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# Direct Fuzzy Tracking Control of a Class of Nonaffine Stochastic Nonlinear Systems with Unknown Dead-Zone Input ⋆

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**Abstract:** In this paper, a direct adaptive fuzzy tracking control scheme is presented for a class of stochastic nonaffine uncertain nonlinear systems with unknown dead-zone input. Based on the first-type fuzzy logic system's online approximation capability, a direct adaptive fuzzy tracking controller is developed by using the backstepping approach. It is proved that the design scheme ensures that all the error variables are bounded in probability while the mean square tracking error becomes semiglobally uniformly ultimately bounded(SGUUB) in an arbitrarily small area around the origin. Simulation results show the effectiveness of the control scheme.

Keywords: Nonlinear system control; Adaptive control; Robust controller synthesis; fuzzy control; stochastic system; dead-zone input.

## 1. INTRODUCTION

Recently, the adaptive control of uncertainty nonlinear system has been extensively studied. As a breakthrough in nonlinear control area, a recursive design procedure, backstepping approach, was presented to obtain global stability and asymptotic tracking for a large class of nonlinear system, mostly the strict-feedback system(Krstic et al. [1995]). In addition, some adaptive fuzzy or neural network controllers, which ensured the stability of the closed-loop systems, have also been constructed for nonlinear systems with unknown nonlinear functions (Chen et al. [2007]; Wang et al. [2006]; Zhang et al. [2007a]) by means of the backstepping approach.

Nevertheless, relatively fewer results have been obtained for the nonaffine feedback systems. The nonaffine feedback system has a more representative form than the strict-feedback systems, and there are many systems falling into this category(Wang et al. [2006]). Several special cases of pure-feedback systems are studied by combined the backstepping technique with the neural network or fuzzy system function approximation property in (Wang et al. [2002]; Ge et al. [2002]), while they are still affine in control $u$, the paper of Yu et al. [2004] considered the external disturbances or unmoulded dynamics in some pure-feedback systems, the completely pure-feedback system are investigated in the work of Wang et al. [2006] via small gain theory. On the other hand, some backstepping-based control laws (Deng et al. [1997]; Deng et al. [2000]; Arslan et al. [2002]; Arslan et al. [2003]; Pan [2002]; Liu et al. [2004]; Ji et al. [2002]; Liu et al. [2003]; Wu et al. [2007]; Liu et al. [2007]) for stochastic strict-feedback systems that include a Wiener process have been developed to guarantee stability, known as stability in probability. Combined problem of the control of stochastic strict-feedback nonlinear system with nonlinear uncertainties is firstly studied in the paper of Psillakis et al. [2005].

Nonsmooth nonlinear characteristics such as dead-zone, backlash, hysteresis are common in actuator and sensors such as mechanical connections, hydraulic actuators and electric servomotors. Dead-zone is one of the most important non-smooth nonlinearities in many industrial processes (Tao et al. [1996], Lewis et al. [1999]). Its presence severely limits system performance, and its study has been drawing much interest in the control community for a long time (Zhou et al. [2006], Zhou et al. [2007], Zhang et al. [2007a], Zhang et al. [2007b]). In the paper of Zhang et al. [2007a], an adaptive neural controller was developed for a class of uncertain multi-input multi-output nonlinear state time-varying delay systems in triangular control structure with unknown nonlinear dead-zones and gain signs.

In this paper, we consider adaptive fuzzy tracking controller for a class of stochastic nonaffine uncertain nonlinear systems with dead-zone input. With the mean value theorem, we change the nonaffine nonlinear system into the structure that backstepping can handle. As in the work of Zhang et al. [2007a], the dead-zone output is represented as a simple linear system with a static time-varying gain and bounded disturbance by introducing characteristic function. The first type fuzzy system is employed

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to approximate the unknown nonlinear system. Extensive stability analysis proves that all the error variables are bounded in probability while the mean square tracking error becomes semiglobally uniformly ultimately bounded in an arbitrarily small area around the origin.

This paper is organized as follows. The preliminaries and problem formulation are presented in Section 2. In Section 3, a systematic procedure for the synthesis of the adaptive fuzzy tracking controller is developed. In Section 4, simulation example is used to demonstrate the effectiveness of the proposed scheme. Finally, the conclusion is given in Section 5.

## 2. PRELIMINARIES AND PROBLEM FORMULATION

### *2.1 Stochastic Stability*

The following notation will be used throughout the paper. $R_+$ denotes the set of all nonnegative real numbers, $R^n$ denotes the real $n-$dimensional space. For a given vector or matrix $X$, $X^T$ denotes is transpose, $Tr\{X\}$ denotes its trace when $X$ is square, and $|X|$ denotes the Euclidean norm of a vector $X$. $C^i$ denotes the set or all functions with continuous $i$th partial derivatives. $\mathscr{K}$ denotes the set of all functions: $R_+ \to R_+$, which are continuous, strictly increasing and vanishing at zero, $\mathscr{K}_\infty$ denotes the set of all functions which are of class $\mathscr{K}$ and unbounded; $\mathscr{K}\mathscr{L}$ denotes the set of all functions $\beta(s,t) : R_+ \times R_+ \to R_+$ which is of class $\mathscr{K}$ for each fixed $t$, and decreases to zero as $t \to \infty$ for fixed $s$.

Consider the following stochastic nonlinear system:

$$dx = f(x)dt + h(x)d\omega, \quad \forall x \in R^n \tag{1}$$

where $x \in R^n$ is the state of the system, $\omega$ is $r-$dimensional independent standard Wiener process defined on a probability space $(\Omega, \mathscr{F}, P)$. $f(\cdot)$, $h(\cdot)$ are locally Lipschitz in $x$.

**Definition 1** For any given $V(x) \in C^2$, associated with stochastic system (1), we define the differential operator $\mathscr{L}$ as follows:

$$\mathscr{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2} Tr\{\frac{\partial^2 V}{\partial x^2} h(x)h^T(x)\}. \tag{2}$$

**Definition 2** For the stochastic system (1) with $f(0) = h(0) = 0$, the equilibrium $x(t) = 0$ is
(i) globally stable in probability if for any $\epsilon > 0$, there exists a class $\mathscr{K}$ function $\gamma(\cdot)$ such that $P\{|x(t)| < \gamma(|x_0|)\} \geq 1 - \epsilon, \forall t \geq 0, \forall x_0 \in R^n - \{0\}$;
(ii) globally asymptotically stable in probability if $\forall \epsilon > 0$, there exists a class $\mathscr{K}\mathscr{L}$ function $\beta(\cdot,\cdot)$ such that $P\{|x(t)| < \beta(|x_0|, t) \geq 1 - \epsilon, \forall t \geq 0, \forall x_0 \in R^n - \{0\}$.

*Lemma 1.* (Psillakis et al. [2005]) Consider the stochastic nonlinear system (1). If there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V : R^n \to R$, and constants $c_1 > 0$, $c_2 > 0$ such that

$$\mathscr{L}V(x) \leq -c_1 V(x) + c_2, \tag{3}$$

then (i) the system has a unique solution almost surely and (ii) the system is bounded in probability.

### *2.2 Fuzzy System*

In this paper, we adopt the singleton fuzzifier, product inference and the center-defuzzifier to reduce the following fuzzy rule:
$R_i$: IF $x_1$ is $F_1^i$ and $\cdots$ and $x_n$ is $F_n^i$, THEN $y$ is $B^i$ $(i = 1, 2, \cdots, N)$.
where $x = [x_1, \cdots, x_n]^T \in R^n$ and $y$ are the input and output of the fuzzy system, respectively. Since the strategy of singleton fuzzification, center-average defuzzification and product inference is used, the output of the fuzzy system can be formulated as

$$y(x) = \frac{\sum_{i=1}^N \theta_i \Pi_{j=1}^n \mu_{F_j^i}(x_j)}{\sum_{i=1}^N [\Pi_{j=1}^n \mu_{F_j^i}(x_j)]}, \tag{4}$$

where $\mu_{F_j^i}(x_j)$ is the membership of $F_j^i$, and $\theta_i = \max_{y \in R} \mu_{B_i(y)}$. Let $\xi_j(x) = \frac{\prod_{i=1}^n \mu_{F_j^i}(x_j)}{\sum_{j=1}^N [\prod_{i=1}^n \mu_{F_j^i}(x_j)]}$, $\xi(x) = [\xi_1(x), \xi_2(x), \cdots, \xi_N(x)]^T$ and $\theta = [\theta_1, \cdots, \theta_N]^T$. Then the fuzzy logic system above can be expressed as follows:

$$y(x) = \theta^T \xi(x). \tag{5}$$

If all membership functions are taken as Gaussian functions, then the following lemma holds.

*Lemma 2.* Let $f(x)$ be a continuous function defined on a compact set $\Omega$. Then for any constant $\epsilon > 0$, there exists a fuzzy logic system (5) such that

$$\sup_{x \in \Omega} |f(x) - \varphi(x)| = \sup_{x \in \Omega} |f(x) - \theta^T \xi(x)| \leq \epsilon. \tag{6}$$

Define the ideal estimation parameter as

$$\theta^* = \arg \min_{\theta \in \Theta} [\sup_{x \in \Omega} |f(x) - \theta^T \xi(x)|],$$

where $\Theta$ and $\Omega$ denote the sets of suitable bounds on $\theta$ and $x$.

### *2.3 Problem*

Consider a SISO nonaffine nonlinear systems in the following form:
Plant:

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = x_3 dt \\ \vdots \\ dx_{n-1} = x_n dt \\ dx_n = F(x,u)dt + h^T(x)d\omega \\ y = x_1 \end{cases} \tag{7}$$

Dead-zone:

$$u = D(v) = \begin{cases} g_r(v) & \text{if} \quad v \geq b_r, \\ 0 & \text{if} \quad b_l < v < b_r, \\ g_l(v) & \text{if} \quad v \leq b_l, \end{cases} \tag{8}$$

where $x = (x_1, \cdots, x_n)^T, y \in R$ are state variables and output, respectively. $\omega$ is $r-$dimensional independent standard Wiener process defined on a probability space $(\Omega, \mathscr{F}, P)$. $F(x,u)$ is unknown smooth function. $u \in R$ is the output of the dead-zone. $v(t) \in R$ is the input to the dead-zone, $b_l$ and $b_r$ are the unknown parameters of the dead-zone. The reference signal is $y_d(t)$, and $y_d(t), y_d^{(1)}(t), \cdots, y_d^{(n)}(t)$ are smooth and bounded.
For the unknown dead-zone input, we make the following assumptions

*Assumption 1.* The dead-zone output, $u$, is not available.

*Assumption 2.* The dead-zone parameters, $b_r$ and $b_l$, are unknown bounded constants, but their signs are known, i.e., $b_r > 0$ and $b_l < 0$.

*Assumption 3.* The functions, $g_l(v)$ and $g_r(v)$, are smooth, and there exist unknown positive constants, $k_{l0}, k_{l1}, k_{r0}$, and $k_{r1}$ such that

$$0 < k_{l0} \le g_l^{'}(v) \le k_{l1}, \quad \forall v \in (-\infty, b_l] \tag{9}$$

$$0 < k_{r0} \le g_r^{'}(v) \le k_{r1}, \quad \forall v \in [b_r, +\infty) \tag{10}$$

and $\beta_0 \le \min\{k_{l0}, k_{r0}\}$ is a known positive constant, where $g_l^{'}(v) = \frac{dg_l(z)}{dz}|_{z=v}$ and $g_r^{'}(v) = \frac{dg_r(z)}{dz}|_{z=v}$.

Based on Assumption 3, the dead-zone (8) can be rewritten as follows as shown Zhang et al. [2007a]:

$$u = D(v) = K^T(t)\Phi(t)v + d(v), \tag{11}$$

where

$$\Phi(t) = [\varphi_r(t), \varphi_l(t)]^T, \tag{12}$$

$$\varphi_r(t) = \begin{cases} 1 & \text{if } \ v(t) > b_l, \\ 0 & \text{if } \ v(t) \le b_l, \end{cases} \tag{13}$$

$$\varphi_l(t) = \begin{cases} 1 & \text{if } \ v(t) < b_r, \\ 0 & \text{if } \ v(t) \ge b_r, \end{cases} \tag{14}$$

$$K(t) = [K_r(v(t)), K_l(v(t))]^T \tag{15}$$

$$K_r(v(t)) = \begin{cases} 0 & \text{if } \ v(t) \le b_l, \\ g_{ir}^{'}(\xi_r(v(t))) & \text{if } \ b_l < v(t) < +\infty, \end{cases} \tag{16}$$

$$K_l(v(t)) = \begin{cases} g_l^{'}(\xi_l(v(t))) & \text{if } \ -\infty < v(t) < b_r, \\ 0 & \text{if } \ v(t) \ge b_r, \end{cases} \tag{17}$$

$$d(v) = \begin{cases} -g_r^{'}(\xi_r(v))b_r & \text{if } \ v \ge b_r, \\ -[g_l^{'}(\xi_l(v)) + g_r^{'}(\xi_r(v))]v & \text{if } \ b_l < v < b_r, \\ -g_l^{'}(\xi_l(v))b_l, & \text{if } \ v \le b_l \end{cases} \tag{18}$$

and $|d(v)| \le p^*$, $p^*$ is an unknown positive constant with $p^* = (k_{r1} + k_{l1}) \max\{b_r, -b_l\}$.

**Remark 1** There are many results for the case of linear dead-zone outside the deadband, but equation (11) is to capture the most realistic situation. As shown in Zhang et al. [2007a], we known that $K^T(t)\Phi(t) \in [\beta_0, k_{l1} + k_{r1}] \subset (0, +\infty)$.

The control objective is to design an adaptive fuzzy controller $v(t)$ for the system (7) such that the output $y$ follows the specified desired trajectory $y_d$ with guaranteeing that the system is bounded in probability.

Define $g(x,u) = \int_0^1 \frac{\partial F(x,\xi)}{\partial \xi}|_{\xi = su} ds$, which is an unknown nonlinear function satisfying the following assumption.

*Assumption 4.* There exist positive constants $g_0, g_1$ such that $0 < g_0 \le g(x,u) \le g_1, \forall x \in \Omega_x \subset R^n$, where $\Omega_x$ is a compact set.

**Remark 2** In Assumption 4, although $g(\cdot)$ appears to be similar with the affine terms in a strict-feedback system, a major difference lies in that $g(\cdot)$ is a function of $u$, and thus, it is still a nonaffine term in character. Furthermore, we don't need the assumption that $g(\cdot)$ is independent from $x_n$.

## 3. CONTROL DESIGN AND STABILITY ANALYSIS

In this section, we will use backstepping to design an adaptive fuzzy controller. First, we introduce the error variables

$$z_i = x_i - \alpha_{i-1}(z_{[i-1]}, y_d^{[i-1]}), \qquad i = 1, 2, \cdots, n \tag{19}$$

where $\alpha_{i-1}(z_{[i-1]}, y_d^{[i-1]})$ to be given in the following steps, $\alpha_0 = y_d$, $\alpha_n = u$, $z_{[i-1]} = [z_1, \cdots, z_{i-1}]^T$, $y_d^{[i-1]} = [y_d, \dot{y}_d, \cdots, y_d^{i-1}]^T$, $z = z_{[n]}$.

***step*** 1 The derivation of $z_1$ is

$$dz_1 = dx_1 - \dot{y}_d dt = (z_2 + \alpha(z_1, y_d, \dot{y}_d) - \dot{y}_d)dt.$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{z_1^2}{2}, \tag{20}$$

the time derivative of $V_1$ is

$$\begin{aligned} dV_1 &= z_1 dz_1 \\ &= z_1(z_2 + \alpha_1(z_1, y_d, \dot{y}_d) - \dot{y}_d)dt. \end{aligned} \tag{21}$$

Select virtual control $\alpha_1(z_1, y_d, \dot{y}_d)$ as

$$\alpha_1(z_1, y_d, \dot{y}_d) = -k_1 z_1 + \dot{y}_d, \tag{22}$$

with design constat $k_1 > 0$, it is easy to get

$$dV_1 \le (z_1 z_2 - k_1 z_1^2)dt. \tag{23}$$

***step*** 2 The derivation of $\alpha_1$ is $d\alpha_1 = \frac{\partial \alpha_1}{\partial z_1} dz_1 + \frac{\partial \alpha_1}{\partial y_d} dy_d + \frac{\partial \alpha_1}{\partial y_d^{(1)}} dy_d^{(1)}$, so the derivation of $z_2$ is $dz_2 = (z_3 + \alpha_2 - d\alpha_1)dt = [z_3 + \alpha_2 - (\frac{\partial \alpha_1}{\partial z_1} dz_1 + \frac{\partial \alpha_1}{\partial y_d} dy_d + \frac{\partial \alpha_1}{\partial y_d^{(1)}} dy_d^{(1)})]dt$.

Consider the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} z_2^2, \tag{24}$$

the time derivative of $V_2$ is

$$\begin{aligned} dV_2 &\le (z_1 z_2 - k_1 z_1^2 + z_2(z_3 + \alpha_2 - d\alpha_1))dt \\ &= [z_1 z_2 - k_1 z_1^2 + z_2(z_3 + \alpha_2 - (\frac{\partial \alpha_1}{\partial z_1} dz_1 + \frac{\partial \alpha_1}{\partial y_d} dy_d \end{aligned}$$

$$+\frac{\partial \alpha_1}{\partial y_d^{(1)}} dy_d^{(1)}))]dt. \tag{25}$$

Select virtual control $\alpha_2(z_1, z_2, y_d, \dot{y}_d, \ddot{y}_d)$ as

$$\alpha_2(z_1, z_2, y_d, \dot{y}_d, \ddot{y}_d) = -k_2 z_2 - z_1 + \frac{\partial \alpha_1}{\partial z_1} dz_1 + \frac{\partial \alpha_1}{\partial y_d} y_d^{(1)} + \frac{\partial \alpha_1}{\partial \dot{y}_d} y_d^{(2)}, \tag{26}$$

with design constat $k_2 > 0$, it is easy to get

$$dV_2 \leq (z_2 z_3 - k_1 z_1^2 - k_2 z_2^2)dt. \tag{27}$$

***step*** $k$ $(3 \leq k \leq n-1)$ A similar procedure is employed recursively for each step $k$. The derivation of $\alpha_{k-1}$ is

$$d\alpha_{k-1} = (\sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j + \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)})dt$$

then the derivation of $z_k$ is

$$\begin{aligned} dz_k &= [z_{k+1} + \alpha_k - d\alpha_{k-1}]dt \\ &= [z_{k+1} + \alpha_k - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j - \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)}]dt, \end{aligned}$$

choose the Lyapunove candidate functions and the virtual control laws as follows,

$$V_k = V_{k-1} + \frac{1}{2} z_k^2, \tag{28}$$

$$\alpha_k = -k_k z_k - z_{k-1} + \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial z_j} dz_j + \sum_{j=0}^{k-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)}, \tag{29}$$

and we get

$$dV_k \leq (z_k z_{k+1} - \sum_{j=1}^{k} k_j z_j^2)dt. \tag{30}$$

***step*** $n$ The derivation of $z_n$ is

$$\begin{aligned} dz_n &= dx_n - d\alpha_{n-1} \\ &= F(x,u)dt + h^T(x)d\omega - (\sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial z_j} dz_j \\ &\quad + \sum_{j=0}^{n-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)})dt, \end{aligned} \tag{31}$$

use the mean value theorem, we get

$$\begin{aligned} dz_n &= F(x,0)dt + g(x,u)udt + h^T(x)d\omega \\ &\quad -(\sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial z_j} dz_j + \sum_{j=0}^{n-1} \frac{\partial \alpha_{k-1}}{\partial y_d^{(j)}} y_d^{(j+1)})dt, \end{aligned} \tag{32}$$

choose Lyapunov candidate as

$$V_n = V_{n-1} + \frac{z_n^4}{4}, \tag{33}$$

then

$$\begin{aligned} \mathscr{L}V_n &\leq -k_1 z_1^2 - k_2 z_2^2 - \cdots - \frac{k_n}{2} z_n^4 + \frac{3}{2} z_n^2 h^T h + z_n[z_{n-1} \\ &\quad + \frac{k_n}{2} z_n^3 + z_n^2(F(x,0) + g(x,u)u - \dot{\alpha}_{n-1})]. \end{aligned}$$

Consider the following inequalities:

$$\frac{3}{2} z_n^2 h^T(x)h(x) \leq \frac{1}{\zeta} + \frac{9}{16} \zeta \|h(x)\|^4 z_n^4 \quad \forall \zeta > 0 \tag{34}$$

$$z_n z_{n-1} \leq \frac{1}{\xi} + \frac{1}{4} (\frac{3\xi}{4})^3 z_{n-1}^4 z_n^4 \tag{35}$$

So,

$$\begin{aligned} \mathscr{L}V_n &\leq \frac{1}{\zeta} + \frac{1}{\xi} - \sum_{j=1}^{n-1} k_j z_j^2 - \frac{k_n}{2} z_n^4 + z_n^3\{F(x,0) + g(x,u)u \\ &\quad -\dot{\alpha}_{n-1} + [\frac{9}{16} \zeta \|h(x)\|^4 + \frac{1}{4} (\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2}] z_n\}, \end{aligned}$$

Then use the first type fuzzy systems to approximate $\bar{f}(x) = g_0^{-1}(F(x,0) - \dot{\alpha}_{n-1} + \frac{9}{16} \zeta \|h(x)\|^4 z_n)$

$$\bar{f}(x) = \theta^{*T} \xi(x) + \epsilon_f, \tag{36}$$

where $\epsilon_f$ is approximation error, according to Lemma 2, there exist $\epsilon_M > 0$ such that $|\epsilon_f| < \epsilon_M$. Let $\hat{\theta}$ is the estimation of $\theta^*$, and $\tilde{\theta} = \hat{\theta} - \theta^*$.
We define the overall Lyapunov function as

$$V = V_n + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \tag{37}$$

and choose the adaptive law as

$$\dot{\hat{\theta}} = \Gamma[g_0 \xi(x) z_n^3 - \sigma(\hat{\theta} - \theta^0)], \tag{38}$$

with gain matrix $\Gamma > 0$, then

$$\begin{aligned} \mathscr{L}V &\leq \frac{1}{\zeta} + \frac{1}{\xi} - \sum_{j=1}^{n-1} k_j z_j^2 - \frac{k_n}{2} z_n^4 + z_n^3\{g_0(\theta^{*T} \xi(x) + \epsilon) \\ &\quad + g(x,u)u + [\frac{1}{4} (\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2}] z_n\} + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}, \end{aligned}$$

on the other hand

$$\begin{aligned} -\tilde{\theta}^T \sigma(\hat{\theta} - \theta^0) &\leq -\frac{1}{2} \|\tilde{\theta}\|^2 + \frac{1}{2} \|\theta^* - \theta^0\| \\ &\leq -\frac{\sigma \lambda_{min}(\Gamma)}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2} \|\theta^* - \theta^0\|^2 \end{aligned} \tag{39}$$

So, we get

$$\begin{aligned} \mathscr{L}V &\leq \frac{1}{\zeta} + \frac{1}{\xi} - 2\mu_1 V + z_n^3\{g_0 \hat{\theta}^T \xi(x) + g(x,u)u \\ &\quad + [\frac{1}{4} (\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2} + \frac{1}{4} z_n^2] z_n\} + \mu_2. \end{aligned} \tag{40}$$

Let $\mu_1 = \min_{1 \leq i \leq n}\{(1 - \epsilon_i)k_i, \sigma \lambda_{min}(\Gamma)\}$, $\mu_2^0 = \frac{1}{2} \|\theta^* - \theta^0\|^2 + g_0 \epsilon_M^2$. It is easy to see $\mu_1 > 0$ and $\mu_2^0 > 0$. Choose the control law $v$ as follows:

$$v = -\frac{1}{\beta_0}(-u_r - u_f). \tag{41}$$

where

$$u_f = -\hat{\theta}^T\xi(x)\tanh(\frac{z_n\hat{\theta}^T\xi(x)}{\delta}), \tag{42}$$

$$u_r = -g_0^{-1}[\frac{1}{4}(\frac{3\xi}{4})^3 z_{n-1}^4 + \frac{k_n}{2} + \frac{1}{2}z_n^2 + \frac{g_1^2}{4}z_n^2]z_n, \tag{43}$$

with $\delta$ is a positive constant. Using the inequality

$$0 \le |x| - xtanh(\frac{x}{\delta}) \le 0.2785\delta \tag{44}$$

Then

$$\begin{aligned} & z_n^3(g_0\hat{\theta}^T\xi(x) + g(x,u)u_f) \\ & = z_n^2(g_0 z_n\hat{\theta}^T\xi(x) - g(x,u)z_n\hat{\theta}^T\xi(x)\tanh(\frac{z_n\hat{\theta}^T\xi(x)}{\delta})) \\ & \le z_n^2 g_0(|z_n\hat{\theta}^T\xi(x)| - z_n\hat{\theta}^T\xi(x)\tanh(\frac{z_n\hat{\theta}^T\xi(x)}{\delta})) \\ & \le 0.2785\delta z_n^2 g_0 \\ & \le \kappa + \frac{1}{3}z_n^6, \end{aligned} \tag{45}$$

where $\kappa = \frac{2}{3}(0.2785 g_0\delta)^{3/2}$ is a positive constant,

$$z_n^3 g(x,u)d(v) \le \frac{g_1^2}{4}z_n^6 + p^{*2} \tag{46}$$

From (40), it holds true that

$$\mathscr{L}V \le -2\mu_1 V + \mu_2. \tag{47}$$

with $\mu_2 = \mu_2^0 + \kappa + p^{*2}$.

The main result on the asymptotic stability of the closed-cloop system is summarized in the following theorem.

*Theorem 3.* For the stochastic uncertain nonaffine nonlinear systems (7) satisfying Assumptions 1-4, with control law (41) and adaptive law (38), then the tracking error is bounded and the mean square tracking error enters inside the region

$$\Omega = \{y(t) \in R | E[(y(t) - y_d(t))^2] \le \frac{2\mu_2}{\mu_1}, \quad \forall t \ge T_1\} \tag{48}$$

wherein it remains for all time thereafter, and the variable $T_1$ will be given later.

**Proof** From (47) and Lemma 1, it is easy to get that the system is bounded in probability and the mean value of the Lyapunov function satisfies

$$\frac{d}{dt}[E(V)] \le -2\mu_1 E[V(t)] + \mu_2. \tag{49}$$

So,

$$\begin{aligned} E[V(t)] & \le e^{-2\mu_1 t}V_n(0) + \mu_2\int_0^t \le e^{-2\mu_1(t-\tau)}d\tau \\ & \le e^{-2\mu_1 t}V_n(0) + \frac{\mu_2}{2\mu_1}, \qquad \forall t \ge 0 \end{aligned} \tag{50}$$

there exists a time $T_1$

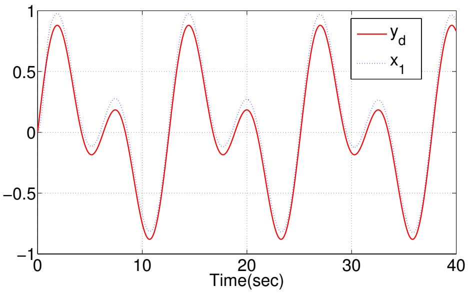


Fig. 1. The response of $x_1$

$$T_1 = \max\{0, \frac{1}{2\mu_1}ln[\frac{2\mu_1 V_n(0)}{\mu_2}]\}. \tag{51}$$

such that

$$E[(y(t) - y_d(t))^2] \le 2E[V(t)] \le \frac{2\mu_2}{\mu_1}. \quad \square \tag{52}$$

## 4. COMPUTER SIMULATION

Consider the following nonlinear systems:

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = (x_1x_2 + 1.1u + \cos u)dt + \frac{1}{3}(x_2 + \sin x_1)d\omega \\ y = x_1 \end{cases} \tag{53}$$

where $D(v)$ as follows:

$$u = D(v) = \begin{cases} 1.5(v-2) & \text{if } v \ge 2, \\ 0 & \text{if } -1.5 < v < 2.5, \\ v + 0.5 & \text{if } v \le -0.5. \end{cases} \tag{54}$$

with initial conditions $x_1(0) = 0.5$, $x_2(0) = 0.1$, and the reference signal $y_d = 0.5(\sin t + \sin(0.5t))$. It is easy to get $g_0 = 0.1, g_1 = 2.1, \beta_0 = 1$. In the simulation, the fuzzy membership functions are defined as

$\mu_{F_j^1}(x) = \exp[\frac{-(x+2)^2}{4}]$, $\mu_{F_j^2}(x) = \exp[\frac{-(x+1)^2}{4}]$,
$\mu_{F_j^3}(x) = \exp[\frac{-(x)^2}{4}]$, $\mu_{F_j^4}(x) = \exp[\frac{-(x-2)^2}{4}]$,
$\mu_{F_j^5}(x) = \exp[\frac{-(x-1)^2}{4}]$.

We choose the virtual control law and the control law as

$$\alpha_1 = -k_1 z_1 + \dot{y}_d, \tag{55}$$

$$\begin{aligned} v = & -\hat{\theta}^T\xi(x)\tanh(\frac{z_2\hat{\theta}^T\xi(x)}{\delta}) - 10[\frac{1}{4}(\frac{3\xi}{4})^3 + \frac{k_2}{2} + \frac{1}{2}z_2^2 \\ & + \frac{g_1^2}{4}z_2^2]z_2, \end{aligned} \tag{56}$$

where $z_1 = y - y_d = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $k_1 = k_2 = 2$, $\delta = 0.2$, $\xi = 2$. The adaptive law

$$\dot{\hat{\theta}} = \Gamma[\xi(x)z_2^3 - \sigma(\hat{\theta} - \theta^0)], \tag{57}$$

where $\Gamma = 2I$, $\sigma = 2$, $\theta^0$ is randomly taken in the intervals $[-1, 1]$. The simulation results are shown in Figure 1 and Figure 2. From Figure 1, it can be seen that fairly good tracking performance is obtained.

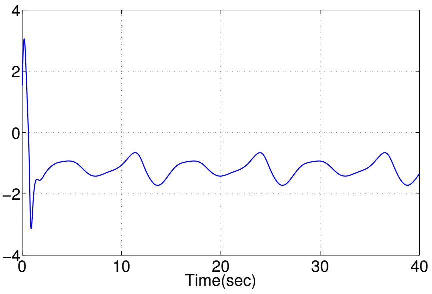


Fig. 2. Control signal

## 5. CONCLUSION

In brief, a novel adaptive fuzzy control scheme has been presented for a class of nonaffine uncertain nonlinear systems with unknown dead-zone input, which is driven by unknown covariance noise inputs. The proposed control scheme ensures that all the error variables are bounded in probability while the mean square tracking error becomes SGUUB in an arbitrarily small area around the origin.

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