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Determining Fixturing Points for Complex Objects ⋆

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Abstract: This paper presents a method to design fixtures for complex objects in robotized environments. The object is described with a set of points on its surface. First, an initial form-closure fixture is found with an iterative algorithm. The initial fixture is then improved with an iterative approach relying on a geometrical reasoning that efficiently looks for a locally optimum fixture; the quality of the fixture is measured considering the largest perturbation wrench that the fixture can resist, with independence of its direction. Once a locally optimum fixture has been reached, independent contact regions are computed to provide robustness in front of the locator positioning errors or to allow variations in the locators position. The proposed approach can also be applied to compute fixtures when one or several locators are fixed beforehand. The procedure has been implemented and application examples are included in the paper.

Keywords: fixture design, frictionless grasps, independent contact regions.

1. INTRODUCTION

Manufacturing or inspection operations on industrial workpieces usually require the immobilization of the object to resist external disturbances, specially when a robot or any other machine must perform an action on them; such problem is known as fixture layout design. In fixture design, a set of clamps and locators must be placed such that the position of the contact points on the object ensure its immobility; this property is commonly called form-closure or kinematical immobility, and it is mostly used when the task requires a robust grasp not relying on friction (Bicchi, 1995). When friction is taken into account so that the forces applied at the contact points ensure the object immobility, the object fulfills the force-closure property, which is used in grasping and manipulation of objects with a lower number of frictional contacts.

The problems related to fixture design and grasp synthesis on 3D objects, with different number of fingers and satisfying the form or force-closure condition, have been extensively investigated. Several algorithms have been proposed for 3D polyhedral objects or objects with smooth curved surfaces (Ponce et al., 1997; Zhu and Wang, 2003). More recently the synthesis of fixtures/grasps for 3D discretized objects has been addressed. A strategy based on random generation of grasps was shown to be quick and efficient to generate good grasps on arbitrary objects (Borst et al., 2003); the complexity of such grasp planner depends on the form of the object, but the generated grasps are not optimal. An algorithm for fixture synthesis on discrete objects was proposed by minimizing the workpiece positioning errors due to uncertainties in the position of the locators and in the geometry of the workpiece (Wang and Pelinescu, 2001). An algorithm to generate a form-closure grasp with 7 frictionless contact points was proposed (Ding et al., 2001), although it can be trapped in local minima; this algorithm was extended to find one force-closure grasp with frictional or frictionless contact points (Liu et al., 2004).

Most of the approaches mentioned above require precise finger placements; however, in a real execution the actual and the theoretical fixture/grasp may differ due to positioning errors. The concept of independent contact regions (ICRs) was introduced in order to provide robustness in front of these errors (Nguyen, 1988). ICRs are regions on the object boundary such that a finger positioned on each of them assures a form-closure (FC) grasp, with independence of the exact position of each finger. The determination of ICRs was initially addressed for polygonal and polyhedral objects (Ponce and Faverjon, 1995; Ponce et al., 1997). The ICRs have also been used to determine contact regions for the grasp of 3D objects based on initial examples, although the results depend on the choice of the example (Pollard, 2004). Recently, the computation of ICRs for 2D discrete objects with four contact points has also been addressed (Cornellà and Suárez, 2005); however, the determination of ICRs on 3D discrete objects has not been efficiently tackled yet.

This paper presents an approach to design form-closure (FC) fixtures on general 3D workpieces with 7 frictionless contacts representing the constraints provided by locators and clamps. The object surface is approximated with a triangular mesh with a high number of faces or with a set of surface points and their corresponding normal direction; this representation allows dealing with complex objects, and with objects whose boundary is only known at a finite set of points, as in the case of the aerodynamic design

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of an airfoil (Liu et al., 2004). The proposed integrated approach comprises three phases. The first phase finds at least one FC fixture, and the second phase optimizes the initial fixture to get a locally optimum one; the optimization procedure is an oriented search that looks for the fixture that resists the largest perturbation wrench, with independence of the perturbation direction. Finally, the third phase computes the ICRs from the locally optimum fixture, assuring a FC fixture with a controlled minimum quality. The algorithms are based on geometrical procedures that avoid using a costly FC test, simplifying the overall complexity of the approach. The proposed approach is easily extended to deal with $n > 7$ frictionless contacts without increasing the computational complexity. It can also deal with the more simple case where k contacts are fixed beforehand, looking for the location and ICRs for the $n - k$ missing contacts.

This paper is organized as follows. Section 2 provides the basic assumptions and required background, including the form-closure test and quality measure used in the paper. Section 3 presents the algorithms to compute the initial and locally optimum fixture, and the independent contact regions. The algorithms have been implemented, and Section 4 shows the results of their application to different objects. Finally, Section 5 summarizes the approach, compares it with previous works, and presents future works.

2. PROBLEM OVERVIEW

2.1 Object and contact models

The object surface is represented with a large set Ω of points, described by position vectors $\boldsymbol{p}_i$ measured with respect to a reference system located in the object's center of mass (CM). Each point has an associated unitary normal direction $\hat{\boldsymbol{n}}_i$ pointing towards the interior of the object. It is assumed that the number of points in Ω is large enough to accurately represent the surface of the object, and that each point on the surface of the object has three neighboring points.

Seven frictionless contacts are necessary and may be sufficient to hold a 3D object with a FC fixture, provided that the object has no rotational symmetries (Bicchi, 1995); the frictionless assumption is realistic in fixturing, as the fixture requires total kinematical restriction. With frictionless contact points, the grasp forces can only be applied in the direction normal to the object surface. A unitary force $\hat{\boldsymbol{n}}_i$ applied on the object at the point $\boldsymbol{p}_i$ generates a torque $\boldsymbol{\tau}_i = \boldsymbol{p}_i \times \hat{\boldsymbol{n}}_i$ with respect to CM. The force and the torque are grouped together in a wrench vector given by $\boldsymbol{\omega}_i = (\hat{\boldsymbol{n}}_i, \boldsymbol{\tau}_i)^T$.

For a given fixture $F = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_7\}$, the wrenches produced by the forces applied at the contact points on the object are grouped in a wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots, \boldsymbol{\omega}_7\}$. Each physical point $\boldsymbol{p}_i$ in Ω has a corresponding wrench $\boldsymbol{\omega}_i$ in the wrench space; when it is clear, both of them will be used to indicate a fixture constraint (in general, the same wrench can be produced at different contact points).

2.2 Form-closure condition

A necessary and sufficient condition for the existence of a FC fixture is that the origin of the wrench space lies strictly inside the convex hull (CH) of the contact wrenches (Murray et al., 1994). Let F be a fixture with a set W of contact wrenches, $\mathcal{I}$ the set of strictly interior points of $CH(W)$, and H a supporting hyperplane of $CH(W)$ (i.e. a hyperplane containing one of the facets of $CH(W)$). The origin O of the wrench space satisfies $O \in \mathcal{I}$ iff any $P \in \mathcal{I}$ and O lie in the same half-space for every H of $CH(W)$. Thus, checking whether a given point $P \in \mathcal{I}$ and the origin O lie in the same half-space defined by each supporting hyperplane H of $CH(W)$ is enough to prove whether O lies inside $CH(W)$, i.e. to prove whether the fixture F is FC. P is chosen as the centroid of the contact wrenches, which is always an interior point of $CH(W)$. Then, the FC test checks whether the centroid P and the origin O lie on the same side for all the supporting hyperplanes of $CH(W)$.

2.3 Fixture quality measure

This work uses as quality measure the largest perturbation wrench that the fixture can resist, with independence of the perturbation direction (Ferrari and Canny, 1992); this is one of the most popular grasp quality measures. Geometrically, the quality is the radius of the largest ball centered at the origin O of the wrench space and fully contained in $CH(W)$, i.e. it is the distance from O to the closest facet of $CH(W)$.

A theoretical upper limit on the quality measure for a particular object can be established to evaluate the fixtures generated by the design procedure. A 6-dimensional hyperplane is described with $\sum_{j=1}^{6} e_j x_j = e_0$, with $\boldsymbol{e} = (e_1, \ldots, e_6)$ the vector normal to the hyperplane. The distance to the origin is given by $D = |e_0| / \|\boldsymbol{e}\|$.

The upper quality limit problem is stated as follows. Given a wrench set of 7 non-collinear points $\boldsymbol{\omega}_i = (x_{i1}, x_{i2}, \ldots, x_{i6}) \in \Re^6$, the following max-min problem should be solved

$$\max \left\{ \min \left(\frac{|e_{01}|}{\|\boldsymbol{e_1}\|}, \frac{|e_{02}|}{\|\boldsymbol{e_2}\|}, \cdots, \frac{|e_{07}|}{\|\boldsymbol{e_7}\|} \right) \right\} \tag{1}$$

subject to

$$\lambda_1 \boldsymbol{\omega}_1 + \lambda_2 \boldsymbol{\omega}_2 + \cdots + \lambda_7 \boldsymbol{\omega}_7 = 0 \tag{2}$$
$$\lambda_1 + \lambda_2 + \cdots + \lambda_7 = 0 \tag{3}$$
$$0 < \lambda_j \leq 1,\ j = 1, 2, \ldots, 7 \tag{4}$$
$$x_{i1}^2 + x_{i2}^2 + x_{i3}^2 = 1 \tag{5}$$
$$-\tau_{4min} \leq x_{i4} \leq \tau_{4max} \tag{6}$$
$$-\tau_{5min} \leq x_{i5} \leq \tau_{5max} \tag{7}$$
$$-\tau_{6min} \leq x_{i6} \leq \tau_{6max} \tag{8}$$

with $\boldsymbol{e}_i$ and e_{0i} the parameters for each one of the 7 supporting hyperplanes of $CH(W)$. Constraints (2), (3) and (4) assure that the origin O lies strictly inside $CH(W)$, constraint (5) normalizes the force component of the wrench $\boldsymbol{\omega_i}$ ($\hat{\boldsymbol{n}}_i$ lies on the surface of the sphere $\mathcal{S}^2$), and constraints (6), (7) and (8) provide the limits for the torque components ($\boldsymbol{\tau}_i$ lies inside a parallelepiped in $\Re^3$).

3. FIXTURE POINT DETERMINATION

3.1 Search of an initial fixture

An initial FC fixture is determined based on geometric reasoning, avoiding the inclusion of an explicit FC test in

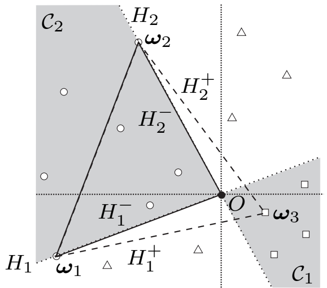

Fig. 1. Synthesis of a FC fixture. The convex hull of the wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \mathbf{0}\}$ (in continuous lines) defines the supporting hyperplanes H_1 and H_2 that contain the origin. The convex set $\mathcal{C}_1$ contains 4 points (depicted as white squares), thus the algorithm provides 4 FC fixtures, one of them illustrated with the convex hull in discontinuous lines.

the algorithm. First, a set S^1 of 6 random points is selected from Ω, and the convex hull $CH(W)$ of the selected points plus the origin O of the wrench space is computed. Let H_l be a supporting hyperplane of $CH(W)$ containing the origin, and let H_l^- be the closed half-space defined by H_l that contains $CH(W)$; $l = 1, \ldots, 7$. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be the intersection of all of the half-spaces H_l^+ and H_l^-, respectively, as illustrated in Fig. 1 for a hypothetical 2D wrench space (the actual wrench space is 6-dimensional). If there is at least one wrench in $\mathcal{C}_1$, it will provide a FC fixture when added to S^1 (Section 2.2). If $\mathcal{C}_1$ is empty, the algorithm iteratively replaces one of the wrenches in S^1 and performs another search of points in the new $\mathcal{C}_1$ until it contains at least one point, i.e. until it finds at least one FC fixture. The steps in the algorithm are:

Algorithm 1: Search of a FC fixture

(1) Generate a random set $S^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_6\}$, $k = 1$
(2) Build $W^k = S^k \cup \{O\}$
(3) Compute $CH(W^k)$
(4) Find $\mathcal{C}_1 = \left\{\boldsymbol{\omega} \mid \boldsymbol{\omega} \in H_1^+ \cap \ldots H_7^+\right\}$ and $\mathcal{C}_2 = \{\boldsymbol{\omega} \mid \boldsymbol{\omega} \in H_1^- \cap \ldots H_7^-\}$
(5) If $\mathcal{C}_1 \neq \emptyset$ then return $F = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_c\}$, with $\boldsymbol{\omega}_c$ randomly picked from $\mathcal{C}_1$
Else
Pick up a $\boldsymbol{\omega}_j \notin \mathcal{C}_1 \cup \mathcal{C}_2$
Form S^{k+1} by replacing a $\boldsymbol{\omega}_i \in S^k$ such that $d_{L_2}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_j)$ be a minimum. Go to Step 2.
Endif

The algorithm finishes when one FC fixture is obtained, and it provides as many FC fixtures as points lie in $\mathcal{C}_1$ in that iteration. When $\mathcal{C}_1$ is empty, any combination of 6 wrenches in $\mathcal{C}_2$ (including the wrenches in S^k) will not yield a FC fixture, thus all of these possible combinations are left out for subsequent searches. The wrenches in $\mathcal{C}_2$ are labeled as explored wrenches to progressively explore the search space and assure the completeness of the algorithm.

The computational complexity of Algorithm 1 is hard to estimate, as it has an heuristic nature; however, some remarks can be pointed out. Step 3 requires the computation of a convex hull, which is $\mathcal{O}(N \log N)$; however, only the supporting hyperplanes are required, which can be easily computed as every 6-point combination defines one of the 7 facets of $CH(W)$. Step 4 requires the classification of the

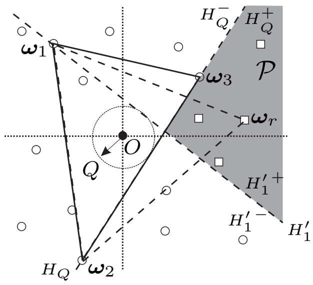

Fig. 2. Optimization procedure in a 2D wrench space. The set $\mathcal{P}$ of wrenches that improve the actual quality (depicted as white squares in the gray area) is defined by the hyperplanes H_1' and $H_2' = H_Q$. The fixture for the next iteration cycle, $F^k = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_r\}$ is also shown.

N-points in Ω with respect to the 7 hyperplanes (described by a linear equation). The total number of iterations required to determine a FC fixture (or to decide if there is no solution at all) depends on the random choice of Step 1 and in the random replacement in Step 5.

3.2 Optimization of the initial fixture

The optimization algorithm improves the fixture quality for the initial FC fixture obtained with the previous algorithm, looking for the fixture that resists the largest perturbation wrench with independence of its direction (Section 2.3). The steps in the algorithm are:

Algorithm 2: Optimization of an initial FC fixture

(1) Find an initial FC fixture $F^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_7\}$, $k = 1$
(2) Determine H_Q such that $D = |e_0| \,/\, \|\boldsymbol{e}\|$ is a minimum. The current fixture quality is $Q^k = D_q$
(3) Build $\mathcal{T} = \{\tilde{\boldsymbol{\omega}}_m,\ m = 1, \ldots, 6 \mid \|\tilde{\boldsymbol{\omega}}_1\| \leq \ldots \leq \|\tilde{\boldsymbol{\omega}}_6\|\}$; $\tilde{\boldsymbol{\omega}}_m$ lie on H_q
(4) Initialize $m = 1$. Compute the 6 hyperplanes H_l' containing every possible 5-point combination in $F^k - \{\tilde{\boldsymbol{\omega}}_m\}$ and lying to a distance Q^k from the origin O.
(5) Build $\mathcal{P} = \left\{\boldsymbol{\omega} \mid \boldsymbol{\omega} \in H_1'^+ \cap \ldots \cap H_6'^+\right\}$
If $\mathcal{P} = \emptyset$ and $m \neq 6$
Let $m = m + 1$. Go to Step 4
Elseif $\mathcal{P} = \emptyset$ and $m = 6$
A local maximum has been reached; return F^k
Elseif $\mathcal{P} \neq \emptyset$
Find $\boldsymbol{\omega}_r \in \mathcal{P}$ such that $\prod D\left(\boldsymbol{\omega}_r, H_l'\right)$ is a maximum
Replace $\tilde{\boldsymbol{\omega}}_m$ with $\boldsymbol{\omega}_r$. Go to Step 2
Endif

Steps 4 and 5 are the more complex steps in Algorithm 2. Step 4 requires the computation of 6 hyperplanes H_l' to find the points that improve the actual fixture quality (Fig. 2). A supporting hyperplane H_l is defined by 6 points, one of them being $\tilde{\boldsymbol{\omega}}_m$. The parameters of H_l' are computed from 5 linear equations (5 points lying on the hyperplane) and one non linear equation ($\|e_l\| = 1/Q^k$). These 6 equations admit 2 solutions; the hyperplane searched is the one leaving O and $\tilde{\boldsymbol{\omega}}_m$ in different half-spaces. Step 5 requires, in the worst case (the local maximum) 6 classification cycles of the N points in Ω with respect to 6 hyperplanes. The local maximum implies there are no more points that, combined with the point $F^k - \mathcal{T}$, improve the actual quality Q^k. The total number of iterations required to reach the local maximum depends on the number of local

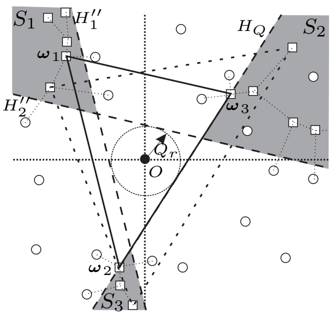

Fig. 3. Search of the ICRs in a 2D wrench space. The hyperplanes H_Q, H_1' and H_2' define the search zones S_1, S_2 and S_3, depicted in gray. The points within each ICR are depicted as white squares. An instance of a fixture with quality higher than Q_r is also shown.

maximums in the wrench space, which is related with the object shape.

3.3 Determination of the independent contact regions

The independent contact regions (ICRs) ensuring a minimum fixture quality Q_r are computed for the locally optimum FC fixture obtained with the previous algorithm. Several fixtures may be formed when placing a finger in different positions inside its corresponding ICR; any of these fixtures must satisfy $O \in CH(W)$ and have a quality $Q > Q_r$. The proposed approach is based on these geometrical conditions, as illustrated in Fig. 3. For a given FC fixture, the quality Q_r is fixed by F_Q, the facet of the convex hull closest to the origin. Six hyperplanes H_k'' (two in the hypothetical two-dimensional wrench space), parallel to the remaining facets of the convex hull and tangent to the ball of radius Q_r are then considered. These hyperplanes define S_i, the search zone containing the ICR for each wrench $\boldsymbol{\omega}_i$; S_i is the intersection of the half-spaces containing the wrench $\boldsymbol{\omega}_i$. The ICR is the set of neighbor points of $\boldsymbol{p}_i$ whose wrenches fall into the search zone S_i.

The procedure can also be applied to generate ICRs with points that produce a lower fixture quality $Q_r = \alpha Q_i$, with Q_i the quality of the initial fixture and $0 < \alpha < 1$. When $\alpha \to 0$, the ICRs contain FC fixtures without a lower limit on the fixture quality. In fact $\alpha = 0$ is a forbidden value, as it does not assure that any $CH(W)$ will strictly contain the origin O. The steps in the algorithm are:

Algorithm 3: Determination of ICRs

(1) Find a locally optimum FC fixture, $F_o = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_7\}$
(2) Fix the minimum acceptable quality $Q_r = \alpha Q$
(3) Build the hyperplanes H'' such that $D_{H''} = Q_r$
(4) Let $S_i = \bigcap H_k''$ such that $\boldsymbol{\omega}_i \in H_k''$, $i = 1, \ldots, 7$
(5) Initialize $I_i = \{\boldsymbol{p}_i\}$. Label the points in I_i as open
(6) Check the wrenches of the neighbor points $\boldsymbol{p}_{kn}$ for every open point $\boldsymbol{p}_k \in I_i$
If $\boldsymbol{\omega}_{kn} \in S_i$, then $I_i = I_i \cup \{\boldsymbol{p}_{kn}\}$, and label $\boldsymbol{p}_{kn}$ as open
Label $\boldsymbol{p}_k$ as closed
(7) If there are open points in any I_i, go to Step 6. Otherwise, the algorithm finishes, and returns the sets of points I_i, $i = 1, \ldots, 7$, i.e. the ICRs for each finger.

Algorithm 3 is computationally very simple. The hyperplanes H_k'' are computed for the corresponding facets of $CH(W)$. Let H_{F_l} be the hyperplane containing the facet F_l, described as $\boldsymbol{e}_l \cdot \boldsymbol{x} = e_{0l}$; the hyperplane H_l'' parallel to H_{F_l} but lying to a distance $D = Q_r$ from the origin is $\boldsymbol{e}_l \cdot \boldsymbol{x} = e_{0l}'$, with $e_{0l}' = Q_r \, \|\boldsymbol{e}_l\|$. Step 6 is the more complex step in the algorithm; every checked point requires its classification with respect to 7 hyperplanes.

3.4 Fixtures with some contacts fixed beforehand

A particular extension of the proposed approach deals with the fixture design for objects where a predefined number of contacts is fixed beforehand, for instance when there exist several locator pins on the workpiece, or for a workpiece lying on a surface. If k contact points are provided, the previous algorithms are used to determine the missing $7-k$ contacts. Algorithms 1 and 2 remain unchanged when applied to this particular case; they just must take into account that the points fixed beforehand can not change in any iteration.

In Algorithm 3, the independent contact regions for the fixed contact points are not computed (as it is assumed that its location is precisely determined). The hyperplanes H_l'' in Step 4 (defining the search zones S_i for the ICRs of the remaining points) are computed for the corresponding facet F_l of $CH(W)$, $l = 1, \ldots, 7$, according to the following considerations:

- The facet F_l is formed by 6 wrenches; m of them are fixed points (with $m \leq k$) and $6-m$ are the wrenches obtained with Algorithm 2.
- H_l'' must contain the m fixed points (m linear equations), must be parallel to the polytope formed by the $6-m$ remaining points ($6-m-1$ linear equations) and must lie at a distance $D = Q_r$ from the origin (one nonlinear equation).
- These 6 equations admit 2 solutions; the hyperplane required is the one leaving O and the wrench not belonging to F_l in different half-spaces.

The other steps in Algorithm 3 remain unchanged. Note that when there exist points fixed beforehand, larger ICRs are obtained for the rest of the contacts.

The proposed approach can also easily deal with forbidden contact regions on the object, for instance, regions where the geometrical features are very delicate. The points lying in the forbidden regions are just erased from the original set Ω, and Algorithm 1 will indicate if, in the absence of such regions, a FC fixture is still possible. If at least one FC fixture exists, then a locally optimum fixture and its corresponding ICRs can be computed.

4. EXAMPLES

The proposed approach has been implemented using Matlab on a Pentium IV 3.2 GHz computer. The performance of the algorithm is illustrated using two objects; the considered contact points $\boldsymbol{p}_i$ on the object surface are the centroids of the triangles in the mesh, with the corresponding directions normal to the triangles.

The first object is a parallelepiped described with a mesh of 3422 triangles. This simple figure makes more difficult the search of the first FC fixture, as the initial random selection of 6 points may place all the fingers on just one or the two larger faces (the probability of placing a finger

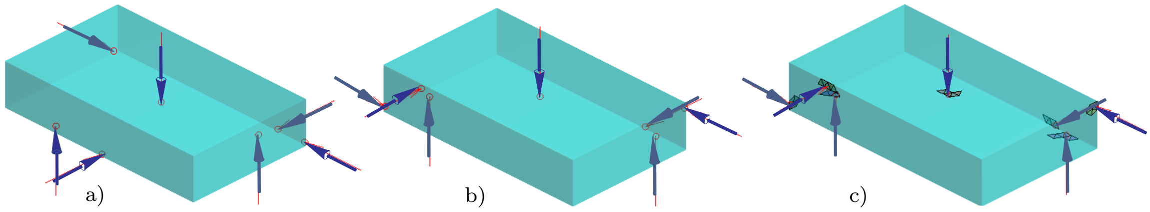

Fig. 4. Example on a parallelepiped: a) Initial FC fixture, $Q = 0.0098$ (Algorithm 1), b) Locally optimum FC fixture, $Q = 0.292$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.219$ (Algorithm 3).

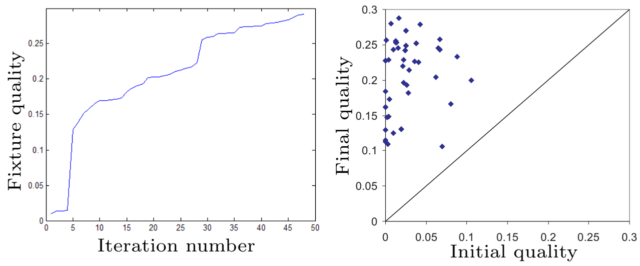

Fig. 5. Performance in the search of a locally optimum FC fixture for the parallelepiped: a) Increase in the fixture quality, b) Initial vs. final quality.

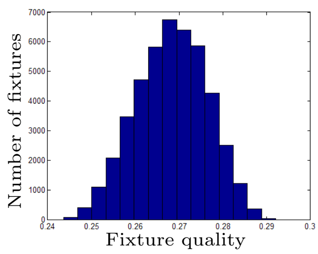

Fig. 6. Histogram with the quality distribution for all the possible fixtures within the ICRs of the parallelepiped for $Q_r = 0.219$ ($\alpha = 0.75$).

on those faces is greater than on the others). Fig. 4 shows an instance of the results obtained with the proposed approach. The first FC fixture is shown in Fig. 4a; the time elapsed to obtain this fixture was 0.49 seconds in 5 iterations. Algorithm 1 provides other 18 possible FC fixtures. The locally optimum FC fixture, shown in Fig. 4b, was obtained with Algorithm 2 in 24.3 s and 48 iterations. Fig. 4c shows the corresponding independent contact regions, obtained with Algorithm 3 and $\alpha = 0.75$ in 0.17 s.

Fig. 5a plots the fixture quality in the optimization phase; the quality always increases until it finds the locally optimum fixture (which depends on the initial fixture). In the example, the initial fixture quality is 0.0098, and the locally optimum fixture quality is 0.2921; the quality improvement factor is 29.8. The upper quality limit, obtained by solving the optimization problem in eq. (1) (Section 2.3) is 0.32; the locally optimum fixture quality is 91.3% of the quality limit. To obtain a better insight into the performance of the whole process, 50 locally optimum fixtures were computed. The correlation between the initial and final qualities is shown in Fig. 5b. The average qualities give an idea of the behavior of the algorithm, they are 0.024 and 0.215 for the initial and locally optimum FC fixtures, respectively; the average improvement factor is 9.

The points within the ICRs for the example may be combined to provide 45000 different fixtures; Fig. 6 shows the quality distribution for all these possible fixtures. For lower minimum fixture qualities, the size of each ICR grows, as illustrated in Fig. 7 for the same instance and minimum fixture qualities given by $\alpha = 0.5$ and $\alpha = 10^{-5} \approx 0$. Fig. 8 shows the application of Algorithm 3 to compute the ICRs for the optimum fixture shown in Fig. 4b, but with the 2 contacts in the bottom face fixed beforehand. Note that the ICRs for the movable contacts are larger than in the previous case for the same minimum quality.

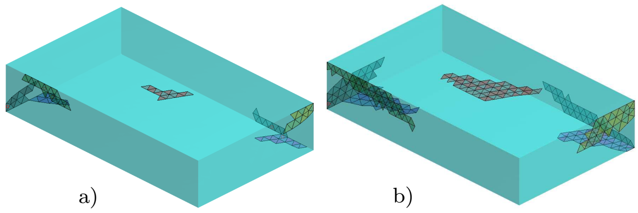

Fig. 7. ICRs on the parallelepiped with a minimum quality of: a) $Q_r = 0.146$ ($\alpha = 0.5$), b) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

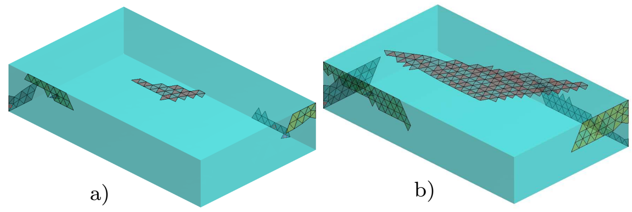

Fig. 8. ICRs on the parallelepiped with 2 contacts fixed beforehand: a) $Q_r = 0.146$ ($\alpha = 0.5$), b) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

The second object is a workpiece discretized with 3222 triangles. Fig. 9 shows the results for the ICR search; the first FC fixture was found with no iterations in 0.08 seconds, the locally optimum fixture was obtained after 33 iterations in 12.4 s and the ICRs (with $\alpha = 0.75$) were computed in 0.17 s. The fixture qualities are 0.0066 and 0.278 for the initial and locally optimum FC fixtures, respectively, with an improvement factor of 42.1. The upper quality limit is 0.34; the locally optimum fixture quality is 81.7% of the quality limit. The points within the ICRs provide 6480 different fixtures. Fig. 10 shows the ICRs for lower quality ratios: $\alpha = 0.5$ and $\alpha = 10^{-5}$.

5. DISCUSSION

This paper proposes an approach to obtain locally optimum FC fixtures and independent contact regions on 3D discretized objects with 7 frictionless contacts that ensure a FC fixture with a controlled minimum quality. The procedure is composed by 3 algorithms. Algorithm 1 looks for an initial FC fixture in the wrench space; the search space is progressively covered until finding a FC fixture, or until all of the space has been covered and

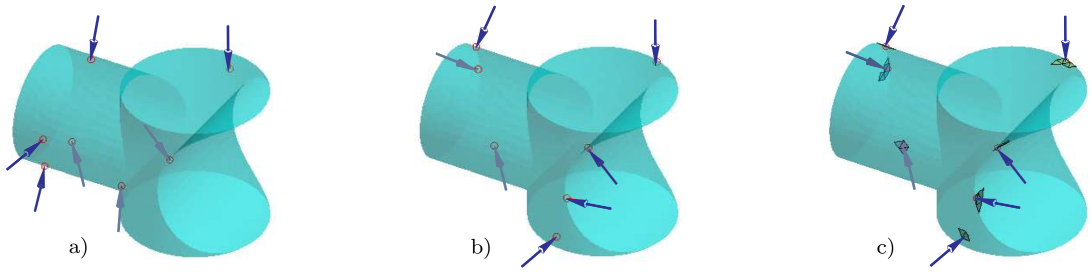

Fig. 9. Example on a workpiece: a) Initial FC fixture, $Q = 0.0066$ (Algorithm 1), b) Locally optimum FC fixture, $Q = 0.278$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.208$ (Algorithm 3).

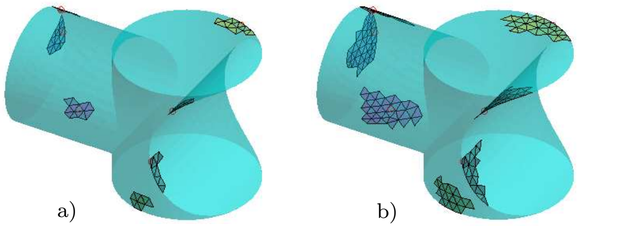

Fig. 10. ICRs on the workpiece with different minimum quality: a) $Q_r = 0.139$ ($\alpha = 0.5$), b) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

no FC fixture is found. The procedure is computationally simpler than the algorithm in (Liu et al., 2004), as it does not include an explicit FC test in the algorithm; the FC condition is embedded in the search process. Moreover, the algorithm finds one or more FC fixtures, depending on the number of wrenches lying in the subset $\mathcal{C}_1$ when a solution is found; if there are several FC fixtures, they can be classified according to the quality measure to pick the best candidate. The time elapsed to obtain one FC fixture is significantly lower with our approach.

The initial fixture is optimized with Algorithm 2. Most of the algorithms presented in the literature for FC grasp synthesis in discretized 3D objects focus only on getting one FC grasp, regardless of whether it is optimal or not. Some works in fixturing consider an optimization criterion in the design process (e.g. minimization of the workpiece positioning errors (Wang and Pelinescu, 2001)), but none of them considers the largest perturbation wrench as a quality measure, despite its popularity in the grasp community.

The independent contact regions around the contact locations of the locally optimum FC fixture are computed with Algorithm 3. A work in this line presents a procedure to compute a family of grasps for 3D objects that keep a fraction of the grasp quality in an initial example (Pollard, 2004); however, the selection of a good initial example remains as a critical step. This initial grasp is provided here with a procedure assuring a locally optimum grasp, and then a minimum quality is guaranteed. A previous work of the authors (Roa and Suarez, 2007) presented the basis for the computation of ICRs on discretized 3D objects; the algorithms proposed there have been simplified here and the computational complexity and running times have been significantly lowered down.

The algorithms of our approach have been implemented and the execution results illustrate the relevance and efficiency of the approach, which is easily extended to determine ICRs for more than 7 fingers or to consider the case with k contact locations fixed beforehand.

The extension of the approach to the case of frictional contacts in grasp applications is more complex, as the frictional model is nonlinear. In this case, the linearization of the friction cone requires suitable modifications in the algorithms, which are currently under development.

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