

Probabilistic and Self-organized Strategies to Coordinate Multiple Robotic Pursuers in the Pursuit of an Adversarial Evaders

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Abstract: this paper addresses the problem of coordinating multiple robotic pursuers in tracking and catching an adversarial evader in a dynamic environment. We assume that the adversarial evader can be detected independently by one pursuer but two pursuers are needed for a successful capture. We aim to reduce the capture time of the evader. Therefore, we model the motion of the evader by the probabilistic method and incorporate the model into directing the motion of the pursuers. In addition, we keep the pursuer communicating with at least another pursuer so that the evader found can be known immediately by another pursuer and then a quick capture can be produced by these two pursuers. By combining the two issues above, the evader can be detected and captured as quickly as possible. Finally, we present the simulation results to demonstrate the performance of our algorithm in an indoor environment. The results show that our method can greatly reduce the capture time of the evader.

1. INTRODUCTION

The following paper examines the problem of controlling a swarm of autonomous agents in the pursuit of an adversarial evader, with the aim of reducing the capture time of the evader. The problem was originally called pursuit-evasion games. The applications of such a problem are as diverse as they are numerous. In search and rescue operations, it is often necessary to search an urban environment for survivors who may be moving randomly. In military applications, human or mechanized infantry often need to locate and track friendly or hostile targets in a partially known or completely unknown environment. Moreover, toxic waste or mine cleanup are also common applications of the pursuit-evasion problem. Research becomes the focus in the structure because of the extensive applications of the pursuit-evasion problem.

In the previous literature on the pursuit-evasion problem, the evader is often assumed to be non-adversarial, and can be detected and caught independently by any pursuer. The capture event occurs once the evader is detected (G. Hollinger, 2007). However, we assume the evader to be adversarial in this paper. This assumption is consistent with the task of tracking and catching a hostile target in an urban environment. Under this assumption, the evader can be detected independently by one pursuer but two pursuers are needed for a successful capture. Therefore, a strongly coordinated algorithm is required for the pursuers to track and catch the evader successfully. Furthermore, without loss of generality, we assume that the evader moves randomly in the environment. For the task of finding a moving survivor, we can utilize the environment clues, such as smoke, voices,

and heat gradients. If such information is known, it can easily be incorporated into our framework by modifying the probability transition matrix.

Under these assumptions above, we aim to reduce the capture time of the evader. We have to detect the evader as quickly as possible and then organize enough pursuers to realize the successful capture of the evader. Therefore, we formulated the pursuit-evasion problem with a probabilistic framework. Then we modeled the random motion of the evader by using the probabilistic method, and incorporated the model into the probabilistic framework, in order to reduce the expected detection time of the evader. In addition, we kept the pursuer communicating with at least another pursuer in order to realize the successful capture of the evader. When the evader is found by a pursuer, another pursuer can know this information immediately and a collective capture behavior can be produced by these two pursuers. By combining the two issues above, we can greatly reduce the capture time of the evader. The methods in this paper are also easily applicable to the domain where two or more robots are needed to perform a subtask. For example, when a pursuer finds a stationary object or a moving target, it can quickly announce the adjacent pursuers and then a quick capture can be produced. Another advantage of our method is that it can greatly lower the requirement of the communication capability of the pursuers.

The remainder of this paper is organized as follows. Section 2 describes related work in the field of pursuit-evasion games. Section 3 provides a mathematical definition of the pursuit-evasion problem, with multiple robotic pursuers tracking and catching a mobile, adversarial evader. Section 4 describes our

pursuit-evasion coordination algorithm, including map discretization, modeling of the pursuers and evader, communication relationship of the pursuers, and cost functions for path planning. Section 5 presents the simulation results in an indoor environment. Finally, Section 6 draws conclusions and discusses the future work.

2. RELATED WORK

The pursuit-evasion problem has been deeply studied in the fields of mathematics, computer science, and robotics. P. Cheng provides a short survey of previous work on pursuit-evasion games. He briefly summarizes the fundamental methods to solve the pursuit-evasion problem and discusses the remaining questions in the area of pursuit-evasion games (2003). The earliest work on pursuit-evasion problem was done by Parsons. In his work, the region where the pursuit took place was abstracted to be a finite collection of nodes, and the allowed motion for the pursuers and evaders were represented by edges connecting the nodes. Parsons studied how to determine the minimum number of guards necessary to catch an adversarial evader with arbitrary speed (1976). However, the solutions are always overly conservative, because the evader cannot move at an arbitrary speed due to the limits of mechanical systems.

The probabilistic pursuit-evasion game framework is first pointed out by J. Hespanha to solve a game involving multiple pursuers and one randomly moving evader (1999). R. Vidal extends the probabilistic framework to solve a game with multiple pursuers and multiple randomly moving evaders (2001). Then R. Vidal provides a general overview and main idea of the research using probabilistic method to solve the pursuit-evasion games. This paper considers many practical conditions, such as limited range and sensor uncertainty, differential motion models of various players, unknown environments and exogenous disturbance to dynamics of players (2002). Recently, L. Guibas and S. LaValle develop pursuit-evasion strategies for multiple pursuers in polygonal environments. Their algorithm discretizes the polygonal environments into conservative visibility regions and then uses an information space approach to develop complete algorithms that guarantee capture in simple environments (1999). B. Gerkey extends these methods to cases where the pursuer has a limited field-of-view (2006). These algorithms are very difficult to extend to complex environments because of the sheer number of cells (often very small) necessary in a conservative visibility discretization. In the areas of urban surveillance and urban search and rescue, B. Ferris, D. Hahnel, and D. Fox use a particle filter with Gaussian processes to track humans by using wireless signal strength. In their algorithm, the environment is discretized into a mixed graph of 1D hallways and 2D rooms (2006). This paper will follow the principle of discretizing the environments into a mixed graph involving 1D hallways and 2D rooms.

G. Hollinger studied the pursuit-evasion problem involving multiple pursuers and a non-adversarial evader. He formulated the problem by using a probabilistic framework, and incorporated the evader's movement model into the

coordination algorithm in order to minimize the expected capture time of the evader (2007). However, the algorithm assumes that the pursuers can communicate with each other whenever re-planning is needed. This assumption is not available when the pursuers are far away from each other or move in a complex environment. Another assumption in the algorithm is that the evader can be detected and caught independently by any pursuer. Therefore, the algorithm is not available in the problem where two or more pursuers are needed for a successful capture of the evader. This paper makes a concerted effort to address this problem. The solutions to the problem are to keep the robots maintaining a mobile robot network so that the evader found can be known immediately by other pursuers and then a collective capture can be produced by these pursuers. C. Clark proposed the concept of a dynamic robot network in order to share the environment information obtained by the limited range sensors installed on robots (2004). J. Vazquez provided a distributed multi-robot exploration method by maintaining a mobile network. The algorithm considers the constraint of short-range communication, which often occurs in decentralized systems. By maintaining a mobile network, the robots can immediately exchange the environment information in order to keep the environment map consistent all the time (2004). In this paper, we consider the limited communication range of pursuers, and keep the pursuer communicating with at least another pursuer in order to reduce the capture time of the evader that two pursuers are needed for a successful capture.

3. PROBLEM DEFINITION

To formulate the pursuit-evasion problem, we must develop representations of the environment and the locations of the pursuers and the evader. For simplicity, we assume that both space and time are quantized. The region where the pursuit takes place is then regarded as a finite collection of cells $\mathcal{X} := \{1, 2, \dots, K, n_c\}$ and all events are taken place on a set of discrete time $T := \{1, 2, \dots, K\}$. In the pursuit-evasion problem considered in this section, we assume that n_p pursuers try to find and catch a single randomly moving evader. The evader is assumed to be adversarial. The evader can be detected independently by any pursuer but two pursuers are needed for a successful capture. Assume that the state of the i^{th} pursuer at time t is known to be $x_i^p(t)$, and the state of the evader at any time t is known with a certain probability to be $x_e(t)$. To define the state of the evader, let p be a row vector such that $p = [p_0, p_1, \dots, p_K, p_{n_c}]$ where values $p_1 \dots p_{n_c}$ represent the probability that the evader is in the corresponding cell. Let the value p_0 represent the probability that the evader has already been captured by the pursuers.

We assume that the maximum communication range between the pursuers is d_{max} and the preventive communication range

between the pursuers is d_{pre} . The value d_{pre} is designed to avoid the total loss of communication between a pair of pursuers. We define g_{ij} as the distance between the i^{th} pursuer and the j^{th} pursuer ($i \neq j$), and define $g_i = \min_{j \neq i} \{g_{ij}\}$ as the minimum distance from the i^{th} pursuer to other pursuers. The communication zone of the i^{th} pursuer is defined as the area of $g_i < d_{max}$.

Now, define a detection event at time t as the occurrence of $x_i^p(t) = x^e(t)$ for any pursuer i . A capture event is defined as follows: the evader is captured successfully at time t if $\exists i$ such that $x_i^p(t) = x^e(t)$ and $g_i < d_{max}$. According to the definition above, the evader is detected when it occupies the same cell with any pursuer, and the capture event occurs or the evader is captured successfully when a pursuer occupies the same cell with the evader as well as the pursuer stays within the communication zone.

The pursuers' goal is to minimize the expected time of reaching a capture event. Thus, the pursuers seek to maximize the probability that the evader is in the detection state at any given time t , as well as to keep themselves within the communication zone. Therefore, the coordination problem is then defined as the determination of paths for the pursuers in order to maximize the probability of detection and to keep the pursuer within the communication zone at any given time t .

4. PURSUIT-EVASION COORDINATION ALGORITHM

In this section, we present our pursuit-evasion coordination algorithm, which is used to coordinate multiple pursuers to track and catch a randomly moving evader. The evader can be detected independently by any pursuer but two pursuers are needed for a successful capture. First, we give the method for discretizing the environments. Then we model the motion of the pursuers and evader by the probabilistic method on a discretized map. Followed by is the method to keep communication relationship of the pursuers. Finally, we provide a method to design the cost functions, which are used to plan paths for the pursuers. The pursuers can capture the evader in minimum time when they move along these paths.

4.1 Map Dcretization

According to the inherent characteristics of an indoor environment, we have discretized the indoor map into convex hallways or rooms as cells. Such discretization is so simple that it can be performed by hand, even for large maps. Fig.1 shows an example of a small house map involving nine cells. Taking into account the cell adjacency in a discretized map yields an undirected graph that can be searched by the pursuers. Fig.2 shows the undirected map derived from the house map. According to the undirected graph, a probabilistic search can be performed to determine the paths of the

pursuers, which can guide the pursuers in tracking and catching the randomly moving evader as quickly as possible. Such a discretization is easily expanded to a scenario having a large map.

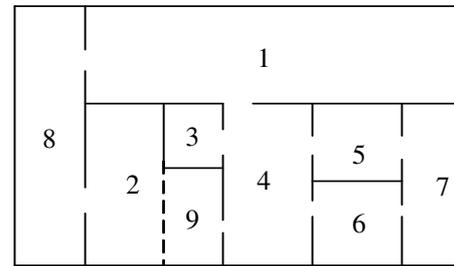


Fig.1. small house map used for pursuit-evasion simulation

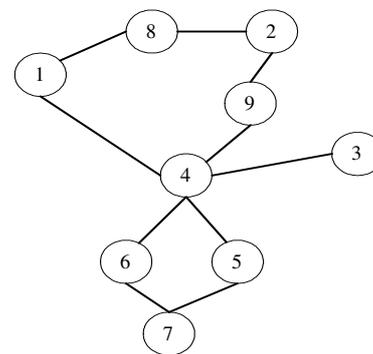


Fig.2. undirected graph built from house discretization

According to the undirected map, we can describe the states of the pursuers and formulate the motion of the evader (see the next section for more details). After discretizing the environment into cells, we assume that the evader is detected when it occupies the same cell with any pursuer. Our discretization method yields far fewer cells so that it can be applied to large and complex environments. In the future, we will work on the implementation of automatic discretization of the environments.

4.2 Modelling of the Pursuers and the Evader

The behavior of the pursuers is determined by the state of the evader and the state of other pursuers. To integrate the motion model of the evader into our pursuit-evasion framework, we model the motion of the evader. As presented in Section 3, the location of the evader is represented by a vector $p = [p_0, p_1, \mathbf{K}, p_{n_c}]$ where the value p_0 represents the probability that the evader is detected by the pursuers, and values $p_1 \mathbf{K} p_{n_c}$ represent the probability that the evader is in the corresponding discretized cell. After discretization of the environment, the evader can move between the cells. Based on a specific motion model of the evader, we can assign probabilities to each of these movements and define a probability transition matrix that properly disperses the evader's probable location. Then we can apply the matrix P at time t to yield a new evader state vector at

time $t + 1$ as in Equation 1.

$$p(t+1) = p(t)P \quad (1)$$

For example, if we assume that the evader moves randomly and that the evader remains still or moves to any adjacent cell with an equal probability at the next step, the probability transition matrix for the environment in Fig.1 would be:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1/3 & 0 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & 1/6 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

We can mathematically represent a detection event on that state vector by defining a matrix that moves a certain probability from all cells visible from the i^{th} pursuer's current cell $x_i^p(t)$ to the detection state. The appropriate detection matrix $C_{x_i^p(t)}$ for cell $x_i^p(t)$ is applied at time t to yield $p(t+1) = p(t)C_{x_i^p(t)}$. For instance, if we assume that the pursuer cannot see through doorways, the detection matrix for a pursuer in cell 8 of the environment in Fig.1 would be the 10×10 identity with $C_{x_i^p(t)=8}(9,1) = 1$.

However, if we assume that the pursuer can also see the adjacent cells, the detection matrix then would be 10×10 identity with $C_{x_i^p(t)=8}(9,1) = C_{x_i^p(t)=8}(2,1) = C_{x_i^p(t)=8}(3,1) = 1$. (Note, here we assume that the pursuers can detect with probability 1.)

We can produce a new evader state vector by integrating the detection matrix and the probability transition matrix as in Equation 2.

$$p(t+1) = p(t)PC_{x_i^p(t)} \quad (2)$$

In larger environments, multiple pursuers are required to search for a single evader. Similarly, we can yield the new evader state vector as in Equation 3.

$$p(t+1) = p(t)P \prod_{i=1}^N C_{x_i^p(t)}, \quad (3)$$

where N represents the number of the robotic pursuers that can communicate with each other.

4.3 Communication Relationship of the Pursuers

In this paper, we assume that the evader can be detected independently by any pursuer but two pursuers are needed for

a successful capture. We keep the pursuer communicating with at least another pursuer so that two pursuers can be organized immediately to perform the capture task once the evader is detected.

We assume that the maximum communication range between the pursuers is d_{max} and the preventive communication range between the pursuers is d_{pre} . The value d_{pre} is designed to avoid the total loss of communication between a pair of pursuers. We define g_{ij} as the distance between the i^{th} pursuer and the j^{th} pursuer ($i \neq j$), and define $g_i = \min_{j \neq i} \{g_{ij}\}$ as the minimum distance from the i^{th} pursuer to other pursuers. The communication zone is defined as $g_i < d_{max}$ beyond which the communication between the i^{th} pursuer and any other pursuers is broken. As presented in Section 3, a capture event is defined as follows: the evader is captured successfully at time t if $\exists i$ such that $x_i^p(t) = x^e(t)$ and $g_i < d_{max}$. The capture event occurs or the evader is captured successfully when a pursuer occupies the same cell with the evader as well as the pursuer can communicate with at least another pursuer. Therefore, the pursuers attempt to cause themselves within the communication zone in order to make the capture event occur as quickly as possible.

4.4 Cost Functions

In this section, we develop a heuristic cost function to determine the paths of the pursuers, which can guide the pursuers to capture the adversarial evader as quickly as possible. We take the cost of two aspects into consideration. One is searching for the evader. The other is keeping the pursuers within the communication zone. They are respectively represented by $S(x)$ and $G(x)$. Then the heuristic cost function is designed as in Equation 4.

$$C(x) = \gamma_1 S(x) + \gamma_2 G(x) \quad (4)$$

Where x represents the cell that the pursuer is about to enter, γ_1 and γ_2 are positive weighting constants. In this paper, we define $S(x) = 1 - p_x(t)$ as the probability of failing to capture the evader when moving into a cell,

$$\text{and } G(x) = \begin{cases} 0 & g_i \leq d_{pre} \\ |g_i - d_{pre}| & d_{pre} < g_i < d_{max} \\ \infty & g_i \geq d_{max} \end{cases}$$

The cost in the area $g_i < d_{pre}$ is zero, the cost in the area $d_{pre} < g_i < d_{max}$ is $|g_i - d_{pre}|$, and the cost in the area $g_i \geq d_{max}$ is infinite. $S(x)$ makes the evader be detected as quickly as possible; $G(x)$ causes the pursuers within the communication zone. According to the definition

of the capture event, $C(x)$ enables the capture event to occur in minimum time.

5. SIMULATION RESULTS

In this section, we present the simulation results of pursuit-evasion games in order to test the performance of our algorithm. We developed a multi-agent pursuit-evasion simulation in Matlab on a 2.8GHz Pentium 4 processor. In our simulation, we have assumed two pursuers (P1 and P2) and one evader (E). The evader is assumed to be adversarial, and can be detected by any pursuer but needs two pursuers for a successful capture. Furthermore, we assume an indoor environment, which involves 54 rooms with the same size. Each room of the environment has an entrance to its adjacent rooms. The evader remains still or moves to the adjacent cells with an equal probability. Fig.3 shows the trajectories of the pursuers and the evader in a pursuit-evasion trial.

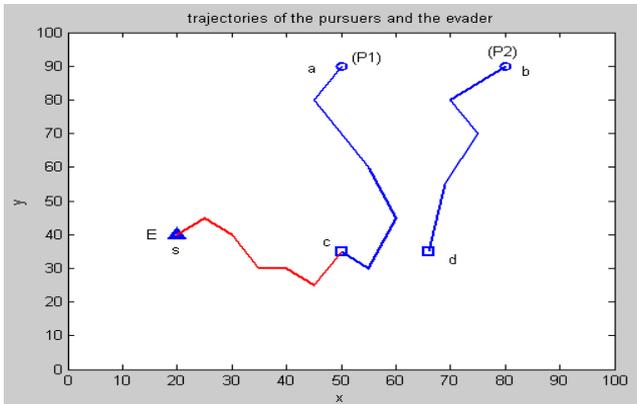


Fig.3 the trajectories of two pursuers (P1 and P2) and one evader (E) in a pursuit-evasion trial. The pursuers initially locate at positions a and b, respectively. The blue lines represent the trajectories along which the pursuers move. The red line represents the randomly moving trajectory of the evader that starts at position s. The evader is detected by pursuer P1 and captured by both pursuers at position c.

In our simulation, the evader E moves randomly along the red line from the starting position s until the captured position c. The pursuers P1 and P2 start to track the evader at positions a and b, respectively. Both pursuers can move towards the evader by incorporating the motion model of the evader. In addition, the pursuers are not far away from each other due to the consideration of the communication relationship of the pursuers. The evader is detected at position c by pursuer P1. Then the detection information can be known immediately by pursuer P2 because the two pursuers are within the communication zone. Therefore, pursuer P1 and P2 can be organized immediately and a collective capture behavior can be produced by them in order to capture the evader successfully at position c. The method to produce the collective capture behavior is beyond the scope of this paper. We will make a concerted effort on the collective capture behavior in the future.

In order to further demonstrate the performance of our

pursuit-evasion coordination algorithm, two other coordination methods were added to the results for comparison. In the random method, the robot moves randomly between the cells. Another method is proposed by G. Hollinger (2007). This method incorporates the evader's movement model, but does not consider the communication relationship with other pursuers. We have tested three methods in three kinds of environments which involve 25 rooms, 40 rooms and 54 rooms, respectively. The average capture time of the evader is gotten for each method. The results are shown in Fig.4. (Note: we also test the stationary method in which the robot remains in its starting cell. The capture time of the evader is much longer than other methods. The results obtained by the stationary method are left out of the figure in order to better show trends in the other results.)

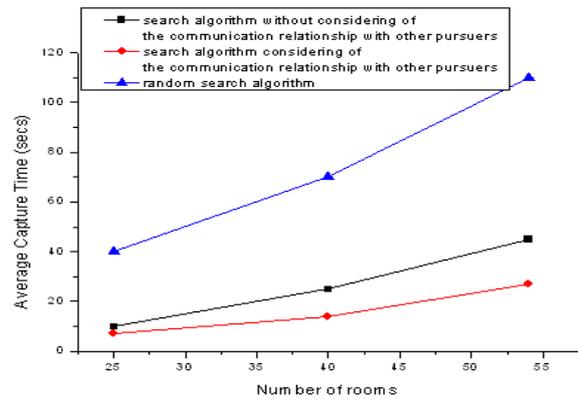


Fig.4 average search time for three kinds of search algorithm. The blue line represents the average search time of random search algorithm; the black line represents the search algorithm integrated the motion model of the evader without considering the communication relationship with other pursuers; the red line represents our search algorithm that considers both the motion model of the evader and the communication relationship with other pursuers.

The average capture time increases as the working areas expand for all three search algorithms. In comparison to the random search method, we can greatly reduce the capture time of the evader by incorporating the evader's movement model. The average capture time can be further reduced by considering the communication relationship of the pursuers. That is because we assume that the adversarial evader can be detected independently by any pursuer but two pursuers are needed for a successful capture. The duration of a task is defined as the capture time not the detection time. Without considering the communication relationship of the pursuers, the evader can be detected but cannot be captured successfully because no other pursuers can cooperate to perform the successful capture. That is to say, the evader may be detected by many times but is still not captured successfully. The results show that the algorithm is effective under our assumption, which is reasonable for real-world applications. In many applications described in Section 1, we need to detect the mobile evader as well as to perform the evader by two or more pursuers (for example, to transport or push). By keeping the pursuers within the communication

zone, we can organize enough pursuers to perform a successful capture immediately after the evader is detected. However, the capture chance is often lost without keeping the pursuers within the communication zone, because the evader is moving all the time. Therefore, the capture time can be further reduced by considering the communication relationship of the pursuers.

Our method can easily extend to the scenario in which the evader needs more than two pursuers for a successful capture. In this situation, we keep enough pursuers communicating with each other. Then the pursuers within the network can self-organize the resources needed for a successful capture. However, the key problem introduced by the extension is the management of the mobile network when it involves too many pursuers.

The algorithm proposed in this paper is also demonstrated in physical robots. Three robots are involved in our experiments. One is considered as the evader. The other two are considered as the pursuers. The experiments are performed in an indoor environment shown in Fig.5.



Fig.5. physical robot experiments involving three robots. The two robots in the front are considered as the pursuers. The robot at the back is considered as the evader.

In the experiments, the evader can be detected by one pursuer and captured successfully by both pursuers. Our future work will concentrate on the optimization of the algorithm.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have addressed the problem of coordinating multiple robotic pursuers in tracking and catching an adversarial evader that moves randomly. We assume that the evader can be detected independently by any pursuer but two pursuers are needed for a successful capture. Based on the assumption, we then provide a probabilistic and self-organized algorithm in order to reduce the capture time of the evader. In our algorithm, the pursuers can effectively detect the evader by incorporating the evader's movement model. In addition, the pursuer that detected the evader can immediately organize two pursuers to capture the evader, because each pursuer keeps communicating with at least another pursuer. Therefore, our method can detect the evader immediately and self-organize enough pursuers to capture the evader successfully when a central control does not exist.

For future work, we intend to examine our pursuit-evasion coordination algorithm on large and complex environment where the obstacles are involved. The pursuers may enter into the broken communication zone due to the obstacles. Thus we plan to study the methods of returning the pursuers back to the communication zone as quickly as possible. Furthermore, we plan to extend our method to the scenario where more than two pursuers are needed for a successful capture. In this situation, the key point is how to quickly self-organize enough pursuers to capture the evader once the evader is detected by some pursuer. Finally, we plan to study the collective behavior of capture, by which the evader can be captured successfully.

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