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Development and Application of a Sliding Mode based Diagonal Recurrent Cerebellar Model Articulation Controller

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Abstract: This paper presents a sliding-mode-based diagonal recurrent cerebellar model articulation controller (SDRCMAC) for multiple-input-multiple-output (MIMO) uncertain nonlinear systems. Sliding mode technology is used to reduce the dimension of the control system. Two learning stages are adopted to train the SDRCMAC and to improve the stability of the control system. Lyapunov stability theorem and Barbalat's lemma are adopted to guarantee the asymptotical stability of the system. Performance is illustrated on a two-link robotic control and motor control of the human arm in the sagittal plane.

1. INTRODUCTION

In control engineering application, control of multiple-input-multiple-output (MIMO) uncertain nonlinear systems is a challenging task. Neural networks have been suggested as a powerful building strategy (Huang, Huang & Chiou, 2003). Cerebellar model articulation controller (CMAC), a non-fully connected associative memory network, has good generalization capability and fast learning property (Albus, 1975). For acquiring the derivative information of input and output variable, a CMAC with differentiable Gaussian receptive field basis function has been developed (Chiang & Lin, 1996); and a recurrent CMAC has been presented to solve dynamic problems (Wai, Lin & Peng, 2004). Some applications of CMAC for complex dynamic systems have been presented (Miller et al., 1990; Peng & Lin, 2007; Lin, Chen & Chen, 2007; Yeh, 2007). However, though CMAC can accurately learn the inverse mapping of the plant, it is hard to maintain the stability when a CMAC is solely used in the control systems.

Sliding mode control (SMC) is an effective robust control approach for MIMO systems (Slotine & Li, 1991; Hung, Gao & Hung, 1993). However, the control chattering in the SMC may result in unforeseen instabilities. Some methods have been proposed to solve the problem (Man, Paplinski & Wu, 1994; Feng, Yu & Man, 2002).

It this study, a sliding-mode-based diagonal recurrent cerebellar model articulation controller (SDRCMAC) is proposed. The sliding mode technology is used to integrate the feedback information and reduce the dimension of the controller. A coarse-tuning stage is to enable the output behaviour of the SDRCMAC to approximate control surface of a SMC controller. A fine-tuning stage follows to improve the stability of the control system.

While the new SDRCMAC should be applicable to a range of MIMO uncertain nonlinear systems, our specific motivation is the biological movement control. Such research is useful in the study of how the brain controls the movements (Todorov, 2004). Yet, there is a lack of efficient methods to handle realistic biomechanical control problems. The characteristics of these problems are high dimension nonlinear dynamics, control constraints, complex performance criteria.

The organization of this paper is described as follows. Section 2 presents problem formulation. The sliding mode controller is described in Section 3. The SDRCMAC control system is constructed in Section 4. Simulation results are shown in Section 5. And, conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

Consider an MIMO nonlinear uncertain system described by the following:

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = F(X,t) + B(X,t)u + d(t). \end{cases} \tag{1}$$

where $u \in R^m$, and $X = [x_1, x_2]^T \in R^{2n}$, $x_1, x_2 \in R^n$ represent the control input, and the state vector, respectively. $F(X,t) \in R^n$ and $B(X,t) \in R^{n\times m}$ are bounded real continuous functions. $d(t)$ represents the bounded disturbance and model uncertainty. Let the Euclidean norm $\| d(t) \| \le \kappa D < D$, κ is a positive decimal fraction, and D is a positive constant.

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The object of the control system is to design a suitable control law such that the state vector X can track a specified reference trajectory $X_d = [x_{1d}, x_{2d}]^T \in R^{2n}$.

3. SLIDING MODE CONTROLLER

We define the following sliding mode vector as

$$S = [s_1, s_2, \cdots, s_n]^T = CE = [C_1 \quad C_2][e_1 \quad e_2]^T. \quad (2)$$

where $C_1 = diag(c_1, \ldots, c_n) \in R^{n\times n}$ is a positive matrix, $C_2 \in R^{n\times n}$ is a unit matrix, $E = X_d - X$ is the tracking error vector. The SMC law is defined as

$$u = inv(B(X))\left(-F(X) + \dot{x}_{2d} + C_1(\dot{x}_{1d} - \dot{x}_1) + Dsat(S) + \dot{S} + C_1 sat(S)\right). \quad (3)$$

where $inv(B(X))$ represents the inverse of the matrix $B(X)$, $sat(s)$ is a saturation function, which is defined as follows:

$$sat(s) = \begin{cases} 1 & s/\delta > 1; \\ s/\delta & -1 \le s/\delta \le 1; \\ -1 & s/\delta < -1. \end{cases} \quad (4)$$

where $\delta > 0$ is the layer thickness, $|s/\delta| < \kappa$ is assumed.

Consider a Lyapunov function as

$$V_1 = \frac{1}{2}(S - sat(S)\delta)^T (S - sat(S)\delta) = \frac{1}{2} S_1^T S_1. \quad (5)$$

Differentiating (5) with respect to time, we have

$$\begin{aligned} \dot{V}_1 &= S_1^T \dot{S} = S_1^T (C_1(\dot{x}_{1d} - \dot{x}_1) + \dot{x}_{2d} - \dot{x}_2) \\ &= S_1^T(-Dsat(S) - d(t) - \dot{S} - C_1 sat(S)) \\ &= -\frac{1}{2} S_1^T C_1 sat(S) - \frac{1}{2} S_1^T Dsat(S) - \frac{1}{2} S_1^T d(t). \end{aligned} \quad (6)$$

In the case of $|S/\delta| > 1$, $S_1 sat(S) = |S_1|$, then $\dot{V}_1 < 0$; in the case of $|S/\delta| \le 1$, $S_1 = 0$, $\dot{V}_1 = 0$. It can be shown that the error $|E| < \delta$ as $t \to \infty$ for any initial error. Thus, the SMC law (3) can guarantee the stability of the system (1).

4. SDRCMAC CONTROLLER

In the SDRCMAC control system, there are two learning stage, the coarse-tuning learning stage and the fine-tuning learning stage.

4.1 SDRCMAC Architecture

The architecture of the SDRCMAC is shown in Fig. 1, in which T denotes the delay time. The network is composed of input space, association memory space with recurrent units, receptive field space, weight memory space and output space. The input of the SDRCMAC is the signed sliding mode vector S through (2). Firstly, $S \in R^n$ is normalized, the input space is quantized into discrete regions (called elements), and the number of elements is Ne. Next, S is mapped into the association memory space through receptive basis functions, where the space consists of $n \times Na$ blocks, a complete block is formed by Nr elements, Na is the number of blocks relative to each input. Thirdly, the association memory matrix $A \in R^{n\times Na}$ is mapped into the receptive field space through multidimensional receptive field functions. Lastly, the receptive field vector $Rs \in R^{Nr}$ is projected onto weight matrix $W \in R^{m\times Nr}$ to computer the output $Y \in R^m$. The SDRCMAC consists of two primary functions that are performed in the association memory space and the receptive field space, respectively.

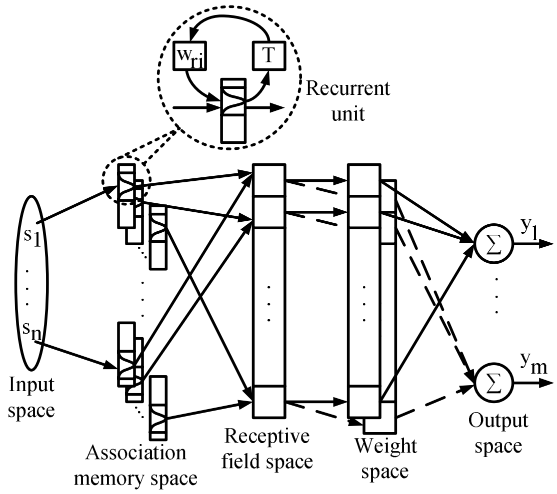

Fig. 1. Architecture of a SDRCMAC

1) Receptive basis function: The Gaussian function is adopted here, which can be represented as

$$\alpha_{ij}^h = \exp\left(-\frac{(s_{rij} - \lambda_{ij})^2}{\sigma_{ij}^2}\right), i = 1, \ldots, n; h = 1, \ldots, Nr; j = 3p + h. \quad (7)$$

where α_{ij}^h represents the j th block of the i th input s_i with the mean λ_{ij} and variance σ_{ij} in the h th layer, and $p = 0, \ldots, ceil\left[\frac{Ne-h}{Nr}\right]$. The mean and variance can be expressed in the vector form $\Lambda \in R^{n\times Na}$ and $\Sigma \in R^{n\times Na}$. The input s_{rij} in time step k is represented as

$$s_{rij}(k) = s_i(k) + w_{rij}\alpha_{ij}^h(k - T). \quad (8)$$

where w_{rij} is the recurrent weight, $\alpha_{ij}^h(k-T) \triangleq \alpha_{ijT}^h$ denotes the value of $\alpha_{ij}^h(k)$ through time delay T. The recurrent weight matrix can be expressed as $Wr \in R^{n\times Na}$. Fig. 2 depicts the schematic diagram of a 2-dimension SDRCMAC.

2) Multidimensional receptive field function: Receptive fields are formed by blocks, named as b_1b_2 and d_1d_2 in Fig. 2. The multidimensional receptive field function is defined as

$$r_h = \prod_{i=1}^{n} \sum_{j=1}^{3p+h} \alpha_{ij}^h, \quad p = 0,1,\ldots,ceil\left[\frac{Ne-h}{Nr}\right], \quad h = 1,2,\ldots,Nr. \qquad (9)$$

In this SDRCMAC scheme, no receptive field is formed by the combination of blocks in different layers, such as b_1 and d_2. Thus, the number of receptive fields is Nr. This kind of composition reduces the memory requirement, and makes nearby inputs can produce similar outputs, which provide local generation to SDRCMAC.

The output of the SDRCMAC is expressed as $Y = W \bullet Rs$, where the lth element in Y is $y_l = \sum_{h=1}^{Nr} w_{lh} r_h$. In the two-dimension case (Fig. 2), the output is the sum of receptive fields b_1b_2, d_1d_2 and e_1f_2 when the input is $(0.17, 0.67)$.

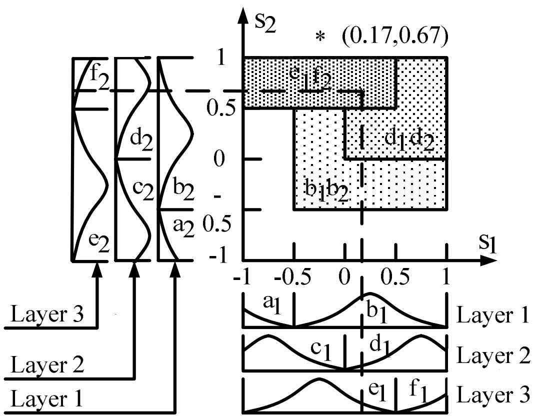

Fig. 2. Two-dimension SDRCMAC with $Nr = 3$ and $Ne = 4$.

4.2 Coarse-tuning Stage

The purpose of this stage is to enable the output behaviour of the SDRCMAC to approximate the control surface of the SMC controller. The control system is shown in Fig. 3(a). The control law $u(t)$ is the sum of the SDRCMAC output $u_C(t)$ and the SMC output $u_S(t)$. $u(t)$ is used as the target output, the error function is defined as

$$E_1(k) = \frac{1}{2}\left(u_C(k) - u(k)\right)^T \left(u_C(k) - u(k)\right) = \frac{1}{2}\sum_{l=1}^{m} u_{Sl}(k)^2. \qquad (10)$$

Initially, the SDRCMAC weight matrixes are set as zero matrixes. And then, according to the gradient descent method, these weight matrixes are updated at each time step by the following learning rules

$$\begin{aligned}
\dot{w}_{lh} &= -\eta_{w1}\frac{\partial E_1}{\partial w_{lh}} = \eta_{w1} u_{Sl} r_h; \\
\dot{\lambda}_{ij} &= -\eta_{\lambda 1}\frac{\partial E_1}{\partial \lambda_{ij}} = \eta_{\lambda 1}\sum_{l=1}^{m}\left(u_{Sl} w_{lh}\right)\frac{\partial r_h}{\partial \lambda_{ij}}; \\
\dot{\sigma}_{ij} &= -\eta_{\sigma 1}\frac{\partial E_1}{\partial \sigma_{ij}} = \eta_{\sigma 1}\sum_{l=1}^{m}\left(u_{Sl} w_{lh}\right)\frac{\partial r_h}{\partial \sigma_{ij}}; \\
\dot{w}_{rij} &= -\eta_{r1}\frac{\partial E_1}{\partial w_{rij}} = -\eta_{r1}\sum_{l=1}^{m}\left(u_{Sl} w_{lh}\right)\frac{\partial r_h}{\partial w_{rij}}.
\end{aligned} \qquad (11)$$

where η_{w1}, $\eta_{\lambda 1}$, $\eta_{\sigma 1}$, and η_{r1} are positive constants, the subscript h can be derived from the subscript j, and

$$\begin{aligned}
\frac{\partial r_h}{\partial \lambda_{ij}} &= 2\left(\prod_{k=1,\neq i}^{n}\sum_{j=1}^{3p+h}\alpha_{kj}^h\right)\alpha_{ij}^h\frac{(s_{rij}-\lambda_{ij})}{\sigma_{ij}^2}; \\
\frac{\partial r_h}{\partial \sigma_{ij}} &= 2\left(\prod_{k=1,\neq i}^{n}\sum_{j=1}^{3p+h}\alpha_{kj}^h\right)\alpha_{ij}^h\frac{(s_{rij}-\lambda_{ij})^2}{\sigma_{ij}^3}; \\
\frac{\partial r_h}{\partial w_{rij}} &= -2\left(\prod_{k=1,\neq i}^{n}\sum_{j=1}^{3p+h}\alpha_{kj}^h\right)\alpha_{ij}^h\alpha_{ijT}^h\frac{(s_{rij}-\lambda_{ij})}{\sigma_{ij}^2}.
\end{aligned} \qquad (12)$$

The gradient descent method can guarantee the convergence of the parameters λ_{ij}, σ_{ij}, and w_{rij}, and the output of the receptive field basis functions are limited in $[0,1]$. Therefore, the stability of the control system will not be destroyed due to the adaptive learning rules shown in (11).

4.3 Fine-tuning Stage

The objective of this stage is to improve the system stability. The control system is shown in Fig. 3(b). Learning rules are derived from the gradient of $S\dot{S}$ with respect to parameters in the SDRCMAC.

$$\begin{aligned}
\dot{w}_{lh} &= -\eta_{w2}\frac{\partial S\dot{S}}{\partial w_{lh}} = \eta_{w2}\sum_{i=1}^{n}(s_i b_{il}) r_h; \\
\dot{\lambda}_{ij} &= -\eta_{\lambda 2}\frac{\partial S\dot{S}}{\partial \lambda_{ij}} = \eta_{\lambda 2}\sum_{i=1}^{n}\sum_{l=1}^{m}(s_i b_{il} w_{lh})\frac{\partial r_h}{\partial \lambda_{ij}}; \\
\dot{\sigma}_{ij} &= -\eta_{\sigma 2}\frac{\partial S\dot{S}}{\partial \sigma_{ij}} = \eta_{\sigma 2}\sum_{i=1}^{n}\sum_{l=1}^{m}(s_i b_{il} w_{lh})\frac{\partial r_h}{\partial \sigma_{ij}}; \\
\dot{w}_{rij} &= -\eta_{r2}\frac{\partial S\dot{S}}{\partial w_{rij}} = \eta_{r2}\sum_{i=1}^{n}\sum_{l=1}^{m}(s_i b_{il} w_{lh})\frac{\partial r_h}{\partial w_{rij}}.
\end{aligned} \qquad (13)$$

where η_{w2}, $\eta_{\lambda 2}$, $\eta_{\sigma 2}$ and η_{r2} are positive constants. The adaptive rules (13) can be expressed in vector form as

$$\dot{W} = \eta_{w2} B^T S R s^T ; \qquad \dot{\Lambda} = \eta_{\lambda 2} tr(S^T B W) r_{h\Lambda} ;$$
$$\dot{\Sigma} = \eta_{\sigma 2} tr(S^T B W) r_{h\Sigma} ; \quad \dot{W}r = \eta_{r2} tr(S^T B W) r_{hWr} . \tag{14}$$

where $tr(\bullet)$ is defined as $tr(S^T S) = \sum_{i=1}^{n} s_i^2$, $r_{h\Lambda} = \{\partial r_h / \partial \lambda_{ij}\}_{n\times Na}$, $r_{h\Sigma} = \{\partial r_h / \partial \sigma_{ij}\}_{n\times Na}$, and $r_{hWr} = \{\partial r_h / \partial w_{rij}\}_{n\times Na}$. The parameters update equations are given by

$$\begin{aligned}
W(k+1) &= W_{coarse-tuning}(k) + \dot{W} \\
\Lambda(k+1) &= \Lambda_{coarse-tuning}(k) + \dot{\Lambda} \\
\Sigma(k+1) &= \Sigma_{coarse-tuning}(k) + \dot{\Sigma} \\
Wr(k+1) &= Wr_{coarse-tuning}(k) + \dot{W}r.
\end{aligned} \tag{15}$$

where $W_{coarse-tuning}$, $\Lambda_{coarse-tuning}$, $\Sigma_{coarse-tuning}$, and $Wr_{coarse-tuning}$ are the final SDRCMAC parameters at the coarse-tuning stage; the behaviour of the sliding mode controller is implicit in these parameters. Since the learning error will not accumulated in the fine-tuning stage, the instability caused by the continued learning after the tracking error has been reduced can be solved by (15).

4.4 Stability Analysis

Lyapunov analysis is employed to investigate the stability of the SDRCMAC control system. In the coarse-tuning stage, the SMC law (3) can guarantee the stability of the control system according to the stability analysis in Section 3. In the following, the stability of the control system in the fine-tuning stage will be proved.

For the stability analysis, we assume the optimal parameter matrixes $\bar{W}$, $\bar{\Lambda}$, $\bar{\Sigma}$, and $\bar{W}r$ of the SDRCMAC exists, which makes the SDRCMAC output to approximate the SMC law (3) with an error smaller than ξ , ξ is a positive number.

$$\max\left(\bar{u}(S,\bar{W},\bar{\Lambda},\bar{\Sigma},\bar{W}r) - u_S\right) < \xi. \tag{16}$$

where $\bar{u}(S,\bar{W},\bar{\Lambda},\bar{\Sigma},\bar{W}r) \triangleq \bar{W}\bar{R}s$, Then, $u_S = \bar{W}\bar{R}s + \Xi$, where $\Xi \in R^m$. The SDRCMAC output can be written as $u(S,\hat{W},\hat{\Lambda},\hat{\Sigma},\hat{W}r) \triangleq \hat{W}\hat{R}s$. According to (1), (3) and (16), the following equation can be derived

$$\begin{aligned}
\dot{S} &= -\frac{1}{2}\left(Dsat(S) + C_1 sat(S) + d(t)\right) + B\left(u_S - u\right) \\
&= -\frac{1}{2}\left(Dsat(S) + C_1 sat(S) + d(t)\right) + B\left(\bar{W}\bar{R}s + \Xi - \hat{W}\hat{R}s\right) \\
&= -\frac{1}{2}\left(Dsat(S) + C_1 sat(S) + d(t)\right) + B\left(\tilde{W}\hat{R}s + \hat{W}\tilde{R}s + \tilde{W}\tilde{R}s\right).
\end{aligned} \tag{17}$$

where $\tilde{W} = \bar{W} - \hat{W}$ and $\tilde{R}s = \bar{R}s - \hat{R}s$. Taylor linearization technique is employed to transform the nonlinear function into a partially linear form

$$\begin{aligned}
\tilde{R}s &= \left[\frac{\partial Rs}{\partial \Lambda}\right]\Bigg|_{\Lambda=\hat{\Lambda}} \tilde{\Lambda} + \left[\frac{\partial Rs}{\partial \Sigma}\right]\Bigg|_{\Sigma=\hat{\Sigma}} \tilde{\Sigma} + \left[\frac{\partial Rs}{\partial W_r}\right]\Bigg|_{Wr=\hat{W}r} \tilde{W}r + H \\
&= \left[tr\left(\tilde{\Lambda}^T r_{h\Lambda}\big|_{\Lambda=\hat{\Lambda}}\right)\right] + \left[tr\left(\tilde{\Sigma}^T r_{h\Sigma}\big|_{\Sigma=\hat{\Sigma}}\right)\right] + \left[tr\left(\tilde{W}r^T r_{hWr}\big|_{Wr=\hat{W}r}\right)\right] \\
&= R_\Lambda + R_\Sigma + R_{Wr} + H.
\end{aligned} \tag{18}$$

where $\tilde{\Lambda} = \bar{\Lambda} - \hat{\Lambda}$, $\tilde{\Sigma} = \bar{\Sigma} - \hat{\Sigma}$, $\tilde{W}r = \bar{W}r - \hat{W}r$, H is a high-order term, and $R_\Lambda, R_\Sigma, R_{Wr} \in R^{Nr}$.

Choose the Lyapunov function as

$$V_2 = S^T S + \frac{1}{\eta_{w2}} tr(\tilde{W}^T \tilde{W}) + \frac{1}{\eta_{\lambda 2}} tr(\tilde{\Lambda}^T \tilde{\Lambda}) + \frac{1}{\eta_{\sigma 2}} tr(\tilde{\Sigma}^T \tilde{\Sigma}) + \frac{1}{\eta_{r2}} tr(\tilde{W}r^T \tilde{W}r). \tag{19}$$

Differentiating (19) with respect to time and using (15), (17) and (18) yields

$$\begin{aligned}
\dot{V}_2 &= S^T \dot{S} + \frac{1}{\eta_{w2}} tr(\tilde{W}^T \dot{\tilde{W}}) + \frac{1}{2\eta_{\lambda 2}} tr(\tilde{\Lambda}^T \dot{\tilde{\Lambda}}) + \frac{1}{2\eta_{\sigma 2}} tr(\tilde{\Sigma}^T \dot{\tilde{\Sigma}}) + \frac{1}{2\eta_{r2}} tr(\tilde{W}r^T \dot{\tilde{W}}r) \\
&= -S^T(C_1 sat(S) + Dsat(S) + d(t)) + S^T B(\tilde{W}\hat{R}s + \hat{W}\tilde{R}s + \tilde{W}\tilde{R}s + \Xi) \\
&\quad \frac{1}{\eta_{w2}} tr(\tilde{W}^T \dot{\hat{W}}) - \frac{1}{\eta_{\lambda 2}} tr(\tilde{\Lambda}^T \dot{\hat{\Lambda}}) - \frac{1}{\eta_{\sigma 2}} tr(\tilde{\Sigma}^T \dot{\hat{\Sigma}}) - \frac{1}{\eta_{r2}} tr(\tilde{W}r^T \dot{\hat{W}}r) \\
&= -S^T(C_1 sat(S) + Dsat(S) + d(t)) - tr(\tilde{W}^T B^T S Rs^T) \\
&\quad + S^T B(\tilde{W}\hat{R}s + \hat{W}\tilde{R}s + \tilde{W}\tilde{R}s + \Xi) - tr(\tilde{\Lambda}^T tr(S^T B W) r_{h\Lambda}) \\
&\quad - tr(\tilde{\Sigma}^T tr(S^T B W) r_{h\Sigma}) - tr(\tilde{W}r^T tr(S^T B W) r_{hWr}) \\
&= -S^T(C_1 sat(S) + Dsat(S) + d(t)) + S^T B\tilde{W}\hat{R}s + S^T B(\tilde{W}\tilde{R}s + \Xi) \\
&\quad + S^T B\hat{W}(R_\Lambda + R_\Sigma + R_{Wr} + H) - tr(\tilde{\Lambda}^T tr(S^T B\hat{W}) r_{h\Lambda}) \\
&\quad - tr(\tilde{\Sigma}^T tr(S^T B\hat{W}) r_{h\Sigma}) - tr(\tilde{W}r^T tr(S^T B\hat{W}) r_{hWr}) - tr(\tilde{W}^T B^T S\hat{R}s^T) \\
&= -S^T(C_1 sat(S) + Dsat(S) + d(t)) + S^T B\tilde{W}\hat{R}s - tr(\tilde{W}^T B^T S\hat{R}s^T) \\
&\quad + S^T B\hat{W}R_\Lambda - tr(\tilde{\Lambda}^T tr(S^T B\hat{W}) r_{h\Lambda}) + S^T B\hat{W}R_\Sigma - tr(\tilde{\Sigma}^T tr(S^T B\hat{W}) r_{h\Sigma}) \\
&\quad + S^T B\hat{W}R_{Wr} - tr(\tilde{W}r^T tr(S^T B\hat{W}) r_{hWr}) + S^T B(\hat{W}H + \tilde{W}\tilde{R}s + \Xi) \\
&= -S^T(C_1 sat(S) + Dsat(S) + d(t)) + S^T B\Delta
\end{aligned} \tag{20}$$

where the approximation error term $\Delta = \hat{W}H + \tilde{W}\tilde{R}s + \Xi$ is assumed to be bounded by $\|B\Delta\| \le \kappa\|C_1\| < \|C_1\|$. Then, (20) becomes

$$\dot{V}_2 \le -S^T C_1 sat(S) + \|S\|\|B\Delta\| - S^T Dsat(S) + \|S\|\|d(t)\| \tag{21}$$

In the case of $|S/\delta| > 1$, $\dot{V}_2 \le -\|S\|(\|C_1\| - \|B\Delta\|) - \|S\|(D - \|d(t)\|) < 0$; in the case of $|S/\delta| < 1$, $\dot{V}_2 \le -\|S\|(\kappa\|C_1\| - \|B\Delta\|) - \|S\|(\kappa D - \|d(t)\|) \le 0$. Since $\dot{V}_2$ is negative semidefinite that is $V_2 \le V_2(0)$, it implies that variables S, $\tilde{W}$, $\tilde{\Lambda}$, $\tilde{\Sigma}$, and $\tilde{W}r$ in V_2 are bounded. Let function

$$\begin{aligned} L &\equiv S\left(\kappa\|C_1\|-\|B\Delta\|\right)+S\left(\kappa D-\|d(t)\|\right) \\ &\le \|S\|\left(\kappa\|C_1\|-\|B\Delta\|+\kappa D-\|d(t)\|\right) \\ &\le -\dot{V}_2. \end{aligned} \tag{22}$$

and integrate L with respect to time, it can be shown that $\int_0^t L d\tau \le V_2 - V_2(0)$. Because $V_2(0)$ is bounded and V_2 is non-increasing and bounded, the following result can be shown $\lim_{t\to\infty}\int_0^t L d\tau < \infty$. In addition, since $\dot{L}$ is bounded by Barbalat's lemma, it can be shown that $\lim_{t\to\infty} L = 0$. That is, $S \to 0$ as $t \to \infty$. As a result, the control system is asymptotically stable.

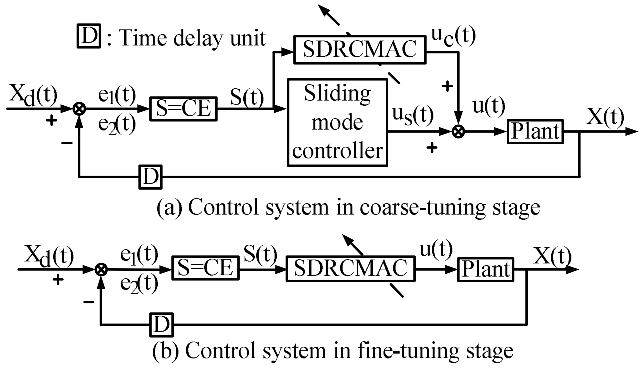

Fig. 3. SDRCMAC control system

5. SIMULATION RESULTS

In this section, the proposed SDRCMAC is applied to control two control problems. The control objective is to let the system state X track the reference trajectory X_d. The SDRCMAC used in these systems is characterized as follows: $Ne = 4$; $Nr = 3$; $Na = 6$. The receptive fields are selected to cover the input space $\{[-1,1],[-1,1]\}$ along each input dimension. Therefore, the initial values of the parameters for the receptive field basis functions in the coarse-tuning stage are $[\lambda_{i1},\lambda_{i2},\lambda_{i3},\lambda_{i4},\lambda_{i5},\lambda_{i6}] = [-1.25,-0.75,-0.25,0.25,0.75,1.25]$, and $\sigma_{ij} = 0.75$. The weight W and Wr in coarse-tuning stage are initialized from zero matrixes, and the initial parameters in the fine-tuning stage are chosen as the final parameters in the coarse-tuning stage.

5.1 A Two-link Robotic Arm

The dynamic equations for a two-link robotic arm are from (Sun & Wang, 2006). The initial state are $\theta_0 = [0\ \ 0]^T$ deg, $\dot{\theta}_0 = [0\ \ 0]^T$ deg/sec, and the reference trajectories are set as $\theta_d(t) = [30+10\sin t\ \ \ 60+30\sin t]^T$ deg. In order to study the robustness of the proposed controller, assume the robotic arm has external disturbance $\tau_d = \{\exp(-0.1t)\} \in R^2$. The control parameters are chosen as $\delta = 0.3$, $C_1 = diag(10,10)$, $\kappa = 0.3$, $D = 5$, $\eta_{wi} = 0.6$, $\eta_{\lambda i} = \eta_{\sigma i} = \eta_{ri} = 0.1$, where $i = 1,2$. These parameters are chosen through trial and error to achieve satisfactory performance. For comparison, the fuzzy CMAC (FCMAC) (Sun & Wang, 2006), the SMC and the SDRCMAC are used in the simulation. Fig. 4(a) and (b) show the angle tracking responses of these three methods. The results of the SDRCMAC and the SMC scheme are all satisfactory, and slightly better than that of the FCMAC. Fig. 4(c) and (d) show the angle velocity tracking responses of the SMC and the SDRCMAC to more compare these two methods. These figures illustrate that the SDRCMAC controller can more smoothly track the reference trajectory.

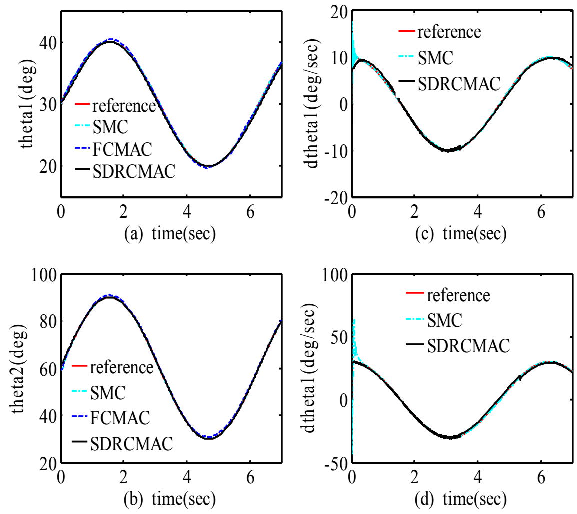

Fig. 4. Trajectory tracking response of the robotic arm

5.2 A Model of the Human Arm

The model of the two-link six-muscle human arm in the sagittal plane is rather complex, we lack the space to describe it in detail, see (Liu, Wang & Zhu, 2007). The initial state are $\theta_0 = [0\ \ 0]^T$ deg, $\dot{\theta}_0 = [0\ \ 0]^T$ deg/sec, and the reference trajectories are set as $\theta_d(t) = [60\sin t\ \ \ 60-60\cos t]^T$ deg. In order to study the robustness of the proposed controller, assume some pulse signals as external disturbance are added into the human arm, the pulse amplitude is 1, period is $3.5s$, and the width is 15% of the period. The control parameters are chosen as $\delta = 0.35$, $C_1 = diag(10,10)$, $\kappa = 0.2$, $D = 7$, $\eta_{wi} = \eta_{\lambda i} = \eta_{\sigma i} = \eta_{ri} = 0.33$, where $i = 1,2$. These parameters are chosen through trial and error to achieve satisfactory performance. For comparison, the EDRNN proposed in (Liu, et al., 2007) and the SDRCMAC are used in the simulation. The angle tracking results are shown in Fig. 5(a) and (b). These figures illustrate that the two methods can all implement the trajectory tracking control of the human arm in the sagittal plane. Fig. 5(c) and (d) show that the angle velocity tracking responses of the EDRNN and the SDRCMAC. The results illuminate that the robustness of the proposed SDRCMAC is better than the EDRNN scheme when the disturbances exist in the human arm. And the training time of the SDRCMAC is extremely less than the EDRNN.

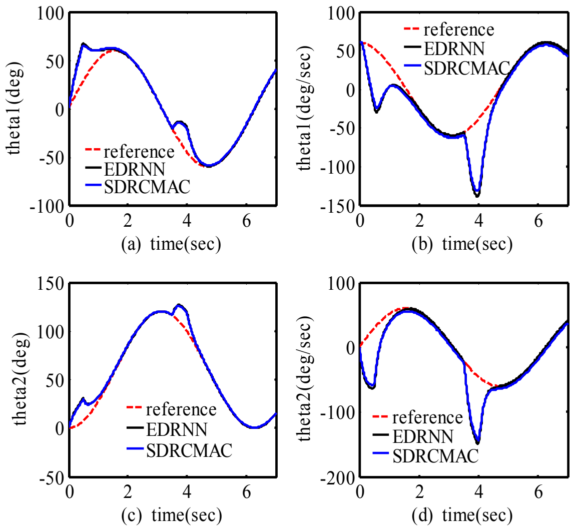

Fig. 5. Trajectory tracking responses of the human arm

6. CONCLUSIONS

In this paper, a SDRCMAC with two learning stages has been proposed to control MIMO uncertain nonlinear systems. Through the coarse-tuning stage, the output behaviour of the SDRCMAC can approximate the control surface of the SMC controller. At the fine-tuning stage, the adaptive laws are derived from the stable convergence feature of the SMC. According to Lyapunov stability theorem and Barbalat's lemma, the asymptotical stability of the SDRCMAC is guaranteed. Finally, the SDRCMAC is applied to implement trajectory tracking control of a two-link robotic arm and the human arm in the sagittal plane. The simulation results demonstrate that the smoothness of the SDRCMAC is better than the SMC; compared with the EDRNN, the robustness is better and the consumed time is less.

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