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A novel method of Stability analysis for networked control systems ⋆

Xi-Ming Sun [*,**] **Guo-Ping Liu** [*] **David Rees** [*] **Wei Wang** [**]

[*] *Faculty of Advanced Technology, University of Glamorgan, Pontypridd, CF37 1DL, United Kingdom (Email: wrsxm@126.com)*
[**] *Research Center of Information and Control, Dalian University of Technology, Dalian 116024, P. R. China*

Abstract: This paper studies the problem of stability analysis for a class of networked control systems (NCSs), whose control gain is assumed to be known. A switched delay system model different from existing ones is obtained based on the event-time-driven scheme. To solve the stability problem of the new NCSs' model, a new approach based on the Lyapunov functional exponential estimation (LFEE) method is introduced. Using this method, sufficient conditions are developed to guarantee exponential stability of the considered system. The results obtained are less conservative than existing ones, as is shown from both theory and example.

Keywords: Networked control systems, switched systems, delay systems, event-time-driven mode, exponential stability.

1. INTRODUCTION

Recent advances in networked technology have lead to an increased use of control systems where the control loops are closed through a network channel. Such systems are referred to as Networked control systems (NCSs), which has been a popular research topic in recent years due to their low cost, reduced weight and power requirement, simple installation and maintenance (Nesic and Teel [2004a,b], Tabbara et al. [2007], Carnevale et al. [2007], Liu et al. [2007, 2006], Zhang et al. [2001], Park et al. [2002]). However, due to the insertion of communication channels, induced delay and packet dropout are almost unavoidable, and this may seriously deteriorate the performance of the system. Thus it is important to deal with induced delay and packet dropout issues (Walsh et al. [2001, 2002], Kim et al. [2003], Yue et al. [2004, 2005]). Just as is pointed out in Zhang et al. [2001], to handle delay one might formulate control strategies based on the study of delay-differential equations. Traditional approach in dealing with delay in the time domain mainly uses two methods: the first is Razumikhin method, the second is Lyapunov-Krasovskii method Hale and Lunel [1993], Gu et al. [2003]. At present, many papers on delay systems which do not consider the effect of a network give delay-dependent results based on the second method (He et al. [2006], Gao and Wang [2003]). These two methods are also mainly used in the study of NCSs. For example, Zhang et al. [2001], Walsh et al. [2002] give the calculation method for the maximum allowable transfer interval (MATI) based on Razumikhin method. But the results obtained using this method are often very conservative. Lyapunov-Krasovskii method is used in Kim et al. [2003], Yue et al. [2005] to obtain the maximum allowable delay bound (MADB).

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In the study of NCSs, it is often assumed that the actuator and controller are event-driven, (for example refer to Zhang et al. [2001], Yue et al. [2004, 2005]. But for the reason of networked-induced delay and packet dropout, a larger delay bound arising from control input will be often encountered due to the applying this event-driven mode. Once the large delay bound appears, system may become unstable, or even though it is still stable, exiting method can not provide stability criterion. How to deal with the large delay case and analyze system stability are still an open problem to date.

In this paper, we adopt a new mode of actuator and controller: event-time-driven mode, which is different to event-driven mode adopted in Yue et al. [2004] and Yue et al. [2005]. Event-time-driven mode means that the controller and actuator data will be updated once the new packet comes and this new data will be held during the intervening time interval. But this new data will be only held during a certain intervening time interval. If at the end of this time interval, new packet has not yet come, the data will be changed to zero automatically and will be held until the new data comes; if during this time interval, a new packet comes, then event-time-driven is the same as event-driven. Under this mode, a switched delay system is obtained, which may include an unstable subsystem. This switched delay system is completely different from those in Sun et al. [2006] because all subsystems in Sun et al. [2006] are stable. To deal with this switched delay system, a new method called Lyapunov functional exponential estimation (LFEE) is adopted in this paper. Under this LFEE method, explicit conditions to guarantee exponential stability of the considered system are developed. In addition, compared with existing literature, the explicit decay form of system state is also given. The given example shows the effectiveness of the proposed method.

The paper is organized as follows. Section II gives system description and preliminaries. Section III devolves stability criterion based on LFEE method and gives some remarks. Section IV provides a numerical example to show the effectiveness of the proposed method. Conclusions are summarized in Section V.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following NCS proposed in Yue et al. [2004] and Yue et al. [2005]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu, \ t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}), \\ u(t) = Kx(t - \tau_k), t \in \{i_k h + \tau_k, k = 1, 2, \ldots\}, \end{cases} \tag{1}$$

where matrices A, B and K are known constant matrices; h is the sampling period; $i_k (k = 1, 2, 3, \ldots)$, are some integers and $\{i_1, i_2, i_3, \ldots\} \subset \{1, 2, 3, \ldots\}$; $\tau_k \geq 0$ is the time delay, which denotes the time from instant $i_k h$ where sensor notes sample sensor data from a plant to the instant when actuators transfer data to the plant. Let $i_1 h + \tau_1 = t_0$, and thus $\bigcup_{k=1}^{\infty} [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}] = [t_0, +\infty)$. Similar to the paper Yue et al. [2004] and Yue et al. [2005], we also assume that $u(t) = 0$ before the first control signal reaches the plant and there is a constant $h_2 > 0$ such that $(i_{k+1} - i_k)h + \tau_{k+1} \leq h_2$. Also, $i_{k+1}h + \tau_{k+1} \geq i_k h + \tau_k$. h_2 can be used to reflect the allowable bound on the amount of the data dropout and networked induced delays, and is called the maximum allowable transfer interval (MATI).

Let us review the stability analysis method for system (1) in existing literature (for example, see Yue et al. [2004] and Yue et al. [2005]). First, system (1) is rewritten as a delay system

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)), t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}), \forall k \in \bar{N}, \tag{2}$$

where $d(t) = t - i_k h$. Then the methods used to study delay systems (for example, Lyapunov functional method Gu et al. [2003]), can be applied to system (2). Some concrete conditions in the form of linear matrix inequalities can be given to obtain the maximum delay bound h_1 Zhang et al. [2001], Yue et al. [2004, 2005]. However, it needs the restriction $d(t) \leq h_1, t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$ for $\forall k \in \bar{N}$, as means that the upper bound of $d(t)$, that is $(i_{k+1} - i_k)h + \tau_{k+1}$, must less than h_1 for $\forall k \in \bar{N}$. Noticing the feature of networked control systems, the restriction is an ideal case. Due to the effect of delay and packet dropout, the case $h_1 < d(t) < h_2$ (see Fig. 1), may be encountered more frequently, which is called large delay case. If large delay case holds in interval $[i_k h + \tau_k, i_{k+1}h + \tau_{k+1}), \forall k \in \bar{N}$, then system stability will not be guaranteed based on existing methods (Zhang et al. [2001], Yue et al. [2004] and Yue et al. [2005]) or system itself becomes unstable. In this situation, controller $u(t) = kx(i_k h), t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$, that is $u(t) = kx(t - d(t)), d(t) = t - i_k h$, may deteriorate system performance due to the effect of lager delay in control input. Thus, once the large delay appears, it may bring better results to use zero control signal instead of the previous control signal with large delay. Based on this idea, we adopt event-time-driven mode in this paper. Under this mode, system (1) can be described as the following switched delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(i_k h), t \in [i_k h + \tau_k, j_k), \\ \dot{x}(t) = Ax(t), t \in [j_k, i_{k+1}h + \tau_{k+1}) \end{cases} \tag{3}$$

where

$$j_k = \begin{cases} i_{k+1}h + \tau_{k+1}, \ if \ (i_{k+1} - i_k)h + \tau_{k+1} \leq h_1 \\ i_k h + h_1, \ if \ (i_{k+1} - i_k)h + \tau_{k+1} > h_1. \end{cases} \tag{4}$$

Remark 1. In (4), h_1 can be given based on exiting methods (for example, see, Zhang et al. [2001], Yue et al. [2004, 2005]).

Inspired by the concept of unavailability rate of the controller in Zhai and Lin [2004], we introduce the following concepts.

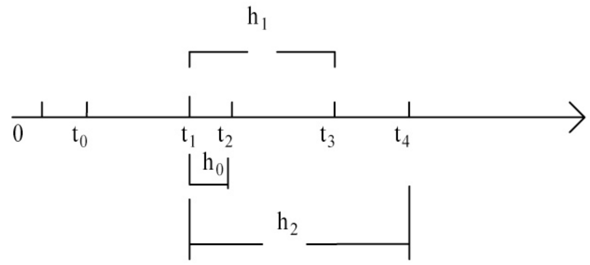

Fig. 1. Relationship between h_0, h_1 and h_2, where, $t_1 = i_k h, t_2 = i_k h + \tau_k, t_3 = i_k h + h_1, t_4 = i_{k+1}h + \tau_{k+1}$.

Definition 1. The time $t \in \cup_{l=1}^{+\infty} [j_l, i_{l+1}h + \tau_{l+1})$ is called unavailable time, and the time $t \in \cup_{l=1}^{+\infty} [i_k h + \tau_k, j_k)$ is called available time. Using $T^+(t_1, t_2)$ and $T^-(t_1, t_2)$ to denote the length of total unavailable time and available time over time interval $[t_1, t_2)$, respectively, the ratio $\frac{T^+(t_0,t)}{T^-(t_0,t)}$ over interval $[t_0, t)$ is called average unavailable rate in NCSs. Suppose $t \in [i_l h + \tau_l, i_{l+1}h + \tau_{l+1})$, the maximum unavailable rate is defined as $\frac{T_m^+(t_0,t)}{T_m^-(t_0,t)} = max\{\frac{i_{k+1}h + \tau_{k+1} - j_k}{j_k - i_k h - \tau_k}, k \in \{1, 2 \ldots, l\}\}$.

Existing literature do not consider the case of unavailable time. In this paper, we will address the problem: under what size of unavailable rate, system (3) can still maintain exponential stability.

Lemma 1. For any constant matrix $Z \in R^{n \times n}$, and $Z > 0$, scalars $h_1 \geq 0$, $\alpha_1 > 0$, and any $t \in [0, +\infty)$, vector function $\omega : [t - h_1, t] \to R^n$, such that the integrations in the following are well defined, then

$$\begin{aligned} &\left(\int_{-h_1}^{0} \int_{t+\theta}^{t} \omega(s) ds d\theta\right)^T aZ \int_{-h_1}^{0} \int_{t+\theta}^{t} \omega(s) ds d\theta \\ &\leq \int_{-h_1}^{0} \int_{t+\theta}^{t} \omega^T(s) e^{\alpha_1 \theta} Z \omega(s) ds d\theta; \end{aligned}$$

where $a = \frac{\alpha_1^2}{h_1 \alpha_1 e^{\alpha_1 h_1} - e^{\alpha_1 h_1} + 1}$.

Proof. See the appendix.

3. STABILITY ANALYSIS

First consider the system,

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(i_k h), t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}), \\ x(t) = \varphi(t), t \in [t_0 - h_1, t_0). \end{cases} \tag{5}$$

Assume $T^+(t_0, t) = 0$, that is $(i_{k+1} - i_k)h + \tau_{k+1} \leq h_1$ for $\forall k \in \{1, 2 \ldots\}$.

For convenience, $V(t)$ is used to denote Lyapunov functional candidate instead of $V(t, x_t)$. Choose a Lyapunov functional candidate to be

$$V(t) = x^T(t) P x(t) + \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) e^{\alpha(s-t)} Z \dot{x}(s) \, \mathrm{d}s \mathrm{d}\theta \tag{6}$$

where P and Z are positive definite matrices to be determined.

Lemma 2. Given constant $\alpha_1 > 0$, $h_1 > 0$, and matrix K, if there exist matrices $P > 0, Z > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, and any matrices Y, T with appropriate dimensions such that

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & h_1 A^T Z \\ * & \varphi_{22} & h_1 K^T B^T Z \\ * & * & -h_1 Z \end{bmatrix} < 0, \quad (7)$$

$$\Theta = \begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & e^{-\alpha_1 h_1} Z \end{bmatrix} \geq 0, \quad (8)$$

then, along the trajectory of the system (5), we have

$$V(t) \leq e^{-\alpha(t-t_0)} V(t_0), \quad (9)$$

where $\varphi_{11} = A^T P + PA + Y + Y^T + h_1 X_{11} + \alpha_1 P$,
$\varphi_{12} = PBK - Y + T^T + h_1 X_{12}$,
$\varphi_{22} = -T - T^T + h_1 X_{22}$.

Proof. See the appendix.

Lemma 3. Given matrices $P > 0, Z > 0$, and scalar $\alpha_1 > 0, h_1 > 0$, if there exist scalar $\alpha_2 > 0$, and appropriate matrices $M_i (i = 1, 2, 3)$ such that the following LMI holds,

$$\begin{bmatrix} \Theta_1 & -M_1 + M_2^T & (\alpha_1 + \alpha_2) a h_1 Z + M_3^T & M_1 \\ * & -M_2 - M_2^T & -M_3^T & M_2 \\ * & * & -(\alpha_1 + \alpha_2) a Z & M_3 \\ * & * & * & -e^{-\alpha_1 h_1} Z \end{bmatrix} < 0, \quad (10)$$

then for Lyapunov functional candidate (6), along the trajectory of $\dot{x}(t) = Ax(t), x(\theta) = \varphi(\theta), \theta \in [t_0 - h_2, t_0]$, it holds that

$$V(t) \leq e^{\alpha_2 (t-t_0)} V(t_0), \quad (11)$$

where

$$\Theta_1 = PA + A^T P + h_1 A^T Z A - \alpha_2 P + h_1^2 a Z_1 + M_1 + M_1^T,$$

$$a = \frac{\alpha_1^2}{h_1 \alpha_1 e^{\alpha_1 h_1} - e^{\alpha_1 h_1} + 1}.$$

Proof. See the appendix.

Now, we are in the position to give the main result of this section.

Theorem 1. Given scalars $h_1 > 0$, $\alpha_i > 0 (i = 1, 2)$ and matrix K, if there exist matrices $P > 0$, $Z > 0$ and $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, and any appropriate matrices Y, T, M_1, M_2 and M_3 such that LMIs (7) and (8) and (10) hold, then under the average unavailable rate

$$\frac{T^+(t_0, t)}{T^-(t_0, t)} \leq \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}, \alpha^* \in (0, \alpha_1) \quad (12)$$

system (3) is exponentially stable and

$$\|x(t)\| \leq \sqrt{\frac{V(t_0)}{\lambda_{min}(P)}} e^{-0.5\alpha^* t}. \quad (13)$$

Proof. See the appendix.

The basic principle of Theorem 1 is that by estimating decay rate α_1 of Lyapunov functional candidate $V(t)$ in available time interval and estimating growth rate α_2 of $V(t)$ in unavailable time interval, search for the conditions for exponential stability. Thus, this method is called Lyapunov functional exponential estimation (LFEE) method.

Remark 2. In Yue et al. [2004] and Yue et al. [2005], the form of exponential stability is not given because the developed LMIs' condition in such papers does not contain any information about decay degree. In Theorem 1, decay degree $0.5\alpha^*$ can be determined by LMIs (7), (8) and (10).

Remark 3. Theorem 1 gives the calculating method for $T^+(t_0, t)$ during the NCSs' running in interval $[t_0, t)$. Notice that from the unavailable rate (12), it holds that

$$\frac{T^+(t_0, t)}{t - t_0} \leq \frac{\alpha_1 - \alpha^*}{\alpha_1 + \alpha_2}.$$

Thus, if $T^+(t_0, t)$ is not more than $\frac{\alpha_1 - \alpha^*}{\alpha_1 + \alpha_2}$ of the whole time $t - t_0$, system (3) is exponentially stable.

Remark 4. By Corollary 1, noticing that $\frac{T_m^+(t_0,t)}{T_m^-(t_0,t)} \leq \frac{h_2 - h_1}{h_1 - h_0}$, where $h_0 \geq \tau_k \geq 0$ (h_0 can be seen in Fig. 1.), we can let $\frac{h_2 - h_1}{h_1 - h_0} \leq \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}$, then MATI can be obtained as $h_2 = h_1 + \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}(h_1 - h_0)$. Existing papers (Kim et al. [2003], Yue et al. [2004, 2005]) fail to analyze the unavailable time and concentrate on the size of h_1, and neglect the important part $\frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}(h_1 - h_0)$, and may lead to much conservativeness.

Remark 5. To obtain h_2, we can first solve the LMI (7) and (8) to obtain h_1. Then according to obtained P, Z, solve LMI (10) to get α_2 and then calculate $h_2 = h_1 + \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}(h_1 - h_0)$ according to Remark 4. Notice that a smaller α_2 will lead to a lager h_2. Thus to get a larger MATI h_2, we can adopt the following method to minimize α_2.

$$\begin{array}{c} Minimize \ \ \alpha_2 \\ S.t. \quad LMI(10). \end{array}$$

Also, it can be seen that the maximum h_1 does not necessarily means the maximums of h_2 since h_2 is also affected by α_1, α_2 and α^*.

4. A NUMERICAL EXAMPLE

Consider the following NCS coming from Zhang et al. [2001] and Yue et al. [2004]

$$\dot{x}(t) = Ax(t) + Bu(i_k h), \ \ t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \quad (14)$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$, $k = 1, 2, \ldots$. According to the switching state feedback control proposed in this paper, a new model is described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(i_k h), \ \ t \in [i_k h + \tau_k, j_k), \\ \dot{x}(t) = Ax(t), \ \ t \in [j_k, i_{k+1} h + \tau_{k+1}), k = 1, 2, \ldots, \end{cases} \quad (15)$$

where j_k is defined as (4). We will employ the same controller as in Zhang et al. [2001] and Yue et al. [2004], that is $u(t) = [-3.75 \ -11.5] x(t)$. Based on Theorem 1 and Remark 5, given $h_1 = 0.69; \alpha_1 = 0.67$, it is obtained that $\alpha_2 = 0.8676$. Therefore, if without considering the effect of induced-delay, that is $\tau_k = 0$, then MATI $h_2 = (1 + \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha^*}) h_1 = 1.2089$ for given $\alpha^* = 0.01$ from Remark 5, and corresponding decay degree is $0.5\alpha^* = 0.005$ based on Theorem 1. If the upper bound of τ_k is known, for example, constant delay case, $\tau_k = 0.1$, thus $h_0 = 0.1$, and MATI $h_2 = 1.0432$ for given $\alpha^* = 0.002$. Moreover, the decay degree is $0.5\alpha^* = 0.5 * 0.002 = 0.001$. The MATI obtained by different methods are given in Table 1. It is obvious our method is much less conservative. Simultaneously, using the method in this paper, decay degree of system state can also be explicitly given for different MATI.

Table 1. MATI for different methods

Methods	Zhang et al. [2001]	Kim et al. [2003]	Yue et al. [2004]	Yue et al. [2005]	Theorem 1
MATI	4.5×10^{-4}	0.7805	0.8695	0.8871	1.2089

5. CONCLUSION

A new model of NCSs has been developed in this paper under the event-time-driven scheme. To deal with the problems of stability analysis for such systems, LFEE method is presented, which is less conservative than existing methods since the case of unavailable time is sufficiently considered. Also, decay degree can be explicitly given. An example is given to show the effectiveness of the proposed method.

6. APPENDIX

Proof of Lemma 1. By Schur complement, it is true that

$$\begin{bmatrix} \omega^T(s)e^{\alpha_1\theta}Z\omega(s) & \omega^T(s) \\ * & e^{-\alpha_1\theta}Z^{-1} \end{bmatrix} \geq 0 \tag{16}$$

Integrating inequality (16) from $t+\theta$ to t leads to

$$\begin{bmatrix} \int_{t+\theta}^{t}\omega^T(s)e^{\alpha_1\theta}Z\omega(s)ds & \int_{t+\theta}^{t}\omega^T(s)ds \\ * & -\theta e^{-\alpha_1\theta}Z^{-1} \end{bmatrix} \geq 0 \tag{17}$$

Integrating inequality (17) from $-h_1$ to 0 leads to

$$\begin{bmatrix} \int_{-h_1}^{0}\int_{t+\theta}^{t}\omega^T(s)e^{\alpha_1\theta}Z\omega(s)dsd\theta & \int_{-h_1}^{0}\int_{t+\theta}^{t}\omega^T(s)dsd\theta \\ * & \int_{-h_1}^{0}-\theta e^{-\alpha_1\theta}Z^{-1}d\theta \end{bmatrix} \geq 0 \tag{18}$$

Noticing that

$$\int_{-h_1}^{0}-\theta e^{-\alpha_1\theta}d\theta = \frac{1}{\alpha_1^2}(h_1\alpha_1e^{\alpha_1h_1}-e^{\alpha_1h_1}+1),$$

the proof is completed by Schur complement.

Proof of Lemma 2.

Let $d(t)=t-i_kh$ for $t\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1})$, then system (5) can be rewritten into the following form

$$\begin{cases} \dot{x}(t)=Ax(t)+BKx(t-d(t)), t\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1}), \\ x(t)=\varphi(t), t\in[t_0-h_1, t_0). \end{cases} \tag{19}$$

Notice that $0\leq d(t)\leq h_1$. From Lemma 1 in Sun et al. [2006], it holds that $V(t)\leq e^{-\alpha_1(t-\tau_k)}V(i_kh+\tau_k)$ for any $k\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1})$. Thus it is obtained that

$$\begin{aligned} V(t) &\leq e^{-\alpha_1(t-(i_kh+\tau_k))}V(i_kh+\tau_k) \\ &\leq e^{-\alpha_1(t-(i_{k-1}h+\tau_{k-1}))}V(i_{k-1}h+\tau_{k-1}) \\ &\leq \ldots \leq e^{-\alpha_1(t-t_0)}V(t_0). \end{aligned}$$

This completes the proof.

Proof of Lemma 3. From the Leibniz-Newton formula, the following equations are true

$$2\xi(t)^TM\left[x(t)-x(t-h_1)-\int_{t-h_1}^{t}\dot{x}(s)ds\right]=0, \tag{20}$$

where $\xi(t)=\left[x^T(t)\ x^T(t-h_1)\ \int_{t-h_1}^{t}x^T(s)ds\right]^T$,

$$M=\left[M_1^T\ M_2^T\ M_3^T\right]^T.$$

Based on Lemma 1, the following inequalities hold

$$\begin{aligned} &-\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)e^{\alpha_1\theta}Z\dot{x}(s)dsd\theta \\ &\leq -\left[h_1x^T(t)-\int_{t-h_1}^{t}x^T(s)ds\right]aZ \\ &\quad\times\left[h_1x(t)-\int_{t-h_1}^{t}x(s)ds\right] \end{aligned} \tag{21}$$

Noting that (20)-(21), and calculating the derivative of Lyapunov functional candidate (6) along the trajectory of system $\dot{x}=Ax(t)$ lead to

$$\begin{aligned} &\dot{V}(t)-\alpha_2V(t) \\ &= 2x^T(t)P\dot{x}(t)+h_1\dot{x}^T(t)Z\dot{x}(t) \\ &\quad-\int_{t-h_1}^{t}\dot{x}^T(s)e^{\alpha_1(s-t)}Z\dot{x}(s)ds-\alpha_2x^T(t)Px(t) \\ &\quad-(\alpha_1+\alpha_2)\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)e^{\alpha_1(s-t)}Z\dot{x}(s)dsd\theta \\ &\leq 2x^T(t)P\dot{x}(t)+h_1\dot{x}^T(t)Z\dot{x}(t)-\alpha_2x^T(t)Px(t) \\ &\quad-\int_{t-h_1}^{t}\dot{x}^T(s)e^{-\alpha_1h_1}Z\dot{x}(s)ds \\ &\quad-(\alpha_1+\alpha_2)\left[h_1x^T(t)-\int_{t-h_1}^{t}x^T(s)ds\right]aZ \\ &\quad\times\left[h_1x(t)-\int_{t-h_1}^{t}x(s)ds\right] \\ &\quad+2\xi(t)^TM\left[x(t)-x(t-h_1)-\int_{t-h_1}^{t}\dot{x}(s)ds\right] \\ &= x^T(t)(A^TP+AP+h_1A^TZA-\alpha_2P)x(t) \\ &\quad+\xi^T(t)\Delta\xi(t)-2\xi^T(t)M\int_{t-h_1}^{t}\dot{x}(s)ds \\ &\quad-\int_{t-h_1}^{t}\dot{x}^T(s)e^{-\alpha_1h_1}Z\dot{x}(s)ds \\ &\leq \xi^T(t)\Pi\xi(t) \end{aligned} \tag{22}$$

where,

$$\begin{aligned} \Delta = &-(\alpha_1+\alpha_2)\begin{bmatrix} h_1 \\ 0 \\ -1 \end{bmatrix}aZ[h_1\ 0\ -1] \\ &+M[1\ -1\ 0]+[1\ -1\ 0]^TM^T, \end{aligned}$$

$$\Pi = \begin{bmatrix} \Theta_1 & -M_1+M_2^T & (\alpha_1+\alpha_2)ah_1Z+M_3^T \\ * & -M_2-M_2^T & -M_3^T \\ * & * & -(\alpha_1+\alpha_2)aZ \end{bmatrix} +2Me^{\alpha_1h_1}Z^{-1}M^T.$$

From (10), by Schur complement, it is true that $\Pi<0$. Thus, it holds from (22)

$$\dot{V}(t)-\alpha_2V(t)\leq 0.$$

Multiplying $e^{-\alpha_2t}$ on both sides of this inequality and integrating this inequality from 0 to t lead to (11). This completes the proof.

Proof of Theorem 1. From Lemma 2 and Lemma 3, it holds that

$$V(t) \leq \begin{cases} e^{-\alpha_1(t-i_k h-\tau_k)}V(i_k h+\tau_k), t \in [i_k h+\tau_k, j_k), \\ e^{\alpha_2(t-j_k)}V(j_k), t \in [j_k, i_{k+1}h+\tau_{k+1}), \end{cases}$$

$\forall k \in \{1,2,\ldots\}$. Without loss of generality, we assume that $t \in [j_k, i_{k+1}h+\tau_{k+1})$. Thus, it can seen that

$$\begin{aligned} V(t) &\leq V(j_k)e^{\alpha_2(t-j_k)} \leq V(i_k h+\tau_k)e^{-\alpha_1(j_k-i_k h-\tau_k)}e^{\alpha_2(t-j_k)} \\ &\leq V(j_{k-1})e^{\alpha_2(i_k h+\tau_k-j_{k-1})}e^{-\alpha_1(j_k-i_k h-\tau_k)}e^{\alpha_2(t-j_k)} \\ &\leq \ldots \qquad (23) \\ &\leq V(t_0)e^{-\alpha_1 T^-(t_0,t)+\alpha_2 T^+(t_0,t)} \end{aligned}$$

Noting the average unavailable rate condition (12), we have

$$V(t) \leq e^{-\alpha^*(t-t_0)}V(t_0).$$

It is true that

$$\lambda_{min}(P)\|x(t)\|^2 \leq V(t), \qquad (24)$$

From (23) and (24), (13) follows. Thus, the proof is completed.

REFERENCES

D. Carnevale, A. R. Teel, and D. Nesic. A lyapunov proof of an improved maximum allowable transfer interval for networked control systems. *IEEE Trans. Automat. Contr.*, 52(5):892–897, 2007.

H. Gao and C. Wang. Comments and further results on ‘a descriptor system approach to H∞ control of linear time-delay systems. *IEEE Trans. Automat. Contr.*, 48(3):520–525, 2003.

K. Gu, V. kharitonov, and J. Chen. *Stability of Time-Delay Systems*. Boston, Birkhauser, 2003.

J. K. Hale and S. M. Verduyn Lunel. *Introduction to Functional Differential Equations*. Springer-Verlag, New York, 1993.

Y. He, Q. G. Wang, and C. Lin. An improved H∞ filter design for systems with time-varying interval delay. *IEEE Transactions on Circuits and Systems, Part II*, 53(11):1235–1239, 2006.

D. Kim, Y. Lee, W. Kwon, and H. Park. Maximum allowable delay bounds of networked control systems. *Control Engineering Practice*, 11:1301–1313, 2003.

G. P. Liu, J. X. Mu, D. Rees, and S.C. Chai. Design and stability analysis of networked control systems with random communication time delay using the modified mpc. *International Journal of Control*, 79(4):287–296, 2006.

G. P. Liu, Y. Xia, D. Rees, and W.S. Hu. Design and stability criteria of networked predictive control systems with random network delay in the feedback channel. *IEEE Transactions on Systems, Man and Cybernetics*, 54(3):1282–1297, 2007.

D. Nesic and A. R. Teel. Input output stability properties of networked control systems. *IEEE Trans. Automat. Contr.*, 49:1650–1667, 2004a.

D. Nesic and A. R. Teel. Input to state stability of networked control systems. *Automatica*, 40:2121–2128, 2004b.

H. Park, Y. Kim, D. Kim, and W. Kwon. A scheduling method for network based control systems. *IEEE Transactions on Control Systems Technology*, 10:318–330, 2002.

X. M. Sun, J. Zhao, and D. J. Hill. Stability and L_2-gain analysis for switched delay systems: a delay-dependent method. *Automatica*, 42(10):1769–1774, 2006.

M. Tabbara, D. Nesic, and A. R. Teel. Stability of wireless and wireline networked control systems. *IEEE Trans. Automat. Contr.*, 52(9):1615–1630, 2007.

G. C Walsh, O. Beldiman, and L. G. Bushnell. Asymptotic behavior of nonlinear networked control systems. *IEEE Trans. Autom. Control*, 46:1093–1097, 2001.

G. C Walsh, H. Ye, and L. G. Bushnell. Stability analysis of networked control systems. *IEEE Transactions on Control Systems Technology*, 10:438–446, 2002.

D. Yue, Q. L. Han, and P. Chen. State feedback controller design of networked control systems. *IEEE Transactions on Circuits and SystemsłII: Express Briefs*, 51(11):640–644, 2004.

D. Yue, Q. L. Han, and J. Lam. Network-based robust H∞ control of systems with uncertainty. *Automatica*, 41:999–1007, 2005.

G. S. Zhai and H. Lin. Controller failure time analysis for symmetric H∞ control. *International Journal of Control*, 77(6):598–605, 2004.

W. Zhang, M. S. Branicky, and S. M. Phillips. Stability of networked control systems. *IEEE Contr. Syst. Mag.*, 21(1): 84–99, 2001.