Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Heart Rate Regulation During Exercise with Various Loads: Identification and Nonlinear H_∞ Control ⋆

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Abstract: A model for the heart rate response to treadmill walking exercise is proposed in this paper. The parameters of the model were experimentally identified which involved subjects walking at different speeds. A 2-degree-of-freedom controller was then developed for the regulation of the heart rate response during treadmill exercise. The controller consists of a piecewise LQ and an H_∞ sub-controllers. Experimental results demonstrated that the heart rate of the subjects were regulated by the proposed controller.

Keywords: Biomedical systems; Non-linear systems modelling; Robust control design; Application; Piecewise affine system; Experimental study

1. INTRODUCTION

In order to meet the metabolic demand during exercise, the heart rate (HR) of an exerciser increases. Thus, knowing how HR responds to exercise will improve our understanding of exercise physiology. In addition, it may also be useful for predicting cardiovascular disease mortality (Savonen et al. [2006], Cole et al. [1999]). The understanding of HR response may also lead to an improvement in developing training protocols for athletics and more efficient weight loss protocols for the obese, and in facilitating assessment of physical fitness and health of individuals (Achten and Jeukendrup [2003]). Furthermore, knowing the cardiovascular system responses to the stress induced by physical exercise provides us another perspective on how this system functions. For instance, this may give us some measures for the prevention of cardiac failure from dialysis.

HR response during exercise have been widely studied, e.g. Brodan et al. [1971], Hajek et al. [1980], Rowell [1993], Coyle and Alonso [2001], Su et al. [2007]), among them a number of models have been proposed. Broden et al. Brodan et al. [1971] and Hajek et al. Hajek et al. [1980] modelled the HR response from a regulation point of view. Their models are reliable for short duration exercises, but are not sufficient for explaining long duration exercises. As shown in, e.g. Coyle and Alonso [2001], HR will continue to increase during prolonged exercise. In reference (Su et al. [2007]), exercising HR response was modelled by a Hammerstein system [1]. Besides modelling, they also studied the control of the HR response during exercise.

The ability to control the HR during exercise is of importance in the design of exercise protocols for patients with cardiovascular diseases and in developing rehabilitation exercises to aid patients recovering from cardiothoracic surgery. The control of heart rate response during exercise has been reported in the references (Kawada et al. [1999], Cooper et al. [1998], Su et al. [2007]). Among them, a number of different control strategies or algorithms have been successfully applied, e.g. classical PID control, H_∞ control, and model reference control. Each has its merits or disadvantages and therefore, it is interesting to investigate the usefulness of other control algorithms and techniques that have been developed by the control society.

The objective of this paper is twofold. First, a nonlinear model is proposed to describe the HR response to treadmill walking exercise during both the exercising and the recovery phases. Secondly, using the proposed model, we develop a controller-using the treadmill's speed as a control variable-that regulates the HR during exercise.

2. THE MODEL

In this paper, we propose the following nonlinear state-space control systems to model the HR response to treadmill walking exercise:

$$\begin{aligned} \dot{x}_1(t) &= -a_1 x_1(t) + a_2 x_2(t) + a_2 u^2(t) \\ \dot{x}_2(t) &= -a_3 x_2(t) + \phi(x_1(t)) \\ z(t) &= x_1(t) \\ \phi(x_1(t)) &:= \frac{a_4 x_1(t)}{1 + \exp\big(-(x_1(t) - a_5)\big)} \end{aligned} \tag{1}$$

where $x(0) = [x_1(0) \quad x_2(0)]^T = 0$, $z(t)$ describes the change in HR from rest, and $a_1, ..., a_5$ are positive scalars. The control input $u(t)$ represents the normalised speed of

⋆ This work was supported by the Australian Research Council.
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[1] A system consists of a static nonlinearly cascaded at the input of a linear system.

	Age (yr)	Height (cm)	Weight (kg)	BMI (kg/m^{-2})
mean	29.3	174	68.5	22.5
std	5.9	3.4	12.6	3.4
range	23–38	169–178	53–85	18–27

Table 1. Physical characteristics of the subjects: age, height, weight, and BMI (Body Mass Index)

the treadmill and it is normalised by 8 km/h, assuming the maximum walking speed is 8 km/h. The unit of time t is in minutes.

System (1) can be viewed as a feedback interconnected system, i.e. x_1 in the forward path and x_2 in the feedback path. The component $x_1(t)$ can be viewed as the change of HR due to the neural response to exercise, including both the parasympathetic and the sympathetic neural inputs (see e.g. Rowell [1993]). The component x_2 is utilised in describing the complex slow-acting peripheral effects from, e.g. the hormonal systems, the peripheral local metabolism, and/or the increase in body temperature, etc.. Generally, these effects cause vasodilatation and hence HR needs to be increased in order to maintain the arterial pressure (see McArdle et al. [2007])). So, the feedback signal x_2, which can be thought of as a dynamic disturbance input to the x_1 subsystem, is a reaction to the peripheral local effects. By observing system (1), the input u drives the system nonlinearly, describing the nonlinear increase of the HR in response to the increase in walking speed. It has been observed that there is a curvilinear relationship between aerobic demand and walking speed (see, e.g. McArdle et al. [2007]). The quadratic increase in HR in response to an increase in walking speed was observed in Johnson [2007].

2.1 Experimental Setup

The parameters in system (1) were identified from experimental data. The setup of the experiment is described in this section.

Subject: Six healthy male subjects were studied. The physical characteristics of the subjects are given in Table 1.

Procedure: Each subject completed three exercise sessions in separate occasions. In each session, a subject was requested to walk on a treadmill at a given speed (5km/h, 6km/h, and 7km/h) for 15 minutes with a recovery period of 15 minutes. After three sessions, each subject completed the treadmill walking exercise at the three different speeds.

Data acquisition: In this study, the Powerjog fully motorised medical grade treadmill was used. The HR of the subjects was monitored by the wireless Polar system and recorded by LabVIEW. To remove noises, the HR measurements were filtered using the moving average with a 5-second window.

Parameter estimation: Using the measured HR data of all subjects and the Levenberg-Marquardt method, the parameters in system (1) were estimated. Since three sets of input-output measurements were collected for each subject (where the input is the speed of the treadmill and the output is the HR), there were 18 sets of input-output measurements in total. To search for a parameter set that

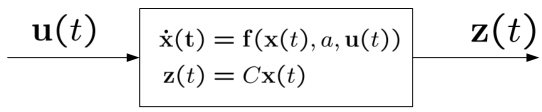

Fig. 1. A Multi-input-multi-output system.

gives a good fit to all the measurements, we estimated the parameters by using all the measurements simultaneously. In other words, we estimated the parameters of the following multi-input multi-output system (see Figure 1):

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), a, \mathbf{u}(t)) \\ \mathbf{z}(t) &= C\mathbf{x}(t), \quad \mathbf{x}(0) = 0 \end{aligned} \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^{36}$, $\mathbf{u} = [u_1 \quad u_2 \dots u_{18}]^T \in \mathbb{R}^{18}$ and $\mathbf{z} = [z_1 \quad z_2 \dots z_{18}]^T \in \mathbb{R}^{18}$ and $a = [a_1 \quad a_2 \dots a_5]^T \in \mathbb{R}^5$. The measurement matrix C was defined as:

$$C_{i,j} = \begin{cases} 1 & \text{if } j = 2i - 1 \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

for $i = 1, 2, \dots, 18$ and $j = 1, 2, \dots, 36$. To make the estimation more robust, the output $z_i(t)$ from the input $u_i(t)$ was defined as $z_i(t) = (HR_i(t) - 74)/4$, where $HR_i(t)$ is the absolute HR at time t, 74 bpm is the average resting HR for all the subjects (resting HR was estimated from the 3-minute resting period before exercise), and 4 bpm is a normalising factor.

For estimation, the objective function was chosen as

$$S(a) = \sum_{i=1}^{N} (\mathbf{z}(t_i) - \hat{\mathbf{z}}(t_i, a))^T (\mathbf{z}(t_i) - \hat{\mathbf{z}}(t_i, a)) \tag{4}$$

where, for $i = 1, 2, \dots, N$, $\mathbf{z}(t_i)$ is the measurement of the output vector at time t_i and $\hat{\mathbf{z}}(t_i, a)$ is the output of system (2) with the parameter vector a. With the objective function (4), the Levenberg-Marquardt method was used to determine an estimate of a which was denoted as $\hat{a} := [\hat{a}_1 \quad \hat{a}_2 \dots \hat{a}_5]^T$ (see, e.g. Bard [1974], Stortelder [1996], Englezos and Kalogerakis [2001]). Based on a linear approximate method (see e.g. Stortelder [1996]), an approximate $100(1-\alpha)\%$ independent confidence interval for each estimate is given by $(\hat{a}_i - \delta a_i, \hat{a}_i + \delta a_i)$, for $i = 1, 2, \dots, 5$, with

$$\delta a_i = \sqrt{\frac{p}{Nm - p} S(\hat{a}) \mathcal{F}_\alpha(p, Nm - p) [A^{-1}]_{i,i}} \tag{5}$$

where $\mathcal{F}_\alpha(p, Nm - p)$ denotes the upper α quantile for Fishers $\mathcal{F}$-distribution with p and $Nm - p$ degrees of freedom and the scalar $[A^{-1}]_{i,i}$ is the (i, i) diagonal element of the inverse matrix A that is defined as

$$A := \sum_{i=1}^{N} G'(t_i) C' C G(t_i), \quad G(t_i) := \left. \frac{\partial \mathbf{f}}{\partial a} \right|_{a = \hat{a}, t = t_i}. \tag{6}$$

In this study, $N = 180$, $m = 18$ and $p = 5$. An α level of 0.05 was used for obtaining the confidence intervals of parameter estimates. Table 2 summaries the estimated parameters of the model (1).

3. CONTROLLER DESIGN

In the second part of this paper, a controller design is proposed for the regulation of HR. The controller essentially controls the speed of the treadmill and in turn controls the HR during treadmill exercise. System (1) is first written in a state-space form as follows:

Parameter estimates, $\hat{a}$ (δa)				
$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$
1.84 (0.36)	24.32 (4.36)	6.36×10^{-2} (1.95×10^{-2})	3.21×10^{-3} (6.84×10^{-4})	8.32 (0.44)

Table 2. Estimated parameter $\hat{a}$ and parameter variation δa.

$$\begin{aligned} \dot{\eta}(t) &= A\eta(t) + B_1\Phi(\eta_1(t)) + B_2 g(u(t)) \\ y(t) &= C\eta(t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} A &= \begin{bmatrix} -3.07 \times 10^{-2} & 2.7 \times 10^{-2} \\ 0 & -1.06 \times 10^{-3} \end{bmatrix}, \ \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 2.7 \times 10^{-2} \\ 0 \end{bmatrix}, \ C' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ \Phi(\eta_1(t)) &:= \frac{8.03 \times 10^{-4}\eta_1(t)}{1 + \exp\bigl(-15(\eta_1(t) - 5.55 \times 10^{-1})\bigr)}, \\ g(u(t)) &:= u^2(t). \end{aligned} \tag{8}$$

To obtain (7), we have defined the normalised output $y(t) = \eta_1(t) := x_1(t)/15$ (i.e. $y(t) := (\mathrm{HR}(t) - \mathrm{HR}_{\mathrm{rest}})/60$). The time unit of t in (7) is in seconds, instead of minutes as in (1).

System (7) is a nonlinear system with nonlinearity $\Phi(\eta_1)$ and nonlinear control input $g(u)$. To overcome the control input nonlinearity, a transformed input

$$v(t) = g(u(t)) \tag{9}$$

is defined. As for the nonlinear function $\Phi(\eta_1)$, it can be approximated by a piecewise linear function

$$\gamma(\eta_1) = \begin{cases} 0 & \text{if } \eta_1 \le 0.419 \\ 1.52 \times 10^{-3}\eta_1 - 6.34 \times 10^{-4} & \text{if } \eta_1 > 0.419. \end{cases} \tag{10}$$

In fact, $\gamma(\eta_1)$ is obtained by linearising the function $\Phi(\eta_1)$ at $\eta_1 = 0$ and 0.5. As a result, system (7) can be approximated by a piecewise affine system (see e.g. Rantzer and Johansson [2000]).

In this paper, we adopt a two-degree-of-freedom (2-DOF) controller consisting of a piecewise LQ feedforward and a H_∞ feedback controllers, as shown in Figure 2, for the control and regulation of heart rate responses.

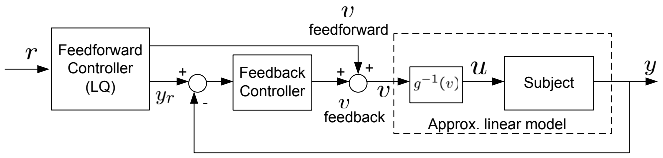

Fig. 2. Control configuration

3.1 LQ Feedforward Controller Design

First, we design a feedforward controller using the piecewise LQ optimal control technique of Rantzer and Johansson [2000]. In doing so, we also incorporate an integral action in the controller, see e.g., Burl [1999].

Define two partitions of the state space as shown in Figure 3:

$$\begin{aligned} X_1 &:= \{[\eta_1 \ \ \eta_2]' \in \mathbb{R}^2 \big| \ \eta_1 < 0.419\} \\ X_2 &:= \{[\eta_1 \ \ \eta_2]' \in \mathbb{R}^2 \big| \ \eta_1 \ge 0.419\}. \end{aligned} \tag{11}$$

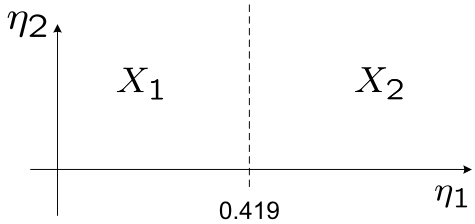

Fig. 3. Partition of state space.

Next, define

$$\bar{A}_i = \begin{bmatrix} A_i & 0_{2\times 1} & a_i \\ -C & 0 & 0 \\ 0_{1\times 2} & 0 & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B_2 \\ 0 \\ 0 \end{bmatrix}, \ \bar{\eta}(t) = \begin{bmatrix} \eta(t) \\ e(t) \\ 1 \end{bmatrix} \tag{12}$$

for $\eta \in X_i$ and $i = 1, 2$, where

$$\begin{aligned} A_1 &= \begin{bmatrix} -3.07 \times 10^{-2} & 2.7 \times 10^{-2} \\ 0 & -1.06 \times 10^{-3} \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -3.07 \times 10^{-2} & 2.7 \times 10^{-2} \\ 1.52 \times 10^{-3} & -1.06 \times 10^{-3} \end{bmatrix}, \\ a_1 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0 \\ -6.39 \times 10^{-4} \end{bmatrix}, \\ e(t) &= \int_0^t (r - C\eta(t))dt, \end{aligned} \tag{13}$$

and r is the constant reference input. Therefore, we have

$$\begin{aligned} \dot{\bar{\eta}}(t) &= \bar{A}_i\bar{\eta}(t) + \bar{B}v(t), \quad \text{for } \eta \in X_i \\ y(t) &= \bar{C}\bar{\eta}(t). \end{aligned} \tag{14}$$

where $\bar{C} = [C \ 0 \ 0]$. Then, the control problem is to minimise the following cost function:

$$J = \int_0^\infty (\bar{\eta}'(t)\bar{Q}\bar{\eta}(t) + v'(t)Rv(t))dt \tag{15}$$

for any given $\bar{Q} \ge 0$ and $R > 0$. In the control design, the matrix $\bar{Q}$ and the value of R were chosen as follows:

$$\bar{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = 350. \tag{16}$$

The minimising control law is

$$v(t) = L_i\bar{\eta}(t), \qquad \eta \in X_i, i = 1, 2 \tag{17}$$

where

$$\begin{aligned} L_1 &= \begin{bmatrix} -8.8 \times 10^{-1} & -9.7 \times 10^{-1} & 3.8 \times 10^{-2} & 0 \end{bmatrix} \\ L_2 &= \begin{bmatrix} -9.1 \times 10^{-1} & -1.0 & 3.8 \times 10^{-2} & 1.9 \times 10^{-2} \end{bmatrix}. \end{aligned} \tag{18}$$

The gains L_1 and L_2 (18) are obtained by using PWLTool, a MATLAB Toolbox for Piecewise Linear Systems (see Hedlund and Johansson [1999]).

In turn, the LQ feedforward controller is in the form:

$$\begin{aligned} \dot{\bar{\eta}}(t) &= \bar{A}_i\bar{\eta}(t) + \bar{B}v(t) + B_r r, \quad \text{for } \eta \in X_i \\ y_r(t) &= \bar{C}\bar{\eta}(t), \quad v(t) = L_i\bar{\eta}(t) \end{aligned} \tag{19}$$

where $\bar{\eta}(0) = [0 \ 0 \ 0 \ 1]'$, $B_r = [0 \ 0 \ 1 \ 0]'$ and r is the reference input. In other words, the input to this feedforward controller is the reference r and the output are the feedforward control $v(t)$ and the "smoothed" reference $y_r(t)$. It is clear that the LQ controller is a switching controller (see e.g. Savkin and Evans [2002]), here the control law is chosen depending on the state $\eta(t)$.

3.2 H_∞ Controller Design

Next, we design a feedback controller based on the H_∞ control technique (see e.g. Petersen et al. [2000], Petersen and Savkin [1999], Moheimani et al. [1998]). We first linearise the system (7) and then formulate the control problem as a mixed sensitivity problem (see e.g. Skogestad and Postlethwaite [1996], Zhou and Doyle [1998] for details). In a mixed sensitivity problem, the idea is to choose some weighing functions, namely $W_1(s)$, $W_2(s)$ and $W_3(s)$ to satisfy the control objectives.

The system (7) was first linearised at $x_0 = [0.5 \;\; 0.115]^T$, $v_0 = 0.452$, and the transfer function of the linearised model is given by

$$G(s) = \frac{0.027s + 2.86 \times 10^{-5}}{s^2 + 0.0317s - 8.65 \times 10^{-6}}. \tag{20}$$

Then, the weighting functions were chosen as

$$W_1(s) = \frac{0.1(s + 0.083)}{(s + 8.33 \times 10^{-5})},\; W_2(s) = \frac{70(s + 6.25 \times 10^{-4})}{(s + 4.38)},$$
$$W_3(s) = \frac{100(s + 0.066)}{(s + 8.33)}. \tag{21}$$

The weighting function $W_1(s)$ is chosen as a high-gain lowpass filter approximating an integral action in order to ensure good tracking accuracy. A first-order high-pass filter is chosen for $W_2(s)$ to limit the controller bandwidth and magnitude. The weighting function $W_3(s)$ is chosen to accommodate any multiplicative modelling error. The inverse of the weighting functions $W_i(s)$, $i = 1, 2, 3$, are shown in Figure 4.

By using MATLAB Robust Control Toolbox, we then obtain a robust controller

$$K(s) = \frac{8.91 \times 10^{-3}s^4 + 0.11s^3 + 0.33s^2 + 0.01s + 2.6 \times 10^{-6}}{s^5 + 9.1s^4 + 6.43s^3 + 0.42s^2 + 4.61 \times 10^{-4}s + 3.56 \times 10^{-8}} \tag{22}$$

and $\gamma = 0.9$ so that $|S(jw)| \le \gamma/|W_1(jw)|$, $|T(jw)| \le \gamma/|W_3(jw)|$ and $|K(jw)S(jw)| \le \gamma/|W_2(jw)|$ for all ω. The functions S and T are the sensitivity and the complementary sensitivity functions, respectively (see Figure 4).

4. CONTROLLER VERIFICATION

By using the control design presented in Section 3, a controlled treadmill system was implemented for the heart rate regulation (see Figure 5).

The Powerjog fully motorised treadmill was connected to the controller via an RS232 serial port. The heart rate was collected in a similar way as in the stage of identification of the model (see Section 2.1). Except that here the computer (LabVIEW) collected heart rate signal from the wireless Polar system every 6 seconds. Also, an exponential smoothing with filter coefficient $\alpha = 0.75$ was employed on-line (see Diggle [1990]).

As for the controller, it was implemented in LabVIEW. The feedforward controller (19) was pre-computed offline, whereas the robust feedback controller (22) was discretized using the zero-order-hold method with a sampling period $T = 6$ seconds. The control signal was then sent to the treadmill via the serial port.

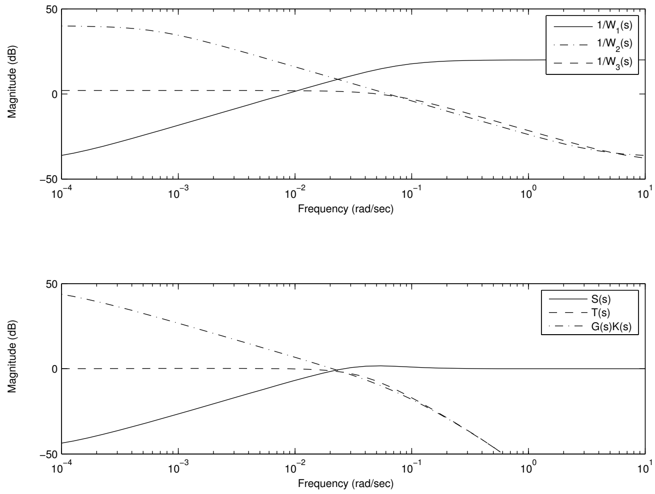

Fig. 4. Inverse weighting functions, sensitivity, complementary sensitivity, and loop gain.

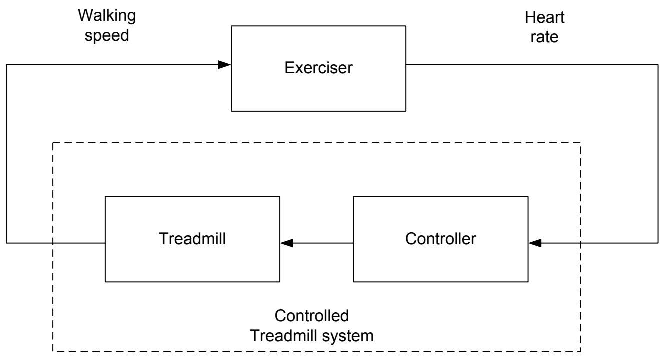

Fig. 5. Controlled Treadmill System.

To validate the controller, the 6 subjects participated in system identification were requested to exercise on the treadmill and prescribed with 2 sets of pre-defined exercise heart rate profiles. The goal was to regulate subjects' heart rate according to the profile. The pre-defined heart rate profiles may be viewed as prescribed training or exercise protocols. The first profile had 3 10-minute stages involving 2 heart rate levels, namely 100bpm and 115bpm. The first 10-minute stage with heart rate 100bpm was considered as a warm-up period, the second 10-minute stage with heart rate 115bpm was the exercise period, and the last stage with heart rate 100bpm was the cool-down period.

The second exercise heart rate profile also includes the warm-up, exercise and cool-down stages. The differences between the first and the second profiles were: 1) the warm-up period was 3-minute long with gradually increase of heart rate to 110bpm from rest; 2) the heart rate level during the exercise phase was 110bpm; 3) the cool-down period was 7 minutes long with heart rate gradually decreasing to the subjects' recovery heart rate from the exercise phase. As shown in Figure 6, the heart rate of each subject closely followed the first exercise heart rate profile and regulated at the pre-defined levels, i.e. 100 and

115 bpm, by using the proposed controller. Similar results were obtained for the second heart rate profile that are shown in Figure 7.

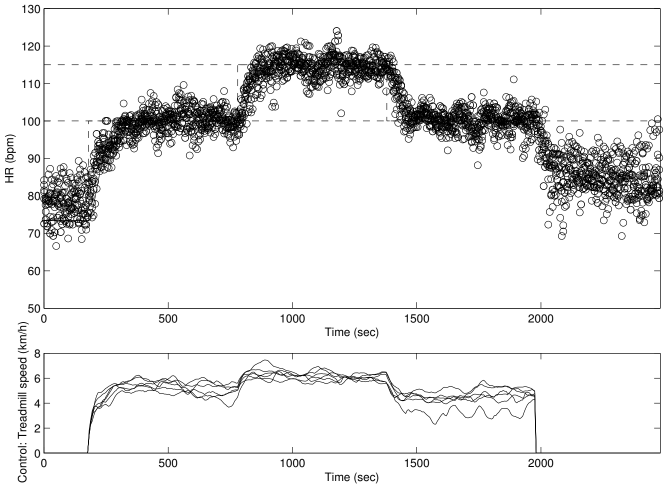

Fig. 6. Regulation of heart rate at 100bpm and 115bpm for all 6 subjects.

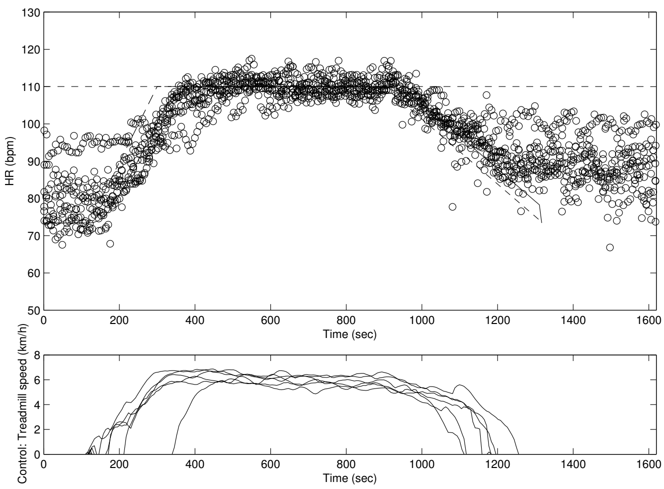

Fig. 7. Heart rate regulation at 110bpm for all 6 subjects with gradually warm-up and cool-down periods.

5. CONCLUDING REMARKS

A nonlinear model describing the heart rate response to the treadmill walking exercise was proposed. The proposed model is a feedback interconnected system. The subsystem in the forward path may be used to describe the neural or the central response, whereas the feedback subsystem may be utilised to describe the peripheral local response. Moreover, the model would be useful in describing the interactions between these two responses. Using the nonlinear model, a controller was developed for the regulation of heart rate during treadmill exercise. The controller consists of a piecewise LQ feedforward and an H_∞ feedback controller. Experimental results demonstrated that the proposed controller had the ability to regulate heart rate for all the experimental subjects. By applying the controller, the heart rate of the subjects could follow two pre-defined heart rate profiles that may represent two kinds of exercise protocols.

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