

## Pantograph Dynamics and Control of Tilting Train

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**Abstract:** The modeling methodology of tilting train is studied. The vehicle and pantograph are considered as nonlinear multiple rigid body systems, and the catenary is modeled by FEM method. The electrical mechanical actuators for carbody and pantograph tilting control are also considered in the tilting train model. The method of on-line curving detection, filtering and control is adopted for the tilting train. The passive and active controls for the pantograph lateral motions during tilting train curve negotiation are analyzed. The pantograph guideway on top of the carbody is designed, and the control strategy for the pantograph active control is investigated. It is known that the pantograph lateral and vertical displacement and the fluctuation of pantograph-catenary contact force can be minimized by the use of active control.

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### 1. INTRODUCTION

Together with the rapid development of modern control and computer technologies, the active suspension control technologies have been increasingly applied in railway vehicle systems in order to improve the dynamic performance [Goodall,1997; Bruni, 2007]. Tilting train is one of the examples of active secondary suspensions with carbody tilting control during curve negotiation. Tilting the carbody inwards on curve can reduce the unbalanced lateral acceleration experienced by passengers, which can permits higher speeds and provides a variety of operational benefits [Harris, 1998; Cheli, 2001; Zolotas, 2002]. Therefore, the application of carbody tilting technology is an efficient way to increase train speed and to shorten travelling time for existing railways.

The study on pantograph-catenary interaction is an important issue for efficient current collection of pantograph-catenary system [Zhai, 1998; Zhang, 2002]. Active pantograph vibration controls were also investigated in the past years in order to improve the vibration behaviour [Wu, 1997; Diana, 1998]. Because the tilting train and conventional train run in the same railway lines with the same catenary systems, the pantograph of the tilting train faces more serious dynamic problems compared with the one of conventional train. When the carbody tilts, the pantograph head will result in large lateral displacement, which may exceed the pantograph normal working range, for example,  $\pm 500\text{mm}$ . Thus the pantograph can not satisfy the carbody tilting condition if without lateral displacement control [Zhang, 2004; Luo, 2007].

In this paper, the electrical mechanical actuators are adopted for carbody tilting and pantograph moving control. The lateral acceleration and roll velocity of the leading bogie of the first car in the train set are measured by using acceleration sensor and gyroscope. Then the measured signals are filtered and modified to generate the control signals for carbody

tilting control and pantograph control. The time delay of the control signals caused by the measuring and control system can be compensated by using forecast method [Luo, 2007]. The models of the tilting train and the pantograph-catenary system are built in the paper. The passive and active controls for the pantograph lateral motions during curve negotiation and the dynamic performance are investigated.

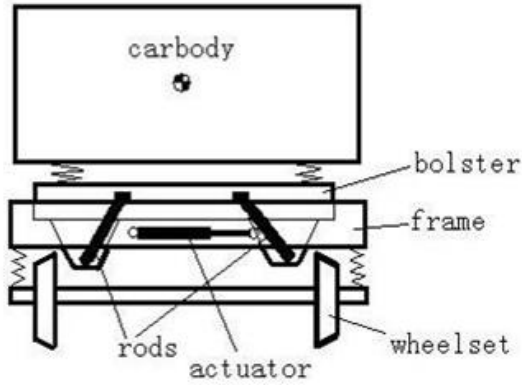
### 2. DYNAMIC MODELS

#### 2.1 Tilting Train Model

The dynamic model of a tilting passenger car with 34 degrees of freedom (DOF) is set up. The degrees of freedom are longitudinal, lateral, vertical, roll, pitch and yaw motions of carbody and each bogie frame, longitudinal, lateral, yaw and spin motions of each wheelset. The motion of the tilting bolster is also considered, which is related to the controlled actuator displacement. Then the model of a train set with three cars is built by considering the coupler forces between cars, and the traction force and resistant force. The schematic diagram of the tilting car is shown as Fig.1. The four swing arm mechanism is located between the bogie frame and the tilting bolster. The electrical mechanical actuator is mounted between the bogie frame and bolster. According to the curving signals, the electric mechanical actuator pushes or pulls the bolster and further drives the carbody to tilt a certain angle. The general equation of the tilting train is written as:

$$M\ddot{X} + C\dot{X} + KX = P(X, \dot{X}, t) + F_p \quad (1)$$

where  $M$ ,  $C$ ,  $K$  are the mass, damping and stiffness matrices,  $X$  the displacement vector,  $P$  the nonlinear force vector,  $F_p$  the force between the car body and the pantograph.



### 2.2 Pantograph Model

The nonlinear pantograph model with two degrees of freedom is shown in Fig.2. The frame rotational angle  $\alpha$  and the head vertical displacement  $z$  are the two independent variables. In the frame,  $m_1, m_2, m_3, J_1, J_2, J_3, l_1, l_2, l_3$  and  $l_4$  are the masses, inertia moments and lengths of the bars. The value  $\delta$  is not changed, and  $\gamma$  and  $\beta$  are changing with  $\alpha$ . The  $m$  denotes the mass of the head, and  $c, k, u$  indicate the vertical damping, stiffness and friction coefficient between the head and the frame.  $z_c, \dot{z}_c, \ddot{z}_c$  are the excitation inputs from the carbody to the pantograph.  $F_c$  represents the dynamic force between the pantograph and the catenary, and  $M_p$  is the pantograph rising moment. Then the dynamic equations of the pantograph is derived as below by using Lagrange equation

$$f_1(\alpha)\ddot{\alpha} + f_2(\alpha)\dot{\alpha}^2 + f_3(\alpha)\dot{\alpha} + f_4(\alpha)\text{sgn}(\dot{\alpha}) + f_5(\alpha, \dot{\alpha}) = f_6(\alpha)\ddot{z}_c \quad (2)$$

$$m\ddot{z} + c(\dot{z} - \dot{z}_E) + k(z - z_E) + u\text{sgn}(\dot{z} - \dot{z}_E) = -F_c \quad (3)$$

Where  $f_i(\alpha)(i=1\sim 6)$  are the coefficients changing with  $\alpha$ . and the parameters of pantograph [Zhang, 2002].

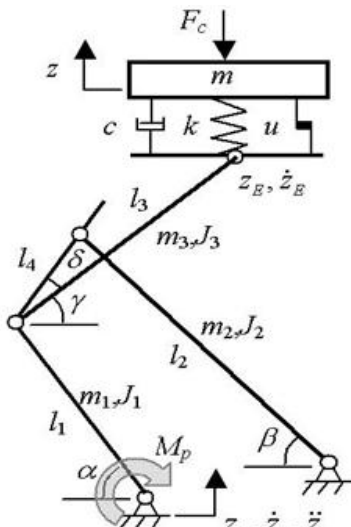


Fig.2 Pantograph model

### 2.3 Catenary FEM Model

The catenary structure with 3 span sections is shown in Fig.3 (a). The catenary is constituted by contact wire, dropper and support wire. The wires are supported by the tower. The catenary is a wire structure with anti-bending stiffness, especially when there is large tension on the wires. Thus the catenary can be meshed by 2-D elastic beam FEM element.

The catenary FEM model can be expressed as

$$M_e \ddot{q} + C_e \dot{q} + K_e q = Q \quad (4)$$

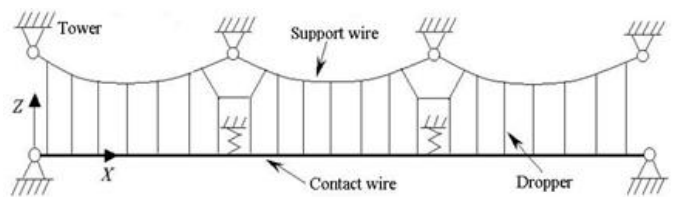
Where  $M_e, C_e$  and  $K_e$  indicate the generalized mass, damping and stiffness matrices respectively.  $q$  is the generalized variable, and  $Q$  the generalized force acting on the catenary.

In order to reduce the DOF of the catenary and the simulation time, the FEM element length of the contact wire is chosen as 1.0m, and only the cross points of the other wires are considered as nodes. The DOF of the catenary with 12 span sections is 2664. When the force  $F_c$  is exerted on the contact wire at the distance  $x$ , we get a static displacement  $z(x)$ . Then the static stiffness of the catenary is  $k(x) = z(x) / F_c$ , as shown in Fig.3(b). If the train speed is not very high, this model can be used to simulate the catenary. The Wilson-0 algorithm is adopted in the dynamic simulation.

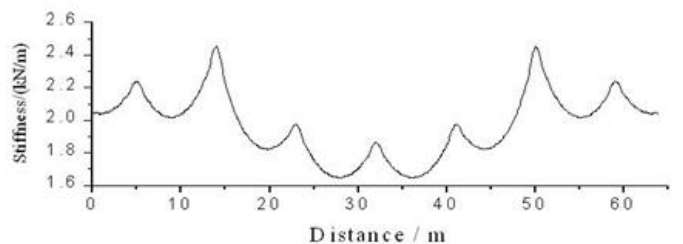
The contact force between the pantograph and the catenary is calculated as

$$F_c(x, t) = \begin{cases} 0 & z(t) \leq z_j(x, t) \\ k(x)[z(t) - z_j(x, t)] & z(t) > z_j(x, t) \end{cases} \quad (5)$$

Where  $x$  is the longitudinal distance of the contact point at time  $t$ . The  $z(t)$  and  $z_j(x, t)$  are the vertical displacements on the contact point of the head and the catenary, here the contact stiffness  $k(x)=82300\text{N/m}$ . If the catenary vibration is not considered,  $k(x)$  can take the value as shown in Fig.3(b).



(a) Catenary structure



(b) Catenary static stiffness

Fig.3 Structure and stiffness of catenary

### 3. PANTOGRAPH CONTROL MODELS

#### 3.1 Carbody Tilting Control

A lateral acceleration sensor and a gyroscope are located on the leading bogie of the first car to obtain the tilting signals, and an electric mechanical actuator is mounted on each bogie to tilt the carbody with maximum angle  $8^\circ$ . The carbody tilting control system is depicted by Fig.4. The tilting train can negotiate curves with a speed about 35% higher than the conventional train.

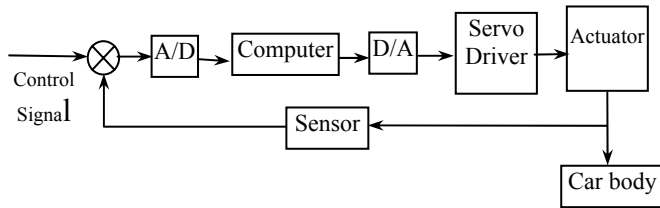


Fig.4 Electric mechanical servo system

#### 3.2 Pantograph Lateral Control

Three lateral control strategies for pantograph are considered:

1) Passive control with four-bar linkage (see Fig.5): The base of pantograph is connected with the car body by a four-bar linkage. One node of the four-bar linkage is connected to the car body and the bogie frame by a three-bar linkage. Because the bogie frame does not tilt during curving, the pantograph can maintain in the position corresponding to track center line.

2) Active control with four-bar linkage: The base of pantograph is also connected with the car body by a four-bar linkage (see Fig.5). But the linkage is not connected with the bogie frame instead of driving by an electric mechanical actuator.

3) Active control with guideway (see Fig.6): the base of pantograph is connected with the car body through a guideway. And the lateral movement of the pantograph is controlled by an electric mechanical actuator.

##### 3.2.1 Passive control with four-bar linkage

The mechanical system is shown in Fig.5, in which points A, D and N are fixed on car body, point S is fixed on the bogie frame. The optimized parameters of system are[Zhang,2004]:

- Height of AD to rail top: 4300mm
- AB=620mm, BC=1175mm
- CD=620mm, AD=1425mm

The coordinates of each point in the track coordinate system are:

$$R(-1082.8, 3209.8), N(-784.6, 3058.0), J(-899.8, 3731.4)$$

##### 3.2.2 Active control with four-bar linkage

By fixing the point J on the carbody and replacing the linkage JK by an electric mechanical actuator as shown in Fig.5, the pantograph movement can be actively controlled. When the carbody does not tilt, point K and point J are in the same horizontal plane. The pantograph control signal can be obtained by modifying the carbody control signal.

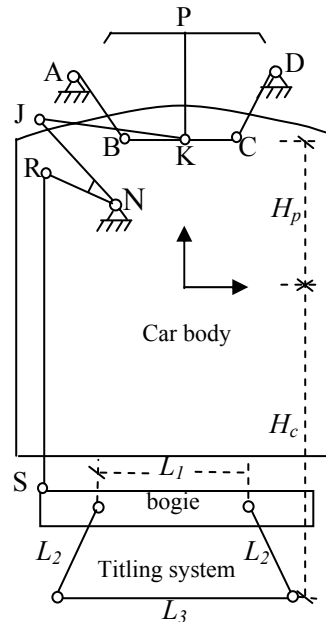


Fig.5 passive control pantograph

##### 3.2.3 Active control with guideway

As shown in Fig.6, the guideway is located on the roof of the carbody, and the base of the pantograph is positioned on the guideway by rollers. Point J is on the car body and point K is on the base. The JK represents the actuator. The control signal is also modified from the carbody tilting signal. The shape of the guideway is designed based on the motion of carbody top when carbody tilts, and to maintain the movement of pantograph head (point P) as small as possible. In the carbody coordinate system, the coordinate of point K is:

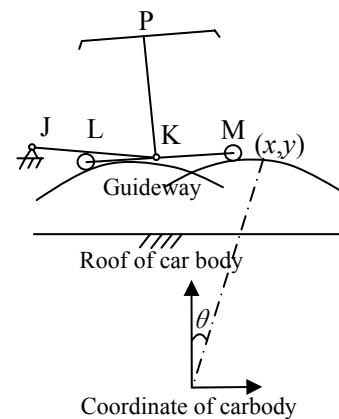


Fig.6 Pantograph with guideway

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} -L_1 \cos \theta_1 - \sqrt{\left(\frac{L_3}{2}\right)^2 + H_c^2} \cos[\arctan\left(\frac{2H_c}{L_3}\right) + \theta] \\ H_p - L_1 \sin \theta_1 - \sqrt{\left(\frac{L_3}{2}\right)^2 + H_c^2} \sin[\arctan\left(\frac{2H_c}{L_3}\right) + \theta] \end{pmatrix} \quad (6)$$

where

$$\theta_1 = 2\pi - \arccos\left(\frac{-\sqrt{L_3^2 + L_1^2 - 2L_1L_3 \cos \theta}}{2L_2}\right) + \arctan\left(\frac{L_3 \sin \theta}{L_3 \sin \theta - L_1}\right)$$

The parameters  $L_1, L_2, L_3, H_p$  and  $H_c$  are shown in Fig.5. The variable  $\theta$  is shown in Fig.6. The coordinates of point L and M can be easily derived from the coordinate of point K.

#### 4. NUMERICAL SIMULATION RESULTS

The track condition used for the simulation is:

- 120m straight+100m transition+100m constant curve
- +100m transition curve+100m straight
- Curve radius: 800m, superelevation: 0.1m
- Speed: 160km/h

The track irregularity inputs are also considered in the simulation. The train-set is composed of a motor car + a trailer car + a trailer car. The total DOF of the train-set and the pantograph is  $34 \times 3 + 3 = 105$ . The pantograph is located on the roof of the first car or the second car, above the rear bogie of the car. The static force  $F_0$  between the pantograph and the contact wire is 80N. There are five working conditions calculated in this paper.

The carbody tilting signals and tilting angles of the train set are shown in Fig.7. The tilting signal of the 1st carbody can be obtained by using a forecast method. When the train negotiates the curve with 160km/h, the vertical and lateral displacements of the tilting carbody are larger than non-tilting carbody, as shown in Fig.8. The lateral displacement of the pantograph head is larger than its working range when the carbody tilts, as shown in Fig.9.

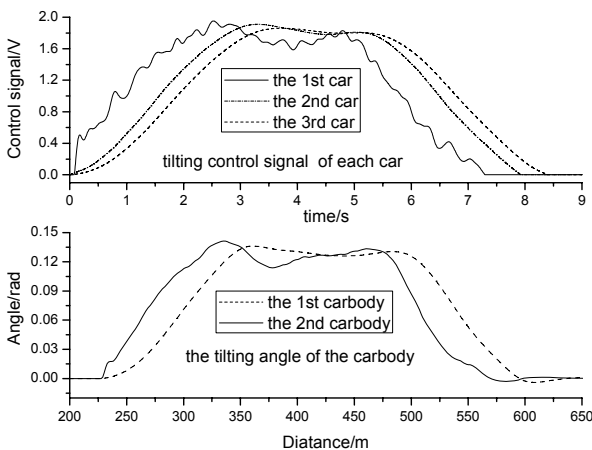


Fig.7 Control signal and tilting angle of tilting train

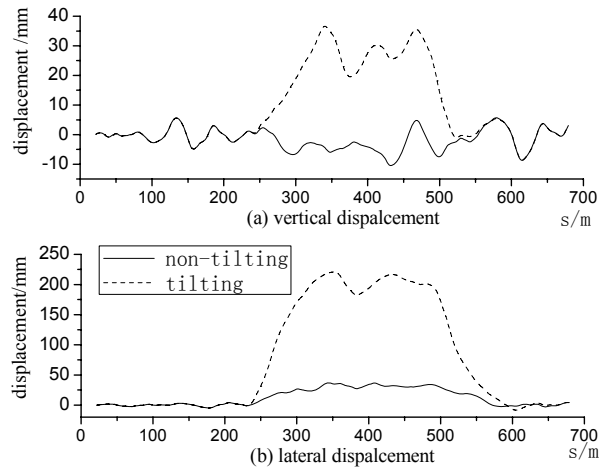


Fig.8 Displacements of carbody center of gravity

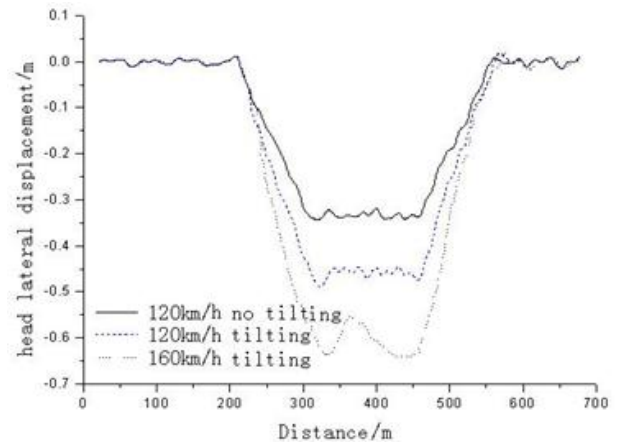


Fig.9 Lateral displacement of pantograph head

Three schemes of pantograph control strategy are simulated in this paper. The passive four-bar linkage system is simple and reliable, and can be mounted on the top of every carbody. The active four-bar linkage system needs control signal and control system, thus the first carbody is not suitable for placing the pantograph. Active pantograph control is able to keep the lateral displacement small, although the system is more complex. Even though the active four-bar linkage system is simple than the guideway system, it cannot reduce the pantograph vertical displacement while reducing the lateral displacement.

##### 4.1 Passive Four-Bar Linkage System

During curve negotiation, the pantograph head results in small lateral displacement and certain vertical displacement. The lateral displacements of the pantograph head are almost the same which less than 25cm when the pantograph is on the top of the first carbody or the second carbody. But the vertical displacement of pantograph head can reach to 12.5cm. In the passive control case, the pantograph-catenary contact point can always keep in the pantograph working range. The simulation results are illustrated in Fig.10 and Fig.11.

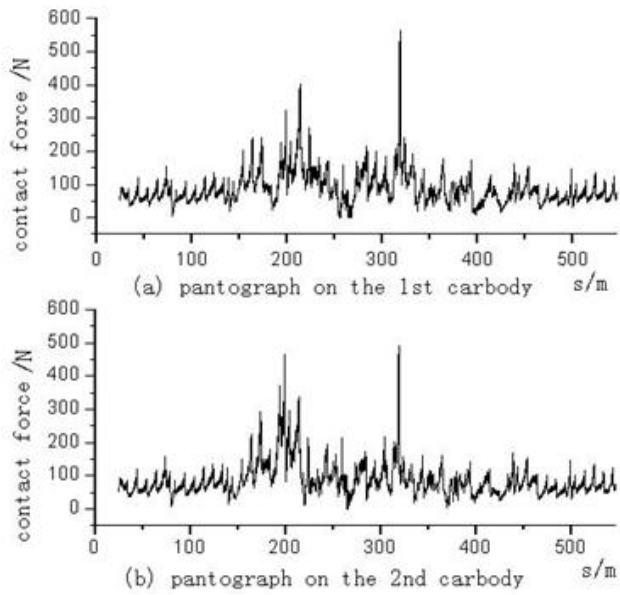


Fig.10 Contact force of the pantograph-catenary

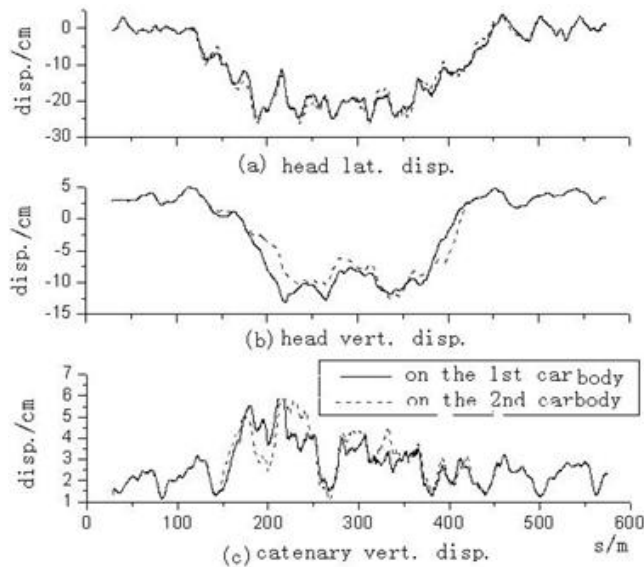


Fig.11 Displacement of pantograph-catenary contact point

#### 4.2 Active Four-Bar Linkage System

The contact force and displacement of pantograph in this active case are almost the same as the passive case. The pantograph lateral displacement is smaller when the pantograph is on the second carbody. The simulation results are shown in Fig.12 and Fig. 13.

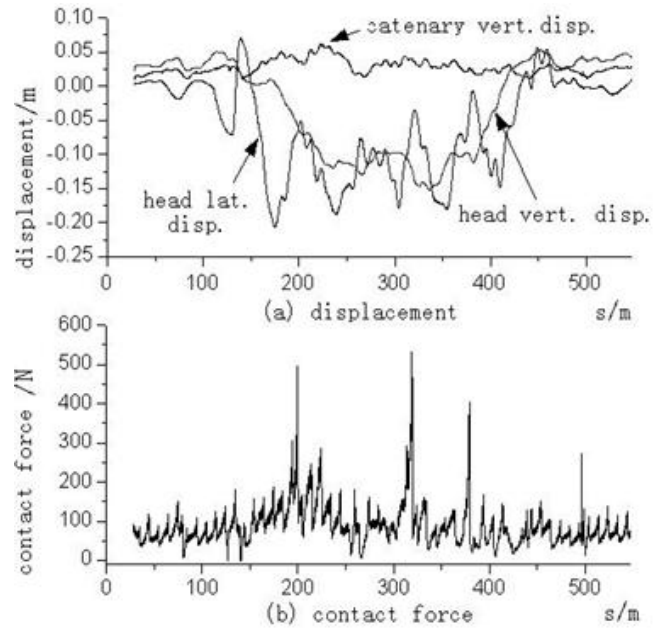


Fig.12 Results of pantograph on the first carbody

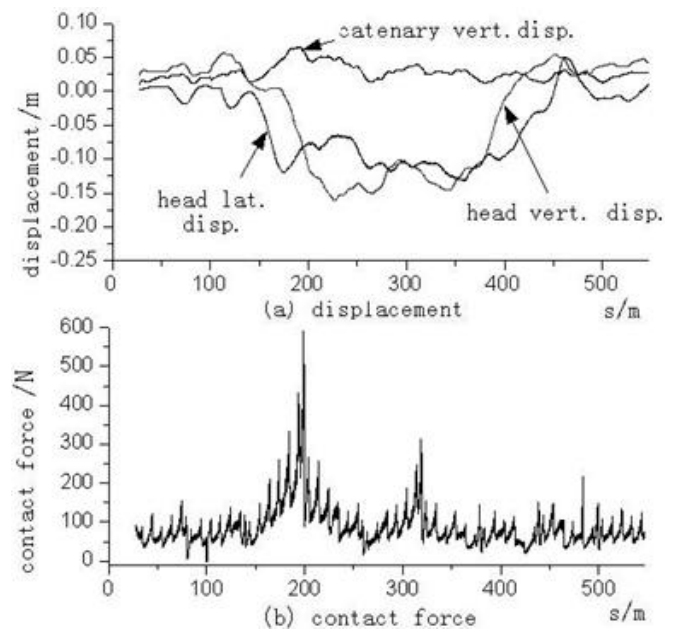


Fig.13 Results of pantograph on the second carbody

#### 4.3 Active Guideway System

The design principle of the pantograph guideway is to realize minimum lateral and vertical displacements of pantograph simultaneously. The simulation results are shown in Fig.14 with the pantograph on the second carbody. It can be seen that the lateral displacements of the pantograph head are smaller than 20cm and the vertical displacements of the pantograph head smaller than 7cm. The contact forces are less than 200N.

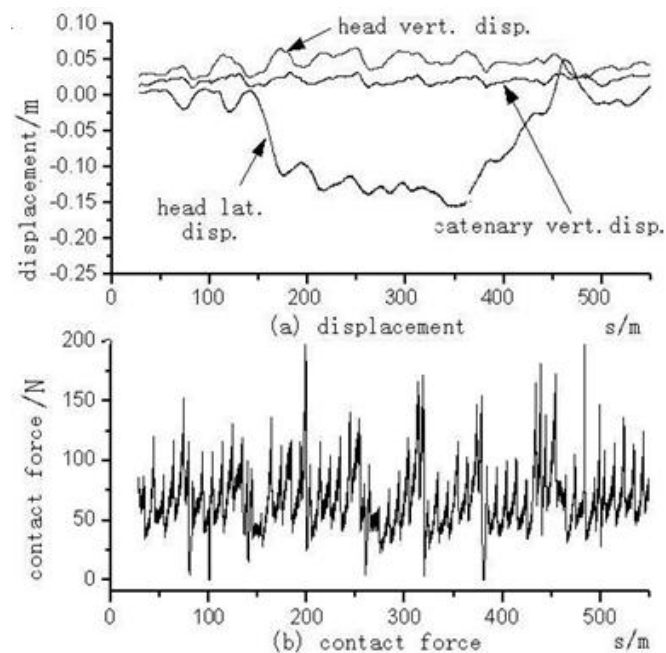


Fig.14 Pantograph displacement and contact force

## 5. CONCLUSIONS

The tilting train model and pantograph-catenary model are set up. The vehicle and the pantograph are considered as nonlinear multiple rigid body systems, and the catenary is modelled by FEM method. The electrical mechanical actuators for carbody and pantograph tilting control are considered. The method of on-line curving detection, filtering and control is adopted for the tilting train. When the tilting train is passing through curves, the lateral and vertical displacements of carbody and pantograph head will increase due to carbody tilting motion. Thus pantograph lateral control is necessary in order to satisfy the current collecting performance. Three schemes for pantograph motion control, either passive or active, are studied in this paper. The contact force between pantograph and contact wire, the displacements of pantograph head are simulated by using numerical method. It can be seen from the results that the use of pantograph control for tilting train can improve the pantograph/catenary contact performance, and scheme of the active control with guideway can achieve better performance.

## ACKNOWLEDGEMENT

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