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Performance of reconfiguration structures based on the constrained control

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Abstract: An approach is proposed for reconfigurable control structure design to obtain a system tolerant to state-variable sensor faults. The method is based on discrete-time constrained control design techniques for linear systems with state constraints, defined by linear equalities. Degradation in steady state performance is dealt with fixing that state variable which is associated with sensor fault to zero value. Since that control design can be viewed as a specific pole-assignment problem, reconfigured LQ control structures, as well as stabilizing reconfiguration structure are introduced.

1. INTRODUCTION

Automated diagnosis has been one of the more fruitful applications in sophisticated control systems, with potential significance for domains in which diagnosis of systems must proceed while the system is operative and testing opportunities are limited by operational considerations. The real problem is usually to fix the system with faults so that system can continue its mission for some time with some limitations of functionality. Consequently, diagnosis is a part of larger problem known as fault detection, identification and reconfiguration (FDIR). The practical benefits of an integrated approach to FDIR seem to be considerable, especially when knowledge of available fault isolations and system reconfigurations is used to reduce the cost and increase the reliability and utility of control and diagnosis.

In the last years many significant results have spurred interest in problem of determining control laws for systems with constraints. One approach to the problem of finding the optimal control results is technique dealing with system constraints directly. If this constrained problem is solvable, then one can modify optimized linear quadratic control performance index to adapt it for constraints. A special form of this constrained problem can be formulated with the goal to optimize state feedback controller parameters while the system state variables satisfy the equality constraints. These principles were presented e.g. in Ko and Bitmead (2007).

In this paper the reconfigurable structure is suggested where a control law from an admissible set of constrained LQ control laws is singled out. It is assumed, that system is free of actuator faults, and according to the performance of FDIR the state variable sensor faults detection and isolation is available to take equivalent control law for a occurred sensor fault. This method is an adaptation of methodology given in Ko and Bitmead (2005) and can be noted as an extension to the pseudo-inverse methods (PIM), presented e.g. in Staroswiecki (2005), as well as degraded reference models, used in Zhang and Jiang (2003).

Based on the discrete-time linear system state description, as well as on nominal LQ performance index, the generalized performance indexes were formulated, which are associated with the standard forms of algebraic Riccati equations for time-invariant discrete LQ control. In addition for given sensor fault structures reconfigured LQ control structures, as well as stabilizing reconfiguration structure are defined. Finally numerical example is shown in this paper to demonstrate the role of singularities in the design procedure.

2. LQ CONTROL TASK

Generally, a discrete-time linear dynamic system can be described by set of equations

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i) \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i) \tag{2}$$

where $\boldsymbol{q}(i) \in I\!R^n$, $\boldsymbol{u}(i) \in I\!R^r$, $\boldsymbol{y}(i) \in I\!R^m$ and matrices $\boldsymbol{F} \in I\!R^{n\times n}$, $\boldsymbol{G} \in I\!R^{n\times r}$, $\boldsymbol{C} \in I\!R^{m\times n}$ are finite valued.

For such system (1), (2) the optimal control design task is to determine the control law

$$\boldsymbol{u}(i) = -\boldsymbol{K}(i)\boldsymbol{q}(i) \tag{3}$$

that minimizes the quadratic cost function

$$J_N = \boldsymbol{q}^T(N)\boldsymbol{Q}^{\bullet}\boldsymbol{q}(N) + \sum_{i=0}^{N-1} s\big(\boldsymbol{q}(i),\boldsymbol{u}(i)\big) \tag{4}$$

$$\begin{aligned} & s\big(\boldsymbol{q}(i),\boldsymbol{u}(i)\big) = \\ &= \boldsymbol{q}^T(i)\boldsymbol{Q}\boldsymbol{q}(i) + 2\boldsymbol{q}^T(i)\boldsymbol{S}\boldsymbol{u}(i) + \boldsymbol{u}^T(i)\boldsymbol{R}\mathbf{u}(i) = \\ &= \begin{bmatrix} \boldsymbol{q}^T(i) & \boldsymbol{u}^T(i) \end{bmatrix} \boldsymbol{J}_J \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}(i) \end{bmatrix} \end{aligned} \tag{5}$$

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$$\boldsymbol{J}_J = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & \boldsymbol{R} \end{bmatrix} \geq 0, \quad \boldsymbol{Q} - \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T \geq 0 \tag{6}$$

where matrices $\boldsymbol{Q} \geq 0 \in I\!\!R^{n\times n}$, $\boldsymbol{Q}^{\bullet} \geq 0 \in I\!\!R^{n\times n}$ and $\boldsymbol{R} > 0 \in I\!\!R^{m\times m}$ have full row rank, $\boldsymbol{S} \in I\!\!R^{n\times r}$ satisfies (6) and $\boldsymbol{K}(i) \in I\!\!R^{n\times r}$ is the optimal control gain matrix.

3. CONSTRAINED CONTROL

Using control law (3) the steady-state equilibrium control equation takes the form

$$\boldsymbol{q}(i+1) = (\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})\boldsymbol{q}(i) \tag{7}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i) \tag{8}$$

There is considered a design constraint

$$\boldsymbol{q}(i) \in \mathcal{N}(\boldsymbol{D}) = \{\boldsymbol{q} : \boldsymbol{D}\boldsymbol{q} = \boldsymbol{0}\} \tag{9}$$

where the state-variable vectors have to satisfy equalities

$$\boldsymbol{D}\boldsymbol{q}(i+1) = \boldsymbol{D}(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})\boldsymbol{q}(i) = \boldsymbol{0} \tag{10}$$

$$\boldsymbol{D}(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}) = \boldsymbol{0} \tag{11}$$

$$\boldsymbol{D}\boldsymbol{F} = \boldsymbol{D}\boldsymbol{G}\boldsymbol{K} \tag{12}$$

respectively, and $\mathcal{N}(\boldsymbol{D})$ is the constrain subspace. It is supposed that states to be constrained in the null space of $\boldsymbol{D}$, and matrix $(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})$ is a stable matrix (all eigenvalues of $(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})$ lie in the unit circle in the complex plane $\mathcal{Z}$. Under these conditions the state stays within the constrain surface, i.e. $\boldsymbol{q}(i) \in \mathcal{N}(\boldsymbol{D})$ and $\boldsymbol{F}\boldsymbol{q}(i) \in \mathcal{N}(\boldsymbol{D})$.

All solutions of $\boldsymbol{K}$ are

$$\boldsymbol{K} = (\boldsymbol{D}\boldsymbol{G})^{\ominus 1}\boldsymbol{D}\boldsymbol{F} + \boldsymbol{K}^{\circ} - (\boldsymbol{D}\boldsymbol{G})^{\ominus 1}\boldsymbol{D}\boldsymbol{G}\boldsymbol{K}^{\circ} \tag{13}$$

where $\boldsymbol{K}^{\circ}$ is an arbitrary matrix with appropriated dimension and

$$(\boldsymbol{D}\boldsymbol{G})^{\ominus 1} = (\boldsymbol{D}\boldsymbol{G})^T\big(\boldsymbol{D}\boldsymbol{G}(\boldsymbol{D}\boldsymbol{G})^T\big)^{\dagger} \tag{14}$$

is the Moore-Penrose pseudoinverse of $\boldsymbol{D}\boldsymbol{G}$. One can therefore express (13) as

$$\boldsymbol{K} = \boldsymbol{M} + \boldsymbol{N}\boldsymbol{K}^{\circ} \tag{15}$$

where

$$\boldsymbol{M} = (\boldsymbol{D}\boldsymbol{G})^{\ominus 1}\boldsymbol{D}\boldsymbol{F} \tag{16}$$

and

$$\boldsymbol{N} = \boldsymbol{I}_m - (\boldsymbol{D}\boldsymbol{G})^T\big(\boldsymbol{D}\boldsymbol{G}(\boldsymbol{D}\boldsymbol{G})^T\big)^{\dagger}\boldsymbol{D}\boldsymbol{G} \tag{17}$$

is the projection matrix (the orthogonal projector onto the null space $\mathcal{N}(\boldsymbol{D}\boldsymbol{G})$ of $\boldsymbol{D}\boldsymbol{G}$). This results in

$$\begin{aligned} \boldsymbol{u}(i) &= -\boldsymbol{M}\boldsymbol{q}(i) + \boldsymbol{N}\big(-\boldsymbol{K}^{\circ}\boldsymbol{q}(i)\big) = \\ &= -\boldsymbol{M}\boldsymbol{q}(i) + \boldsymbol{N}\boldsymbol{u}^{\circ}(i) \end{aligned} \tag{18}$$

where

$$\boldsymbol{u}^{\circ}(i) = -\boldsymbol{K}^{\circ}\boldsymbol{q}(i) \tag{19}$$

(see e.g. Ko and Bitmead (2007), Krokavec and Filasová (2007)).

4. CONSTRAINED LQ CONTROL

The systems under consideration are discrete-time linear MIMO dynamic systems described by (1), (2). It is supposed, that all state variables are measurable, and constraints take forms

$$\boldsymbol{q}(i) \in \mathcal{N}(\boldsymbol{d}_h^T) = \{\boldsymbol{q} : \boldsymbol{d}_h^T\boldsymbol{q} = 0\}, \quad h \in \{1, 2, \dots, n\} \tag{20}$$

$$\boldsymbol{d}_h^T = [\,0 \cdots 0 \;1_h\; 0 \cdots 0\,] \tag{21}$$

Using identity $\boldsymbol{q}(i) = \boldsymbol{q}(i)$ and (18), the system transform was introduced

$$\begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}(i) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{M}_h & \boldsymbol{N}_h \end{bmatrix} \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} = \boldsymbol{T}_h \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} \tag{22}$$

$$\boldsymbol{T}_h = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{M}_h & \boldsymbol{N}_h \end{bmatrix} \tag{23}$$

$$\boldsymbol{M}_h = (\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T\boldsymbol{F} \tag{24}$$

$$\boldsymbol{N}_h = \boldsymbol{I} - (\boldsymbol{d}_h^T\boldsymbol{G})^T\big(\boldsymbol{d}_h^T\boldsymbol{G}(\boldsymbol{d}_h^T\boldsymbol{G})^T\big)^{\dagger}\boldsymbol{d}_h^T\boldsymbol{G} \tag{25}$$

to describe modified control law representation.

Theorem 1. For a system given in (1), (2) with equality constraints (20), (21), the performance index (4), (5), and gain matrices (24), (25) solution to the constrained LQ control is given by

$$\boldsymbol{u}(i) = -\boldsymbol{K}_h\boldsymbol{q}(i) = -(\boldsymbol{M}_h + \boldsymbol{N}_h\boldsymbol{K}_h^{\circ})\boldsymbol{q}(i) \tag{26}$$

where

$$\boldsymbol{K}_h^{\circ} = (\boldsymbol{G}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})^T \tag{27}$$

and $\boldsymbol{N}_h$ is a regular matrix.

Here $\boldsymbol{P}(i)$ is a solution of the discrete Riccati equation

$$\begin{aligned} \boldsymbol{P}(i-1) &= \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} + \boldsymbol{Q}^{\circ} - \\ (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ}+\boldsymbol{S}_h^{\circ})&(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ}+\boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ}+\boldsymbol{S}_h^{\circ T}) \end{aligned} \tag{28}$$

and

$$\boldsymbol{F}_h^{\circ} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{M}_h \tag{29}$$

$$\boldsymbol{G}_h^{\circ} = \boldsymbol{G}\boldsymbol{N}_h \tag{30}$$

$$\boldsymbol{Q}_h^{\circ} = \boldsymbol{Q} + \boldsymbol{M}_h^T\boldsymbol{R}\boldsymbol{M}_h - \boldsymbol{S}\boldsymbol{M}_h - \boldsymbol{M}_h^T\boldsymbol{S}^T \tag{31}$$

$$\boldsymbol{R}_h^{\circ} = \boldsymbol{N}_h^T\boldsymbol{R}\boldsymbol{N}_h \tag{32}$$

$$\boldsymbol{S}_h^{\circ} = (\boldsymbol{S} - \boldsymbol{M}_h^T\boldsymbol{R})\boldsymbol{N}_h \tag{33}$$

Proof. Since system (1), (2) is linear in $\boldsymbol{q}(i)$ the quadratic Lyapunov function can be of the form

$$v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i)\boldsymbol{P}(i-1)\boldsymbol{q}(i) \tag{34}$$

where $\boldsymbol{P}(i-1) \in I\!\!R^{n\times n}$ is a symmetric positive definite matrix and $\mathbf{P}(-1) = \mathbf{P}(0)$. If Lyapunov function takes form (34), its difference is

$$\Delta v(\boldsymbol{q}(i), \boldsymbol{u}(i)) = v(\boldsymbol{q}(i+1)) - v(\boldsymbol{q}(i)) \tag{35}$$

$$\Delta v(\boldsymbol{q}(i), \boldsymbol{u}(i)) = [\,\boldsymbol{q}^T(i)\;\boldsymbol{u}^T(i)\,]\,\boldsymbol{J}_V(i)\begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}(i) \end{bmatrix} \tag{36}$$

respectively, where

$$\boldsymbol{J}_V(i) = \begin{bmatrix} \boldsymbol{F}^T\boldsymbol{P}(i)\boldsymbol{F} - \boldsymbol{P}(i-1) & \boldsymbol{F}^T\boldsymbol{P}(i)\boldsymbol{G} \\ (\boldsymbol{F}^T\boldsymbol{P}(i)\boldsymbol{G})^T & \boldsymbol{G}^T\boldsymbol{P}(i)\boldsymbol{G} \end{bmatrix} \tag{37}$$

and Lyapunov function at the time instant $N-1$ takes value

$$V_{N-1} = \sum_{i=0}^{N-1} \Delta v(\boldsymbol{q}(i), \boldsymbol{u}(i)) \tag{38}$$

which, in turn, is equivalent to

$$V_{N-1} = \boldsymbol{q}^T(N)\boldsymbol{P}(N-1)\boldsymbol{q}(N) - \boldsymbol{q}^T(0)\boldsymbol{P}(0)\boldsymbol{q}(0) \tag{39}$$

Adding (38) to (4) and subtracting (39) from (4) the performance index for control law can be brought to the form

$$J_N = \boldsymbol{q}^T(0)\boldsymbol{P}(0)\boldsymbol{q}(0) + \sum_{i=0}^{N-1} p\big(\boldsymbol{q}(i), \boldsymbol{u}(i)\big) \tag{40}$$

where $\boldsymbol{P}(N-1) = \boldsymbol{Q}^{\bullet}$ and

$$\begin{aligned} p\big(\boldsymbol{q}(i), \boldsymbol{u}(i)\big) &= s\big(\boldsymbol{q}(i), \boldsymbol{u}(i)\big) + \Delta v\big(\boldsymbol{q}(i), \boldsymbol{u}(i)\big) = \\ &= \left[\, \boldsymbol{q}^T(i) \;\; \boldsymbol{u}^T(i) \,\right] \boldsymbol{J}(i) \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}(i) \end{bmatrix} \end{aligned} \tag{41}$$

$$\boldsymbol{J}(i) = \boldsymbol{J}_J + \boldsymbol{J}_V(i) \tag{42}$$

Using (22), (23) the performance index (40) can be equivalently rewritten to the form

$$J_N = \boldsymbol{q}^T(0)\boldsymbol{P}(0)\boldsymbol{q}(0) + \sum_{i=0}^{N-1} p\big(\boldsymbol{q}(i), \boldsymbol{u}^{\circ}(i)\big) \tag{43}$$

where

$$\big(\boldsymbol{q}(i), \boldsymbol{u}^{\circ}(i)\big) = \left[\, \boldsymbol{q}^T(i) \;\; \boldsymbol{u}^{\circ T}(i) \,\right] \boldsymbol{J}_h^{\circ}(i) \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} \tag{44}$$

$$\boldsymbol{J}_h^{\circ}(i) = \boldsymbol{T}_h^T(\boldsymbol{J}_J + \boldsymbol{J}_V(i))\boldsymbol{T}_h \tag{45}$$

$$\begin{aligned} \boldsymbol{T}_h^T\boldsymbol{J}_J\boldsymbol{T}_h &= \begin{bmatrix} \boldsymbol{Q}_h^{\circ} & \boldsymbol{S}_h^{\circ} \\ \boldsymbol{S}_h^{\circ T} & \boldsymbol{R}_h^{\circ} \end{bmatrix} = \\ = &\begin{bmatrix} \boldsymbol{Q} + \boldsymbol{M}_h^T\boldsymbol{R}\boldsymbol{M}_h - \boldsymbol{S}\boldsymbol{M}_h - \boldsymbol{M}_h^T\boldsymbol{S}^T & (\boldsymbol{S} - \boldsymbol{M}_h^T\boldsymbol{R})\boldsymbol{N}_h \\ \boldsymbol{N}_h^T(\boldsymbol{S} - \boldsymbol{M}_h^T\boldsymbol{R})^T & \boldsymbol{N}_h^T\boldsymbol{R}\boldsymbol{N}_h \end{bmatrix} \end{aligned} \tag{46}$$

$$\begin{aligned} &\boldsymbol{T}_h^T\boldsymbol{J}_V(i)\boldsymbol{T}_h = \\ &\begin{bmatrix} \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} - \boldsymbol{P}(i-1) & \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} \\ \boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} & \boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} \end{bmatrix} \end{aligned} \tag{47}$$

where

$$\boldsymbol{F}_h^{\circ} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{M}_h \tag{48}$$

$$\boldsymbol{G}_h^{\circ} = \boldsymbol{G}\boldsymbol{N}_h \tag{49}$$

Thus, an equivalent standard form of (45) is

$$\boldsymbol{J}_h^{\circ}(i) = \begin{bmatrix} \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} - \boldsymbol{P}(i-1) + \boldsymbol{Q}_h^{\circ} & \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ} \\ (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})^T & \boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ} \end{bmatrix} \tag{50}$$

Accepting this generalized weighting matrix there exists such optimal control law $\boldsymbol{u}^{\circ}(i) = -\boldsymbol{K}^{\circ}(i)\boldsymbol{q}(i)$ satisfying conditions

$$\begin{aligned} \boldsymbol{0} &= \frac{\partial p(\boldsymbol{q}(i), \boldsymbol{u}^{\circ}(i))}{\partial \boldsymbol{u}^{\circ T}(i)} = \left[\boldsymbol{0}^T \;\; \boldsymbol{I}^T\right] \boldsymbol{J}_h^{\circ}(i) \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} = \\ &= \left[(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})^T \;\; \boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ}\right] \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} \end{aligned} \tag{51}$$

$$\begin{aligned} \boldsymbol{0} &= \frac{\partial p(\boldsymbol{q}(i), \boldsymbol{u}^{\circ}(i))}{\partial \boldsymbol{q}^T(i)} = \left[\boldsymbol{I}^T \;\; \boldsymbol{0}^T\right] \boldsymbol{J}_h^{\circ}(i) \begin{bmatrix} \boldsymbol{q}(i) \\ \boldsymbol{u}^{\circ}(i) \end{bmatrix} = \\ &= (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} - \boldsymbol{P}(i-1) + \boldsymbol{Q}_h^{\circ})\boldsymbol{q}(i) + \\ &\quad + (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})\boldsymbol{u}^{\circ}(i) \end{aligned} \tag{52}$$

that

$$\boldsymbol{K}_h^{\circ}(i) = (\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})^T \tag{53}$$

$$\boldsymbol{P}(i-1) = \boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} + \boldsymbol{Q}_h^{\circ} - (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})\boldsymbol{K}_h^{\circ}(i) \tag{54}$$

i.e. $\boldsymbol{P}(i) = \boldsymbol{P}^T(i) > 0$ is a solution of discrete Riccati equation (28), and resulting solution to the LQ problem with state equality constraints is given by the optimal control law

$$\boldsymbol{u}(i) = -\big(\boldsymbol{M}_h + \boldsymbol{N}_h\boldsymbol{K}_h^{\circ}(i)\big)\boldsymbol{q}(i) \tag{55}$$

It is clear, that (53), (54) imply (28), where existence of (28) is conditioned by the inequality

$$\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ} = \boldsymbol{N}_h^T\big(\boldsymbol{G}^T\boldsymbol{P}(i)\boldsymbol{G} + \boldsymbol{R}\big)\boldsymbol{N}_h > 0 \tag{56}$$

Supposing that $\boldsymbol{N}_h$ is a regular matrix, then using (49) and (56) one can obtain

$$\begin{aligned} &(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} + \boldsymbol{S}_h^{\circ T}) \\ &= (\boldsymbol{F}_h^{\circ T}\boldsymbol{P}(i)\boldsymbol{G} + \boldsymbol{S}_{0h}^{\circ})(\boldsymbol{G}^T\boldsymbol{P}(i)\boldsymbol{G} + \boldsymbol{R})^{-1}(\boldsymbol{G}^T\boldsymbol{P}(i)\boldsymbol{F}_h^{\circ} + \boldsymbol{S}_{0h}^{\circ T}) \end{aligned} \tag{57}$$

where

$$\boldsymbol{S}_{0h}^{\circ} = \boldsymbol{S} - \boldsymbol{M}_h^T\boldsymbol{R} \tag{58}$$

$$\boldsymbol{R} > 0, \quad \boldsymbol{G}^T\boldsymbol{P}(i)\boldsymbol{G} + \boldsymbol{R} > 0, \quad \det \boldsymbol{N}_h \neq 0 \tag{59}$$

Therefore for $\boldsymbol{R} > 0$ and $\det \boldsymbol{N}_h \neq 0$ solution (28) exists. This concludes the proof.

It is clear, that the existence of constrained LQ control over infinite-time horizon is related to the algebraic Riccati equations

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{F}_h^{\circ T}\boldsymbol{P}\boldsymbol{F}_h^{\circ} + \boldsymbol{Q}^{\circ} - \\ &(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ} + \boldsymbol{S}_h^{\circ})(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ} + \boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}\boldsymbol{F}_h^{\circ} + \boldsymbol{S}_h^{\circ T}) \end{aligned} \tag{60}$$

The constant gain state feedback controller for infinite-time horizon and control law (26) is given by a steady-state solution $\boldsymbol{P}$ of (60) with $\boldsymbol{R} > 0$ and $\det \boldsymbol{N}_h \neq 0$.

Corollary 1. Since

$$\begin{aligned} \boldsymbol{F}_{ch}^{\circ} &= \boldsymbol{F}_h^{\circ} - \boldsymbol{G}_h^{\circ}\boldsymbol{K}_h^{\circ} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{M}_h - \boldsymbol{G}\boldsymbol{N}_h\boldsymbol{K}_h^{\circ} = \\ &= \boldsymbol{F} - \boldsymbol{G}\boldsymbol{M}_h - \boldsymbol{G}(\boldsymbol{K}_h - \boldsymbol{M}_h) = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}_h \end{aligned} \tag{61}$$

one can see that the eigenvalues spectrum $\rho(\boldsymbol{F}_{ch})$ of the closed-loop system matrix $\boldsymbol{F}_{ch} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}_h$ is the same as the eigenvalues spectrum of the designed closed-loop system matrix $\boldsymbol{F}_{ch}^{\circ}$, obtained in modified LQ control design.

Corollary 2. For any non-zero row vector $\boldsymbol{d}_h^T$, satisfying (20), is

$$\begin{aligned} \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}_h &= \boldsymbol{F} - (\boldsymbol{G}\boldsymbol{M}_h + \boldsymbol{G}\boldsymbol{N}_h\boldsymbol{K}_h^{\circ}) = \\ &= \boldsymbol{F} - \boldsymbol{G}(\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T\boldsymbol{F} - \boldsymbol{G}(\boldsymbol{I}_m - (\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T\boldsymbol{G})\boldsymbol{K}_h^{\circ} = \\ &= (\boldsymbol{I}_n - \boldsymbol{G}(\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T)(\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}_h^{\circ}) \end{aligned} \tag{62}$$

Since (11) implies $\boldsymbol{d}_h^T(\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_h)=\boldsymbol{0}^T$, it is evident, that matrices $\boldsymbol{F}_{ch}=\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_h$, as well as

$$\boldsymbol{V}_h=(\boldsymbol{I}_n-\boldsymbol{G}(\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T) \tag{63}$$

are singular matrices.

Corollary 3. If $\boldsymbol{d}_h^T$ takes structure (21) then the following is yielded

$$\boldsymbol{G}(\boldsymbol{d}_h^T\boldsymbol{G})^{\ominus 1}\boldsymbol{d}_h^T=[\,\boldsymbol{0}\,\cdots\,\boldsymbol{0}\;\boldsymbol{v}_h\;\boldsymbol{0}\,\cdots\,\boldsymbol{0}\,] \tag{64}$$

$$\boldsymbol{v}_h=[\,v_{1h}\,\cdots\,v_{h-1,h}\;1\;v_{h+1,h}\,\cdots\,v_{n,h}\,]^T \tag{65}$$

and using (62) (with notation $\boldsymbol{F}_{ch}^{\bullet}=\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_h^{\circ}$) it can be shown that

$$\begin{aligned}\det(z\boldsymbol{I}_n-\boldsymbol{V}_h\boldsymbol{F}_{ch}^{\bullet})&=\det\big(z\boldsymbol{I}_n-(\boldsymbol{F}_{ch}^{\bullet}-\boldsymbol{v}_h\boldsymbol{f}_{ch}^{\bullet T})\big)=\\&=z\det\big(z\boldsymbol{I}_{n-1}-\boldsymbol{I}_{n\oslash h}(\boldsymbol{F}_{ch}^{\bullet}-\boldsymbol{v}_h\boldsymbol{f}_{ch}^{\bullet T})\boldsymbol{I}_{n\oslash h}^T\big)=\\&=z\det\big(z\boldsymbol{I}_{n-1}-\boldsymbol{W}_{vh}^{\bullet}\big)\end{aligned} \tag{66}$$

where $\boldsymbol{f}_{ch}^{\bullet T}$ is the h-th row of $\boldsymbol{F}_{ch}^{\bullet}$, $\boldsymbol{I}_{n\oslash h}$ can be obtained by deleting the h-th row of identity matrix $\boldsymbol{I}_n$, and $\boldsymbol{W}_{vh}^{\bullet}$ is h-th principal minor of $\boldsymbol{F}_{vh}^{\bullet}=\boldsymbol{F}_{ch}^{\bullet}-\boldsymbol{v}_h\boldsymbol{f}_{ch}^{\bullet T}$.

Corollary 4. The characteristic polynomial of constrained LQ control, designed for $\boldsymbol{d}_h^T$ having structure (21), is

$$P(z)=\det(z\boldsymbol{I}_n-\boldsymbol{F}_{ch})=z\det\big(z\boldsymbol{I}_{n-1}-\boldsymbol{W}_h\big) \tag{67}$$

where $\boldsymbol{W}_h$ is the h-th principal minor of $\boldsymbol{F}_{ch}$.

Equality (61), (67), (66) implies, that eigenvalue spectrum of $\boldsymbol{F}_{ch}$ is

$$\begin{aligned}\rho(\boldsymbol{F}_{ch})&=\rho(\boldsymbol{F}_{ch}^{\circ})=\\&=\{0,\rho(\boldsymbol{W}_h)\}=\{0,\rho(\boldsymbol{W}_h^{\circ})\}=\{0,\rho(\boldsymbol{W}_{vh}^{\bullet})\}\end{aligned} \tag{68}$$

where $\rho(\cdot)$ denotes the eigenvalue spectrum of a square matrix.

5. RECONFIGURABLE CONTROL

Following the design consideration outlined in Sections 3 and 4, and for selected vector $\boldsymbol{d}_h^T$ having structure (21) the reconfigurable control can be designed. It is supposed that system is with no actuator fault and a single fault of sensor is interpreted by condition (20). This interpretation means that sensor outputs is stuck at fixed value equal zero because of a malfunction and the measurement is $\boldsymbol{q}_h=0$.

Theorem 2. For a system given in (1), (2) with equality constraints (20), (21), the performance index (4), (5), and gain matrices (24), (25) solution to the reconfigurable LQ control is given by

$$\boldsymbol{u}(i)=-\boldsymbol{K}_{Rh}\boldsymbol{q}(i)=-(\boldsymbol{M}_h+\boldsymbol{N}_h\boldsymbol{K}_{Rh}^{\circ})\boldsymbol{q}(i) \tag{69}$$

where

$$\boldsymbol{K}_{Rh}^{\circ}=\boldsymbol{K}_h^{\circ}=(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ}+\boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ}+\boldsymbol{S}_h^{\circ})^T \tag{70}$$

Here $\boldsymbol{P}$ is a steady-state solution of the algebraic Riccati equation (60) under conditions given in (59), and matrices $\boldsymbol{F}_h^{\circ}$, $\boldsymbol{G}_h^{\circ}$, $\boldsymbol{Q}_h^{\circ}$, $\boldsymbol{R}_h^{\circ}$, and $\boldsymbol{S}_h^{\circ}$ are the same as in (29)–(33).

Proof. Clear from Theorem 1.

Theorem 3. For a system given in (1), (2) with equality constraints (20), (21), and gain matrices (24), (25), (74) solution to a stabilizing reconfigurable control is given by

$$\boldsymbol{u}(i)=-\boldsymbol{K}_{Sh}\boldsymbol{q}(i)=-(\boldsymbol{M}_h+\boldsymbol{N}_h\boldsymbol{K}_{Sh}^{\circ})\boldsymbol{q}(i) \tag{71}$$

This solution is obtainable using that $\boldsymbol{K}_{Sh}^{\circ}$ for which the desired eigenvalue set of a matrix $\boldsymbol{F}_{Sch}^{\circ}=\boldsymbol{F}_h^{\circ}-\boldsymbol{G}_h^{\circ}\boldsymbol{K}_{Sh}^{\circ}$ is a set of stable variables with one eigenvalue equals to zero. Here used matrices are given in (29) and (30).

Proof. Clear from Corollary 4.

Theorem 4. For a system given in (1), (2) with equality constraints (20), (21), the performance index (4), (5), and gain matrices (24), (25) solution to the reconfigurable LQ control with an integral action is given by

$$\boldsymbol{u}(i)=-\boldsymbol{K}_{Rh}\boldsymbol{q}(i)=-(\boldsymbol{M}_h+\boldsymbol{N}_h\boldsymbol{K}_{Rh}^{\circ}+\boldsymbol{K}_h^{\diamond})\boldsymbol{q}(i) \tag{72}$$

where

$$\boldsymbol{K}_{Rh}^{\circ}=\boldsymbol{K}_h^{\circ}=(\boldsymbol{G}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ}+\boldsymbol{R}_h^{\circ})^{-1}(\boldsymbol{F}_h^{\circ T}\boldsymbol{P}\boldsymbol{G}_h^{\circ}+\boldsymbol{S}_h^{\circ})^T \tag{73}$$

$$\boldsymbol{K}_h^{\diamond}=a_h\boldsymbol{G}^{\ominus 1}\boldsymbol{D}_h \tag{74}$$

Here $\boldsymbol{P}$ is a steady-state solution of the algebraic Riccati equation (60) under conditions given in (59), matrices $\boldsymbol{F}_h^{\circ}$, $\boldsymbol{G}_h^{\circ}$, $\boldsymbol{Q}_h^{\circ}$, $\boldsymbol{R}_h^{\circ}$, and $\boldsymbol{S}_h^{\circ}$ are the same as in (29)–(33), and

$$\boldsymbol{G}^{\ominus 1}=(\boldsymbol{G}\boldsymbol{G}^T)^{\dagger}\boldsymbol{G}^T \tag{75}$$

$$\boldsymbol{D}_h=\operatorname{diag}(\boldsymbol{d}_h^T) \tag{76}$$

$$a_h=-g_h^{-1},\qquad g_h=(\boldsymbol{G}\boldsymbol{G}^{\ominus 1})_h \tag{77}$$

where $(\cdot)_h$ denotes the h-th diagonal element of any square matrix.

Proof. Condition (11) implies for $\boldsymbol{K}_{Rh}=\boldsymbol{K}_h=\boldsymbol{M}_h+$ $+\boldsymbol{N}_h\boldsymbol{K}_h^{\circ}$ that $\boldsymbol{d}_h^T(\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_h)=\boldsymbol{0}^T$, i.e. all elements of the h-th row of $\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_h$ are equal to zero. The additive feedback gain $\boldsymbol{K}_h^{\diamond}$ guaranties, that structure of the matrix $\boldsymbol{G}\boldsymbol{K}_h^{\diamond}$ is

$$-\boldsymbol{G}\boldsymbol{K}_h^{\diamond}=[\,\boldsymbol{0}\,\cdots\,\boldsymbol{0}\;\boldsymbol{g}_h\;\boldsymbol{0}\,\cdots\,\boldsymbol{0}\,] \tag{78}$$

$$\boldsymbol{g}_h=[\,g_{1h}\,\cdots\,g_{h-1,h}\;1\;g_{h+1,h}\,\cdots\,g_{n,h}\,]^T \tag{79}$$

Therefore

$$\begin{aligned}P(z)&=\det\big(z\boldsymbol{I}_n-(\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_{Rh})\big)=\\&=(z-1)\det\big(z\boldsymbol{I}_{n-1}-\boldsymbol{W}_{Rh}\big)=(z-1)\det\big(z\boldsymbol{I}_{n-1}-\boldsymbol{W}_h\big)\end{aligned} \tag{80}$$

where $\boldsymbol{W}_{Rh}$ is the h-th principal minor of $\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}_{Rh}$.

Theorem 5. For a system given in (1), (2) with equality constraints (20), (21), and gain matrices (24), (25), (74) solution to a stabilizing reconfigurable control with an integral action is given by

$$\boldsymbol{u}(i)=-\boldsymbol{K}_{Sh}\boldsymbol{q}(i)=-(\boldsymbol{M}_h+\boldsymbol{N}_h\boldsymbol{K}_{Sh}^{\circ}+\boldsymbol{K}_h^{\diamond})\boldsymbol{q}(i) \tag{81}$$

This solution is obtainable using that $\boldsymbol{K}_{Sh}^{\circ}$ for which the desired eigenvalue set of a matrix $\boldsymbol{F}_{Sch}^{\circ}=\boldsymbol{F}_h^{\circ}-\boldsymbol{G}_h^{\circ}\boldsymbol{K}_{Sh}^{\circ}$ is a stable eigenvalue set with one eigenvalue equals to zero, as well as with an additive gain matrix (74). Here used matrices are given in (29) and (30).

Proof. Clear from Corollary 4 and Theorem 4.

Note that reconfiguration tasks based on constrained state controllers with an integral action have to use additive output feedback.

6. ILLUSTRATIVE EXAMPLES

To demonstrate properties of the proposed approach, a system with two-inputs and two-outputs is used in the example. The parameters of this system are

$$\boldsymbol{F} = \begin{bmatrix} 0.9993 & 0.0987 & 0.0042 \\ -0.0212 & 0.9612 & 0.0775 \\ -0.3875 & -0.7187 & 0.5737 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} 0.0051 & 0.0050 \\ 0.1029 & 0.0987 \\ 0.0387 & -0.0388 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for sampling period $\Delta t = 0.1$ s.

Case 1: Reconfiguration based on constrained LQ control

Assuming the performance index (4), (5) with

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{R} = 0.05 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \boldsymbol{S} = 0.02 \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

and the 3rd sensor fault constraint $\boldsymbol{d}_3^T = [\,0\ 0\ 1\,]$, there were obtained feedback gain matrix parameters

$$\boldsymbol{M}_3 = \begin{bmatrix} -4.9935 & -9.2616 & 7.3930 \\ -0.1029 & 9.2855 & -7.4121 \end{bmatrix}$$

$$\boldsymbol{N}_3 = \begin{bmatrix} 0.5013 & 0.5000 \\ -0.4996 & 0.4987 \end{bmatrix}$$

New design parameters were then recomputed as follows

$$\boldsymbol{F}_3^\circ = \begin{bmatrix} 0.9997 & 0.0995 & 0.0036 \\ -0.0015 & 0.9977 & 0.0483 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

$$\boldsymbol{G}_3^\circ = \begin{bmatrix} 0.0051 & 0.0050 \\ 0.1009 & 0.1007 \\ 0.0000 & 0.0000 \end{bmatrix}$$

$$\boldsymbol{Q}_3^\circ = \begin{bmatrix} 1.2500 & 0.4634 & -0.3701 \\ 0.4634 & 1.8590 & -0.6861 \\ -0.3701 & -0.6861 & 1.5480 \end{bmatrix}$$

$$\boldsymbol{R}_3^\circ = \begin{bmatrix} 0.0025 & 0.0025 \\ 0.0025 & 0.0025 \end{bmatrix}, \ \boldsymbol{S}_3^\circ = \begin{bmatrix} 0.0000 & 0.0000 \\ 0.0200 & 0.0200 \\ 0.0000 & 0.0000 \end{bmatrix}$$

Applying the Matlab function $dare(.)$ to design matrix $\boldsymbol{K}_3^\circ$ the optimal solution was obtained as

$$\boldsymbol{K}_3^\circ = \begin{bmatrix} 7.5872 & 6.0515 & 2.4267 \\ 0.0000 & 4.0000 & -2.0000 \end{bmatrix}$$

and the final feedback gain matrix becomes

$$\boldsymbol{K}_{R3} = \boldsymbol{M}_3 + \boldsymbol{N}_3\boldsymbol{K}_3^\circ = \begin{bmatrix} -1.1902 & -4.2280 & 7.6095 \\ 8.8000 & 14.3061 & -7.1962 \end{bmatrix}$$

The feedback system matrix $\boldsymbol{F}_{Rc3} = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}_{R3}$ and its eigenvalues are

$$\boldsymbol{F}_{Rc3} = \begin{bmatrix} 0.9614 & 0.0487 & 0.0014 \\ -0.7673 & -0.0157 & 0.0048 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}, \mathrm{eig}\,(\boldsymbol{F}_{Rc2}) = \begin{bmatrix} 0.9215 \\ 0.0242 \\ 0.0000 \end{bmatrix}$$

It is easily verified, that using generally some another performance index parameters and $\boldsymbol{d}_1^T = [\,1\ 0\ 0\,]$, as well as $\boldsymbol{d}_2^T = [\,0\ 1\ 0\,]$, one can obtain gain matrices for the rest sensor fault models.

Note that the system matrix $\boldsymbol{F}_\cdot^\circ$ is a singular matrix, therefore the performance index weighting matrices specification can be a non-trivial task.

Case 2: Reconfiguration based on stabilizing constrained control

Assuming the 3rd sensor fault constraint $\boldsymbol{d}_3^T = [\,0\ 0\ 1\,]$, obtained feedback gain matrix parameters were

$$\boldsymbol{M}_3 = \begin{bmatrix} -4.9935 & -9.2616 & 7.3930 \\ 5.0064 & 9.2855 & -7.4121 \end{bmatrix}$$

$$\boldsymbol{N}_3 = \begin{bmatrix} 0.5013 & 0.5000 \\ 0.5000 & 0.4987 \end{bmatrix}$$

If the specified desired closed-loop matrix eigenvalues set is $\rho(\boldsymbol{F}_{c3}) = \{\,0, 0.5, 0.8\,\}$, using the standard Matlab function $place(.)$ to design matrix $\boldsymbol{K}_{S3}^\circ$ the result is

$$\boldsymbol{K}_{S3}^\circ = \begin{bmatrix} 4.9179 & 3.2150 & -6.3395 \\ 4.9591 & 3.2067 & -6.3231 \end{bmatrix}$$

and for given $\boldsymbol{M}_3$ and $\boldsymbol{N}_3$ the final value for the feedback gain matrix becomes

$$\boldsymbol{K}_{S3} = \begin{bmatrix} -0.0216 & -6.0466 & 1.0535 \\ 9.9656 & 12.4922 & -13.7353 \end{bmatrix}$$

Using this feedback gain matrix the closed-loop system matrix $\boldsymbol{F}_{Sc3}$ and its eigenvalues are

$$\boldsymbol{F}_{Sc3} = \begin{bmatrix} 0.9496 & 0.0671 & 0.0675 \\ -1.0026 & 0.3504 & 1.3248 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}, \ \mathrm{eig}\,(\boldsymbol{F}_{Sc3}) = \begin{bmatrix} 0.0 \\ 0.5 \\ 0.8 \end{bmatrix}$$

Analogously for any other desired sets of eigenvalues can be obtained the gain matrices for the rest types of sensor faults.

Case 3: Dynamic properties of closed-loop system reconfiguration

Assuming that the stabilizing control for given system was obtained using the standard Matlab function $place(.)$ for desired closed-loop eigenvalue set $\rho(\boldsymbol{F}_{c3}) = \{\,0.2, 0.5, 0.8\,\}$ and nominal system matrices $\boldsymbol{F}$, $\boldsymbol{G}$. Then the solution of the above task is

$$\boldsymbol{K}_N = \begin{bmatrix} -4.4207 & -7.4955 & 1.7509 \\ 19.5070 & 16.1414 & -1.7903 \end{bmatrix}$$

$$\boldsymbol{F}_{Nc} = \begin{bmatrix} 0.9242 & 0.0562 & 0.0042 \\ -1.4935 & 0.1393 & 0.0740 \\ 0.5398 & 0.1977 & 0.4365 \end{bmatrix}, \ \mathrm{eig}\,(\boldsymbol{F}_{Nc}) = \begin{bmatrix} 0.2 \\ 0.5 \\ 0.8 \end{bmatrix}$$

Then, realizing for the sub-system which is specified by vectors

$$\boldsymbol{c}_1^T = [\,1\ 0\ 0\,], \ \boldsymbol{g}_1^T = [\,0.0051\ 0.1029\ 0.0387\,]$$

the gains of the reference external input signal were computed as

$$G_{N1} = \frac{1}{\boldsymbol{c}_1^T(\boldsymbol{I} - \boldsymbol{F}_{Nc})^{-1}\boldsymbol{g}_1} = 13.2308$$

$$G_{R31} = \frac{1}{\boldsymbol{c}_1^T(\boldsymbol{I} - \boldsymbol{F}_{Sc3}\boldsymbol{X}_3)^{-1}\boldsymbol{g}_1} = 9.7895$$

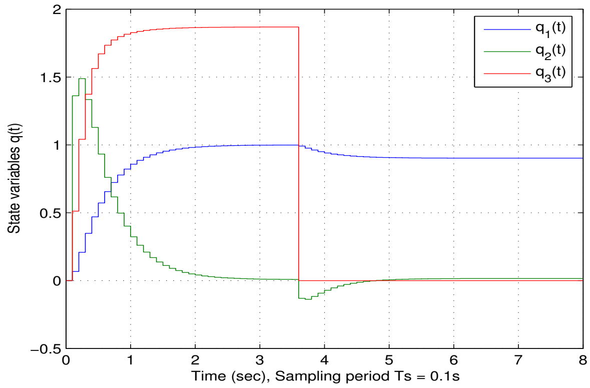

Fig. 1. Step response of the closed-loop system without control reconfiguration for the third sensor fault

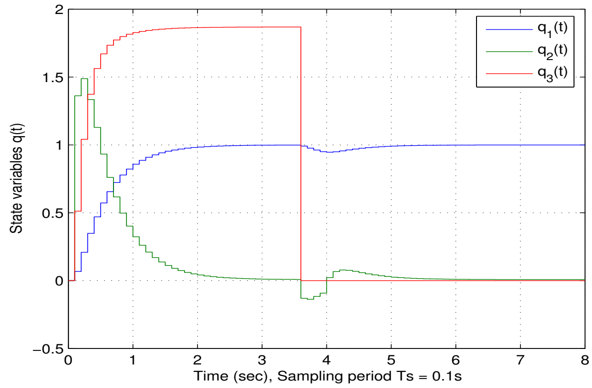

Fig. 2. Step response of the closed-loop system with control reconfiguration and the third variable sensor fault

where

$$\boldsymbol{X}_3 = \mathrm{diag}\,[\,1\;1\;0\,]$$

In Figure 1 and 2, an example is shown of the closed-loop sub-system step response using control laws

$$\boldsymbol{u}(i) = -\boldsymbol{K}_N\boldsymbol{q}(i) + \boldsymbol{G}_N\boldsymbol{u}(i),\;\; \boldsymbol{G}_N = \mathrm{diag}\,[\,13.2308\;\,0\,]$$

$$\boldsymbol{u}(i) = -\boldsymbol{K}_{S3}\boldsymbol{q}(i) + \boldsymbol{G}_{R31}\boldsymbol{u}(i),\;\; \boldsymbol{G}_{R31} = \mathrm{diag}\,[\,9.7895\;\,0\,]$$

Specially, at the Figure 2, one can see a dead time between the fault occurrence time instant and the reconfiguration starting time, which reflect a time consumption for fault detection and isolation.

7. CONCLUDING REMARKS

Based on the state equation, the performance index parameters and the system constraint equations for time-invariant discrete LQ control problem, the generalized Riccati equations for linear equality constrained system, obtained according to the minimum principle, are given in the paper. Obtained solution was used to design a reconfigurable LQ control as well as stabilizing reconfigurable control. The proposed method present some new design features and generalizations where it was emphasized that the advantage offered by the proposed approach is, the state variable associated with a faulty sensor may be fixed at zero value also after that time instant when the reconfigured control is started.

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