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A Siphon-based Deadlock Prevention Policy for a Class of Petri Nets - S^{3}PMR

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Abstract: This paper focuses on the problem of deadlocks in automated flexible manufacturing systems (FMS) where deadlocks are caused by unmarked siphons in their Petri net models. A deadlock prevention policy is proposed for a subclass of Petri nets, S^{3}PMR that can well model a large class of FMS. We distinguish siphons in such a net model by elementary and dependent ones. For each elementary siphon, a monitor is added to the plant model such that it is invariant-controlled. The monitor addition way guarantees that no emptiable control-induced siphon in the resultant net is generated due to the addition of monitors. This novel deadlock prevention policy can usually lead to a more permissive supervisor by adding a relatively much smaller number of monitors and arcs than the existing methods for the design of liveness-enforcing Petri net supervisors. Experimental study validates the result.

1. INTRODUCTION

A deadlock occurs in a flexible manufacturing system (FMS) when parts are blocked and waiting for resources held by others that will never be granted. One way of dealing with deadlock problems is, first, to model an FMS with Petri nets Murata [1989], Zhou and Kurapati [1999], and Hruz and Zhou [2007]. Three basic approaches are used to resolve deadlock problems Zhou and Fanti [2005]. The first is called deadlock detection and recovery Wysk *et al.* [1991]. A deadlock detection approach permits deadlock to occur and does not solve it. Once a deadlock state has been detected, deadlocks are recovered by pre-emptying some of the resources involved in them. The second strategy, namely deadlock avoidance, projects deadlock detection into the future in order to keep the system from committing itself to an allocation that may eventually lead to a deadlock Banaszak and Krogh [1990], Hsien and Chang [1994], Xing et al. [1996], Park and Reveliotis [2001], and Abdallah and ElMaraghy [1998]. Such a strategy, unfortunately, may not eliminate deadlocks completely. Finally, the third approach called deadlock prevention is either to design a system such that deadlocks will never occur or to add a control mechanism on resource requests which prevents deadlocks from occurring.

In FMS context, deadlock prevention is usually achieved either by effective system design or by using an off-line mechanism to control the requests for resources to ensure that deadlocks never occur Xie and Peng [2002] and Zhou and Dicesare [1993]. Monitors or control places and related arcs are often used to achieve such purposes Ezpeleta *et al.* [1995], Abdallah and ElMaraghy [1998], Li and Zhou [2004], Huang *et al.* [2001], Chao [2006], Uzam [2002], and Uzam [2004]. The work of Ezpeleta *et al.* [1995] is usually considered to be the first using structure theory of Petri nets to design monitor-based liveness-enforcing Petri net supervisors for FMS. They defined a subclass of ordinary and conservative Petri nets called System of Simple Sequential Processes with Resources (S^{3}PR) and required the target Petri net to be in that subclass. A monitor is added to every strict minimal siphon such that liveness can be enforced. However, too many monitors and arcs have to be added, leading to a much more complex Petri net supervisor than the originally built Petri net model. Furthermore, the behavior of the system can be rather restrictive.

A deadlock prevention policy is proposed in this paper based on the structure analysis of Petri nets for a class of petri nets that is broader than S^{3}PR, called S^{3}PMR, where deadlocks are related to the unmarked siphons. We first distinguish siphons in such a plant Petri net model by elementary and dependent ones. Then, for each elementary siphon, a monitor is added to the plant model such that the siphon is invariant-controlled. The way to add monitors does not lead to unmarked control-induced siphons in the resultant net. By designing the control depth variables for elementary siphons, dependent siphons can be fully controlled, which leads to a liveness-enforcing Petri net supervisor. It is emphasized that the original net is ordinary and allows more than one different shared resource at each operation stage. Barkaoui and Pradat-Peyre [1996] proposed the concepts of max cs-property and min cs-property. Because the original nets are all ordinary in this paper, we have $max_{p^\bullet}$=1. It is shown that if a marked net is invariant-controlled by adding monitor V_S, then it satisfies the max-controlled siphon property (max cs-property).

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The rest of this paper is organized as follows. Section 2 reviews preliminaries of Petri nets that are used throughout the paper. The deadlock control policy is proposed in Section 3. Section 4 introduces an FMS example to illustrate the applications of the proposed policy. Finally, Section 5 concludes this paper.

2. PRELIMINARIES

As for the standard definitions of deadlock-freeness, liveness, reversibility, and boundedness, the reader is referred to Murata [1989]. Elementary siphons of Petri nets are proposed in Li and Zhou [2004].

Our deadlock prevention policy targets the system modeled by a class of ordinary Petri nets called S^3PMR. This section introduces S^3PMR and RCN-merged net models. An S^3PMR is defined as follows Huang *et al.* [2006].

Definition 1. A process net is a strongly connected state machine (P, T, F, W) with exactly one initially marked place p^0 (idle place) such that each circuit of the net contains p^0. The other places are called operation places.

Definition 2. An S^3PMR net N is a net that results from adding a set R of initially marked places (resource places) to a set of independent process nets.

1) Each resource place r is associated with a set of operation places, $OP(r)$. This implies that these operation places require r.

2) For each transition t, which satisfies $t \in {}^\bullet p$ of some $p \in OP(r)$, there exists an arc from r to t if ${}^\bullet t \cap OP(r) = \emptyset$.

3) For each transition t, which satisfies $t \in p^\bullet$ of some $p \in OP(r)$, there exists an arc from t to r if $t^\bullet \cap OP(r) = \emptyset$.

An S^3PR is an S^3PMR, in which each operation place is associated with an unique resource place, and two consecutive operation places are associated with different resource places.

Let x and y be two nodes of an S^3PMR net N. We will say that x is *previous* to y in N if and only if there exists an elementary path in a circuit C in N such that its length, i.e., the number of nodes, of which is greater than 1 and it does not contain p_i^0. This fact is denoted by $x <_N y$, and the simple path from x to y, denoted by $SP(x, y)$.

Definition 3. Let $N = \bigcirc_{i=1}^{k} N_i = (P \cup P^0 \cup P_R, T, F)$ be an S^3PMR and S be a strict minimal siphon in N, where $S = S_P \cup S_R, S_R = S \cap P_R$, and $S_P = S \backslash S_R$. Let $[S] = (\cup_{r \in S_R} OP(r)) \backslash S$. $[S]$ is called the complementary set of siphon S, and $W_p = max[S](p')\{p' | p' \in SP(p, p_i^0) \cap P_i\}$. If $q \in SP(p, p_i^0) \cap P_i$, then $W_q \leq W_p$.

From the above definition, it is easy to see that W_p means the maximal number of tokens derived from siphon S to complete the process i when a token arrives at place p ($p \in P_i$).

In addition, an S^3PMR has the following properties. (1) $\forall i \in \{1, 2, \ldots, k\}$, $P_i \cup \{p_i^0\}$ is the support of a P-semiflow; and $\forall r \in P_R$, $OP(r) \cup \{r\}$ is the support of a P-semiflow. (2) Given a strict minimal siphon S in N, $[S] \cup S$ is the support of a P-semiflow.

Definition 4. A Resource Control Net (RCN) is a strongly connected state machine (P, T, F, M_0) in which there exists one and only one place $p_r \in P$, called a resource place, such that $M_0(p_r) \neq 0$. The remaining places are called operation places.

By construction, an RCN-merged net is State Machine Decomposable. Each RCN is a state machine component. The following restrictions concerning the merge of RCN's are required.
Restriction 1: At each common transition, there exists at most one input place that is an operation place.
Restriction 2: Common transition subnet should not include resource places.
Restriction 3: The Petri net N^* derived from the integrated model N by removing the resource places is an acyclic graph.
Restriction 4: At any common transition, there is at most one output place that is an operation place.

Theorem 1. Suppose that Restrictions 1-4 hold. Then an RCN-merged net is live and reversible if and only if no siphon can become empty Jeng and Xie [2004].

Theorem 2. Each S^3PMR is an RCN-merged net Huang *et al.* [2006].

Theorem 3. An S^3PMR is live and reversible if no siphon in it can become unmarked Huang *et al.* [2006].

3. DEADLOCK PREVENTION POLICY

Theorem 4. Let (N, M_0), $N = (P, T, F)$, be an S^3PMR net.(a)As a strongly dependent siphon with $\eta_S = \sum_{i=1}^{n} a_i \cdot \eta_{S_i}$, where S_1, S_2, …, and S_n are elementary siphons, S is invariant-controlled if (1) $\forall i \in \{1, 2, \ldots, n\}$, I_i is a P-invariant of N, $\| I_i \|^+ \subseteq S_i$, and $\forall p \in S_i$, $I_i(p) = 1$; (2) $M_0(S) > \sum_{i=1}^{n} \sum_{p \in \|I_i\|^-} (a_i \cdot \mid I_i(p) \mid \cdot M_0(p))$. (b)$S$ be a weakly dependent siphon with $\eta_S = \sum_{i=1}^{n} a_i \cdot \eta_{S_i} - \sum_{j=n+1}^{n+m} a_j \cdot \eta_{S_j}$, where $S_1, S_2, \ldots, S_n, S_{n+1}, \ldots$ and S_{n+m} are elementary siphons, S is invariant-controlled if (1) $\forall i \in \{1, 2, \ldots, n\}$, I_i is a P-invariant of N, $\| I_i \|^+ \subseteq S_i$; (2) $M_0(S) > \sum_{i=1}^{n} \sum_{p \in \|I_i\|^-} (a_i \cdot \mid I_i(p) \mid \cdot M_0(p))$.

Note that the controllability conditions stated in Theorems 4 is sufficient but not necessary. From the basic definition, a siphon S is a potential deadlock if and only if $f(S) = 0$, where $f(S) = min\{M(S) \mid M \in R(N, M_0)\}$. Therefore, siphon S is said to be *controlled* if and only if $f(S) > 0$. Due to a large number of reachable markings, $f(S)$ is difficult to find. To avoid the difficulty, we consider another function $F(S)$ defined as $F(S) = min\{M(S) \mid M = M_0 + [N] \cdot Y, M, Y \geq 0\}$, where M and Y are vectors of real numbers. Relation $M = M_0 + [N] \cdot Y$ is usually called the state equation of (N, M_0). From the basic theory of Petri nets, any reachable marking fulfils the sate equation but the reverse is not true. This implies $F(S) < f(S)$. Hence any siphon with $F(S) > 0$ is not a potential deadlock Chu and Xie [1997].

Proposition 1. Let (N, M_0), $N = \bigcirc_{i=1}^{k} N_i = (P \cup P^0 \cup P_R, T, F)$, be a marked S^3PMR. $\forall S \in \Pi$, add monitor V_S and the augmented net is denoted by (N', M_0'), where $\forall p \in P \cup P^0 \cup P_R$, $M_0'(p) = M_0(p)$, $M_0'(V_S) = M_0(S) - \xi_S$, $1 \leq \xi_S < M_0(S)$. V_S is added such that $I = p_x + \ldots + p_y + p_\alpha + \ldots + p_\beta - V_S$ is a P-invariant of N', where $\{p_x, \ldots, p_y\} = S$, $\{p_\alpha, \ldots, p_\beta\} = [\widehat{S}] = \cup_{i=1}^{k} \{p \mid p <_N$

$p', p' \in [S] \cap P_i, \nexists p'' \in [S], p'' \in SP(p', p_i^0)\}$, and $[\widehat{S}] \cap S = \emptyset$. Then S is invariant-controlled.

Definition 5. Let (N, M_0), be a marked net system and S be a strict minimal siphon of N, S is max-controlled in N, iff there exists a P-invariant I such that $||I||^+ \subseteq S, ||I||^- \cap S = \emptyset$ or $\forall p \in ||I||^- \cap S$, $\max_{p^\bullet} = 1$, and $I^T \cdot M_0 > \sum_{p \in S} [I(p) \cdot (max_{P^\bullet} - 1)]$, where $max_{p^\bullet} = max_{t \in p^\bullet}\{W(p, t)\}$.

Definition 6. (N, M_0) is said to be satisfying the max-controlled siphon property (max cs-property) iff every strict minimal siphon of (N, M_0) is max-controlled.

Algorithm 1- Deadlock Prevention Policy based on Siphon Control

Let $(N, M_0), N = \bigcirc_{i=1}^k N_i = (P \cup P^0 \cup P_R, T, F)$, be a marked S^3PMR.

1) $M_0'(V_S) = M_0(S) - \xi_S$, $\xi_S = 1$.

2) For any source transitions t of N, add an arc (V_S, t) of weight W_p, such that $W_p > 0$, in which $p \in t^\bullet \cap P_i$.

3) For any transitions t that is not a source transition of N, let $p \in^\bullet t$ and $p' \in t^\bullet$. Add an arc (t, V_S) of weight $W_p - W_{p'}$, if $W_p - W_{p'} > 0$.

Theorem 5. Let (N, M_0) be a marked S^3PMR net and S be a strict minimal siphon of N, S is invariant-controlled after adding monitor V_S by Algorithm 1.

Then we develop a method to prevent dependent siphons from being emptied by making its elementary siphons invariant-controlled, which can be achieved by adding monitors to the plant Petri net model.

Algorithm 2 - Deadlock Prevention Policy Based on Elementary Siphons

Let $(N, M_0), N = \bigcirc_{i=1}^k N_i = (P \cup P^0 \cup P_R, T, F)$, be a marked S^3PMR.

Step 1) Find the set of elementary siphons Π_E and the set of dependent siphons Π_D in N. Assume that $\Pi_E = \{S_1, S_2, \ldots, S_m\}$ and $\Pi_D=\{DS_1, DS_2, \ldots, DS_n\}$.

Step 2) By Algorithm 1, add monitors V_{S_1}, V_{S_2}, …, and V_{S_m}. The extended net system is denoted by (N', M_0'), where $\forall i \in \{1, 2, \ldots, m\}$, $M_0'(V_{S_i}) = M_0(S_i) - \xi_{S_i}$, $\xi_{S_i} = 1$.

Step 3) $\Pi_D^C := \emptyset$; $\Pi_D^U := \emptyset$.

Step 4) $i := 1$.

Step 5) **if** $i \geq n + 1$ **then** go to Step 6.

else if DS_i is controlled due to Theorems 4 **then** $\Pi_D^C := \Pi_D^C \cup \{DS_i\}$

else $i := i + 1$; go to Step 5

endif

endif

Step 6) $\Pi_D^U := \Pi_D \backslash \Pi_U^C$.

Step 7) Let $\Pi_D^U = \{DS_1^U, DS_2^U, \ldots, DS_k^U\}$

Step 8) Let $\Pi_D^{U(\alpha)} := \emptyset$ and $\Pi_D^{U(\beta)} := \emptyset$.

Step 9) $j := 1$.

Step 10) **if** $j \geq k + 1$ **then** go to Step 11.

else if $F(DS_j^U) > 0$ **then** $\Pi_D^{U(\alpha)} := \Pi^{U(\alpha)} \cup \{DS_j^U\}$

else $j := j + 1$; go to Step 10

endif

endif

Step 11) $\Pi_D^{U(\beta)} := \Pi_D^U \backslash \Pi_D^{U(\alpha)}$. Let $\Pi_D^{U(\beta)}=\{DS_1^{U(\beta)}, DS_2^{U(\beta)}, \ldots, DS_l^{U(\beta)}\} (l \leq k)$.

Step 12) **if** $\forall i \in \{1, 2, \ldots, m\}, \forall j \in \{1, 2, \ldots, l\}$, S_i is an elementary siphon of $DS_j^{U(\beta)}$

then $DS_j^{U(\beta)}(S_i) = 1$

else $DS_j^{U(\beta)}(S_i) = 0$

endif

Step 13) Let $\gamma_i = \sum_{DS^{U(\beta)} \in \Pi_D^{U(\beta)}} DS^{U(\beta)}(S_i)$, $i \in \{1, 2, \ldots, m\}$.

Step 14). Let γ_x be $max\{\gamma_i \mid i = 1, 2, \ldots, m\}$, where $x \in \{1, 2, \ldots, m\}$.

Step 15) Increase ξ_{S_x} until every siphon S in $\{DS_j^{U(\beta)} \mid DS_j^{U(\beta)}(S_x) = 1, j \in \{1, 2, \ldots, l\}\}$ is either controlled due to Theorems 4 or $F(S) > 0$.

Step 16) $\Pi_D^{U(\beta)} := \Pi_D^{U(\beta)} \backslash \{DS_j^{U(\beta)} \mid DS_j^{U(\beta)}(S_x) = 1, j \in \{1, 2, \ldots, l\}\}$.

Step 17) **if** $\Pi_D^{U(\beta)} = \emptyset$ **then** go to Step 19.

Step 18) $\gamma_x := 0$; go to Step 14.

Step 19) Output (N', M_0').

The proposed approach is to add a control place for each strict minimal siphon such that it can never be emptied without generating new strict minimal siphon Ezpeleta *et al.* [1995]. Note that in Ezpeleta *et al.* [1995] for every strict minimal siphon S, $\xi_S = 1$. In this paper, ξ_S indicates the least number of tokens that siphon S can hold. Obviously, ξ_S is equal to or greater than 1 to achieve a deadlock control purpose. When the above algorithm is applied to an S^3PMR with $1 \leq \xi_S < M_0(S)$, all strict minimal siphons in the original net system (N, M_0) are also controlled. Here a controlled strict minimal siphon means that it can never be emptied. On the other hand, since there is no emptiable control-induced minimal siphon in N', the siphons of (N', M_0') contain no additional control places, i.e., they are the siphons of the original Petri net model N. Therefore, $max_{p^\bullet}$=1 holds. According to the P-invariant-controlled siphons, $|| I ||^+ \subseteq S$ and $I^T \cdot M_0 > 0$ hold. Hence we have $\sum_{p \in S} [I(p) \cdot (max_{P^\bullet} - 1)] = 0$. We can get immediately that $I^T \cdot M_0' > \sum_{p \in S} [I(p) \cdot (max_{P^\bullet} - 1)] = 0$. Thus S is max-controlled. Based on Algorithm 2, we can verify that a dependent siphon is marked. Therefore, each strict minimal siphon of (N', M_0') is max-controlled and (N', M_0') satisfies the max cs-property.

Theorem 6. The net (N', M_0') is live Algorithm 2.

As mentioned above, a strict minimal siphon S in a marked S^3PMR (N, M_0) can be invariant-controlled by adding monitor V_S such that $S \cup \{V_S\} \cup [\widehat{S}]$ is the support of a P-invariant of the resultant net (N', M_0'). The monitor related to S has its output arcs directed to the source transitions of N. This is conservative. It can be verified that (N', M_0') could be also live even if we do not make all monitors have their output arcs directed to the source transitions of N. As shown below, Algorithm 3 can, by re-arranging the output arcs of the control places derived from Algorithm 2, improve the positions of the additional arcs in order that the resultant net remains live and has more permissive behavior than the former. The algorithm is stated as follows.

Algorithm 3 - Optimize the Positions of the Output Arcs

Let $(N, M_0), N = \bigcirc_{i=1}^{k} N_i = (P \cup P^0 \cup P_R, T, F)$, be a marked S^3PMR, S be a strict minimal siphon of N, and $[S]$ be the complementary set of S. $\forall i \in \{1, 2, \ldots, k\}$, if $[S] \cap P_i = \emptyset$, let $B_S^i = \emptyset$; otherwise, let $B_S^i = \{p \mid \forall SP(p_i^0, p_\alpha), p \in SP(p_i^0, p_\alpha) \cap P_i, SP(p_i^0, p_\alpha) \cap [S] = \emptyset, p_\alpha \in^{\bullet\bullet} p_\beta, p_\beta \in [S] \cap P_i\}$. Let $B_S = \cup_{i=1}^{k} B_S^i$. Let $\{V_{S_1}, V_{S_2}, ..., V_{S_m}\}$ be the set of monitors added for the elementary siphons of N, and the extended net system is denoted by (N', M_0').

step 1) Derive the set of monitors $\{V_{S_1}, V_{S_2}, \cdots, V_{S_m}\}$ from Algorithm 2, $a := m$

Step 2) $N_1' = N'$, $M_1' = M_0'$, and m:=1

Step 3) **if** $m \geq a + 1$ **then** $m = m - 1$ go to step 11)

else go to step 4)

endif

Step 4) $i := 1$

Step 5) $p := p_i^0$

Step 6) $p_x := p$

Step 7) **if** $i \geq k + 1$ **then** go to Step 10)

else if $B_{S_m}^i = \emptyset$ **then** i:=i+1; go to Step 5)

else $B_{S_m}^{\triangle} := B_{S_m}$; $B_{S_m} := B_{S_m} \backslash p_x^{\bullet\bullet}$

endif

endif

Step 8) Change the output arcs of V_{S_m} s.t. $[S_m] \cup B_{S_m} \cup V_{S_m}$ is the support of a P-invariant, and the resultant net system is supposed to be (N_m', M_m').

Step 9) **if** The resultant net system is live, **then** change the output arcs of V_{S_m} s.t. $[S_m] \cup B_{S_m} \cup V_{S_m}$ is the support of a P-invariant of N_m'

if $\exists p \in B_{S_m}$, $p \in P_i$

then $\forall p_y \in p_x^{\bullet\bullet}, p := p_y$; go to Step 6)

else $i := i + 1$; go to Step 5)

endif

else $B_{S_m} := B_{S_m}^{\triangle}$; $i := i + 1$; go to Step 5)

endif

Step 10) $m := m + 1$, go to step 3)

Step 11) $N_1' = N_m'$, $M_1' = M_m'$

Step 12) Over

Theorem 7. The net (N_1', M_1') satisfies the max-controlled siphon property (max cs-property).

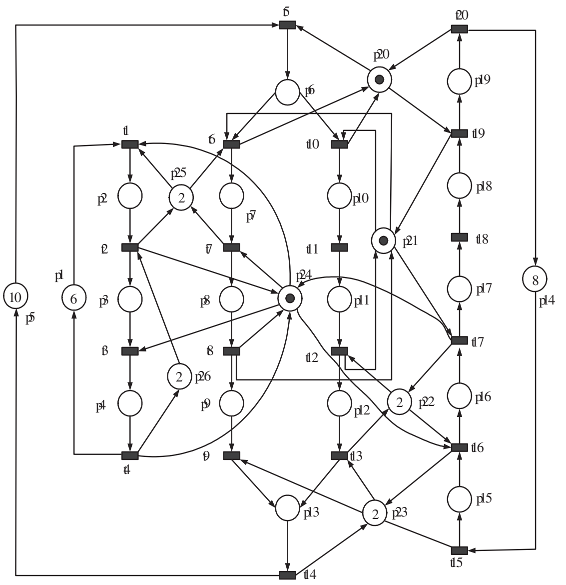

Fig. 1. Petri Net Model for an FMS Cell.

4. AN FMS EXAMPLE

Figure 1 shows the Petri net model of an FMS. The net system is an S^3PMR and contains deadlocks. There are 19 strict minimal siphons and the dependent ones are marked by $*$:S_1^*={p_7, p_8, p_{13},p_{19}, p_{20}, p_{21},p_{22},p_{23}}, S_2^*={p_7,p_8,p_{13}, p_{17}, p_{18}, p_{21}, p_{22}, p_{23}}, S_3={p_{13},p_{16},p_{22}, p_{23}}, S_4={p_7,p_8, p_{10}, p_{11}, p_{19}, p_{20}, p_{21}}, S_5^*={p_7, p_8, p_{12}, p_{19},p_{20}, p_{21}, p_{22}}, S_6={p_7, p_8, p_{12}, p_{17}, p_{18}, p_{21}, p_{22}}, S_7^*={p_4, p_8, p_{13}, p_{19}, p_{20}, p_{21}, p_{22},p_{23},p_{24},p_{26}}, S_8^*={p_4, p_8, p_{12}, p_{19}, p_{20}, p_{21}, p_{22}, p_{24}, p_{26}}, S_9^*={p_4, p_8, p_{13}, p_{17}, p_{18}, p_{21},p_{22},p_{23},p_{24}, p_{26}}, S_{10}^*={p_4, p_8, p_{12}, p_{17}, p_{18},p_{21}, p_{22},p_{24},p_{26}}, S_{11}^*={p_4, p_8, p_{11}, p_{19}, p_{20}, p_{21}, p_{24}, p_{26}}, S_{12}^*={p_4, p_8, p_{11}, p_{17}, p_{18}, p_{21}, p_{24}, p_{26}}, S_{13}={p_4, p_8, p_{11}, p_{16}, p_{24}, p_{26}}, S_{14}^*={p_2, p_4, p_8, p_{13}, p_{19}, p_{20}, p_{21}, p_{22},p_{23},p_{24}}, S_{15}^*={p_2, p_4, p_8, p_{12}, p_{19}, p_{20}, p_{21}, p_{22},p_{24}}, S_{16}^*={p_2, p_4, p_8, p_{13}, p_{17}, p_{18},p_{21},p_{22},p_{23}, p_{24}}, S_{17}={p_2, p_4, p_8, p_{12}, p_{17}, p_{18}, p_{21}, p_{22}, p_{24}}, S_{18}^*={p_2, p_4, p_8, p_{11}, p_{19}, p_{20}, p_{21}, p_{24}}, and S_{19}={p_2, p_4, p_8, p_{11}, p_{17}, p_{18}, p_{21},p_{24}}.

The controllability of dependent siphons due to Theorem 4 is shown in Table 1, where DS denotes the dependent siphon. The relationships between the characteristic T-vectors of dependent siphons and their elementary siphons are as follows:

η_1=η_3+η_4+η_6, η_2=η_3+η_6, η_5=η_4+η_6, η_7=η_3+η_4+η_{13}+η_{17}, η_8=η_4+η_{13}+η_{17}, $\eta_9 = \eta_3$+η_{13}+η_{17}, $\eta_{10} = \eta_{13}$+η_{17}, $\eta_{11} = \eta_4$+η_{13}+η_{19}, $\eta_{12} = \eta_{13}$+η_{19}, $\eta_{14} = \eta_3$+η_4+η_{17}, η_{15}= η_4+η_{17}, η_{16}= η_3+η_{17}, and η_{18}=η_4+η_{19}.

Let us first apply Algorithm 2 to this net system. It is known that there are six elementary siphons. Hence six monitors $V_{S_3}, V_{S_4}, V_{S_6}, V_{S_{13}}, V_{S_{17}}$, and $V_{S_{19}}$ are added to the plant net model. In the plant net, three processes are distinguished with $P_1 = \{p_2, p_3, p_4\}$, $P_2 = \{p_{15} - p_{19}\}$, and $P_3 = \{p_6 - p_{13}\}$. Using Algorithm 3, We have $[S_3] = \{p_{12}, p_{15}\}$. Initially, $B_{S_3} = B_{S_3}^1 \cup B_{S_3}^2 \cup B_{S_3}^3 = \{p_6, p_{10}, p_{11}\}$, where $B_{S_3}^1$=$B_{S_3}^2$=$\emptyset$ and $B_{S_3}^3$={p_6, p_{10}, p_{11}}. We can begin the algorithm from the output arc (V_{S_3}, t_5) with $W(V_{S_3}, t_5) = 1$. First, let B_{S_3}={p_{10}, p_{11}} and $[S_3] \cup B_{S_3} \cup V_{S_3}$ be the support of a P-invariant of N_1'. We re-

Table 1. Marking relationships between dependent and elementary siphons

DS	marking relationships	Ctrl.
S_1	$M_0(S_1) = M_0(S_3) + M_0(S_4) + M_0(S_6)$-3	N
S_2	$M_0(S_2) = M_0(S_3) + M_0(S_6)$-2	N
S_5	$M_0(S_5) > M_0(S_4) + M_0(S_6)$-2	Y
S_7	$M_0(S_7) = M_0(S_3) + M_0(S_4) + M_0(S_{13})$ $+M_0(S_{17})$-4	N
S_8	$M_0(S_8) > M_0(S_4) + M_0(S_{13}) + M_0(S_{17})$-3	Y
S_9	$M_0(S_9) = M_0(S_3) + M_0(S_{13}) + M_0(S_{17})$-3	N
S_{10}	$M_0(S_{10}) > M_0(S_{13}) + M_0(S_{17})$-2	Y
S_{11}	$M_0(S_{11}) > M_0(S_4) + M_0(S_{13}) + M_0(S_{19})$-3	Y
S_{12}	$M_0(S_{12}) > M_0(S_{13}) + M_0(S_{19})$-2	Y
S_{14}	$M_0(S_{14}) = M_0(S_3) + M_0(S_4) + M_0(S_{17})$-3	N
S_{15}	$M_0(S_{15}) > M_0(S_4) + M_0(S_{17})$-2	Y
S_{16}	$M_0(S_{16}) = M_0(S_3) + M_0(S_{17})$-2	N
S_{18}	$M_0(S_{18}) > M_0(S_4) + M_0(S_{19})$-2	Y

arrange the output arc (V_{S_3}, t_5) from t_5 to t_{10}. It can be verified that the addition of V_{S_3} by this way produces no emptiable control-induced siphons. Again, let $B_{S_3} = \{p_{11}\}$ and $[S_3] \cup B_{S_3} \cup V_{S_3}$ be the support of a P-invariant of N'_1. We re-arrange the output arc (V_{S_3}, t_{10}) from t_{10} to t_{11}. Similarly, no emptiable control-induced siphon is produced. Now, let $B_{S_3} = \emptyset$ and $[S_3] \cup V_{S_3}$ be the support of a P-invariant of N'_1. We re-arrange the output arc (V_{S_3}, t_{11}) from t_{11} to t_{12}. We can verify that no siphon that can possibly be unmarked is generated due to the re-arrangement. The processing to V_{S_3} terminates since in this case, B_{S_3} has been empty. Now the incidence relationships between V_{S_3} and the transitions can be finalized by the fact that $[S_3] \cup V_{S_3}$ is the support of a P-invariant of N'_1.

Table 2. Control performance comparison.

control policy	No. of monitors added	No. of arcs added	No. of reachable states
Ezpeleta *et al.* [1995]	19	121	2700
Li and Zhou [2004]	6	29	2700
The proposed method	6	28	3771

Accordingly, by optimizing the positions of the output arcs of monitors V_{S_4}, V_{S_6}, $V_{S_{13}}$, $V_{S_{17}}$, and $V_{S_{19}}$, the resultant net, denoted by (N'_1, M'_1) in the case of no confusion, is found as shown in Figure 2, where $M'_1(V_{S_3})$=3, $M'_1(V_{S_4})$=1, $M'_1(V_{S_6})$=2, $M'_1(V_{S_{13}})$=2, $M'_1(V_{S_{17}})$=3, and $M'_1(V_{S_{19}})$=1 by $\xi_{S_3} = \xi_{S_4} = \xi_{S_6} = \xi_{S_{13}} = \xi_{S_{17}} = \xi_{S_{19}} = 1$.

Now we check the controllability of dependent siphons. We first deal with S_5. Note that $\eta_{S_5} = \eta_{S_4} + \eta_{S_6}$. By the way that V_{S_4} is added, we can see that S_4 is controlled by P-invariant $I_{S_4} = p_7 + p_8 + p_{10} + p_{11} + p_{19} + p_{20} + p_{21} - V_{S_4}$, and S_6 is controlled by P-invariant $I_{S_6} = p_7 + p_8 + p_{12} + p_{17} + p_{18} + p_{21} + p_{22} - V_{S_6}$. By Theorem 4, strongly dependent siphon S_5 is controlled if $M'_1(S_5) > M'_1(V_{S_4}) + M'_1(V_{S_6})$ holds. Considering $M'_1(S_5) = M_0(S_5) = 4, M'_1(V_{S_4}) = M_0(S_4) - \xi_{S_4} = 2 - \xi_{S_4}$, and $M'_1(V_{S_6}) = M_0(S_6) - \xi_{S_6} = 3 - \xi_{S_6}$, we can say that S_5 is controlled when $\xi_{S_4} = \xi_{S_6} = 1$. The controllability of $S_8, S_{10}, S_{11}, S_{12}, S_{15}$, and S_{18} can be accordingly verified.

There are six equalities about the initial marking relationships in Table 1. By Theorem 4, we cannot say that S_1, S_2, S_7, S_9, S_{14}, and S_{16} are controlled. If we let ξ_{S_3}=2, they

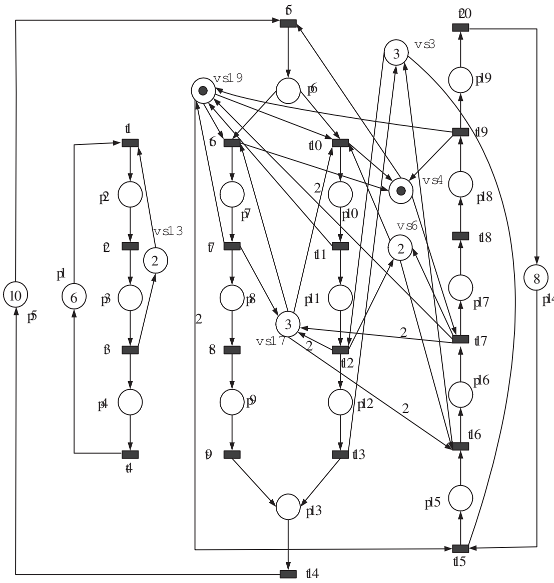

Fig. 2. The supervisor for the FMS without showing the resource places.

become controlled immediately. However, the larger ξ, the more behavior is restricted for the controlled system. Note that the six equalities do not necessarily mean that these dependent siphons are uncontrolled since the controllability conditions stated in Theorem 4 are sufficient but not necessary. It is easy to verify that $\forall S \in \{S_1, S_2, S_7, S_9, S_{14}, S_{16}\}$, we have $F(S) > 0$ and that the least number of tokens in them are all two, respectively. Thus, we do not have to increase ξ_{S_3} to guarantee the controllability of them. As a result, the net in Figure 2 is live. The number of reachable states, as shown in Table 2, is 3771. Our deadlock control policy is more permissive than that of Ezpeleta *et al.* [1995] and Li and Zhou [2004], where, for the same example, both controlled systems have 2700 reachable states, respectively.

5. CONCLUSION

This paper presents a deadlock prevention method for a class of FMS, where the deadlocks are caused by the unmarked siphons in their Petri net models. The FMS is modeled using S^3PMR, which is a special class of Petri nets. The major disadvantage of the siphoned-based deadlock prevention methods is that too many monitors have to be added, which leads to a structurally complex liveness enforcing Petri net supervisor, and the behavior of the modelled system seems much restrictive. This paper shows that by adding a small number of monitors to elementary siphons only, all siphons can be prevented from being unmarked. We use a control policy to ensure that by adding a monitor for each elementary siphon, the siphon is successfully controlled and no emptiable control-induced siphons can be produced. In addition, the examples show that our method can achieve much more permissive supervisors than the existing ones Ezpeleta *et al.* [1995] and Li and Zhou [2004].

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