}] \xrightarrow{k\to\infty} 0. \quad (43)$$

Thus, by (42) and (43), (39) can be obtained, then with (38), (36) follows.

Let us consider the following equation with respect to x.

$$G(k) + \frac{h^+}{\sqrt{2}}(1-\Phi(\alpha^+(k))x) - \frac{\beta^+(k)}{2\sqrt{\pi}x}e^{-\frac{(\alpha^+(k))^2x^2}{2}} = 0. \quad (44)$$

According to Lemma 5, we know that $G(k)$ converges to the G. Then if we can find the root $x(k)$ of (44), it surely converges to $\sqrt{n/\parallel\theta\parallel^2+1}$, so the consistent estimates of $\parallel\theta\parallel$ and $\sigma_v(k) = \sqrt{n+\parallel\theta\parallel^2}$ can be obtained in succession.

The existence and uniqueness of the solution $x(k)$ has been given in (Chen [2006]). Thus, we can use a numerical method to find the solution $x(k)$.

Step 4: Estimating c^+, c^-, d^+, and d^-.

Now we have consistently estimated σ_v. With the help of the results in step 1, it is easy to calculate the values of $c^+(k)$, $c^-(k)$, and $d^+(k)$, $d^-(k)$ as follows:

$$c^+(k) = \frac{\beta^+(k)}{\sigma_v}, \quad d^+(k) = \alpha^+(k)\sigma_v, \quad (45)$$

$$c^-(k) = \frac{\beta^-(k)}{\sigma_v}, \quad d^-(k) = \alpha^-(k)\sigma_v. \quad (46)$$

Theorem 6. By Lemmas 2, 3, and 5, we can easily conclude that if the conditions of Lemma 2 holds, $c^+(k)$, $c^-(k)$, $d^+(k)$, and $d^-(k)$ calculated by (45) and (46) are the consistent estimates of c^+, c^-, d^+, and d^-, respectively. (The proof is obvious and omitted also.)

Step 5: Estimating θ.

At this point, we have obtained all the unknown parameters in the nonlinearity. As in (Chen [2006]), with these results and the output $y(k)$, we can calculate the estimate of the unmeasurable signal $v(k)$ as follows.

$$\hat{v}(k) = \begin{cases} \frac{1}{\bar{c}^+(k)}(h^+(k)+y(k)), & y(k) > m_e \\ 0, & -m_e \le y(k) \le m_e \\ \frac{1}{\bar{c}^-(k)}(y(k)-h^-(k)), & y(k) < -m_e \end{cases} \quad (47)$$

where

$$\bar{c}^+(k) = c^+(k) \vee (1/k), \quad \bar{c}^-(k) = c^-(k) \vee (1/k)$$

are the modifications of $c^+(k)$ and $c^-(k)$ and have the same limits as them respectively.

Define

$$\begin{aligned} z(k) &= (\hat{v}(k) - \sum_{i=1}^{n}u_i(k))I_{[y(k)>m_e \cup y(k)<-m_e]} \\ &= [\theta^T\phi(k)+\varepsilon]I_{[y(k)>m_e \cup y(k)<-m_e]} \\ &= \theta^T\hat{\phi}(k) + \varepsilon(k)I_{[y(k)>m_e \cup y(k)<-m_e]} \end{aligned} \quad (48)$$

where

$$\hat{\phi}(k) = \phi(k)I_{[y(k)>m_e \cup y(k)<-m_e]},$$
$$\varepsilon(k) = \hat{v}(k) - v(k).$$

Then we can estimate the linear subsystem using the Least Squares Algorithm. With arbitrary initial $\theta(0)$ and $P(0) > 0$, the coefficient vector θ can be estimated as:

$$\theta(k) = \theta(k-1) + a(k)P(k)\hat{\phi}(k)(z(k) - \theta^T(k-1)\hat{\phi}(k)) \quad (49)$$

$$P(k+1) = P(k) - a(k)P(k)\hat{\phi}(k)\hat{\phi}^T(k)P(k) \quad (50)$$

$$a(k) = (1-\hat{\phi}^T(k)P(k)\hat{\phi}(k))^{-1} \quad (51)$$

5. SIMULATION

In this section, we apply the above identification algorithm to a numerical simulation with the help of Matlab. Considering the following Wiener system with $n = 2, q = 4$:

$$y(k) = f(v(k)) + e(k),$$

where

$$\begin{aligned} f(v(k)) &= \begin{cases} c^+(v(k)-d^+)+b^+, & v(k) > d^+, c^+ \ge 0 \\ 0, & -d^- \le v(k) \le d^+ \\ c^-(v(k)+d^-)-b^-, & v(k) < -d^-, c^- \ge 0 \end{cases} \\ &= \begin{cases} 1\times(v(k)-1.8)+5, & v(k) > 1.8 \\ 0, & -(-1.75) \le v(k) \le 1.8 \\ 3.6(v(k)+(-1.75))-2.36, & v(k) < -(-1.75) \end{cases} \end{aligned}$$

and

$$v(k) = \sum_{i=1}^{n}u_i(k) + \theta^T\phi(k),$$

$$\begin{aligned} \theta &= [B_1^T, B_2^T, \ldots, B_q^T]^T = [b_{11}, b_{12}, \ldots, b_{qn}]^T \\ &= [0.95, 0.8, 0.5, 0.5, 0, -0.1, -0.45, -0.6]^T \end{aligned}$$

Here we let the parameters $\sigma_e = 1$, and $m_e = 0.5$, which are mentioned in (A.3). With regard to the ε used in (34), we choose the same value (1/13500) used in(Chen [2006]).

On account of the close parameters in θ, in order to make the results appear more clearly, we use two figures to show the estimates of the linear subsystem, as we can see in Fig. 3 and Fig. 4. Fig. 5 shows the estimation results of the nonlinear subsystem. In these pictures the solid lines are the real values as indicated, and the dotted lines are the estimated ones. From these figures we can see clearly that the estimated values in both the nonlinear and linear subsystems all converge to their corresponding real values fast, which confirms the validity of this recursive algorithm for the estimation of Multi-Input Wiener systems in noisy environments.

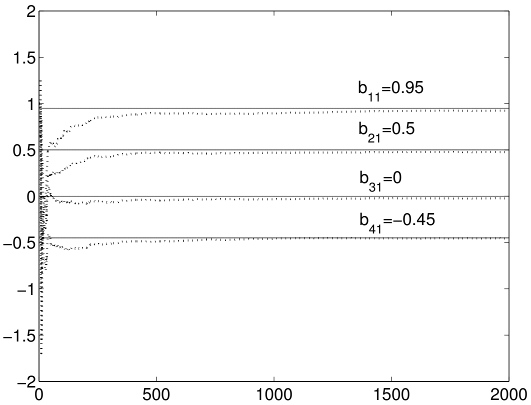

Fig. 3. True values and estimates of $b_{11} \sim b_{41}$

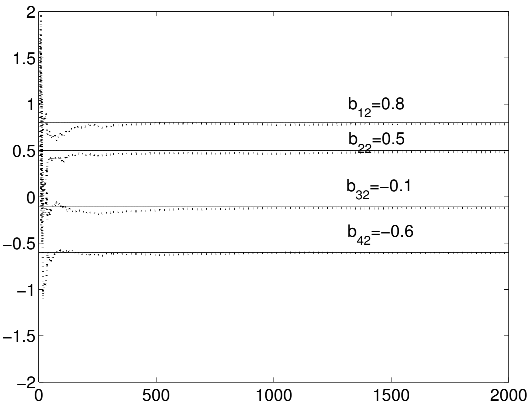

Fig. 4. True values and estimates of $b_{12} \sim b_{42}$

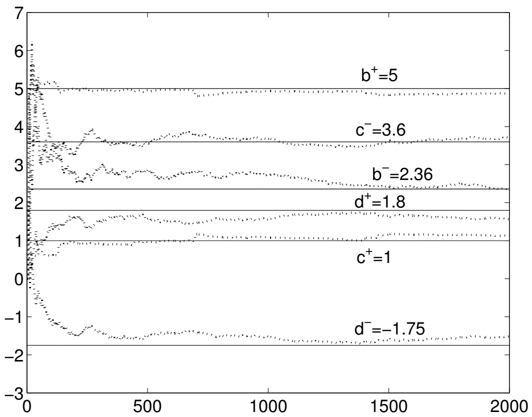

Fig. 5. True values and estimates of the nonlinearity

6. CONCLUSION

Inspired by (Chen [2006]), we have proposed a recursive estimation algorithm in this paper, which proved to have good results for the identification of disturbed Multi-Input Wiener systems with the nonlinearity being a discontinuous asymmetric piece-wise linear function. Although the system we studied is single output, but this method can also be applied to system with multi-dimensional outputs which are independent with each other. However, this method has strong restrictions on the noise signals, so further study is needed to find a better solution.

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